

# Emulating Co-Channel Interference in Wireless Networks Using Equivalent Low-Tap Filters

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**Abstract**—In emulating a multi-node wireless network, received interference can be represented by combining the multipath responses of the interfering links. Each multipath response can be described by a set of mean-squared amplitude of the multipath components and relative delays. The number of filter taps required per link to emulate the actual ('true') channel is a function of the channel bandwidth  $W$  and RMS delay spread  $\tau_{rms}$ . Assuming each per-link channel to have an exponentially decaying power delay profile, this value is about  $4W\tau_{rms}$ . We propose to emulate each link using  $n$  uniformly-spaced taps of equal mean-square gain. For this case, the required number of taps is only  $2W\tau_{rms}$ , while maintaining the important characteristic (i.e., the CDF of total power, taken over the fading) of the true channel. We derive this result analytically and confirm it by simulation. Improving on this 50% reduction in required taps, we further show that the loss in accuracy is significantly low so long as the total number of taps is the order of 16 or more. For large values of  $W\tau_{rms}$ , this can lead to even more reduction in  $n$  and, thus, further limit the cost and complexity of emulators.

**Index Terms**—hardware emulators, multi-node wireless networks, radio interference, tapped-delay-line channels

## I. INTRODUCTION

### A. Motivation

Tactical radio channel simulators need to be able to accurately capture the effect of real-world wireless channels upon communication waveforms. The complexity of emulating multi-node wireless communication networks is directly related to the bandwidth of the underlying channels, and this complexity can become prohibitive to implementation when the underlying communication waveforms have bandwidths of several hundred MHz. The computational tasks needed to simulate a multi-node tactical scenario consists of two separate components: modeling the source-to-receiver channel, and modeling the impact that pairwise interference between nodes can have on the receiver's ability to demodulate transmissions. There is thus a need for techniques that can reduce the computational complexity associated with radio scenario emulation. While improving the computational cost associated with modeling the source-to-receiver channel is important, it only leads to modest gains, and it is the second case, namely modeling the aggregate effect of many other transmitters that act as interference, that can promise significant reduction in the computational complexity needed to accurately emulate a tactical radio scenario.

In this paper, we consider a multi-node wireless network, either military or commercial, in which  $N$  terminals com-

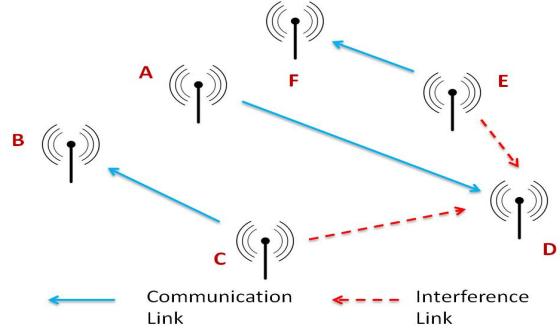


Fig. 1. Network with  $N = 6$  nodes, and three simultaneous communications links. Node D is receiving a desired signal from Node A and  $M = 2$  interferers, from Nodes C and E.

municate dynamically in a peer-to-peer fashion, as shown in Fig. 1. Several nodes may communicate simultaneously over the same frequency channel, either to maximize throughput per unit bandwidth, or because the medium access protocols were not able to resolve physical interference among the transmitters. Hardware and software emulators have been built to evaluate performance in such scenarios, [1]–[3], where each transmitter-to-receiver link can be a dispersive, multipath fading channel. The number of taps needed to characterize any link is related to the *bandwidth-delay spread product*, which is the product of signal bandwidth ( $W$ ) and the RMS delay spread ( $\tau_{rms}$ ) of the multipath channel. Capturing the behavior of all possible links of this network can dominate emulator cost and/or complexity. It is therefore desirable to use the least number of filter taps to represent each of the  $N(N - 1)$  potential links. If most links are over channels for which  $W\tau_{rms}$  is large, then the number of filter taps that must be emulated, and the corresponding cost and complexity, can be prohibitively large.

