Downlink Multiple Antenna Transmitter Optimization on Spatially and Temporally Correlated Channels with Delayed Channel State Information

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Abstract

In this paper we present multiple antenna transmitter optimization schemes that are based on linear transformations and transmit power optimization, while keeping the average transmit power conserved. We consider the downlink of a wireless system with multiple transmit antennas at the base station and a number of mobile terminals (i.e., users) each with a single receive antenna. We consider the maximum achievable sum data rates in the case of (1) zero-forcing spatial pre-filter and (2) modified zero-forcing spatial pre-filter. Using a multiple input single output (MISO) channel model with temporal and spatial correlations, we study the effect of delayed channel state information (CSI) on these schemes. It is seen that as the CSI delay increases, spatially uncorrelated channels perform worse than spatially correlated channels, which is in contrast to the case of zero delay CSI.

***Keywords*: Transmitter beamforming, correlated channels, channel state information.**

I. Introduction

Recent studies are focusing on multiple antenna systems with multiple users (see [1]–[4] and references therein). In this paper we study multiple antenna transmitter optimization (i.e, spatial pre-filtering) schemes that are based on linear transformations and transmit power optimization (keeping the average transmit power conserved). We consider the downlink of a wireless system with multiple transmit antennas at the base station and a number of mobile terminals (i.e., users) each with a single receive antenna. From an information theoretical model the downlink corresponds to the case of a *broadcast channel*. We consider the maximum achievable sum data rates in the case of (1) zero-forcing spatial pre-filter and (2) modified zero-forcing spatial pre-filter [5]. To our best knowledge, most of the previously reported studies assume perfect knowledge of the channel state (i.e., response) at the transmitter. In this paper we study the performance of the transmitter optimization schemes with respect to delayed channel state information (CSI). A multiple input single output (MISO) channel model is introduced modeling temporal and spatial correlations. We show how the performance of the schemes depends on spatial correlations and the CSI delay. It is seen that as the CSI delay increases, spatially uncorrelated channels perform worse than spatially correlated channels, which is in contrast to the case of zero delay CSI.

The paper is organized as follows. In Section II we describe the system model and transmitter optimization schemes. In Section III we introduce a channel model capturing spatial and temporal correlations. Each section contains corresponding numerical examples regarding performance of the schemes under different system scenarios. We conclude in Section IV.

II. System Model and Transmitter Optimization Schemes

The system consists of M transmit antennas and N single-antenna mobile terminals (see Figure 1). In other words each mobile terminal presents a MISO channel as seen from the base station. x_n is the information bearing signal intended for mobile terminal n and y_n is the received signal at the corresponding terminal (for $n = 1, \cdots, N$). The received vector $\mathbf{y} = [y_1, \cdots, y_N]^{\mathrm{T}}$ is

$$\begin{aligned} \mathbf{y} &= \mathbf{HSx} + \mathbf{n}, \\ \mathbf{y} &\in \mathcal{C}^N, \mathbf{x} \in \mathcal{C}^N, \mathbf{n} \in \mathcal{C}^N, \mathbf{S} \in \mathcal{C}^{M\times N}, \mathbf{H} \in \mathcal{C}^{N\times M} \end{aligned} \quad (1)$$

where $\mathbf{x} = [x_1, \cdots, x_N]^{\mathrm{T}}$ is the transmitted vector ($\mathrm{E}[\mathbf{xx}^{\mathrm{H}}] = P_{av}\ \mathbf{I}_{N\times N}$), $\mathbf{n}$ is AWGN ($\mathrm{E}[\mathbf{nn}^{\mathrm{H}}] = N_0\ \mathbf{I}_{N\times N}$), $\mathbf{H}$ is the MIMO channel response matrix, and $\mathbf{S}$ is a transformation (spatial pre-filtering) performed at the transmitter. Note that the vectors $\mathbf{x}$ and $\mathbf{y}$ have the same dimensionality. Further, h_{nm} is the nth row and mth column element of the matrix $\mathbf{H}$ corresponding to a channel between mobile terminal n and transmit antenna m.

Application of the spatial pre-filtering results in the composite MIMO channel $\mathbf{G}$ given as

$$\mathbf{G} = \mathbf{HS}, \quad \mathbf{G} \in \mathcal{C}^{N\times N} \quad (2)$$

where g_{nm} is the nth row and mth column element of the composite MIMO channel response matrix $\mathbf{G}$. The signal received at the nth mobile terminal is

$$y_n = \underbrace{g_{nn}x_n}_{\text{Desired signal for user } n} + \underbrace{\sum_{i=1, i\neq n}^{N} g_{ni}x_i}_{\text{Interference}} + n_n. \quad (3)$$

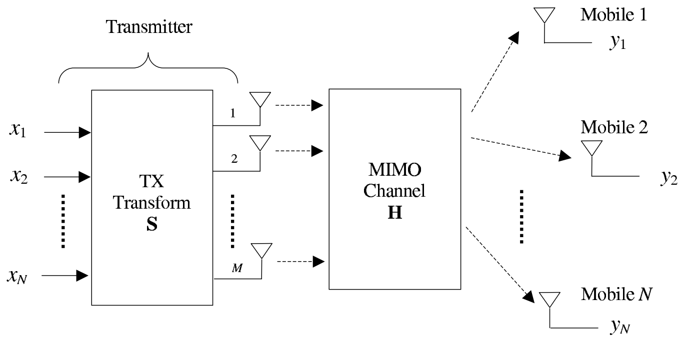

Fig. 1. System model consisting of M transmit antennas and N mobile terminals.

