CHAPTER 4

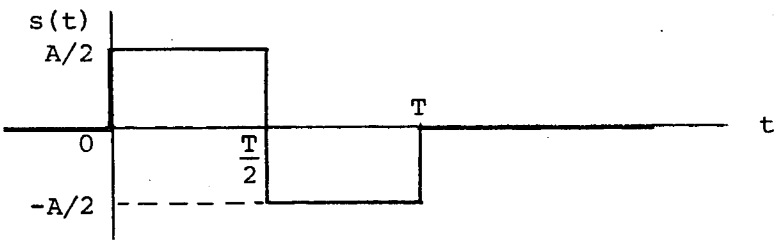
Baseband Pulse Transmission

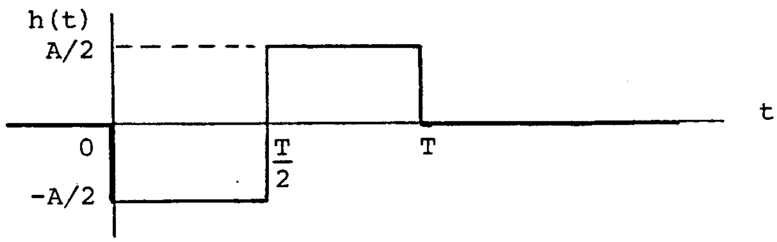
Problem 4.1

(a) The impulse response of the matched filter is

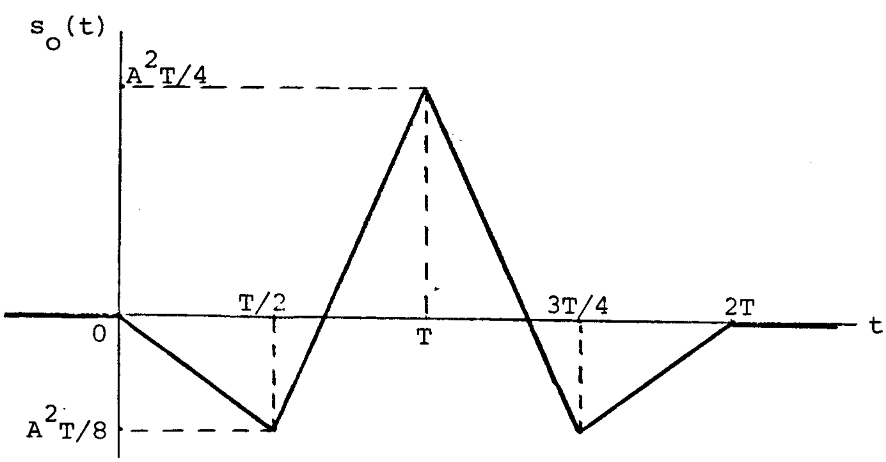
$$h(t) = s(T-t)$$

The s(t) and h(t) are shown below:





(b) The corresponding output of the matched filter is obtained by convolving h(t) with s(t). The result is shown below:



(c) The peak value of the filter output is equal to $A^2T/4$, occuring at t=T.

Problem 4.2

(a) The matched filter of impulse response $h_1(t)$ for pulse $s_1(t)$ is given in the solution to Problem 4.1. The matched filter of impulse response $h_2(t)$ for $s_2(t)$ is given by

$$h_2 = s_2(T-t)$$

which has the following waveform:

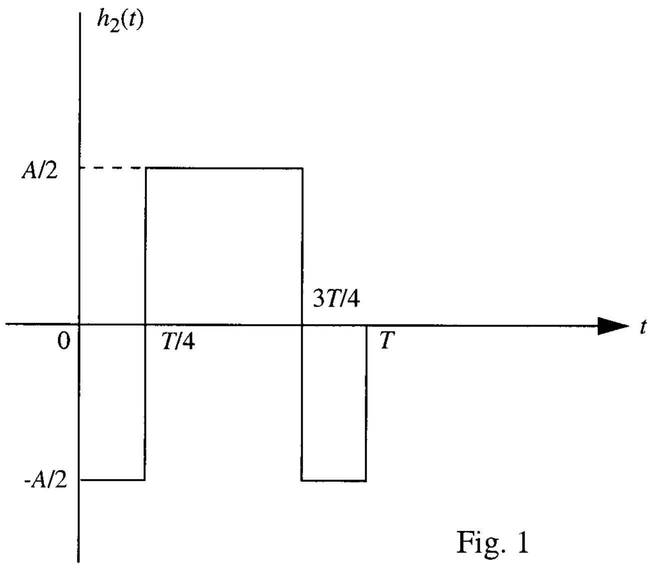


Fig. 1

(b) (i) The response of the matched filter, matched to $s_2(t)$ and due to $s_1(t)$ as input, is obtained by convolving $h_2(t)$ with $s_1(t)$, as shown by

$$y_{21}(t) = \int_0^T s_1(\tau)h_2(t-\tau)d\tau$$

The waveform of the output $y_{21}(t)$ so computed is plotted in Figure 2. This figure also includes the corresponding waveforms of input $s_1(t)$ and impulse response $h_2(t)$.

(ii) Next, the response of the matched filter, matched to $s_1(t)$ and due to $s_2(t)$ as input, is obtained by convolving $h_1(t)$ with $s_2(t)$, as shown by

$$y_{12}(t) = \int_0^T s_2(\tau)h_1(t-\tau)d\tau$$

Figure 3 shows the waveforms of input $s_2(t)$, impulse response $h_1(t)$, and response $y_{12}(t)$.

4.2(b)

(i)