### B. Approach

To estimate the number of taps,  $n$ , needed per link, we consider the following scenario: There are  $M < N$  interference signals at a given receiver, all having the same multipath-averaged power; all  $M$  signals are received over channels with exponential *power delay profiles* (PDPs); the  $M$  channels have independent fading and the same  $W\tau_{rms}$ ; and each channel is represented by a filter with  $n$  taps of equal mean-square gain. Our goal is to determine the total number of per-link filter

TABLE I  
SOME NOTATIONS AND DEFINITIONS

Number of active nodes in the network	$N$
Number of interfering links	$M, M < N$
<i>Power Delay Profile</i> as a function of delay $\tau$	$PDP(\tau)$
Bandwidth	$W$
RMS delay spread of the channel	$\tau_{rms}$
<i>Bandwidth-delay spread product</i>	$W\tau_{rms}$
Mean power for the $k^{th}$ -delayed channel echo	$P_k$
Number of PDP rays per link for true channel	$\hat{n}$
Number of PDP rays per link for approximated channel	$n$
Power spectrum density of the transmitted signal	$S(f)$
Power spectrum density of the received signals	$S_{rec}(f)$
Frequency response of the channel	$H(f)$
Instantaneous power received from a single interfering link	$y$
Instantaneous power received from $M$ interfering links	$z$

taps,  $n$ , needed for a given  $M$  and  $W\tau_{rms}$ , such that the CDF of total instantaneous received interference power, taken over the ensemble of all channel fades, is essentially the same as that for the true (exponential) PDP. We use a combination of analysis and simulation to obtain the required  $n$ . We show that, for any  $M$ , a very close CDF fit is obtained whenever  $n$  is twice  $W\tau_{rms}$ . To implement the true (exponential) channel would require, instead, 3 or 4 times this product, so the proposed emulator approach would yield a tap reduction of up to 50%.

We show further that, so long as the product  $nM$  equals or exceeds the order of 16, the inaccuracy in using the emulator to assess interference is small for any  $W\tau_{rms}$ . For wideband channels, where  $W\tau_{rms}$  is large, this would permit even greater reductions, both in the total number of required taps and in the number of required taps per link.

### C. Organization of the Paper

The paper is organized as follows: Section II introduces the ‘true’ response of the links over which the interfering signals propagate. It is represented by a discrete *power delay profile* (PDP), which has exponentially decaying amplitudes and RMS delay spread  $\tau_{rms}$ . We derive the instantaneous total interference power for this case and derive its mean and variance over the multipath fadings of the  $M$  links. In Section III, we introduce the response of the channels to be emulated, each being an  $n$ -tap response with uniform mean-square tap gains. We derive the mean and variance of total interference power for this channel as well and, by equating them with those for the exponential channel, we determine the required  $n$  as a function of  $W\tau_{rms}$ . Section IV presents results showing how well the CDFs for the two kinds of channel responses compare, and it presents further findings and insights. Notations and definitions used throughout the paper are summarized in Table I.

## II. THE EXPONENTIAL CHANNEL

We now describe the channel impulse response and *power delay profile* of the assumed true channel, which we will use

as a baseline to develop an  $n$ -tap channel approximation in later sections.

### A. Generic Channel Impulse Response

From Nyquist sampling theory, any linear channel of bandwidth  $W$  can be represented by a tapped delay line, with the taps spaced by  $1/W$  or less. Assuming a tap spacing of  $1/W$ , the impulse response for such a channel can be written as

$$h(t) = \sum_{k=0}^{\hat{n}} g_k(t) \delta(t - k/W); \text{ (all channels)}, \quad (1)$$

where  $\hat{n}$  is chosen so that  $\hat{n}/W$  represents the longest delay,  $\tau_{max}$ , among the significant echoes in the channel; and  $g_k(t)$  is the slowly time-varying complex gain for the  $k$ th-delayed channel echo which does not change over the duration,  $\tau_{max}$ , of the impulse response. The value of  $\hat{n}$  should be the smallest integer equal to or greater than  $W\tau_{max}$  where large  $\hat{n}$  implies system implementation with more computational complexity.

