Equivalent Tapped Delay Line Channel Responses with Reduced Taps

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***Abstract*—Typically, a multipath channel response can be characterized as a sum of Rayleigh-fading 'rays', each defined by a time delay and a mean-square amplitude. Therefore, the channel response can be largely described by a *power delay profile* (PDP), which is the set of mean-square ray amplitudes and relative delays. Here, we address the following question: Given an actual (or 'true') PDP, $PDP(\tau)$, which may have many rays, is there a 3-ray (i.e. 3-tap) equivalent response, derivable from $PDP(\tau)$, that can be used to accurately estimate the average bit error rate, <*BER*>, vs. receiver input signal-to-noise ratio, *SNR*? The results reported here give an affirmative answer, e.g., for <*BER*> values down to $= 10^{-4}$, the required *SNR* using a 3-tap equivalent channel response is less than 1.1 dB larger than that required for the 'true' channel. This agreement can be improved upon, suggesting further work on deriving and evaluating equivalent 3-tap channels. We discuss the benefits of such simplifications for hardware emulators as well as for simulation and analysis.**

***Index Terms*—channel models, emulators, power delay profile, tapped delay line**

I. Introduction

A. Background and Motivation

Multipath channel responses come in many varieties, depending on physical environment, path geometry, frequency band and signal bandwidth. In most wireless scenarios, they are typically represented by a multi-tapped delay line, with Rayleigh-fading tap gains, and tap spacings of $1/W$, where W is the signal bandwidth. If the RMS delay spread of the channel is τ_{rms}, the number of significant tap gains is some multiple of $W\tau_{rms}$, which can be quite large for even moderately wideband channels. Here, we investigate whether, for any given channel, a 3-tap model can be found which accurately predicts the average link performance over that channel.

Specifically, we assume a channel can be characterized by a *power delay profile* (PDP) composed of a set of tap delays, $\{T_m\}$, and a set of corresponding mean-square tap gains, $\{P_m\}$, where $m = 0, 1, 2 \ldots$. The tap delays need not be evenly spaced, though we will assume they are in the scenario we study here. Also, the fading statistics of the tap gains can be arbitrary, though here they will be assumed to be complex-Gaussian (Rayleigh fading). We will assume a 'true' PDP that follows a widely used channel model; and we will consider two approaches to 'matching' it with 3-tap equivalent PDPs. The latter are characterized by the sets $\{T_0, T_1, T_2\}$ and $\{P_0, P_1, P_2\}$, where, with no loss in generality, we set $T_0 = 0$ and $P_0 = 1 - P_1 - P_2$.

TABLE I
Some Notations and Definitions

Bandwidth	W
Power Delay Profile as a function of delay τ	$PDP(\tau)$
RMS delay spread of the channel	τ_{rms}
longest delay among significant echoes in the channel (maximum time dispersion in the multipath channel)	τ_{max}
Gain for the mth-delayed channel echo	g_m
Power, $< \lvert g_m \rvert^2 >$, for the m^{th}-delayed channel echo	P_m
Impulse response of the channel	$h(t)$
Ricean K-Factor	K_r
Noise Peaking Factor	NPF, y
Multipath averaged bit-error-rate	<*BER*>
Signal-to-noise ratio	*SNR*

We believe the proposed solution would be of potential value to emulator design, coding of simulation platforms, and coding of analysis programs. For emulators, it simplifies the needed firmware, which leads to lower cost. For coding of either simulation or analysis programs, it both simplifies the coding and allows the code to be universal, i.e., the same lines of code for all channel conditions (*power-delay profiles*); the only thing that changes from one channel to another is the set of input parameters $\{P_1, P_2, T_1, T_2\}$.

The advantage of reduced-tap responses is even more prominent in emulator design, especially those built for large systems with many nodes. For example, given a system with N nodes, there will be not only N direct paths in a given scenario, but $N(N-2)$ potentially interfering paths as well. In such cases, the cost of the taps can dominate the cost of the emulator system. Assuming the number of taps needed for the 'true' channel is $5W\tau_{rms}$ and the channel is only moderately wideband, e.g., $W\tau_{rms} = 2$, the reduction in taps can produce significant cost savings.