Pulse $s_1(t)$

Filter $h_2(t)$

$h_2(t) = s_2(T-t)$

Filter response

$$\int_0^T s_1(\tau)\, h_2(t-\tau)\, d\tau$$

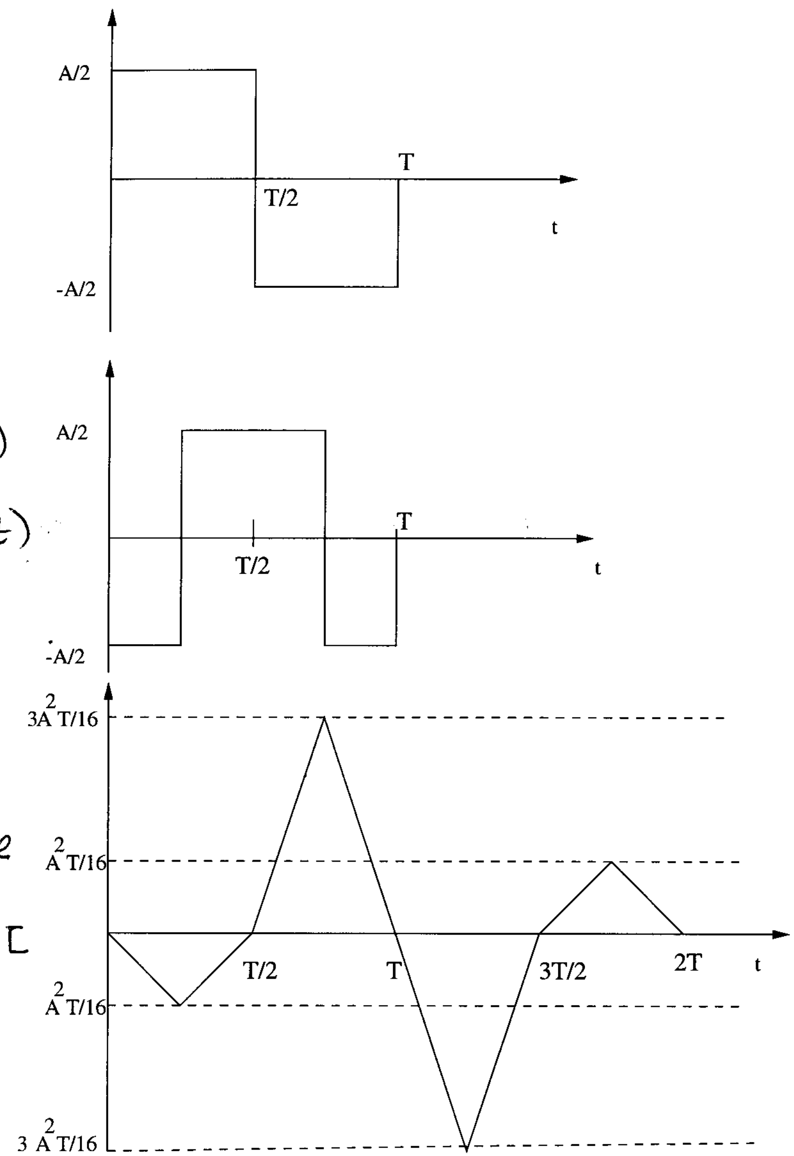


Fig. 3

4.2(b)

(ii)

Pulse $s_2(t)$

Filter $h_1(t)$

Filter matched to Pulse $s_1(t)$

Filter response

$$\int_0^T s_2(\tau)\, h_1(t-\tau)\, d\tau$$

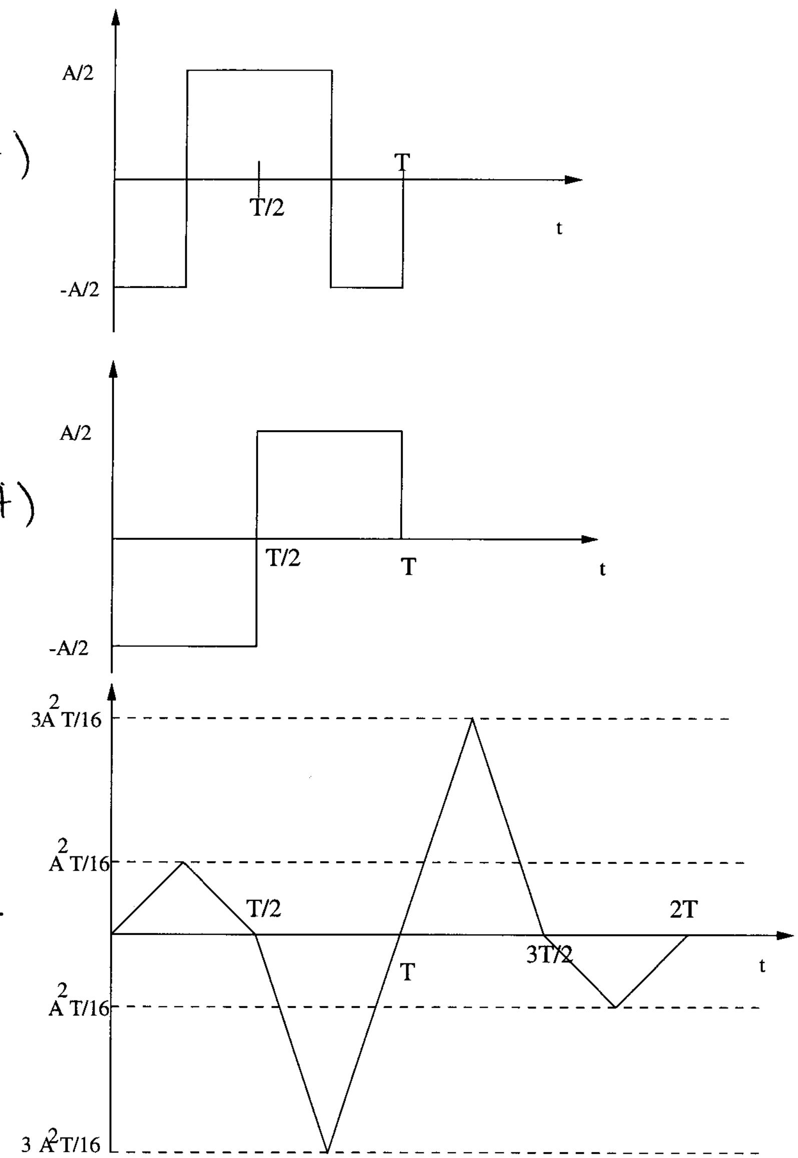


Fig. 3

Note that $y_{12}(t)$ is exactly the negative of $y_{21}(t)$. However, in both cases we find that at $t = T$, both outputs are equal to zero, as shown by

$$y_{21}(T) = y_{12}(T) = 0$$

For n pulses $s_1(t)$ $s_2(t),\ldots,s_n(t)$ that are orthogonal to each other over the interval [0,T], the n-dimensional matched filter has the following structure:

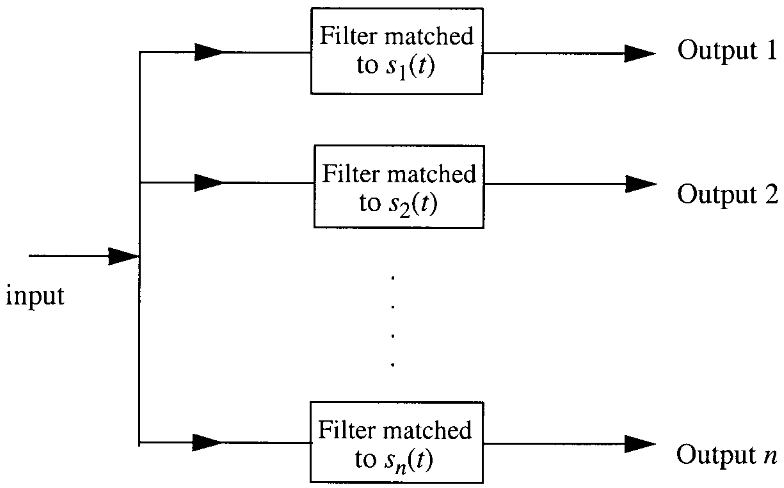


Fig. 4

Problem 4.3

Ideal low-pass filter with variable bandwidth. The transfer function of the matched filter for a rectangular pulse of duration τ and amplitude A is given by

$$H_{opt}(f) = \text{sinc}(fT)\exp(-j\pi fT) \quad (1)$$

The amplitude response $|H_{opt}(f)|$ of the matched filter is plotted in Fig. 1(a). We wish to approximate this amplitude response with an ideal low-pass filter of bandwidth B. The amplitude response of this approximating filter is shown in Fig. 1(b). The requirement is to determine the particular value of bandwidth B that will provide the best approximation to the matched filter.