In the above representation, the interference is the signal that is intended for other mobile terminals than terminal n. As said earlier, the matrix $\mathbf{S}$ is a spatial pre-filter at the transmitter. It is determined based on optimization criteria that we address later in the text and has to satisfy the following constraint

$$\mathrm{trace}\left(\mathbf{S}\mathbf{S}^{\mathrm{H}}\right) \leq N \tag{4}$$

which keeps the average transmit power conserved. We represent the matrix $\mathbf{S}$ as

$$\mathbf{S} = \mathbf{A}\mathbf{P}, \quad \mathbf{A} \in \mathcal{C}^{M\times N}, \mathbf{P} \in \mathcal{C}^{N\times N} \tag{5}$$

where $\mathbf{A}$ is a linear transformation and $\mathbf{P}$ is a diagonal matrix. $\mathbf{P}$ is determined such that the transmit power remains conserved. Considering different forms of the matrix $\mathbf{A}$ we study the following solutions.

1) Zero-forcing (ZF) spatial pre-filtering scheme where $\mathbf{A}$ is represented by

$$\mathbf{A} = \mathbf{H}^{\mathrm{H}}(\mathbf{H}\mathbf{H}^{\mathrm{H}})^{-1}. \tag{6}$$

As can be seen, the above linear transformation is zeroing the interference between the signals dedicated to different mobile terminals, i.e., $\mathbf{H}\mathbf{A} = \mathbf{I}_{N\times N}$. x_n are assumed to be circularly symmetric complex random variables having Gaussian distribution $\mathcal{N}_{\mathcal{C}}(0, P_{av})$. Consequently, the maximum achievable data rate (capacity) for mobile terminal n is

$$R_n^{\mathrm{ZF}} = \log_2\left(1 + \frac{P_{av}|p_{nn}|^2}{N_0}\right) \tag{7}$$

where p_{nn} is the nth diagonal element of the matrix $\mathbf{P}$ defined in (5). In (6) it is assumed that $\mathbf{H}\mathbf{H}^{\mathrm{H}}$ is invertible, i.e, the rows of $\mathbf{H}$ are linearly independent.

2) Modified zero-forcing (MZF) spatial pre-filtering scheme that assumes

$$\mathbf{A} = \mathbf{H}^{\mathrm{H}}\left(\mathbf{H}\mathbf{H}^{\mathrm{H}} + \frac{N_0}{P_{av}}\mathbf{I}\right)^{-1}. \tag{8}$$

In the case of the above transformation, in addition to the knowledge of the channel $\mathbf{H}$ the transmitter has to know the noise variance N_0. x_n are assumed to be circularly symmetric complex random variables having Gaussian distribution $\mathcal{N}_{\mathcal{C}}(0, P_{av})$. The maximum achievable data rate (capacity) for mobile terminal n now becomes

$$R_n^{\mathrm{MZF}} = \log_2\left(1 + \frac{P_{av}|g_{nn}|^2}{P_{av}\sum_{i=1,i\neq n}^{N}|g_{ni}|^2 + N_0}\right). \tag{9}$$

While the transformation in (8) appears to be similar in form to a MMSE linear receiver, the important difference is that the transformation is performed at the transmitter. Using the virtual uplink approach for transmitter beamforming (introduced in [3], [4]) we present the following proposition.

Proposition 1: If the nth diagonal element of $\mathbf{P}$ is selected as

$$p_{nn} = \frac{1}{\sqrt{\mathbf{a}_n^{\mathrm{H}}\mathbf{a}_n}} \quad (n = 1, \cdots, N) \tag{10}$$

where $\mathbf{a}_n$ is the nth column vector of the matrix $\mathbf{A}$, the constraint in (4) is satisfied with equality. Consequently, the achievable downlink rate R_n^{MZF} for mobile n is identical to its corresponding virtual uplink rate when an optimal uplink linear MMSE receiver is applied. □

See Appendix for a definition of the corresponding virtual uplink and a proof of the above proposition.