To summarize, we present two new methods for matching a 3-tap PDP to a 'true' one with many taps. One matches the first 4 moments of the 'true' PDP and produces excellent performance agreement up to about $W\tau_{rms} = 2$; the other is an even simpler method that produces excellent agreement for arbitrarily large values of $W\tau_{rms}$. The 3-tap approach reduces the cost of large-scale emulators. Also, it simplifies computer coding for analysis and simulation programs: The same coding applies to all channel conditions, requiring only channel-specific changes in the input parameters.

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B. Organization

Section II describes related work. Section III introduces a widely used power delay profile for multipath fading channels, which we use here to represent the true channel and from which we derive a multi-tap channel impulse response. In Section IV, we introduce a 3-tap channel that matches the true channel in the first four moments of the PDP and might therefore serve as an equivalent. We also introduce an alternative 3-tap channel response, which we consider along with the moment-matching channel response. In Section V, we describe a method to compute and compare receiver performance over the true and 3-tap channels (specifically, their bit-error-rates averaged over fading) to assess the accuracy of 3-tap equivalents as a function of bandwidth. Section VI gives numerical results, and Section VII discusses further work. Notations and definitions used throughout the paper are partly summarized in Table I.

II. Related Work

Analysis/simulation of tapped-delay-line channel models is a well studied problem and several approaches can be found in the literature. Hoeher [1] studies the discrete-time multipath channel to compute tap-gain values using a delay-weight-and-sum method. Several studies [2]–[4] investigate challenges of computational complexity and simulation accuracy while implementing tapped-delay-line channel models on hardware. Borries et al. [3] proposes a scalar resolution technique in simulator modules to achieve accuracy, dymanic range and speed for FPGA-based hardware simulators. Kahrs and Zimmer [4] discusses multipath signal generation for implementation on a digital signal processor based on replacing complex-valued fading functions with aggregate noise functions which can be generated by using Doppler filtering. Mehlfuhrer and Rupp [5] propose a framework for path reduction based on an ad hoc path combining method and takes an optimization approach to match total power, mean delay, and the RMS delay spread specified by the tapped-delay-line model. Tripathi, et al. [6] employ tap reduction schemes in the channel estimation algorithm to reduce complexity of the receiver, based on maximizing the correlation between the band-limited channel response functions of the estimated and reduced taps. They also propose an ad-hoc approach to find a subset of arbitrarily spaced taps which may receive significant energy.

III. Background

Here, we describe the generic channel impulse response and power delay profile of a true channel, which we will use as a baseline to develop 3-tap channel approximations in later sections.

A. Generic Channel Impulse Response

From Nyquist sampling theory, we know that any linear channel of bandwidth W can be represented by a tapped delay line, with the taps spaced by $1/W$ or less. Assuming a tap spacing of exactly $1/W$, the impulse response for such a channel can be written as

$$h(t) = \sum_m g_m \delta(t - m/W); m = 0, \ldots, M (\text{all channels}) \quad (1)$$

where $\sum_m$ is a summation over m from 0 to M; M is chosen so that M/W represents the longest delay, τ_{max}, among the significant echoes in the channel; and g_m is the complex gain (which can be slowly time-varying) for the mth-delayed channel echo. By 'slowly', we mean that the gains hardly change over the duration, τ_{max}, of the impulse response. The value of M should be the smallest integer equal to or greater than $W\tau_{max}$; if this number is very large, then hardware implementation becomes a major challenge.

Proceeding further with the general case, we now model the temporal variation, $g_m(t)$, of the gain of the mth tap. We assume that the tap gain is a Ricean process with a Ricean K-factor, K_r. If K_r is infinite, then the tap gain is constant (nonfading channel); at the other extreme, if K_r is zero, then the tap gain is a zero-mean complex Gaussian process (Rayleigh fading channel). We will assume the latter case for purposes of this study, but the more general Ricean model for $g_m(t)$ could just as well be assumed.