Proceeding further with the general case, we now model the temporal variation,  $g_k(t)$ , of the gain of the  $k$ th tap. We assume that the tap gain is a Ricean process with a Ricean K-factor,  $K_r$ . If  $K_r$  is infinite, then the tap gain is constant (nonfading channel); at the other extreme, if  $K_r$  is zero, then the tap gain is a zero-mean complex Gaussian process (Rayleigh fading channel). We will assume the latter case for purposes of this study, but the more general Ricean model for  $g_k(t)$  could just as well be considered.

From the above, we can write the  $k$ th tap gain as

$$g_k(t) = (P_k)^{1/2} u_k(t); k = 0, \dots, \hat{n}. \quad (2)$$

where  $P_k$  is the mean-square value of  $g_k(t)$  over time (i.e., it is the average power gain of the channel echo at delay  $k/W$ );  $u_k(t)$  is a complex Gaussian process of zero mean and unit variance; and each  $u_k(t)$  is low-pass-filtered to have the desired Doppler spectrum. The sum of the  $P_k$ ’s is the average path gain, whose negative dB value is the path loss.

### B. The Exponential Power Delay Profile

A simple way to characterize a multipath channel is through the set of power gains  $\{P_k\}$ , i.e., if this set is known, the impulse response in (1) can be determined by substituting the power gain values from this set into (2). A widely accepted assumption [4] is that  $P_k$  decays exponentially with increasing delay,  $k/W$  in (1). Then the  $P_k$ ’s are uniformly-spaced samples of a decaying exponential function of delay called the channel *power delay profile* (PDP). We assume the PDP has unit area and that, for the exponential case, it has the form [5]–[7]

$$PDP(\tau) = \frac{1}{\tau_{rms}} \exp\left(\frac{-\tau}{\tau_{rms}}\right); \tau \geq 0, \quad (3)$$

where  $\tau_{rms}$  is the RMS delay spread of the channel.

We will model the actual channel as a tapped delay line filter, with taps spaced of  $1/W$ , where the  $k$ th tap gain is zero-mean and complex Gaussian with mean-square value proportional to  $PDP(k/W)$ , (3). We call this discrete PDP

with exponentially decaying amplitudes the *true channel*. In using it, an amplitude scaling factor is introduced that forces the sum of the mean-square gains to be 1 (see Section II-D). In the next section, we will describe an  $n$ -tap filter that attempts to approximate it for purposes of emulation, simulation or analysis. Here, we note that the exponential PDP has infinite extent which, strictly speaking, makes the number of taps required to represent the channel  $\hat{n} \rightarrow \infty$ .

### C. Total Instantaneous Interference Power

Assume there are  $M$  interfering links, all having the same transmitted power spectral density,  $S(f)$ , and the same mean-square path loss,  $G$ , to the receiver of the desired signal. For convenience, we set  $G = 1$ . The power spectral density of a single interfering signal at the receiver is the

$$S_{rec}(f) = S(f)|H(f)|^2; 0 \leq f \leq W, \quad (4)$$

where  $H(f)$  is a random multipath fading response whose mean-square value at every  $f$  is 1. For the case of  $\hat{n}$ -taps (multi-ray PDP),  $H(f)$  can be written as

$$H(f) = \sum_{k=0}^{\infty} P_k^{1/2} u_k \exp(-j2\pi fk/W). \quad (5)$$

where  $k/W$  is relative delay;  $P_k$  is the mean power in the  $k$ th ray; all the  $u_k$  are independent and each  $u_k$  is a slowly time-varying complex Gaussian random process. It fluctuates at a rate comparable to the Doppler bandwidth, which is very small compared to the fluctuation rate of the signal (on the scale of the bandwidth,  $W$ ). Thus, we can regard  $H(f)$  in (5) as quasi-static, but still a random function that will vary over time.