Once the matrix $\mathbf{A}$ is selected, the elements of the diagonal matrix $\mathbf{P}$ are determined such that the transmit power remains conserved and the sum rate is maximized. The constraint on the transmit power is

$$\mathrm{trace}\left(\mathbf{A}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{A}^{\mathrm{H}}\right) \leq N. \tag{11}$$

The elements of the matrix $\mathbf{P}$ are selected such that

$$\mathrm{diag}(\mathbf{P}) = [p_{11}, \cdots, p_{NN}]^{\mathrm{T}} = \arg\max_{\mathrm{trace}(\mathbf{A}\mathbf{P}\mathbf{P}^{\mathrm{H}}\mathbf{A}^{\mathrm{H}})\leq N} \sum_{i=1}^{N} R_n. \tag{12}$$

Numerical Results

In the case of perfect CSI being available at the transmitter, the maximum achievable sum data rates for the above schemes are presented in [5]. In [5] the performance is studied for different system scenarios and compared to a number of alternative solutions and performance bounds.

We now present the effects of imperfect channel state knowledge. In practical communication systems, the channel state $\mathbf{H}$ has to be estimated at the receivers, and then fed to the transmitter. Specifically, mobile terminal n feeds back the estimate of the nth row of the matrix $\mathbf{H}$, for $n = 1, \cdots, N$. In the case of a time varying channel, this practical procedure results in noisy and delayed (temporally mismatched) estimates being available to the transmitter to perform the optimization. As said earlier, the MIMO channel is time varying. Let $\mathbf{H}_{i-1}$ and $\mathbf{H}_i$ correspond to consecutive block faded channel responses.

The temporal characteristic of the channel is described using the correlation

$$k = \mathrm{E}\left[h_{(i-1)nm}\, h_{inm}^{*}\right]/\Gamma \tag{13}$$

where $\Gamma = \mathrm{E}[h_{inm}h_{inm}^{*}]$, and h_{inm} is a stationary random process (for $m = 1, \cdots, M$ and $n = 1, \cdots, N$, denoting transmit and receive antenna indices, respectively). Low values of the correlation k correspond to higher mismatch between $\mathbf{H}_{i-1}$ and $\mathbf{H}_i$. Note that the above channel is modeled as a first order discrete Markov process. In the case of Jake's model, $k = J_0(2\pi f_d \tau)$, where f_d is the maximum Doppler frequency and τ is the time difference between $h_{(i-1)nm}$ and h_{inm}. In addition, the above simplified model assumes that there is no spatial correlation. We assume that the mobile terminals feed back $\mathbf{H}_{i-1}$ which is used at the base station to perform the transmitter optimization for the ith block. In other words the downlink transmitter is ignoring the fact that $\mathbf{H}_i \neq \mathbf{H}_{i-1}$. In Figure 2, we present the average rate per user versus the temporal channel correlation k in (13), for $SNR = 10\log(P_{av}/N_0) = 10dB$. From these results we note very high sensitivity of the schemes to the channel mismatch. In this particular case the performance of the ZF and MZF schemes becomes worse than when there is no pre-filtering. See also [6] for a related study of channel mismatch and achievable data rates for single user MIMO systems. Note that the above example and the model in (13) is a simplification that we only use to illustrate the schemes' sensitivity to imperfect knowledge of the channel state. In the following we introduce a detailed channel model incorporating spatial and temporal characteristics.

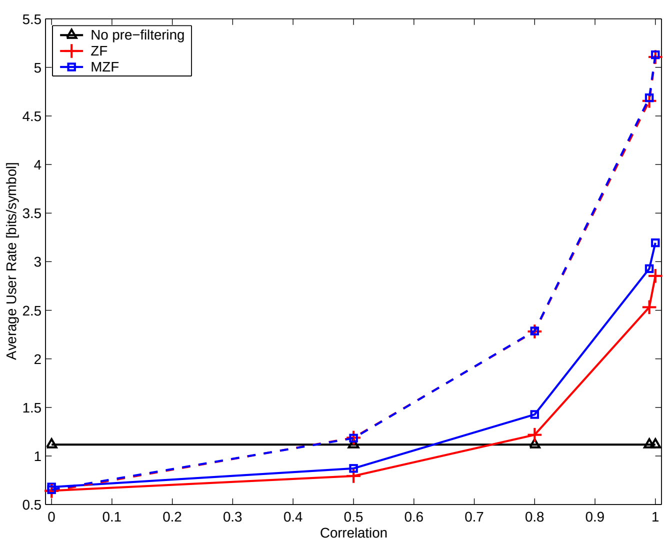

Fig. 2. Average rate per user vs. temporal channel correlation k, $SNR =$ 10dB, $M = 3$ (solid lines), $M = 6$ (dashed lines), $N = 3$, Rayleigh channel.

III. Channel Model

In the following we first address the spatial aspects of the channel $\mathbf{H}$. For each mobile terminal there is a $1 \times M$ dimensional channel between its receive antenna and M transmit antennas at the base station. The MISO channel $\mathbf{h}_n = [h_{n1} \cdots h_{nM}]$ for mobile terminal n $(n = 1, \cdots, N)$ corresponds to the nth row of the channel matrix $\mathbf{H}$, and we assume that it is independent from other channels (i.e., rows of the channel matrix).

