Estimation of Fading Channel Response and System Capacity Considerations

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Abstract

In this paper we analyze the effects of channel response estimation error of a frequency-flat time-varying channel on system capacity. Using a block-fading channel model, where the block length corresponds to the channel coherence time, we consider two pilot based approaches for the estimation. The first approach uses a single pilot symbol per block with variable power, while the second approach uses more than one symbol per block with constant power per symbol. For both cases, we study the effects of the estimation error on the effective signal-to-noise ratio and the resulting system capacity. We also evaluate the impact of the estimation error on a symbol-by-symbol detection scheme with no error correction. Our results show that the first pilot-based approach is less sensitive to the fraction of power allocated to the pilot. Further, both schemes performed quite well under optimum power allocation between the pilot and information bearing signal, more so in the case of long channel coherence times.

I INTRODUCTION

In this paper we analyze how the estimation error of the channel response affects performance of a communication system. We assume a frequency-flat time-varying wireless channel with additive white Gaussian noise (AWGN). The above system may correspond to one subchannel (i.e., carrier) of an OFDM wireless system [1]. We consider two different pilot arrangement schemes and we observe their performance depending on the channel coherence time, signal-to-noise ratio and power allocated to the pilots with respect to the information bearing portion of the signal. The first scheme uses a single pilot symbol per block with variable power, while the second scheme uses more than one symbol per block with constant power per symbol. For the given pilot schemes, in both cases maximum-likelihood (ML) estimate of the channel response is considered [2]. A more detailed description of the system and related assumptions are given in Section II. The capacity (i.e., the Shannon capacity [3, 4]) of the system that consists of the above channel and the channel response estimation is evaluated in Section III. In order to quantify the effects of the estimation error on a symbol-by-symbol detection scheme with no error correction, the symbol-error-rate and throughput of various quadrature amplitude modulation (QAM) schemes is presented in Section IV. We conclude in Section V.

II SYSTEM MODEL

We now introduce the received signal model. After sampling, with sampling rate $T_{smp} = 1/B$, where B is the signal bandwidth, the discrete version of the baseband equivalent of the received signal is

$$r(k) = C(k)d(k) + n(k) \tag{1}$$

where k is the discrete time variable, $C(k)$ is the complex channel response, $d(k)$ is the transmitted complex signal, and $n(k)$ is the additive white Gaussian noise (AWGN) with unit variance. To perform estimation of the channel response $C(k)$, the receiver is using a pilot (training) signal that is a part of the transmitted data. The pilot is sent periodically, every N symbol periods, where the symbol period is $T_{sym} = T_{smp} = 1/B$, satisfying the Nyquist rate. Furthermore, we assume that the channel coherence time is greater or equal than NT_{sym}. This assumption approximates the channel to be constant over at least N symbols ($C \approx C(n)$, for $n = 1, \cdots, N$). In literature, channels with the above property are known as block-fading channels [1]. Without loss of generality, we assume that $E[|C|^2] = 1$. The complex N-dimensional received vector, whose kth component corresponds to the kth symbol interval within the pilot period, is

$$\mathbf{r} = C\mathbf{d} + \mathbf{n}, \quad \mathbf{r} \in \mathcal{C}^N,\ \mathbf{d} \in \mathcal{C}^N,\ \mathbf{n} \in \mathcal{C}^N,\ C \in \mathcal{C} \tag{2}$$

where $\mathbf{n}$ corresponds to the AWGN. Further, the transmitted vector $\mathbf{d}$ is defined as

$$\mathbf{d} = \sum_{i=1}^{N} A_i d_i \mathbf{s}_i \tag{3}$$

where A_i is the amplitude, d_i the unit-variance complex data and $\mathbf{s}_i$ is the corresponding signature (i.e., a waveform). The signatures are mutually orthogonal $\mathbf{s}_i^H \mathbf{s}_j = \delta_{ij}$, where δ_{ij} is the Kronecker delta function. For example, $\mathbf{s}_i$ could be a canonical waveform (i.e., could be a TDMA-like waveform, where $\mathbf{s}_i$ is the unit-pulse at the time instance i). Alternately, $\mathbf{s}_i$ could also be an N-dimensional Walsh sequence spanning all N symbol intervals (i.e., similar to a CDMA-like waveform [5]). We consider the transmitted signal to be comprised of two parts: one is the information bearing signal and the other which is the pilot signal. We can rewrite the equation (3) as

$$\mathbf{d} = \sum_{j=1}^{N-K} A_j d_j \mathbf{s}_j \; + \sum_{l=N-K+1}^{N} A_l d_l \mathbf{s}_l \tag{4}$$