The power in the received signal (conditioned on the  $u_k$ ) from the given interfering link is the integral of  $S_{rec}(f)$ , (4), over the signal bandwidth. We can assume that  $S(f)$  is uniform over the signal bandwidth, as in the most modern radio systems. For simplicity and with no loss in generality, we assume that the uniform value is  $1/W$ , so that the total transmit power is 1. Thus,  $S_{rec}(f)$  simplifies to the integration of  $|H(f)|^2$  over the bandwidth  $W$ . As taps are spaced at integer multiples of  $1/W$ , the integral of  $|H(f)|^2$  reduces to the sum of the ray powers. Thus, instantaneous power received from one interfering link,  $y$ , over a multipath channel with an exponential power delay profile is given by

$$y = \int_f (1/W)|H(f)|^2 df = \sum_{k=0}^{\infty} P_k |u_k|^2. \quad (6)$$

We assume that each interference link has an independent channel response. Thus, the total instantaneous power,  $z$ , of the received interference from  $M$  links is the sum of  $M$  independent and identically distributed (i.i.d.) variables like  $y$ .

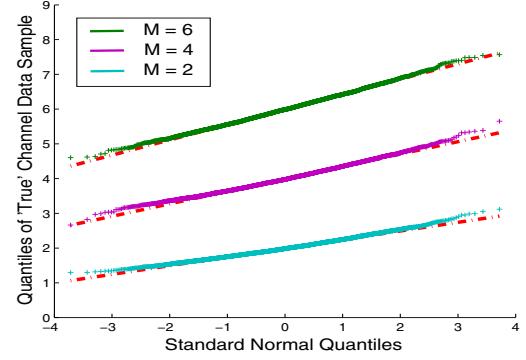


Fig. 2. Quantile-quantile plots of  $z$  for an exponential PDP with  $W\tau_{rms} = 16$  and  $M = 2, 4, 6$ . For each  $M$ , the plot shows the closeness of  $z$  to a Gaussian variable of the same mean and variance (red dashed line).

### D. Analysis of the Exponential PDP

Each normalized path gain for each interference link (e.g., each  $u_k$  in (6)) is a zero-mean, unit-variance, complex Gaussian fading term. Thus,  $|u_k|^2$  is a random variable whose pdf is a decaying exponential of unit mean. We have learned that, as  $W\tau_{rms}$  grows large, the pdf of  $z$  tends towards a near-Gaussian distribution. For example, Fig. 2 compares quantile-quantile plots of the variable  $z$  against those of a Gaussian variable with the same mean and variance; the closeness of each solid curve to the straight dashed line beneath it confirms that  $z$  is near-Gaussian for the conditions shown. For the  $n$ -tap channel with uniform mean-square gains, the same will be true as  $n$  grows large. Therefore, the CDF of total fading power for the two kinds of channels will be very similar so long as  $y$  has the same mean and variance for both [8]. This motivates us to (i) derive the mean and variance of  $z$  for the exponential PDP; (ii) do the same for the  $n$ -tap PDP; and (iii) equate the moments to obtain  $n$  as a function of  $W\tau_{rms}$ . We complete Step (i) here.

To begin, the expression for  $P_k$  is given as

$$P_k = \frac{A}{W\tau_{rms}} \exp\left(\frac{-k}{W\tau_{rms}}\right); k = 0, \dots, \infty, \quad (7)$$

where the factor  $1/W$  is added to average  $|H(f)|^2$  over frequency in a bandwidth  $W$ ; and  $A$  is chosen so that the infinite sum over  $k$  is 1. Using  $\sum_k x^k = 1/(1-x)$  for  $0 < x < 1$  and  $k = 0, 1, \dots, \infty$ , we have

$$A = W\tau_{rms} (1 - \exp(-1/W\tau_{rms})),$$

which gives  $P_k$  as

$$P_k = \left(1 - \exp\left(\frac{-1}{W\tau_{rms}}\right)\right) \exp\left(\frac{-k}{W\tau_{rms}}\right); k = 0, \dots, \infty. \quad (8)$$

We note that, for very large  $W\tau_{rms}$ ,  $A$  approaches 1.