We recall that the maximum value of the output signal, produced by an ideal low-pass filter in response to the rectangular pulse occurs at t = T/2 for $BT \le 1$. This maximum value, expressed in terms of the sine integral, is equal to $(2A/\pi)\text{Si}(\pi BT)$. The average noise power at the output of the ideal low-pass filter is equal to BN_0. The maximum output signal-to-noise ratio of the ideal low-pass filter is therefore

$$(\text{SNR})_o' = \frac{(2A/\pi)^2 \text{Si}^2(\pi BT)}{BN_0} \quad (2)$$

Thus, using Eqs. (1) and (2), and assuming that AT = 1, we get

$$\frac{(\text{SNR})_o'}{(\text{SNR})_o} = \frac{2}{\pi^2 BT}\text{Si}^2(\pi BT)$$

This ratio is plotted in Fig. 2 as a function of the time-bandwidth product BT. The peak value on this curve occurs for BT = 0.685, for which we find that the maximum signal-to-noise ratio of the ideal low-pass filter is 0.84 dB below that of the true matched filter. Therefore, the " best" value for the bandwidth of the ideal low-pass filter characteristic of Fig. 1(b) is B = 0.685/T.

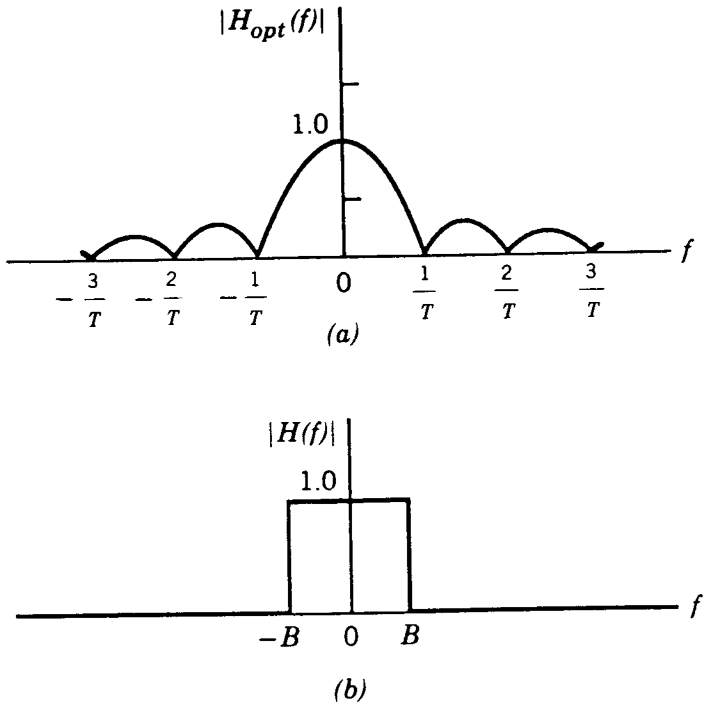


Figure 1

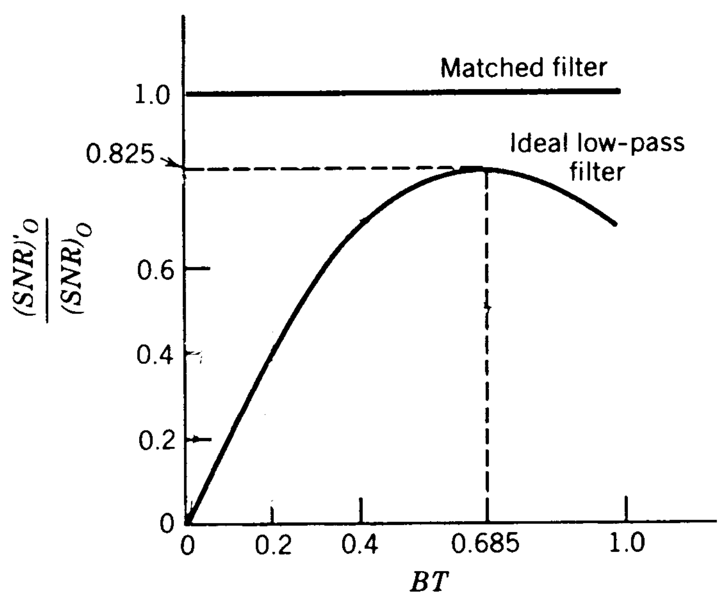
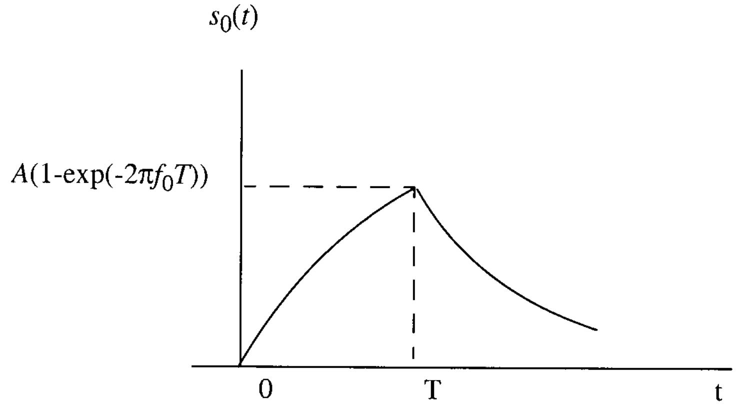


Figure 2

Problem 4.4

The output of the low-pass RC filter, produced by a rectangular pulse of amplitude A and duration T, is as shown below:



The peak value of the output pulse power is

$$P_{\text{out}} = A^2[1 - \exp(-2\pi f_0 T)]^2$$

where f_0 is the 3-dB cutoff frequency of the RC filter.

The average output noise power is

$$N_{\text{out}} = \frac{N_0}{2}\int_{-\infty}^{\infty} \frac{df}{1 + (f/f_0)^2}$$

$$= \frac{N_0 \pi f_0}{2}$$

The corresponding value of the output signal-to-noise ratio is therefore

$$(\text{SNR})_{\text{out}} = \frac{2A^2}{N_0 \pi f_0}[1 - \exp(-2\pi f_0 T)]$$

Differentiating $(\text{SNR})_0$ with respect to $f_0 T$ and setting the result equal to zero, we find that $(\text{SNR})_{\text{out}}$ attains its maximum value at

$$f_0 = \frac{0.2}{T}$$

The corresponding maximum value of $(\text{SNR})_{\text{out}}$ is

$$(\text{SNR})_{0,\max} = \frac{2A^2T}{0.2\pi N_0}[1 - \exp(-0.4\pi)]^2$$

$$= \frac{1.62A^2T}{N_0}$$

For a perfect matched filter, the output signal-to-noise ratio is

$$(\text{SNR})_{0,\text{matched}} = \frac{2E}{N_0}$$

$$= \frac{2A^2T}{N_0}$$

Hence, we find that the transmitted energy must be increased by the ratio 2/1.62, that is, by 0.92 dB so that the low-pass RC filter with $f_0 = 0.2/T$ realizes the same performance as a perfectly matched filter.