In the above $\sum_{j=1}^{N-K} A_j d_j \mathbf{s}_j$ denotes the information bearing signal and $\sum_{l=N-K+1}^{N} A_l d_l \mathbf{s}_l$ denotes the pilot signal. The pilot data (d_l, $l = N-K+1, \cdots, N$) are predefined and known at the receiver. Further, without loss of generality, we assume that $|d_l|^2 = 1\,(l = N-K+1, \cdots, N)$. We also assume that the amplitudes are $A = A_i\,(i = 1, \cdots, N-K)$, and $A_P = A_l\,(l = N-K+1, \cdots, N)$, and they are known at the receiver. We denote the ratio of the amplitudes as $\alpha = A_P/A$.

In this study we consider two different pilot arrangements:

1. $K = 1$ and $A_P \neq A$. For the given average symbol signal-to-noise ratio, denoted as SNR (in dB), the amplitude A is derived from the equation

$$N\,10^{SNR/10} = ((N-1) + \alpha^2)\,A^2 \tag{5}$$

The above implies

$$A_1 = \sqrt{\frac{N}{((N-1)+\alpha^2)} 10^{SNR/10}} \tag{6}$$

In the remainder of the paper, the above pilot arrangement is referred to as case 1.

2. $K \geq 1$ and $A_P = A\,(\alpha = 1)$. For the given average symbol SNR, the amplitude A is

$$A_2 = \sqrt{10^{SNR/10}} \tag{7}$$

In the remainder of the paper, the above pilot arrangement is referred to as case 2.

In both cases we assume that the total transmitted energy (within the pilot period) is the same, but differently distributed between the information bearing portion of the signal and the pilot. Consequently, we observe the performance of the system with respect to the amount of transmitted energy that is allocated to the pilot (percentage wise). This percentage is denoted as μ and is given as

$$\mu = \frac{K\alpha^2}{(N-K) + K\alpha^2} 100 \;\; [\%] \tag{8}$$

We now describe the channel response estimation scheme that is assumed in the system. The estimation is based on averaging the projections of the received signal on $d_l \mathbf{s}_l$ for $l = N-K+1, \cdots, N$ as

$$\widehat{C} = \frac{1}{KA_P} \sum_{l=N-K+1}^{N} (d_l \mathbf{s}_l)^H \mathbf{r}$$

$$= \frac{1}{K} \sum_{l=N-K+1}^{N} (C \; + \; (d_l \mathbf{s}_l)^H \mathbf{n}/A_P)$$

$$= C \; + \; \frac{1}{KA_P} \sum_{l=N-K+1}^{N} (d_l \mathbf{s}_l)^H \mathbf{n} \tag{9}$$

where $\widehat{C}$ denotes the estimate of the channel response C. For a frequency-flat AWGN channel, (9) is the maximum-likelihood estimate of the channel response C, for the given pilot signal [2]. The estimation error is

$$n_e = \frac{1}{KA_P} \sum_{l=N-K+1}^{N} (d_l \mathbf{s}_l)^H \mathbf{n} \tag{10}$$

Having the channel response estimated, the estimate of the transmitted data $d_i\,(i = 1, \cdots, N-K)$ is obtained from the following statistics

$$x_i = \frac{1}{A} \mathbf{s}_i^H \mathbf{r} \tag{11}$$

The above procedure is the matched filtering operation (projection $\mathbf{s}_i^H \mathbf{r}$), resulting in the sufficient statistic x_i [6]. The amplitude A is known at the receiver. Let us now rewrite the above expression (11) using the equations (9) and (10) as

$$x_i = d_i(C + n_e) + \left(\frac{1}{A}\mathbf{s}_i^H \mathbf{n} - d_i n_e\right) = d_i \hat{C} + \left(\frac{1}{A}\mathbf{s}_i^H \mathbf{n} - d_i n_e\right) \tag{12}$$