From the above, we can write the mth tap gain as

$$g_m(t) = (P_m)^{1/2} u_m(t); m = 0, \ldots, M \quad (2)$$

where P_m is the mean-square value of $g_m(t)$ over time (i.e., it is the average power gain of the channel echo at delay m/W); $u_m(t)$ is a complex Gaussian process of zero mean and unit variance; and each $u_m(t)$ is low-pass-filtered to have the desired Doppler spectrum. The sum of the P_m's is the average path gain, whose negative dB value is the path loss.

B. The Power Delay Profile

A simple way to characterize a multipath channel is through the set of power gains $\{P_m\}$, i.e., if this set is known, then combining it with Eq. 2 totally determines the impulse response, Eq. 1. The only information needed, in that case, is how P_m varies with m. A popular assumption is that P_m decays exponentially with increasing delay, m/W in Eq. 1, [7]. We can then say that the P_m's are uniformly-spaced samples of a decaying exponential function of delay. Following tradition, we assume the PDP has unit area and that, for the exponential case, it has the form

$$PDP(\tau) = 1/\tau_{rms} \exp(-\tau/\tau_{rms}); \tau \geq 0 \text{ (True Channel)} \quad (3)$$

where τ_{rms} is the RMS delay spread of the channel. Use of this exponential shape is supported by several reported measurements, e.g., [8], [9], and is widely popular for that and other reasons, such as simplicity of use. We will therefore model the actual channel as a tapped delay line filter, with taps spaced of $1/W$, where the mth tap gain is zero-mean and complex Gaussian with mean-square value proportional to $PDP(m/W)$, Eq. 3. We call this the *true channel* and, in the next section, will describe a 3-tap filter that attempts to approximate it for purposes of emulation, simulation or analysis.

Finally, we note that the particular PDP we are using has infinite extent. The value used for M therefore depends on where the designer/analyst chooses to truncate the PDP: If the choice is to go out, say, to 5 times the RMS delay spread, then M will be the nearest integer for which $M/W = 5\tau_{rms}$, i.e., $M \approx 5W\tau_{rms}$. Including the tap at delay 0, this means that, for $W\tau_{rms} = 1$, the filter would consist of 6 taps.

IV. Channel with Reduced Taps

A. A Moment-matching 3-tap Channel Approximation

An implementation of the tapped-delay line filter above can get large as $W\tau_{rms}$ increases significantly beyond the order of 1, corresponding to significantly greater than 6 taps. What we do here is propose a 3-tap filter that matches the true channel in some important ways. Specifically, the first, second, third and fourth moments of its PDP are the same as those for the true channel. It is easy to show that these four moments for the exponential profile of Eq. 3 are τ_{rms}, $2(\tau_{rms})^2$, $6(\tau_{rms})^3$ and $24(\tau_{rms})^4$, respectively. For a 3-tap channel, the PDP is

$$PDP(\tau) = \sum_m P_m \delta(\tau - T_m); m = 0, 1, 2 \text{ (3-tap Channel).} \tag{4}$$

The design issue thus comes down to finding $\{P_m, T_m\}$ for $m = 0, 1, 2$ such that the first 4 moments of this PDP match those of the true channel's PDP. The solution must satisfy the conditions $P_0 + P_1 + P_2 = 1$ and $T_0 = 0$. Without going through the algebra, the results are as follows:

$$\begin{aligned} (P_0, T_0) &= (0.3333, 0), \\ (P_1, T_1) &= (0.6220, 1.2679\tau_{rms}), \\ (P_2, T_2) &= (0.0447, 4.7318\tau_{rms}). \end{aligned} \tag{5}$$

The time-varying tap gains for the filter emulation can be obtained by forming a set of i.i.d. complex Gaussian processes, $\{u_m(t)\}$, $m = 0, 1, 2$, that have the desired Doppler spectrum, and then applying Eq. 2 and Eq. 5.