Problem 4.5

(i) $p_0 > p_1$

The transmitted symbol is more likely to be 0. Hence, the average probability of symbol error is smaller when a 0 is transmitted than when a 1 is transmitted. In such a situation, the threshold λ in Figs. 4.5(a) and (b) in the textbook is moved to the right.

(ii) $p_1 > p_0$

The transmitted symbol is more likely to be 1. Hence, the average probability of symbol error is smaller when a 1 is transmitted than when a 0 is transmitted. In this second situation, the threshold λ in Figs. 4.5(a) and (b) in the textbook is moved to the left.

Problem 4.6

The average probability of error is

$$P_e = p_1 \int_{-\infty}^{\lambda} f_Y(y|1)dx + p_0 \int_{\lambda}^{\infty} f_Y(y|0)dx \qquad (1)$$

An optimum choice of λ corresponds to minimum P_e. Differentiating Eq. (1) with respect to λ, we get:

$$\frac{\partial P_e}{\partial \lambda} = p_1 f_Y(\lambda|1) - p_0 f_Y(\lambda|0)$$

Setting $\frac{\partial P_e}{\partial \lambda} = 0$, we get the following condition for the optimum value of λ:

$$\frac{f_Y(\lambda_{opt}|1)}{f_Y(\lambda)_{opt}|0)} = \frac{p_0}{p_1}$$

which is the desired result.

Problem 4.7

In a binary PCM system, with NRZ signaling, the average probability of error is

$$P_e = \frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

The signal energy per bit is

$$E_b = A^2 T_b$$

where A is the pulse amplitude and T_b is the bit (pulse) duration. If the signaling rate is doubled, the bit duration T_b is reduced by half. Correspondingly, E_b is reduced by half.

Let $u = \sqrt{E_b/N_0}$. We may then set

$$P_e = 10^{-6} = \frac{1}{2}\,\mathrm{erfc}(u)$$

Solving for u, we get

$$u = 3.3$$

When the signaling rate is doubled, the new value of P_e is

$$P_e' = \frac{1}{2}\,\mathrm{erfc}\left(\frac{u}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\,\mathrm{erfc}(2.33)$$

$$= 10^{-3}.$$

Problem 4.8

(a) The average probability of error is

$$P_e = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

where $E_b = A^2 T_b$. We may rewrite this formula as

$$P_e = \frac{1}{2}\,\text{erfc}\left(\frac{A}{\sigma}\right) \qquad (1)$$

where A is the pulse amplitude at $\sigma = \sqrt{N_0 T_b}$. We may view σ^2 as playing the role of noise variance at the decision device input. Let

$$u = \sqrt{\frac{E_b}{N_0}} = \frac{A}{\sigma}$$

We are given that

$$\sigma^2 = 10^{-2}\ \text{volts}^2, \qquad \sigma = 0.1\ \text{volt}$$

$$P_e = 10^{-8}$$

Since P_e is quite small, we may approximate it as follows:

$$\text{erfc}(u) \approx \frac{\exp(-u^2)}{\sqrt{\pi}\,u}$$

We may thus rewrite Eq. (1) as (with $P_e = 10^{-8}$)

$$\frac{\exp(-u^2)}{2}\sqrt{\pi}u = 10^{-8}$$

Solving this equation for u, we get

$$u = 3.97$$

The corresponding value of the pulse amplitude is

$$A = \sigma u = 0.1 \times 3.97$$

$$= 0.397 \text{volts}$$

(b) Let σ^2_i denote the combined variance due to noise and interference; that is

$$\sigma_T^2 = \sigma^2 + \sigma_i^2$$

where σ^2 is due to noise and σ^2_i is due to the interference. The new value of the average probability of error is 10^{-6}. Hence

$$10^{-6} = \frac{1}{2}\,\text{erfc}\left(\frac{A}{\sigma_T}\right) \qquad (2)$$

$$= \frac{1}{2}\,\text{erfc}(u_T)$$

where

$$u_T = \frac{A}{\sigma_T}$$

Equation (2) may be approximated as (with $P_e = 10^{-6}$)

$$\frac{\exp(-u_T^2)}{2\sqrt{\pi}\, u_T} \simeq 10^{-6}$$

Solving for u_T, we get

$$u_T = 3.37$$

The corresponding value of $\sigma^2{}_T$ is

$$\sigma_T^2 = \left(\frac{A}{u_T}\right)^2 = \left(\frac{0.397}{3.37}\right)^2 = 0.0138 \text{ volts}^2$$

The variance of the interference is therefore

$$\begin{aligned} \sigma_i^2 &= \sigma_T^2 - \sigma^2 \\ &= 0.0138 - 0.01 \\ &= 0.0038 \text{ volts}^2 \end{aligned}$$

Problem 4.9

Consider the performance of a binary PCM system in the presence of channel noise; the receiver is depicted in Fig. 1. We do so by evaluating the average probability of error for such a system under the following assumptions:

1. The PCM system uses an on-off format, in which symbol 1 is represented by A volts and symbol 0 by zero volt.
2. The symbols 1 and 0 occur with equal probability.
3. The channel noise w(t) is white and Gaussian with zero mean and power spectral density $N_0/2$.

To determine the average probability of error, we consider the two possible kinds of error separately. We begin by considering the first kind of error that occurs when symbol 0 is sent and the receiver chooses symbol 1. In this case, the probability of error is just the probability that the correlator output in Fig. 1 will exceed the threshold λ owing to the presence of noise, so the transmitted symbol 0 is mistaken for symbol 1. Since the a priori probabilities of symbols 1 and 0 are equal, we have $p_0 = p_1$ Correspondingly, the expression for the threshold λ simplifies as follows:

$$\lambda = \frac{A^2 T_b}{2} \tag{1}$$

where T_b is the bit duration, and A^2T_b is the signal energy consumed in the transmission of symbol 1. Let y denote the correlator output:

$$y = \int_0^{T_b} s(t)x(t)dt \tag{2}$$

Under hypothesis H_0, corresponding to the transmission of symbol 0, the received signal x(t) equals the channel noise w(t). Under this hypothesis we may therefore describe the correlator output as

$$H_0 : y = A \int_0^{T_b} w(t)dt \tag{3}$$

Since the white noise w(t) has zero mean, the correlator output under hypothesis H_0 also has zero mean. In such a situation, we speak of a *conditional mean*, which (for the situation at hand) we describe by writing

$$\mu_0 = E[Y|H_0] = E\left[\int_0^{T_b} W(t)dt\right] = 0 \tag{4}$$

where the random variable Y represents the correlator output with y as its sample value and W(t) is a white-noise process with w(t) as its sample function. The subscript 0 in the conditional mean μ_0 refers to the condition that hypothesis H_0 is true. Correspondingly, let $\sigma^2{}_0$ denote the *conditional variance* of the correlator output, given that hypothesis H_0 is true. We may therefore write

$$\begin{aligned}\sigma_0^2 &= E[Y^2|H_0] \\ &= E\left[\int_0^{T_b}\int_0^{T_b} W(t_1)W(t_2)dt_1\, dt_2\right]\end{aligned} \tag{5}$$