The data detection is performed using the above statistics. As a common practice, the detection procedure assumes that the channel response is perfectly estimated, and that $\hat{C}$ corresponds to the true channel response. Consequently, the power of the received information bearing signal is $\epsilon = E[|d_i \hat{C}|^2]$. Furthermore, let us denote the second term in the above equation as $\bar{n}_i$, i.e.,

$$\bar{n}_i = \frac{1}{A} \mathbf{s}_i^H \mathbf{n} - d_i n_e \tag{13}$$

Note that $\bar{n}_i$ is the effective noise in the detection process. Assuming statistical independence between $\mathbf{s}_i^H \mathbf{n}$, d_i and n_e, it can be shown that the effective noise is AWGN, for the given d_i. Further, let $\Gamma = E[\bar{n}_i^* \bar{n}_i]$, denote the variance of the effective noise. Γ is a function of A, and can be explicitly written as

$$\Gamma(A) = E\left[|\frac{1}{A}\mathbf{s}_i^H \mathbf{n} - d_i n_e|^2\right] \tag{14}$$

Note that the different pilot arrangements will result in different A (see the equations (6) and (7)), and consequently in different $\Gamma(A)$. Furthermore we define the effective SNR (in dB) as

$$eSNR(A) = 10\, log_{10}\left(\frac{\epsilon}{\Gamma(A)}\right) \qquad (15)$$

In the following, we denote $eSNR_1 = eSNR(A_1)$ (where A_1 is defined in (6)) and $eSNR_2 = eSNR(A_2)$ (where A_2 is defined in (7)).

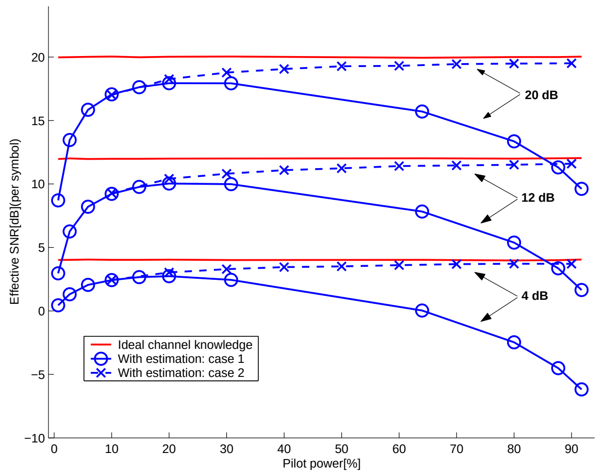

Figure 1: Effective SNR vs. power allocated to the pilot, $SNR = 4, 12, 20$dB, $N = 10$, Rayleigh channel.

In Figure 1, we plot the effective SNR as a function of the allocated power to the pilot (equation (8)), for different values of SNR (corresponding to the channel background noise). In the above example a channel coherence time of 10 symbols is assumed and the channel response is that of a frequency-flat Rayleigh fading channel. The results are shown for the pilot arrangements corresponding to both case 1 and case 2. In the case of the ideal knowledge of the channel response, the effective SNR does correspond to the conventional definition of the SNR (i.e., where the channel background noise is the only noise in the system).

III EFFECT ON SYSTEM CAPACITY

In this section we consider the Shannon capacity [3,4,7] of a system that consists of the block fading channel and the above estimation procedure. Specifically, we evaluate the effect of the effective noise on the system capacity. Recall that the effective noise was assumed to be AWGN. Thus we can directly calculate the system capacity [1] for the two pilot arrangements as follows

1. For $K = 1$ and the amplitude A given in (6), the system capacity is

$$C_1 = \frac{N-1}{N} log_2(1 + 10^{eSNR_1/10}) \qquad (16)$$

The term $(N-1)/N$ is introduced because one signature (i.e., signal dimension) is allocated to the pilot, consequently lowering the system capacity.

2. $K \geq 1$ and the amplitude A given in (7), the system capacity is

$$C_2 = \frac{N-K}{N} log_2(1 + 10^{eSNR_2/10}) \qquad (17)$$

The term $(N-K)/N$ is introduced because K signatures (i.e., signal dimensions) are allocated to the pilot, consequently lowering the system capacity.

Note that the above expressions represent the achievable rates for reliable transmission for the specific estimation procedure assumed in each case. Knowing the channel response perfectly or using a better channel estimation scheme (e.g., decision driven schemes) may result in higher achievable rates.

In Figure 2, we present the system capacity as a function of the power allocated to the pilot (equation (8)). The results correspond to the system observed in the case of Figure 1. For the ideal knowledge of the channel response we apply the conventional capacity formula (corresponding only to the channel background noise), while for case 1 and case 2 the expressions in (16) and (17) are used, respectively (corresponding to the $eSNR_1$ and $eSNR_2$ in Figure 1). For this particular example, even though the effective SNR is different for case 1 and case 2, the system capacity is reaching its maximum at the same $\mu \approx 20\%$. As will be seen in the following set of results, this is not the case for different channel coherence time lengths.