Constraining the analysis to two dimensional (2D) space, the channel $\mathbf{h}_n = [h_n(\mathbf{r}_1) \cdots h_n(\mathbf{r}_M)]$, where $\mathbf{r}_m$ is the position of the transmit antenna m in the 2D plane. The channel response $h_n(\mathbf{r}_m)$ between transmit antenna m and the receive antenna of mobile terminal n, is given as a superposition of plane waves

$$h_n(\mathbf{r}_m) = \int_{-\pi}^{\pi} A(\alpha) e^{-j\mathbf{k}\,\mathbf{r}_m} d\alpha \tag{14}$$

where $\mathbf{k} = [\frac{2\pi}{\lambda}\cos(\alpha+\alpha_n)\ \frac{2\pi}{\lambda}\cos(\alpha+\alpha_n)]$ is the wave vector of a 2D plane wave in the direction corresponding to the angle $\alpha + \alpha_n$. Note that α_n corresponds to the angle of the mobile terminal boresight and it is an instantiation of a real random variable distributed uniformly over the interval $[0\ 2\pi]$. λ is the wavelength of the plane wave with $A(\alpha)$ being a complex value corresponding to its amplitude and its phase. In other words, the channel response $h_n(\mathbf{r}_m)$ in (14) is an infinite sum (integral) of all plane waves at the location $\mathbf{r}_m$. Further, it is assumed here that $A(\alpha)$ has the following statistical property

$$\mathrm{E}[A(\alpha)A(\beta)^{*}] = P(\alpha)\delta(\alpha - \beta) \tag{15}$$

where $P(\alpha)$ is the angular power density of the electromagnetic radiation at the base station. $P(\alpha)$ is also referred to as the power azimuth spectrum [7]. The *rms* angular (i.e., azimuth) spread [8] is defined as

$$AS = \sqrt{\int_{-\pi}^{\pi} \alpha^2 P(\alpha) d\alpha}. \tag{16}$$

For cellular systems, where the relevant scatterers are more likely to be close to the mobile terminal, $P(\alpha)$ is typically modeled as a Gaussian distribution shaped function [8]

$$P(\alpha) = \frac{\kappa}{\sqrt{2\pi}\sigma}\, e^{-\frac{\alpha^2}{2\sigma^2}} \tag{17}$$

where the constant κ is determined from the condition $\int_{-\pi}^{\pi} P(\alpha)d\alpha = 1$. Note that $\sigma \approx AS$ (given in (16)) when $\sigma \ll \pi$. Other distributions such as Laplacian have also been used to model the angular power density (see [7]).

The spatial correlation between two channel responses $h_n(\mathbf{r}_i)$ and $h_n(\mathbf{r}_j)$ corresponding to transmit antennas i and j and mobile terminal n is then given by

$$\phi_{ij} = \mathrm{E}[h_n(\mathbf{r}_i)h_n(\mathbf{r}_j)^{*}] = \int_{-\pi}^{\pi} P(\alpha)\, e^{-j\mathbf{k}(\mathbf{r}_i - \mathbf{r}_j)} d\alpha. \tag{18}$$

For the given $P(\alpha)$ the correlation ϕ_{ij} can be computed numerically from the above expression. The correlation ϕ_{ij} is the ith row and the jth column element of the spatial correlation matrix

$$\mathbf{\Phi}_n = \mathrm{E}[\mathbf{h}_n^{\mathrm{H}}\mathbf{h}_n]. \tag{19}$$

To obtain a spatially correlated row vector (i.e., a MISO channel $\mathbf{h}_n$)

$$\mathbf{h}_n = [n_1 \cdots n_M]\boldsymbol{\Phi}_n^{1/2} \tag{20}$$

where n_i, $i = 1, \cdots, M$, are complex iid random variables with distribution $\mathcal{N}_{\mathcal{C}}(0,1)$. In general, channels with lower angular spread have higher degree of spatial correlations. For example, in the extreme case of $\sigma = 0^o$, the channel has the highest degree of spatial correlations resulting in a single eigenvector of the spatial correlation matrix $\boldsymbol{\Phi}_n$ (i.e., infinitely large condition number of the matrix $\boldsymbol{\Phi}_n$). On the other hand when the channel is spatially uncorrelated $\boldsymbol{\Phi}_n = \mathbf{I}_{M\times M}$ and its condition number is 1.

The temporal evolution of the spatially correlated MISO channel $\mathbf{h}_n$ may be represented as [9]

$$\mathbf{h}_n(t) = [1\cdots 1]\mathbf{D}_n\mathbf{N}_n\boldsymbol{\Phi}_n^{1/2}, \;\; \mathbf{D}_n \in \mathcal{C}^{N_f\times N_f}, \mathbf{N}_n \in \mathcal{C}^{N_f\times M} \tag{21}$$

where $\mathbf{N}_n$ is a $N_f \times M$ dimensional matrix with elements corresponding to complex *iid* random variables with distribution $\mathcal{N}_{\mathcal{C}}(0, 1/N_f)$. $\mathbf{D}_n$ is a $N_f \times N_f$ diagonal Doppler shift matrix with diagonal elements

$$d_{ii} = e^{j\omega_i t} \tag{22}$$

representing the Doppler shifts that affect N_f plane waves and

$$\omega_i = \frac{2\pi}{\lambda} v_n \cos(\gamma_i), \text{ for } i = 1, \cdots, N_f \tag{23}$$

where v_n is the velocity of mobile terminal n and the angle of arrival of the ith plane wave at the terminal is γ_i (generated as $\mathcal{U}[0\ 2\pi]$).