The double integration in Eq. (5) accounts for the squaring of the correlator output. Interchanging the order of integration and expectation in Eq. (5), we may write

$$\begin{aligned}\sigma_0^2 &= \int_0^{T_b}\int_0^{T_b} E[W(t_1)W(t_2)]dt_1\, dt_2 \\ &= \int_0^{T_b}\int_0^{T_b} R_w(T_1 - t_2)dt_1\, dt_2\end{aligned} \tag{6}$$

The parameter $R_w(t_1 - t_2)$ is the *ensemble-averaged autocorrelation function* of the white-noise process W(t). From random process theory, it is recognized that the autocorrelation function and power spectral density of a random process form a Fourier transform pair. Since the white-noise process W(t) is assumed to have a constant power spectral density of $N_0/2$, it follows that the autocorrelation function of such a process consists of a delta function weighted by $N_0/2$. Specifically, we may write

$$R_W(t_1 - t_2) = \frac{N_0}{2}\delta(\tau - t_1 + t_2) \tag{7}$$

Substituting Eq. (7) in (6), and using the property that the total area under the Dirac delta function $\delta(\tau - t_1 + t_2)$ is unity, we get

$$\sigma_0^2 = \frac{N_0 T_b A^2}{2} \tag{8}$$

The statistical characterization of the correlator output is computed by noting that it is Gaussian distributed, since the white noise at the correlator input is itself Gaussian (by assumption). In summary, we may state that under hypothesis H_0 the correlator output is a Gaussian random variable with zero mean and variance $N_0T_bA^2/2$, as shown by

$$f_0(y) = \frac{1}{\sqrt{\pi N_0 T_b}\ A} \exp\left(- \frac{y^2}{N_0 T_b A^2} \right) \tag{9}$$

where the subscript in $f_0(y)$ signifies the condition that symbol 0 was sent.

Figure 2(a) shows the bell-shaped curve for the probability density function of the correlator output, given that symbol 0 was transmitted. The probability of the receiver deciding in favor of symbol 1 is given by the area shown shaded in Fig. 2(a). The part of the y-axis covered by this area corresponds to the condition that the correlator output y is in excess of the threshold λ defined by Eq. (1). Let P_{e0} denote the *conditional probability of error, given that symbol 0 was sent.* Hence, we may write

$$\begin{aligned} p_{10} &= \int_{\lambda}^{\infty} f_0(y)\, dy \\ &= \frac{1}{\sqrt{\pi N_0 T_b}\ a} \int_{A^2T_b/2}^{\infty} \exp\left(- \frac{y^2}{N_0 T_b A^2} \right) dy \end{aligned} \tag{10}$$

Define

$$z = \frac{y}{\sqrt{N_0 T_b}\ A} \tag{11}$$

We may then rewrite Eq. (10) in terms of the new variable z as

$$p_{10} = \frac{1}{\sqrt{\pi}} \int_{\sqrt{A^2T_b/2N_0}}^{\infty} \exp(-z^2)\, dz \tag{12}$$

which may be reformulated in terms of

complementary error function

$$\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty \exp(-z^2)\, dz \tag{13}$$

Accordingly, we may redefine the conditional probability of error P_{e0} as

$$p_{10} = \frac{1}{2}\, \text{erfc}\left(\sqrt{\frac{A^2 T_b}{4N_0}}\right) \tag{14}$$

Consider next the second kind of error that occurs when symbol 1 is sent and the receiver chooses symbol 0. Under this condition, corresponding to hypothesis H_1, the correlator input consists of a rectangular pulse of amplitude A and duration T_b plus the channel noise w(t). We may thus apply Eq. (2) to write

$$H_1 : y = A \int_0^{T_b} [A + w(t)]\, dt \tag{15}$$

The fixed quantity A in the integrand of Eq. (15) serves to shift the correlator output from a mean value of zero volt under hypothesis H_0 to a mean value of A^2T_b under hypothesis H_1. However, the conditional variance of the correlator output under hypothesis H_1 has the same value as that under hypothesis H_0. Moreover, the correlator output is Gaussian distributed as before. In summary, the correlator output under hypothesis H_1 is a Gaussian random variable with mean A^2T_b and variance $N_0T_b^2/2$, as depicted in Fig. 2(b), which corresponds to those values of the correlator output less than the threshold λ set at $A^2T_b/2$, From the symmetric nature of the Gaussian density function, it is clear that

$$p_{01} = p_{10} \tag{16}$$

Note that this statement is only true when the a priori probabilities of binary symbols 0 and 1 are equal; this assumption was made in calculating the threshold λ.

To determine the average probability of error of the PCM receiver, we note that the two possible kinds of error just considered are mutually exclusive events. Thus, with the a priori probability of transmitting a 0 equal to p_0, and the a priori probability of transmitting a 1 equal to p_1, we find that the *average probability of error*, P_e, is given by

$$P_e = p_0 p_{10} + p_1 p_{01} \tag{17}$$

Since $p_{01} = p_{10}$, and $p_0 + p_1 = 1$, Eq. (17) simplifies as

$$P_e = p_{10} = p_{01}$$

or

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\frac{1}{2} \sqrt{\frac{A^2 T_b}{N_0}} \right) \tag{18}$$