B. An Ad-hoc 3-tap Channel Approximation

We considered another plausible 3-tap channel, which is independent of the PDP moments and therefore more general. We will see, moreover, that for the PDP of Eq. 3 at least, it actually produces somewhat better performance predictions. Again subject to the conditions $T_0 = 0$ and $P_0 + P_1 + P_2 = 1$, the set of values for $\{P_m, T_m\}$ for $m = 0, 1, 2$ is

$$\begin{aligned} (P_0, T_0) &= (1/3, 0), \\ (P_1, T_1) &= (1/3, \tau_{rms}), \\ (P_2, T_2) &= (1/3, 2\tau_{rms}). \end{aligned} \tag{6}$$

The PDP for this channel matches that for the true channel in the first moment; almost matches it in the second moment ($1.67\tau_{rms}^2$ instead of $2\tau_{rms}^2$); and falls well short in matching the third and fourth moments. The choice of this second 3-tap channel was basically ad hoc and based on intuition.

V. Evaluation Method

A. Evaluation Metrics

It is easy to compute $H(f)$ for each of the above channels. It is

$$H(f) = \sum_m (P_m)^{1/2} u_m \exp(-j2\pi m f/W); m = 0, \ldots M$$

for the true channel, and similarly for the 3-tap channel.

Note that there is not one frequency response for a channel, but an infinite number, depending on the complex values chosen for the complex Gaussian, normalized tap gains, i.e, $\{u_1, \ldots, u_M\}$ for the true channel, and $\{u_1, u_2, u_3\}$ for the 3-tap channel. So the simulation consists of generating these sets, say, 2000 times; computing a performance metric for each set; and then getting a CDF of the values computed.

What should the computed metric be, and how should we compare it for the two kinds of channels? One metric that is meaningful, easy to compute, and also independent of the particular modulation format, received power and noise level, is the *noise peaking factor* (NPF) that arises when the channel response is fully equalized, i.e., when the receiver filter is adapted to be $1/H(f)$ over the signal bandwidth (this is the *zero-forcing equalizer*). This kind of filter totally neutralizes the multipath response, but it causes an increase in the receiver output noise power. That increase is just the average of $[1/|H(f)|^2]$ over the signal bandwidth and, indeed, it requires no assumptions about the signal modulation, received power, or other link details.

By computing the NPF, which we will denote here by the symbol y, for a few thousand realizations of the sets $\{u_1, u_2, \ldots\}$, we get a population of y-values that can be described by a CDF. Doing this for both kinds of channels, we can see how closely the 3-tap filter predicts the NPF statistics.

We will show in the next section, for a single-carrier link using M-ary quadrature amplitude modulation (M-QAM), how to relate the NPF to the multipath-averaged bit-error-rate, <*BER*> as a function of the link signal-to-noise ratio, *SNR*. This is the ultimate metric we will use to compare true channels with their 3-tap approximations.

Finally, we note that the CDF of y and the resulting curves of <*BER*> vs. *SNR* depend solely on $W\tau_{rms}$. The accuracy of the 3-tap model in predicting performance is likely to decline with increasing $W\tau_{rms}$. We find that in fact it does, but only up to a point and then it levels off.

B. The Simulations

We begin with a 'recipe' for how to compute the CDF of y:

1) Start with the true channel and choose a low value (say, 0.5) for $W\tau_{rms}$.
2) Generate the complex Gaussian set u_m; compute $H(f)$; and compute y, as above.
3) Repeat until there are 2000 samples of y. (Each of these 2000 trials should generate a new, independent sample

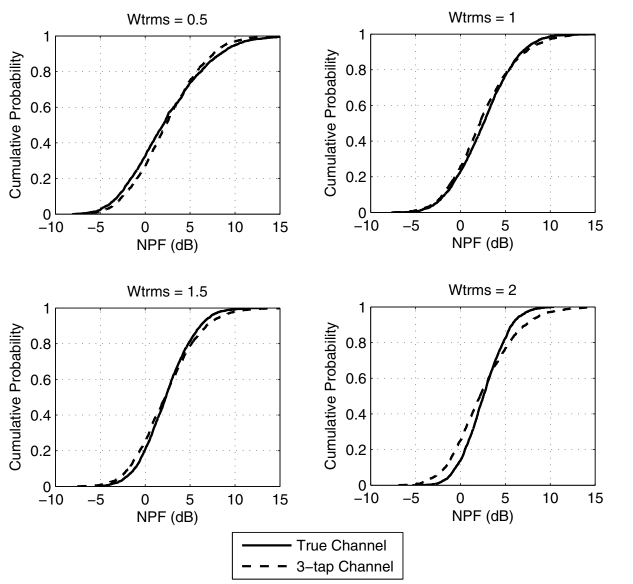

Fig. 1. CDFs of the Noise Peaking Factor (NPF) comparing with 3-tap moment-matching channel