It can be shown that the model in (21) strictly conforms to Jake's model for $N_f \to \infty$. This model assumes that at the mobile terminal the plane waves are coming from all directions with equal probability. With minor modifications, the above model can be modified to capture non-uniform arrival of the plane waves at the terminal. Further, note that each diagonal element of $\mathbf{D}_n$ corresponds to one Doppler shift. The matrix $\mathbf{N}_n\boldsymbol{\Phi}_n^{1/2}$ is introducing spatial correlations at the base station for each Doppler shift. For each mobile terminal, $\mathbf{D}_n$ and $\mathbf{N}_n$ are independently generated.

For $N_f \to \infty$, a more compact representation of the MISO channel model in (21) is

$$\mathbf{h}_n(t) = \left(k_n(t)\,\mathbf{n}_0 + \sqrt{1-k_n(t)^2}\,\mathbf{n}_t\right)\boldsymbol{\Phi}_n^{1/2} \tag{24}$$

where $\mathbf{n}_0 = [n_{01}\cdots n_{0M}]$ and $\mathbf{n}_t = [n_{t1}\cdots n_{tM}]$ with the components n_{0i} and n_{ti} ($i = 1, \cdots, M$) being complex iid random variables with distribution $\mathcal{N}_{\mathcal{C}}(0,1)$. Assuming Jake's model, $k_n(t) = J_0(2\pi v_n t/\lambda)$. It can be shown that the models in (21) and (24) are statistically equivalent as $N_f \to \infty$. In both cases the components of the vector $\mathbf{h}_n(t)$ have a zero mean complex Gaussian distribution and have the same covariance $\mathrm{E}[\mathbf{h}_n(t)^{\mathrm{H}}\mathbf{h}_n(t)] = \boldsymbol{\Phi}_n$ and $\mathrm{E}[\mathbf{h}_n(0)^{\mathrm{H}}\mathbf{h}_n(t)] = k_n(t)\boldsymbol{\Phi}_n$.

Using the above MISO channel model the following channel properties relate temporal and spatial characteristics of the channel.

1) The mean squared distance (MSD) between the MISO channel response $\mathbf{h}_n(t)$ and $\mathbf{h}_n(0)$ is a function of time t and does not depend on the spatial correlation of the channel. In other words

$$\mathrm{MSD}_n(t) = \mathrm{E}[|\mathbf{h}_n(t) - \mathbf{h}_n(0)|^2]. \tag{25}$$

Since $\mathrm{trace}\,(\boldsymbol{\Phi}_n) = M$, it follows that the MSD is

$$\mathrm{MSD}_n(t) = 2M(1 - k_n(t)). \tag{26}$$

2) The average power of the MISO channel response $\mathbf{h}_n(t)$ in the direction of $\mathbf{h}_n(0)$ (i.e., the projection of $\mathbf{h}_n(t)$ on $\mathbf{h}_n(0)$)

$$\zeta(t) = \frac{1}{M}\,\mathrm{E}\left[\left|\frac{\mathbf{h}_n(0)\mathbf{h}_n(t)^{\mathrm{H}}}{\sqrt{\mathbf{h}_n(0)\mathbf{h}_n(0)^{\mathrm{H}}}}\right|^2\right] \tag{27}$$

and it increases with the spatial correlation of the channel. Specifically,

$$\zeta(t) = k_n(t)^2 + (1 - k_n(t)^2)\frac{\sum_{i=1}^{M}\psi_{ni}^2}{M^2} \tag{28}$$

where ψ_{ni} ($i = 1, \cdots, M$) are eigenvalues of the matrix $\boldsymbol{\Phi}_n$. Figure 3 presents $\zeta(t)$ for different spatial correlations of the channel and also a spatially uncorrelated channel (based on model in (21), f_c = 2GHz, v = 30kmph). The results indicate that $\zeta(t)$ is increasing with the spatial correlation (as said earlier, low values of σ correspond to high spatial correlations).