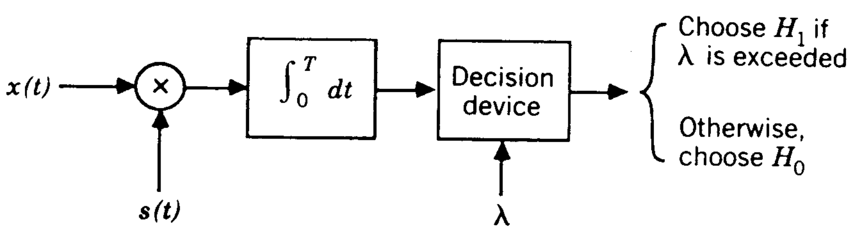


Figure 1

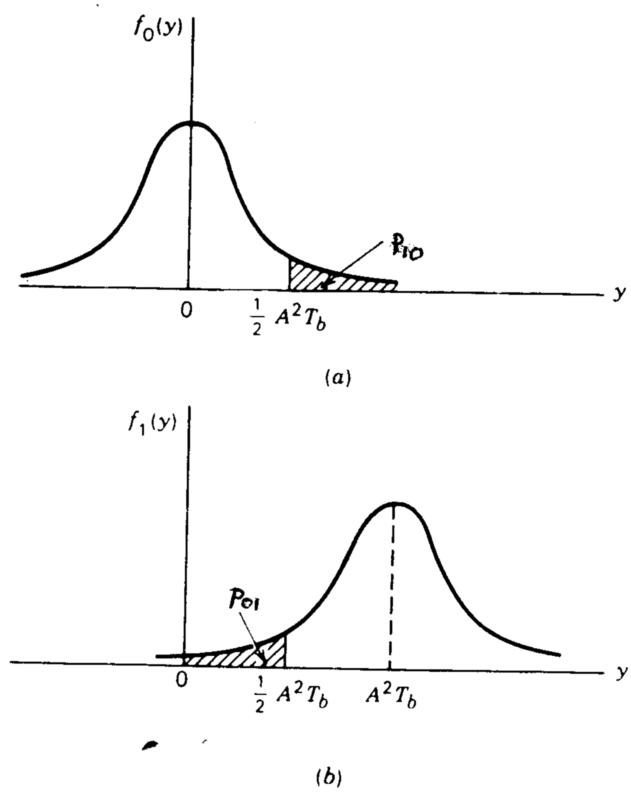


Figure 2

Problem 4.10

For unipolar RZ signaling, we have

Binary symbol 1: $s(t) = +A$ for $0 < t \leq T/2$
and $s(t) = 0$ for $T/2 < t \leq T$

Binary symbol 0: $s(t) = 0$ for $0 < t \leq T$

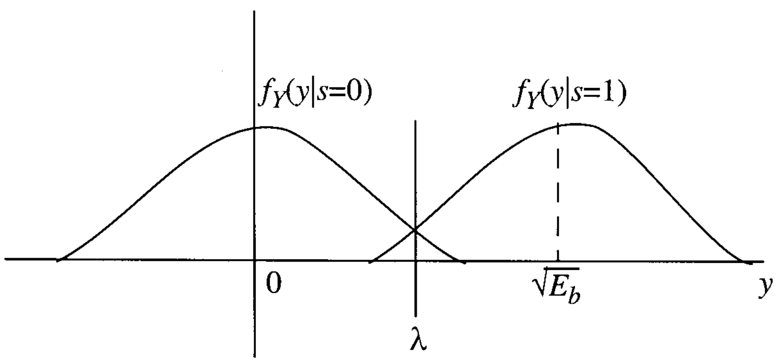
The a priori probabilities of symbols 1 and 0 are assumed to be equal, in which case we have $p_0 = p_1 = 1/2$.

To determine the average probability of error, we consider the two possible kinds of error separately. We begin by considering the first kind of error that occurs when symbol 0 is sent and the receiver chooses symbol 1. In this case, the probability of error is just the probability that the matched filter output will exceed the threshold λ owing to the presence of noise, so the transmitted symbol 0 is mistaken for symbol 1.

$$\text{Energy of symbol 1} = \frac{A^2 T_b}{2} = E_b$$

Energy of symbol 0 = 0

The conditional probability density function of the two signals is given below:



With symbols 1 and 0 assumed to be equiprobable, the optimum threshold is

$$\lambda = \frac{1}{2}\sqrt{E_b} = \frac{1}{2}\sqrt{\frac{A^2 T_b}{2}}$$

Given that symbol 0 was transmitted, the probability of error is simply the probability that $y > \lambda$, as shown by

$$P(\text{error}|0) = \int_{-\infty}^{\infty} f_Y(y|0)dy$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\infty} \exp\left(-\frac{y^2}{N_0}\right) dy$$

Define a new variable z as

$$z = \frac{y}{\sqrt{N_0}}$$

We then have

$$P(\text{error}|0) = \frac{1}{\sqrt{\pi}} \int_{\lambda/\sqrt{N_0}}^{\infty} \exp(-z^2)dz$$

$$= \frac{1}{2}\text{erfc}\left(\frac{\lambda}{\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2 T_b}{2N_0}}\right)$$

Similarly, $P(\text{error}|1) = \int_{-\infty}^{\lambda} f_Y(y|(1)dy$

$$= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\lambda} \exp\left[-\frac{(y - \sqrt{E_b})^2}{N_0}\right] dy$$

Define $z = \dfrac{\sqrt{E_b} - y}{N_0}$, and so write

$$P(\text{error}|1) = \frac{1}{\sqrt{\pi}} \int_{\frac{\sqrt{E_b} - \lambda}{\sqrt{N_0}}}^{\infty} \exp(-z^2)dz$$

$$P(\text{error}|1) = \frac{1}{2}\text{erfc}\left(\frac{\sqrt{E_b}-\lambda}{\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{\sqrt{E_b}}{2\sqrt{N_0}}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2T_b}{2N_0}}\right)$$

The average probability of error is therefore

$$P_e = P(1)P(\text{error}|1) + P(\text{error}|0)P(0)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{E_b/N_0}\right)$$

$$= \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2T_b}{2N_0}}\right) \tag{1}$$

The average probability of error for on-off (i.e., unipolar NRZ) type of encoded signals is

$$\frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{A^2T_b}{N_0}}\right)$$

Comparing this result with that of Eq. (1) for the unipolar RZ type of encoded signals, we immediately see that, for a prescribed noise spectral density N_0, the symbol energy in unipolar RZ signaling has to be doubled in order to achieve the same average probability of error as in unipolar NRZ signaling.

Problem 4.11

Probability of error for bipolar NRZ signal

Binary symbol 1 : $s(t) = \pm A$
Binary symbol 0: $s(t) = 0$
Energy of symbol 1 = $E_b = A^2T_b$

The absolute value of the threshold is $\lambda = \frac{1}{2}\sqrt{E_b} = \frac{1}{2}\sqrt{A^2 T_b}$.

Refering to Fig. 1 on the next page, we may write

$$P(\text{error}|\text{s=-A}) = \frac{1}{\sqrt{\pi N_0}}\int_{-\lambda}^{\lambda} \exp\left[\frac{-(y+\sqrt{E_b})^2}{N_0}\right]dy$$

Let $z = \frac{y+\sqrt{E_b}}{\sqrt{N_0}}$

Then,

$$P(\text{error}|s = -A) = \frac{1}{\sqrt{\pi}}\int_{-\frac{\lambda+\sqrt{E_b}}{\sqrt{N_0}}}^{\frac{\lambda+\sqrt{E_b}}{\sqrt{N_0}}} \exp(-z^2)dz$$

$$= \frac{1}{2}\left[\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) - \text{erfc}\left(\frac{3}{4}\sqrt{\frac{E_b}{N_0}}\right)\right]$$

Similarly, $P(\text{error}|s = +A) = P(\text{error}|s = -A)$

$$P(\text{error}|s = 0) = \frac{2\times 1}{\sqrt{\pi N_0}}\int_{\lambda}^{\infty} \exp\left(\frac{-y^2}{N_0}\right)dy$$

$$= \text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

The average probability of error is therefore

$$P_e = P(s{=}\pm A)P(\text{error}|s{=}\pm A) + P(s{=}0)P(\text{error}|s = 0)$$

The conditional probability density functions of symbols 1 and 0 are given in Fig. 1:

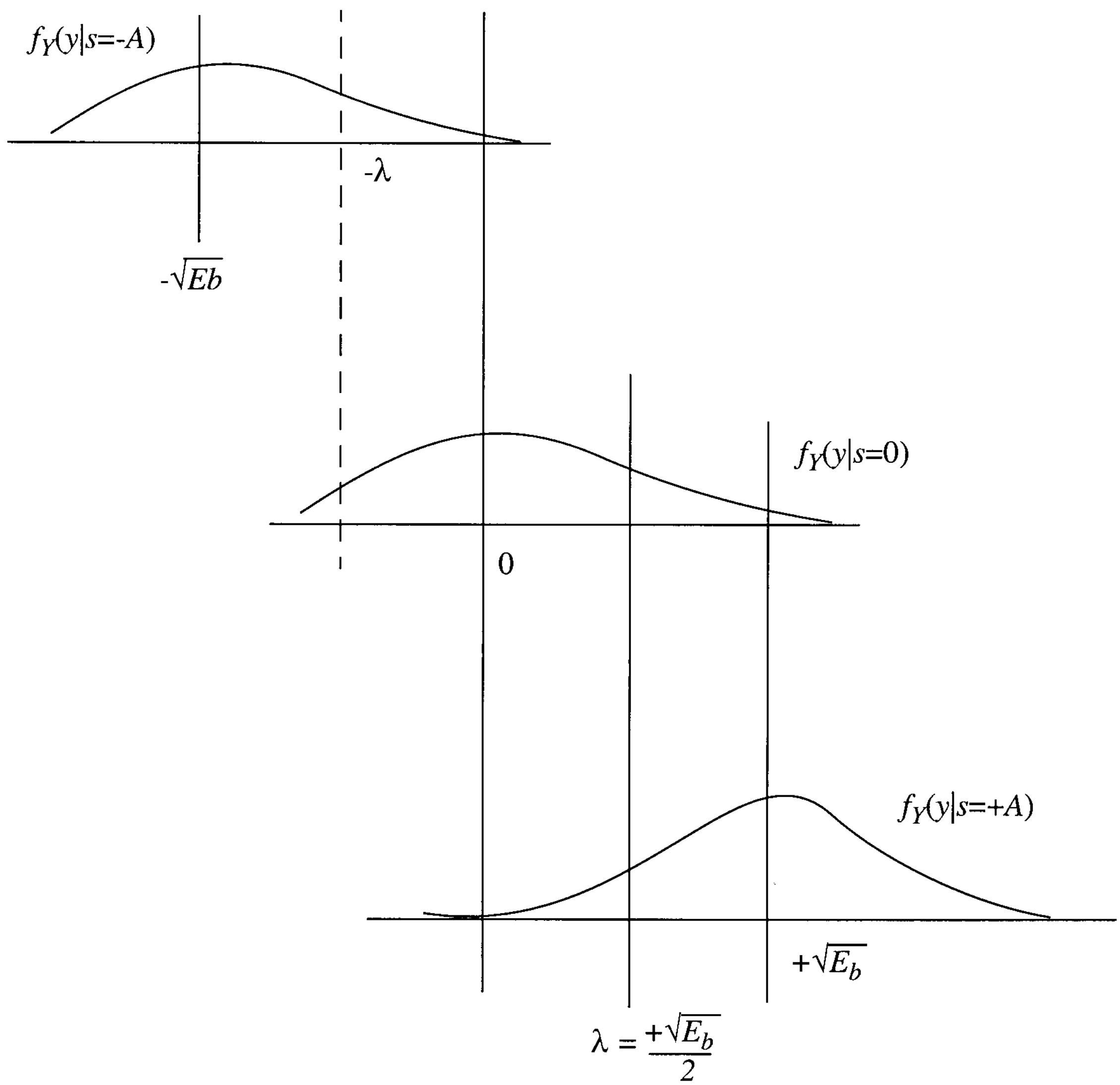

$f_Y(y|s=-A)$
$-\lambda$
$-\sqrt{Eb}$
$f_Y(y|s=0)$
0
$f_Y(y|s=+A)$
$+\sqrt{E_b}$
$\lambda = \frac{+\sqrt{E_b}}{2}$

Figure 1

$$P_e = \frac{1}{2} \times \frac{1}{2}\left[\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) - \text{erfc}\left(\frac{3}{4}\sqrt{\frac{E_b}{N_0}}\right)\right] + \frac{1}{2}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right)$$

$$= \frac{3}{4}\text{erfc}\left(\frac{1}{2}\sqrt{\frac{E_b}{N_0}}\right) - \frac{1}{4}\text{erfc}\left(\frac{3}{4}\sqrt{\frac{E_b}{N_0}}\right)$$

Problem 4.12

The rectangular pulse given in Fig. P4.12 is defined by

$g(t) = \text{rec}(t/T)$

The Fourier transform of $g(t)$ is given by

$$G(f) = \int_{-T/2}^{T/2} \exp(-j2\pi ft)dt$$

$$= T\,\text{sinc}\,(fT)$$

We thus have the Fourier-transform pair

$$\text{rec}(t/T) \rightleftharpoons T\,\text{sinc}\,(fT)$$

The magnitude spectrum $|G(f)|/T$ is plotted as the solid line in Fig. 1, shown on the next page.

Consider next a Nyquist pulse (raised cosine pulse with a rolloff factor of zero). The magnitude spectrum of this second pulse is a rectangular function of frequency, as shown by the dashed curve in Fig. 1.

Comparing the two spectral characteristics of Fig. 1, we may say that the rectangular pulse of Fig. P4.12 provides a crude approximation to the Nyquist pulse.