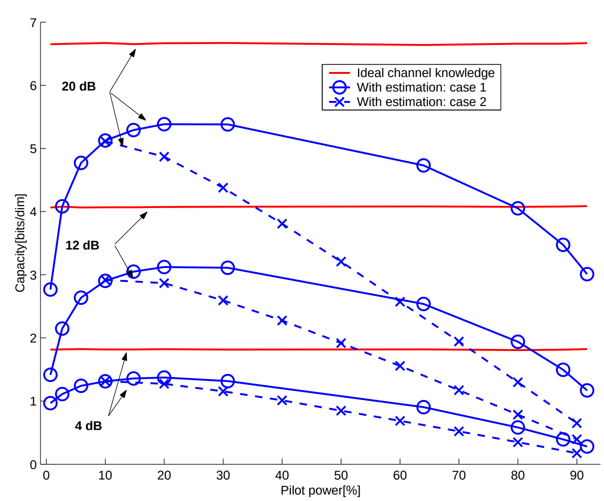

Figure 2: System capacity vs. power allocated to the pilot, $SNR = 4, 12, 20$dB, $N = 10$, Rayleigh channel.

[1]If the effective noise is not distributed as a white Gaussian process, then the above system capacity expressions represent worst-case scenarios.

In Figure 3 and 4, for $SNR = 20$dB, we present the system capacity vs. power allocated to the pilot for different channel coherence time lengths $N = 10, 20, 40, 100$. In case 1 and case 2, by increasing the channel coherence time length we observe that the maximum system capacity is reached for lower percentage of the power allocated to the pilot.

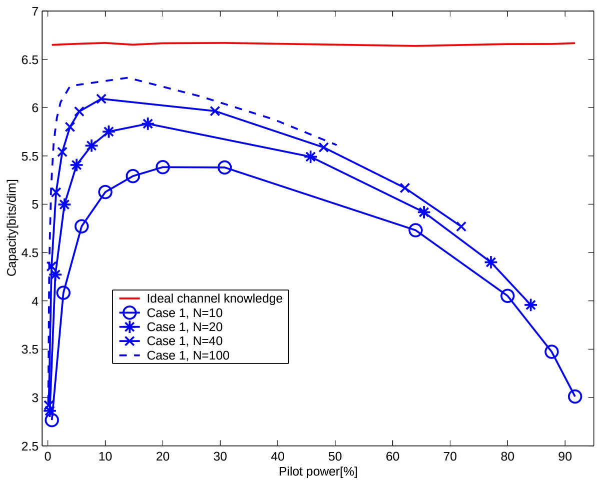

Figure 3: System capacity vs. power allocated to the pilot, case 1, $SNR = 20$dB, for different channel coherence time lengths $N = 10, 20, 40, 100$, Rayleigh channel.

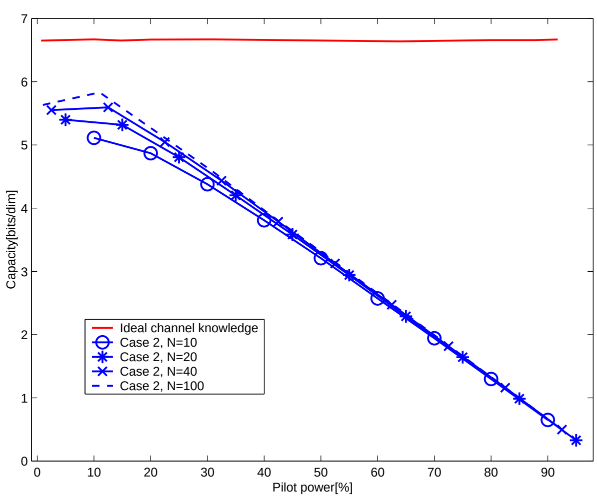

Figure 4: System capacity vs. power allocated to the pilot, case 2, $SNR = 20$dB, for different channel coherence time lengths $N = 10, 20, 40, 100$, Rayleigh channel.

Regarding the system capacity, from the above results we observe that case 1 is less sensitive to the pilot power allocation than case 2 (i.e., in case 2 the system capacity is dropping faster if the allocated power is different than the one that results in the maximum value). Further, for the same channel coherence time lengths, case 1 is reaching the system capacity maximum at a lower power allocated to the pilot than for case 2 (see Figure 3 and 4).