Due to the specific assignment of  $A$  here, we have  $E[y] = 1$ ; thus,  $E[z] = M$ . Variance of  $y$ ,  $\text{var}\{y\} = E[y^2] - E[y]^2$ , requires its first and second moments where we already know

that the first moment is 1. Now, with reference to (6),

$$\begin{aligned} \mathbb{E}[y^2] &= \sum_k P_k^2 \mathbb{E}[|u_k|^4] + \sum_k \sum_{j \neq k} P_j P_k \mathbb{E}[|u_j|^2 |u_k|^2]; \\ &\quad j, k = 0, \dots, \infty. \end{aligned} \quad (9)$$

The mean of  $|u_j|^2 |u_k|^2$  can be evaluated considering the two cases  $j = k$  and  $j \neq k$ .

**Case:**  $k = j$

Since each  $u_k$  is a zero-mean, unit-variance, complex Gaussian,  $x = |u_k|^2$  is an exponential random variate (r.v.) of unit mean with pdf  $P_X(x) = \exp(-x); x > 0$ . Thus,

$$\mathbb{E}[|u_k|^4] = 2.$$

**Case:**  $k \neq j$

Here,  $x_k = |u_k|^2$  and  $x_j = |u_j|^2$  are independent exponential random variable with joint pdf  $P_{X_k, X_j}(x_k, x_j) = \exp(-x_k) \exp(-x_j); x_k > 0, x_j > 0$ ;

$$\mathbb{E}[|u_j|^2 |u_k|^2] = 1.$$

Thus, (9) simplifies to

$$\mathbb{E}[y^2] = \sum_{k=0}^{\infty} P_k^2 + \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} P_j P_k. \quad (10)$$

As  $P_k^2$  and  $P_j P_k$  are known functions of  $W\tau_{rms}$ , we can get closed form expressions for these sums as

$$\begin{aligned} \sum_{k=0}^{\infty} P_k^2 &= \frac{1 - \exp(-1/W\tau_{rms})}{1 + \exp(-1/W\tau_{rms})}; \\ \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} P_j P_k &= 1. \end{aligned} \quad (11)$$

From (10) and (11), an expression for  $\text{var}\{y\}$  is given by

$$\text{var}\{y\} = \frac{1 - \exp(-1/W\tau_{rms})}{1 + \exp(-1/W\tau_{rms})}, \quad (12)$$

and final expressions for mean and variance of total instantaneous interference power  $z$  are given by

$$\begin{aligned} \mathbb{E}[z] &= M\mathbb{E}[y] = M; \\ \text{var}\{z\} &= M\text{var}\{y\} = M \frac{1 - \exp(-1/W\tau_{rms})}{1 + \exp(-1/W\tau_{rms})}. \end{aligned} \quad (13)$$

Finally, we note that for simulation/evaluation purpose, one might choose to truncate the PDP to 4 or 5 times the RMS delay spread. Assuming the latter,  $\hat{n}$  will be the nearest integer for which  $\hat{n}/W = 5\tau_{rms}$ , i.e.,  $\hat{n} \approx 5W\tau_{rms}$ . Including the tap at delay 0, this means that, for  $W\tau_{rms} = 1$ , the filter would consist of 6 taps.

### III. EMULATOR CHANNEL WITH REDUCED TAPS

#### A. The $n$ -tap Channel Approximation

An implementation of the tapped-delay line filter above can get large as  $W\tau_{rms}$  increases significantly beyond the order of 1, corresponding to significantly more than 6 taps. We propose here an  $n$ -tap filter ( $n < \hat{n}$ ) having the PDP

$$PDP(\tau) = \sum_{k=0}^{n-1} P_k \delta(\tau - T_k), \quad (14)$$

where PDP is subject to the conditions  $T_0 = 0$  and  $P_0 + P_1 + \dots + P_{n-1} = 1$ ; and the set of values for  $\{P_k, T_k\}$  for  $k = 0, 1, \dots, n-1$  is

$$\begin{aligned} P_k &= 1/n, \\ T_k &= 2k/W. \end{aligned} \quad (15)$$

The time-varying tap gains for the filter emulation can be obtained by forming a set of i.i.d. complex Gaussian processes,  $\{u_k(t)\}$ ,  $k = 0, 1, \dots, n-1$ , that have the desired Doppler spectrum, and then applying (2). We will see that, with the above choices of tap amplitudes and spacings, an  $n$  can be found which gives excellent agreement with the exponential PDP, both in the CDF of total interference power and in the RMS delay spread.