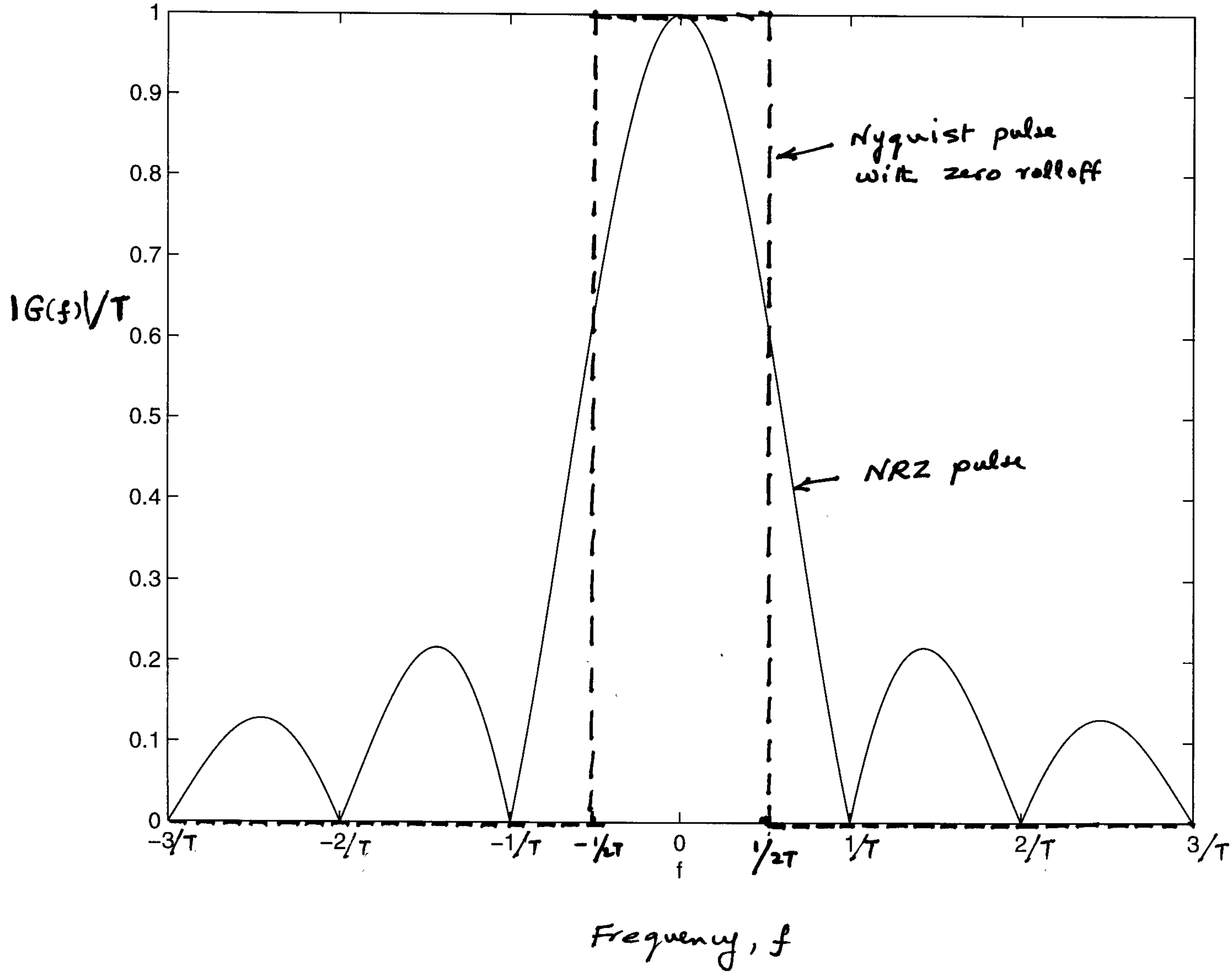


Figure 1 Spectral characteristics

Problem 4.13

Since P(f) is an even real function, its inverse Fourier transform equals

$$p(t) = 2\int_0^{\infty} P(f)\cos(2\pi ft)\,df \tag{1}$$

The P(f) is itself defined by Eq. (7.60) which is reproduced here in the form

$$P(f) = \begin{cases} \dfrac{1}{2W}, & 0 < |f| < f_1 \\ \dfrac{1}{4W}\left\{1 + \cos\left[\dfrac{\pi(|f| - f_1)}{2W - 2f_1}\right]\right\}, & f_1 < f < 2W-f_1 \\ 0, & |f| > 2W-f_1 \end{cases} \tag{2}$$

Hence, using Eq. (2) in (1):

$$p(t) = \frac{1}{W}\int_0^{f_1}\cos(2\pi ft)\,df + \frac{1}{2B}\int_{f_1}^{2W-f_1}\left[1 + \cos\left(\frac{\pi(f-f_1)}{2W\alpha}\right)\right]\cos(2\pi ft)\,df$$

$$= \left[\frac{(\sin(2\pi ft)}{2\pi Wt}\right] + \left[\frac{\sin(2\pi ft)}{4\pi Wt}\right]_{f_1}^{2W-f_1}$$

$$+ \frac{1}{4}W\left[\frac{\sin\left(2\pi ft + \frac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t + \pi/2W\alpha}\right]_{f_1}^{2W-f_1} + \frac{1}{4W}\left[\frac{\sin\left(2\pi ft - \frac{\pi(f-f_1)}{2W\alpha}\right)}{2\pi t - \pi/2W\alpha}\right]_{f_1}^{2W-f_1}$$

$$= \frac{\sin(2\pi f_1 t)}{4\pi Wt} + \frac{\sin[2\pi t(2W-f_1)]}{4\pi Wt}$$

$$- \frac{1}{4W}\,\frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]}{2\pi t - \pi/2W\alpha} + \frac{\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1 t)]}{2\pi t - \pi/2W\alpha}$$

$$= \frac{1}{W}\left[\sin(2\pi f_1 t) + \sin[2\pi t(2W-f_1)]\right]\left[\frac{1}{4\pi t} - \frac{\pi t}{(2\pi t)^2 - (\pi/2W\alpha)^2}\right]$$

$$= \frac{1}{W}\,[\sin(2\pi Wt)\cos(2\pi\alpha W)]\left[\frac{-(\pi/2W\alpha)^2}{4\pi t\,[(2\pi t)^2 - (\pi/2W\alpha)^2}\right]$$

$$= \mathrm{sinc}(2Wt)\cos(2\pi\alpha Wt)\left[\frac{1}{1 - 16\,\alpha^2 W^2\,t^2}\right]$$

Problem 4.14

The minimum bandwidth, B_T, is equal to 1/2T, where T is the pulse duration. For 64 quantization levels, $\log_2 64 = 6$ bits are required.

Problem 4.15

The effect of a linear phase response in the channel is simply to introduce a constant delay τ into the pulse p(t). The delay τ is defined as $-1/(2\pi)$times the slope of the phase response; see Eq. 2.171.

H1

Problem 4.16

The Bandwidth B of a raised cosine pulse spectrum is $2W - f_1$, where $W = 1/2T_b$ and $f_1 = W(1-\alpha)$. Thus $B = W(1+\alpha)$. For a data rate of 56 kilobits per second, W = 28 kHz.

(a) For α = 0.25,

B = 28 kHz x 1.25

= 35 kHz

(b) B = 28 kHz x 1.5

= 42 kHz

(c) B = 49 kHz

(d) B = 56 kHz

Problem 4.17

The use of eight amplitude levels ensures that 3 bits can be transmitted per pulse. The symbol period can be increased by a factor of 3. All four bandwidths in problem 7.12 will be reduced to 1/3 of their binary PAM values.

Problem 4.18

(a) For a unity rolloff, raised cosine pulse spectrum, the bandwidth B equals 1/T, where T is the pulse length. Therefore, T in this case is 1/12kHz. Quarternary PAM ensures 2 bits per pulse, so the rate of information is

$$\frac{2 \text{ bits}}{T} = 24 \text{ kilobits per second.}$$

(b) For 128 quantizing levels, 7 bits are required to transmit an amplitude. The additional bit for synchronization makes each code word 8 bits. The signal is transmitted at 24 kilobits/s, so it must be sampled at

$$\frac{24 \text{ kbits/s}}{8 \text{ bits/sample}} = 3 \text{ kHz.}$$

The maximum possible value for the signal's highest frequency component is 1.5 kHz, in order to avoid aliasing.