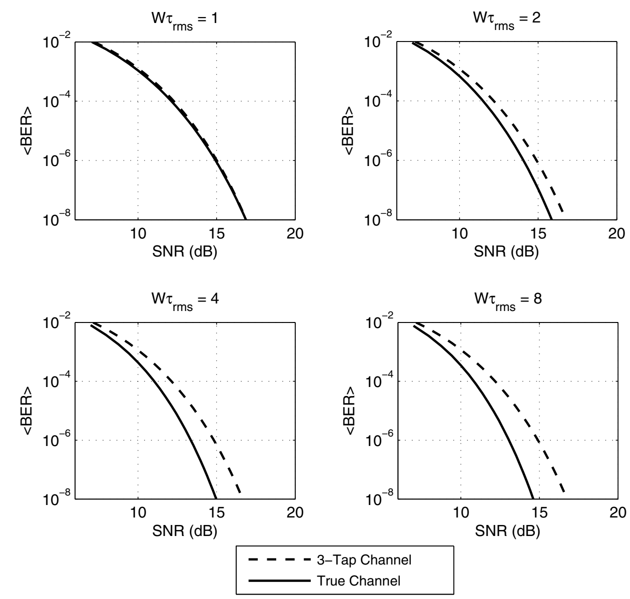

Fig. 2. *BER* vs. *SNR* for 4-QAM Signaling comparing with 3-tap moment-matching channel

of u_m at each m, so the Doppler spectrum is irrelevant for this simulation.)

4) Form a CDF of y, and plot it.
5) Now repeat the above for the 3-tap channel, and compare the two channels.
6) Repeat the above several times, with $W\tau_{rms}$ increasing in each new round.

At some value of $W\tau_{rms}$, the CDFs for the true and 3-tap channel will start to separate. This may denote the limitation of the 3-tap approximation, i.e., the approximation may only be sufficient for $W\tau_{rms}$ up to the value where the CDF separation begins. Some results are shown in Fig.1, which compares CDFs for the true and 3-tap channels, for each of four values of $W\tau_{rms}$. We see that separation between the two CDFs of y start to become significant at $W\tau_{rms} = 2$.

However, there is a further step needed to truly test the 3-tap approximation: Using the set of y-values for a given $W\tau_{rms}$ to generate curves of <*BER*> vs. *SNR*, where <*BER*> is the bit error rate averaged over the channel realizations (e.g., the 2000 realizations generated in the above simulations); and *SNR* is the signal-to-noise ratio at the receiver input. If we include interference, then *SNR* represents the signal-to-(interference-plus-noise) ratio. In either case (with or without interference), we can compute <*BER*> vs. *SNR* curves as follows:

1) Choose a channel type (true or 3-tap) and a low value of $W\tau_{rms}$ such as 0.5.
   - Start with a low value of *SNR*, such as 1 (0 dB).
   - Compute <*BER*> using the set of samples for y and the simple formula shown below.
   - Increment *SNR* in 2-dB steps, up to *SNR*$=100$ (20 dB), computing <*BER*> for each case.
   - Plot <*BER*> vs. *SNR* for the given channel type and value of $W\tau_{rms}$.
2) Repeat 1 for the other channel type.
3) Repeat 1 and 2 for successive doublings of $W\tau_{rms}$.