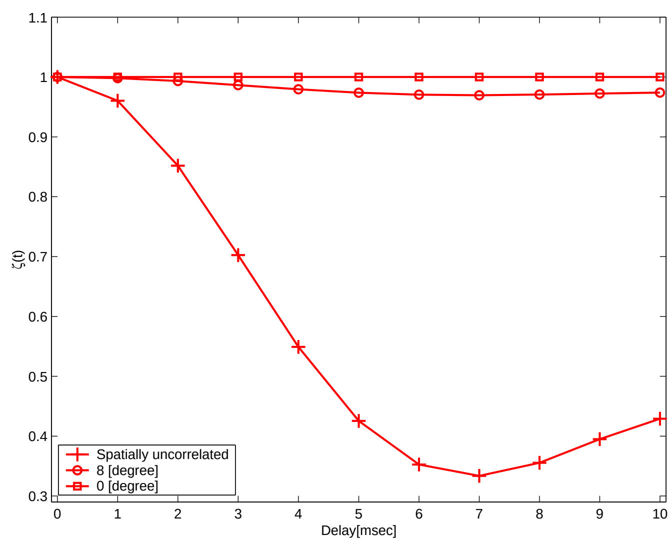

Fig. 3. $\zeta(t)$ for $M = 3$ and channel based on model in (21), f_c = 2GHz, v = 30kmph.

Numerical Results

In Figure 4 and 5 we present average rates per user versus the delay τ of the CSI. The system consists of $M = 3$ transmit antennas and $N = 3$ terminals. The channel is modeled based on (21) (assuming that the carrier frequency is 2GHz and the velocity of each mobile terminal is 30kmph and setting the number of plane waves $N_f = 100$). We assume that the transmit antennas form a proper phased array being spaced $\lambda/2$ apart. Because the ideal channel state $\mathbf{H}(t+\tau)$ is not available at the transmitter, we assume that $\mathbf{H}(t)$ is used instead to perform the transmitter optimization at the moment $t+\tau$. We observe performance for different spatial correlations of the channel and spatially uncorrelated channel. Figure 4 and 5 present average rates for the ZF and MZF scheme, respectively, for $SNR = 10$dB. In all cases, we observe that as the CSI delay increases, spatially uncorrelated channels perform worse than spatially correlated channels, which is a result in contrast to the case of zero delay CSI. In the extreme case of $\sigma = 0^o$, the average rate is hardly affected by the delay of the CSI, while for the spatially uncorrelated channels degradation due to the delay is significant.

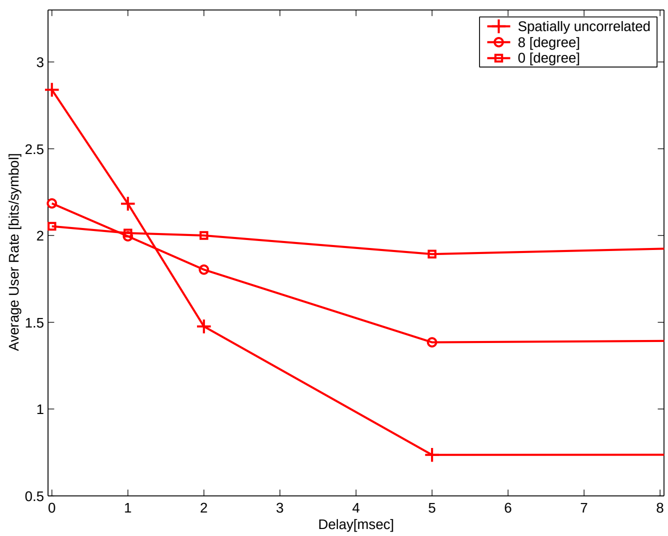

Fig. 4. The ZF scheme, average rate per user vs. CSI delay, $SNR = 10$dB, $M = 3$, $N = 3$, channel based on model in (21), $f_c = 2$GHz, $v = 30$kmph.

In the following we outline the explanation of the above results using a subspace decomposition of the matrix $\mathbf{H}$. Let $\widetilde{C}_n(t)$ for user n denote the subspace spanned by the row vectors of the channel matrix $\mathbf{H}$ other than the nth row. Let us further define the matrix $\mathbf{B}_n(t)$ ($\mathbf{B}_n(t) \in \mathcal{C}^{N-1\times M}$) such that its row vectors correspond to the orthonormal basis that spans $\widetilde{C}_n(t)$. We observe the expected valued of the normalized Frobenius norm ($||\;||^2$) of the product of the basis vectors at the instance t and $t+\tau$

$$\rho_n(\tau) = \frac{\mathrm{E}\left[\left|\left|\mathbf{B}_n(t+\tau)\mathbf{B}_n(t)^{\mathrm{H}}\right|\right|^2\right]}{N-1}. \tag{29}$$

Figure 6 presents $\rho_n(\tau)$ for different spatial correlations of the channel and also a spatially uncorrelated channel. Note that it can be shown that in the static case $\rho_n(\tau) = 1$, while for the case of fully independent $\mathbf{H}(t)$ and $\mathbf{H}(t+\tau)$, $\rho_n(\tau) = (N-1)/M$. These two values represent the upper and lower bound of $\rho_n(\tau)$, respectively. Based on Figure 6, with respect to the temporal variations of the subspace $\widetilde{C}_n(t)$, the case of $\sigma = 0$ is equivalent to the static case (having the upper bound of $\rho_n(\tau)$ for all τ). Furthermore, the spatially uncorrelated channel is approaching the case of independent $\mathbf{H}(t)$ and $\mathbf{H}(t+\tau)$ for large τ (approaching the lower bound of $\rho_n(\tau)$).