#### B. Analysis of the $n$ -tap PDP

The spectral analysis of Section II-C, leading to (6) for the instantaneous power sum,  $y$ , in an interference link, is the same for the  $n$ -tap channel. Thus we can write

$$y = \sum_{k=0}^{n-1} (1/n) |u_k|^2. \quad (16)$$

Since  $x = |u_k|^2$  is an exponential random variate (r.v.) with pdf  $P_X(x) = \exp(-x); x > 0$ , the sum,  $v$ , of  $n$  unit-mean r.v.'s is an  $n$ th-order Gamma r.v. with pdf

$$P_V(v) = \frac{v^{n-1} \exp(-v)}{(n-1)!}; v > 0. \quad (17)$$

From (16), the received power  $y$  is  $v/n$  so that we have

$$P_Y(y) = nP_V(v = ny) = \frac{n^n y^{n-1} \exp(-ny)}{(n-1)!}; y > 0, \quad (18)$$

which indicates the pdf of the received power,  $y$ , due to one interferer. However, there are  $M$  such interfering links and, according to our assumptions, they are all i.i.d. Therefore,  $z$  is a sum of  $n' = nM$  i.i.d. exponential r.v.'s, each having a mean of  $1/n$ . We thus obtain the pdf of  $z$  as

$$P_Z(z) = \frac{n^{n'} z^{n'-1} \exp(-nz)}{(n'-1)!}; z > 0. \quad (19)$$

Here,  $z$  represents the total instantaneous interference power due to  $M$  interfering links and has mean and variance

$$\mathbb{E}[z] = \frac{n'}{n} = M; \text{var}\{z\} = \frac{M}{n}. \quad (20)$$

#### C. Required Number of Taps Per Link, $n$

We now combine (13) and (20) to match the moments,  $\mathbb{E}[z]$  and  $\text{var}\{z\}$ , for the two kinds of channels. The means for the two cases are automatically matched because of our normalizations. Now equating the variances, we obtain

$$n = \frac{1 + \exp(-1/W\tau_{rms})}{1 - \exp(-1/W\tau_{rms})} = \coth\left(\frac{1}{2W\tau_{rms}}\right) \quad (21)$$

which shows  $n$  to be a function of the bandwidth-delay product  $W\tau_{rms}$  only. If  $n$  is not an integer, it can be chosen equal to the integer nearest to the formula (but never less than 1). For

TABLE II  
NUMBER OF TAPS  $n$  AS A FUNCTION OF  $W\tau_{rms}$

$W\tau_{rms}$	$n$ (Eq. (21))	$n$ (Eq. (22))
0.5	1.31	1
0.75	1.72	2
1	2.16	2
2	4.08	4
3	6.06	6
4	8.04	8
8	16.02	8
16	32.01	8
32	64.005	64

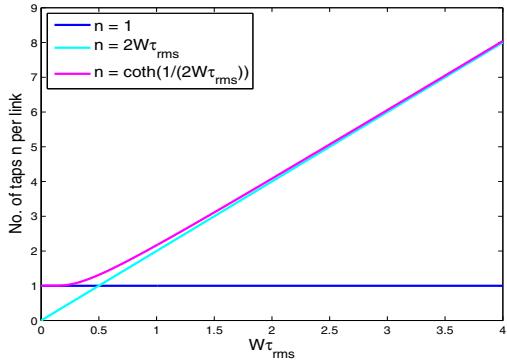


Fig. 3. Number of taps  $n$  per link: Comparison of  $n = \coth(1/(2W\tau_{rms}))$  from analysis  $n = (\max(1, 2 * W\tau_{rms}))$

example, for  $W\tau_{rms} = 0.5$ ,  $n = 1$  tap; for  $W\tau_{rms} = 0.75$  and 1.0,  $n = 2$  taps; and so on (refer Table to II). Also, it is observed that as  $W\tau_{rms}$  increases  $n$  approaches value  $2W\tau_{rms}$ . In light of these observations, we can write a simple yet-accurate approximation for  $n$ , namely,

$$n = Q(\max(1, 2W\tau_{rms})), \quad (22)$$

where  $Q(x)$  means "quantization of  $x$  to the nearest integer". The closeness of (21) and (22) can be discerned from Fig. (3).