A close approximation to the instantaneous bit-error-rate, i.e., the value of *BER* conditioned on the value of the noise peaking factor, y, is given for the case of M-QAM modulation by [10]

$$BER|y = 0.2\exp(\frac{-1.5}{(M-1)}SNR/y); \qquad (7)$$

and <*BER*> is computed as the average of this quantity over the number of y-values computed in the earlier simulation. Note that SNR/y is the receiver output signal-to-noise ratio, which is diminished from its value at the receiver input (*SNR*) by a factor $y \geq 1$, i.e., the noise increase caused by the equalizing filter.

VI. Results and Discussion

Some results of the bit-error-rate computations are shown in Fig. 2. The separation between the solid and dashed curves for a given value of $W\tau_{rms}$ show how well the approximation works for that value. The cases shown are $W\tau_{rms} = 1, 2, 4$ and 8, for *SNR* as high as 20 dB and <*BER*> as low as 10^{-8}. We see that, for $W\tau_{rms}$ up to 2, the *SNR* values required to achieve <*BER*>$= 10^{-8}$, as predicted for the true channel and its 3-tap approximation, differ by less than 1 dB.

Next, we computed results for $W\tau_{rms} = 16$ and 32, and found that these discrepancies remain below 2.7 dB. At the more practical bit-error-rate level <*BER*>$= 10^{-4}$, the discrepancies are even smaller, lying below 1.5 dB. Assuming these are acceptable discrepancies in the prediction of link

TABLE II
DISCREPANCIES (dB) IN *SNR* AT <*BER*>= 10^{-8}

$W\tau_{rms}$	**Moment-matching 3-tap channel**	**Ad-hoc 3-tap channel**
1	0.22	0.67
2	0.91	0.22
4	1.76	0.95
8	2.06	1.42
16	2.46	1.75
32	2.69	1.94
64	2.64	2.04
128	2.61	1.91

performance, we can say that, even for signal bandwidths as high as 250 MHz, the 3-tap channel approximation can be used for RMS delay spreads up to 0.13 microseconds. In most cases, systems with 250-MHz bandwidths will be limited to ranges so short that RMS delay spreads hardly, if ever, exceed this value.

Pursuing the subject further, we considered values for $W\tau_{rms}$ of 64 and 128 and found the discrepancies to be no higher than for $W\tau_{rms} = 32$. In fact, the discrepancy seems to level off asymptotically with increasing $W\tau_{rms}$. The reason is that the variable y in Eq. 7 becomes more and more narrowly distributed about a single value as $W\tau_{rms}$ increases without bound. (The reason, in turn, for this narrowing is that the number of channel 'correlation bandwidths' within the signal bandwidth increases without bound, so that the random variable y converges toward its mean.)

We repeated the above exercise for the ad-hoc 3-tap channel. As given in Table II, beyond the value $W\tau_{rms} = 1$, we find the discrepancies in required *SNR* to be even smaller than for the moment-matching 3-tap channel, though not by much: As $W\tau_{rms}$ increases to very large values, the *SNR* discrepancies for <*BER*> = 10^{-8} level off at about 2.0 dB; at <*BER*>= 10^{-4} they level off at about 1.1 dB.

We are currently developing a more rigorous solution for the 'optimal' 3-tap channel. Meantime, the bottom line of this overall investigation seems clear: For a single-carrier transmission with receiver equalization, a suitable 3-tap approximation to the true channel can be found for any combination of bandwidth and channel delay spread of likely practical interest. Whether this finding applies for other modern-day signaling formats, such as OFDM, remains to be studied.

VII. CONCLUSION

The use of 3-tap channel approximations has been shown to be quite promising. Part of the reason is that, in a well-equalized radio link, the specifics of the multipath channel response have a reduced impact on the details of the performance. That is, the receiver adaptation renders the details of the channel's multipath structure and fading statistics less important to the performance outcome. Even so, further work needed to confirm our current findings would include (1) trying other 'true' channel characteristics, notably, channels that have sparse multipath, as in many ultra-wideband (UWB) scenarios; (2) testing the performance of the 3-tap approximation on other signal formats besides the single-carrier format assumed here, notably, OFDM formats; and (3) devising a more rigorous approach to optimizing reduced-tap equivalent channels.

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