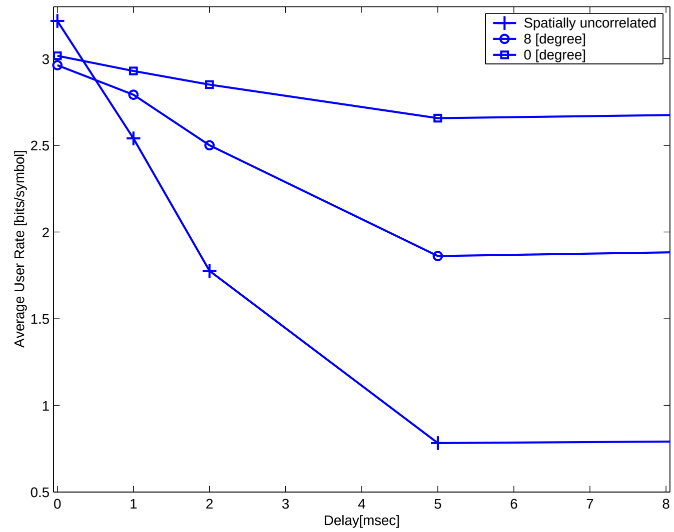

Fig. 5. The MZF scheme, average rate per user vs. CSI delay, $SNR = 10$dB, $M = 3$, $N = 3$, channel based on model in (21), $f_c = 2$GHz, $v = 30$kmph.

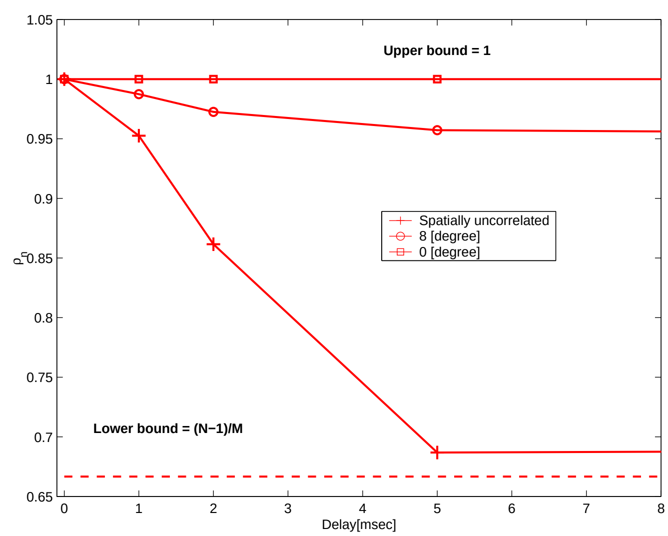

Fig. 6. $\rho_n(\tau)$ for $M = 3$, $N = 3$ and channel based on model in (21), $f_c = 2$GHz, $v = 30$kmph.

What this shows is that under the case of strong spatial correlations, the subspace $\widetilde{C}_n(t)$ is relatively stable (i.e., changing slowly in time). As a result, any transmitter optimization (pre-filtering) scheme is relatively robust to temporal delays in the CSI feedback. For the case of spatially uncorrelated channels, this robustness is lost leading to poorer performance of transmitter optimization.

IV. Conclusion

We have presented a study on multiple antenna transmitter optimization schemes that are based on linear transformations and transmit power optimization. We have presented the performance of the zero-forcing and modified zero-forcing spatial pre-filtering with respect to the delayed CSI. A multiple antenna channel model was introduced modeling temporal and spatial correlations. It was seen that as the CSI delay increases, spatially uncorrelated channels perform worse than spatially correlated channels, which is in contrast to the case of zero delay CSI.

Appendix

Let us describe the corresponding virtual uplink for the system in Figure 1. Let $\bar{x}_n$ be the uplink information bearing signal transmitted from mobile terminal n ($n = 1, \cdots, N$) and $\bar{y}_m$ be the received signal at the mth base station antenna ($m = 1, \cdots, M$). $\bar{x}_n$ are assumed to be circularly symmetric complex random variables having Gaussian distribution $\mathcal{N}_\mathcal{C}(0, P_{av})$. Further, the received vector $\bar{\mathbf{y}} = [\bar{y}_1, \cdots, \bar{y}_M]^{\mathrm{T}}$ is

$$\begin{aligned} \bar{\mathbf{y}} &= \bar{\mathbf{H}}\bar{\mathbf{x}} + \bar{\mathbf{n}} = \mathbf{H}^{\mathrm{H}}\bar{\mathbf{x}} + \bar{\mathbf{n}}, \\ \bar{\mathbf{y}} &\in \mathcal{C}^M, \bar{\mathbf{x}} \in \mathcal{C}^N, \bar{\mathbf{n}} \in \mathcal{C}^M, \bar{\mathbf{H}} \in \mathcal{C}^{M\times N} \end{aligned} \tag{30}$$