#### IV. RESULTS AND DISCUSSION

##### A. The $n$ -tap Channel Approximation

Figure 4 shows CDFs of total interference power,  $z$ , for each of four values of  $W\tau_{rms}$ , namely 1, 2, 4, 8. For each such value, curves are shown for three values of  $M = 2, 4, 6$ . Each curve seen there are actually two curves overlapped, one for the exponential PDP and the other for the  $n$ -tap PDP with  $n$  given by (22). The CDF for the approximated ( $n$ -tap) channel matches that for the exponential channel, for each given  $M$  and  $W\tau_{rms}$ . This confirms the validity of the  $n$ -tap channel approximation.

Another issue is the RMS delay spreads for the two kinds of PDPs. Given the tap spacing of  $2/W$  for the  $n$ -tap channel, we computed RMS delay spread for  $W\tau_{rms}$  values from 1 to infinity. The result is that, over that range, the RMS delay spread for the  $n$ -tap channel increased from  $\tau_{rms}$  (the

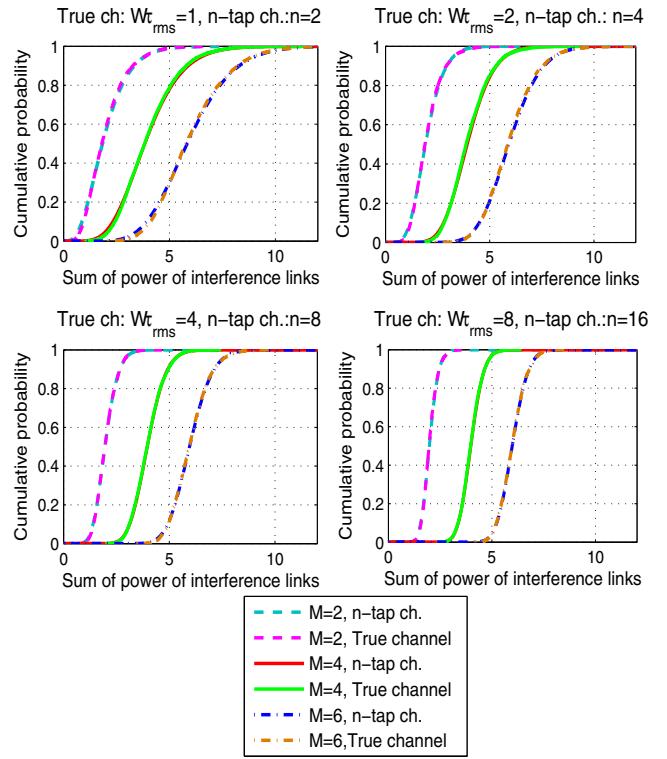


Fig. 4. CDFs of  $z$  for  $n$ -tap channel and true (exponential) channel, when the number of interfering links is  $M = 2, 4, 6$  and  $n = 2W\tau_{rms}$ .

RMS delay spread for the exponential PDP) to  $1.155\tau_{rms}$ , a deviation never greater than 16%.

Some intuitive aspects are worth noting: For  $W\tau_{rms} < 0.5$ , our formula, (22) specifies only one tap. For this condition, the channel is essentially flat-fading (little or no frequency selectivity), so  $n = 1$  is an intuitively obvious solution. We can also see that the delay span of the  $n$ -tap response ranges from  $2\tau_{rms}$  (for  $W\tau_{rms} = 1$ ) to  $4\tau_{rms}$  (as  $W\tau_{rms}$  goes to infinity). This is intuitively satisfying: it suggests that the  $n$ -tap response covers the most significant part of the response of the 'true' channel's exponential PDP.