where $\bar{\mathbf{x}} = [\bar{x}_1, \cdots, \bar{x}_N]^{\mathrm{T}}$ is the transmitted vector ($\mathrm{E}[\bar{\mathbf{x}}\bar{\mathbf{x}}^{\mathrm{H}}] = P_{av}\mathbf{I}_{N\times N}$), $\bar{\mathbf{n}}$ is AWGN ($\mathrm{E}[\bar{\mathbf{n}}\bar{\mathbf{n}}^{\mathrm{H}}] = N_0\mathbf{I}_{M\times M}$) and $\bar{\mathbf{H}} = \mathbf{H}^{\mathrm{H}}$ is the uplink MIMO channel response matrix.

It is well known that the MMSE receiver is the optimal linear receiver for the uplink (*multiple access channel*) [11], [12]. It maximizes the received SINR (and rate) for each user. The decision statistic is obtained after the receiver MMSE filtering as

$$\bar{\mathbf{x}}^{dec} = \mathbf{W}^{\mathrm{H}}\bar{\mathbf{y}} \tag{31}$$

where the MMSE receiver is

$$\mathbf{W} = \left(\left(\mathbf{H}\mathbf{H}^{\mathrm{H}} + \frac{N_0}{P_{av}}\mathbf{I}\right)^{-1}\mathbf{H}\right)^{\mathrm{H}} = \mathbf{H}^{\mathrm{H}}\left(\mathbf{H}\mathbf{H}^{\mathrm{H}} + \frac{N_0}{P_{av}}\mathbf{I}\right)^{-1}. \tag{32}$$

Proof of Proposition 1

Note that $\mathbf{W} = \mathbf{A}$ in (8), for the MZF transmitter spatial pre-filtering. Let us normalize the column vectors of the matrix $\mathbf{W}$ in (32) as

$$\mathbf{W}_{nor} = \mathbf{W}\mathbf{P} \tag{33}$$

where $\mathbf{P}$ is defined in (10). In other words the nth diagonal element of $\mathbf{P}$ is selected as

$$p_{nn} = \frac{1}{\sqrt{\mathbf{w}_n^{\mathrm{H}}\mathbf{w}_n}} \;\; (n = 1, \cdots, N) \tag{34}$$

where $\mathbf{w}_n$ is the nth column vector of the matrix $\mathbf{W}$ (where $\mathbf{w}_n = \mathbf{a}_n$, which is the column vector of $\mathbf{A}$ for $n = 1, \cdots, N$). It is well known that any normalization of the columns of the MMSE receiver in (32) does not change the SINRs. In other words, the SINR for the nth uplink user $(n = 1, \cdots, N)$ is

$$\begin{aligned} SINR_n^{UL} &= \frac{P_{av}|\mathbf{w}_n^{\mathrm{H}}\bar{\mathbf{h}}_n|^2}{P_{av}\sum_{i=1,i\neq n}^{N}|\mathbf{w}_n^{\mathrm{H}}\bar{\mathbf{h}}_i|^2 + N_0\mathbf{w}_n^{\mathrm{H}}\mathbf{w}_n} \\ &= \frac{P_{av}|\mathbf{w}_n^{\mathrm{H}}\bar{\mathbf{h}}_n|^2/(\mathbf{w}_n^{\mathrm{H}}\mathbf{w}_n)}{P_{av}\sum_{i=1,i\neq n}^{N}|\mathbf{w}_n^{\mathrm{H}}\bar{\mathbf{h}}_i|^2/(\mathbf{w}_n^{\mathrm{H}}\mathbf{w}_n) + N_0} \end{aligned} \tag{35}$$

where $\bar{\mathbf{h}}_n$ is the nth column vector of the matrix $\bar{\mathbf{H}}$. Note that $\bar{\mathbf{h}}_n^{\mathrm{H}} = \mathbf{h}_n$ which is the nth row vector of the downlink MIMO channel $\mathbf{H}$. The corresponding downlink SINR when the MZF spatial per-filtering is used (with $\mathbf{P}$ defined in (10)) is

$$SINR_n^{MZF} = \frac{P_{av}|\mathbf{h}_n\mathbf{a}_n|^2/(\mathbf{a}_n^{\mathrm{H}}\mathbf{a}_n)}{P_{av}\sum_{i=1,i\neq n}^{N}|\mathbf{h}_i\mathbf{a}_i|^2/(\mathbf{a}_n^{\mathrm{H}}\mathbf{a}_n) + N_0}. \tag{36}$$

As said earlier, $\mathbf{w}_n = \mathbf{a}_n$ and $\bar{\mathbf{h}}_n^{\mathrm{H}} = \mathbf{h}_n$. Thus, $SINR_n^{MZF} = SINR_n^{UL}$ for $n = 1, \cdots, N$ leading to identical rates which concludes the proof.

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