##### B. An Alternative Approach for $Mn \geq 16$

There is an alternative way of looking at this problem that leads to an even less stringent requirement on the number of taps. It begins with the observation that, for many studies, the model of interference must merely be accurate up to some high level of interference, e.g., up to the 95th percentile. With that in mind, we note that the CDF of  $z$  becomes very steep (i.e., becomes more step-like) as as the total number of emulator taps increases to large values. For combinations of  $M$  and  $W\tau_{rms}$  where  $n' = Mn$  is on the order of 16 or greater, we find that the CDFs for both channels are so steep that their 95th percentiles of total power are no more than 1 dB apart. Thus, as  $M$  and  $W\tau_{rms}$  increase,  $n' = Mn$  need not be bound by the prescription  $n = 2W\tau_{rms}$ ;  $n$  can be smaller, so long as  $Mn$  is 16 or greater, leading to even greater savings in the

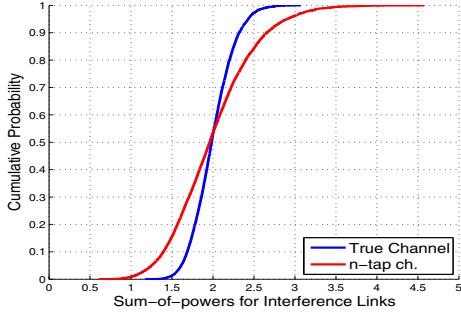


Fig. 5. CDFs of  $z$  for  $n$ -tap channel approximation with  $n = 8$  in comparison with an exponential PDP with  $W\tau_{rms} = 16$  for interfering links  $M = 2$ .

number of emulator taps. We can express this analytically by the prescription

$$\begin{aligned} n &= \min(2W\tau_{rms}, 16/M); \\ n' &= \min(2MW\tau_{rms}, 16). \end{aligned} \quad (23)$$

As an example, consider the realistic case of two interfering links ( $M = 2$ ) and an exponential PDP with  $W\tau_{rms} = 16$ . Using the more conventional approach, the  $1/W$ -spaced channel taps used for each interference link would span a delay interval  $\sim 4\tau_{rms}$ . Hence, the number of taps per link would be  $\sim 4W\tau_{rms} = 64$ ; and for  $M = 2$ , the total would be 128. The above analysis shows that an emulator with a total of 16 taps (8 per link) would suffice to capture the statistics of the total interference power, a reduction in the total and per-link numbers of taps of 8 : 1.

The CDFs of total power for the two channels in this case are shown in Fig. 5. Since  $n$  is not  $2W\tau_{rms}$ , as prescribed in (22), the CDFs do not lie on top of each other (as in Fig. 4). However, they are only 0.8 dB apart at the 95% level. In studying the impact of interference, the effect of this inaccuracy would be trivial.

## V. CONCLUSION

We have shown that  $n$ -tap channels with uniform mean-square tap gains can be used to emulate received interference in a multi-node wireless network. The proposed approach simplifies the design and reduces the complexity and cost of hardware emulators. The key is that, with  $n$  appropriately chosen, the CDF of the total received interference power, taken over the multipath fading, can be made virtually the same as the CDF for the actual links. The similarity extends to the RMS delay spread and delay span of the emulated filters.

A less stringent requirement than matching CDFs of total interference power is to require that the CDF values at and below the 95% level match to within 1 dB. Under this requirement, for  $M$  interfering links, it is sufficient that  $Mn$  equal or exceed the order of 16. This permits the use of only  $X/M$  taps, where  $X \geq 16$ , leading to further reductions in required  $n$  when  $W\tau_{rms}$  is large.

Our analysis is made for the case of an exponential power delay profile on each interfering link. Further work can test

the approach for other PDP shapes, e.g., channels with sparse multipath, and for unequal conditions (PDP and average power) among the interference links.

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