For the given SNR, we define the capacity efficiency ratio η as the ratio between the maximum system capacity in case 1 and case 2 and the system capacity C_0 in the case of the ideal knowledge of the channel response, i.e.,

$$\eta_i = \frac{\max_\mu C_i}{C_0} \quad i = 1, 2 \tag{18}$$

In Figure 5, we show that the capacity efficiency ratio η increases as the channel coherence time is getting longer.

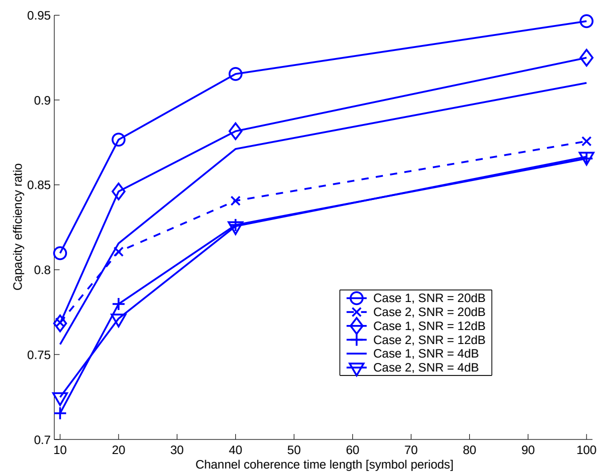

Figure 5: Capacity efficiency ratio vs. channel coherence time length, $SNR = 4, 12, 20$dB, $N = 10, 20, 40, 100$, Rayleigh channel.

IV Effect on QAM with no error correction

While the earlier analysis has focused on evaluating the impact of estimation error on the system (information) capacity, we now present results on the impact on symbol-by-symbol detection and no error correction. Specifically, we consider QPSK (i.e., 4-QAM), 16-QAM and 64-QAM modulation schemes. In general, the symbol-error-rate (SER) for M-ary QAM is given as [8]

$$SER = 1 - \left(1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3S}{2(M-1)}}\right)\right)^2 \tag{19}$$

where S denotes the symbol signal-to-noise ratio. In the case of the ideal knowledge of the channel response $S = 10^{SNR/10}$ (only the backgroung channel noise is present), while in the presence of the estimation errors $S = 10^{eSNR(A)/10}$ (the effective SNR).

For $N = 10$, we select the pilot power that results in the maximum system capacity (from Figure 2 it is $\mu \approx 20\%$) and we present the SER in Figure 6 (for case 1, case 2, and ideal knowledge of the channel response). The performance loss directly corresponds to the difference

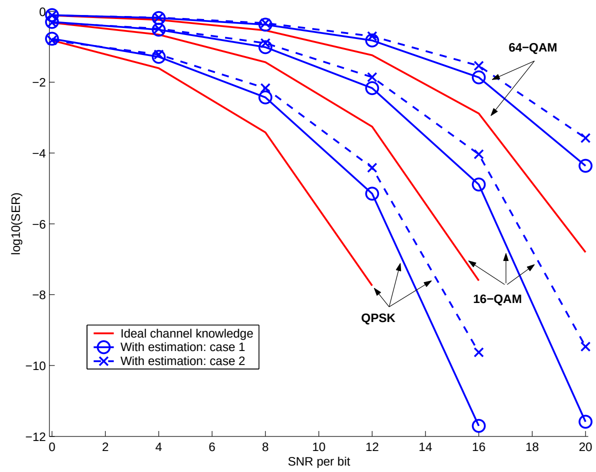

Figure 6: SER vs. SNR, $N = 10$, Rayleigh channel.

between the SNR and the effective SNR (i.e., $eSNR_1$ and $eSNR_2$).

To evaluate the spectral efficiency of the modulation scheme, for given SNR and the above estimation, we define the throughput as

$$T = log_2(M)\frac{N-K}{N}(1-SER) \tag{20}$$

Recall that for case 1, $K = 1$, for case 2, $K = 1, \cdots, N$ and for the ideal knowledge of the channel response $K = 0$. In Figure 7, for QPSK modulation, $N = 10$ and $SNR = 4$dB, the throughput is presented as a function of the pilot power. Note that the maximum throughput for QPSK is 2[bits/dim]. As in the case of the system capacity, we again observe that case 1 is less sensitive to the pilot power allocation than case 2.

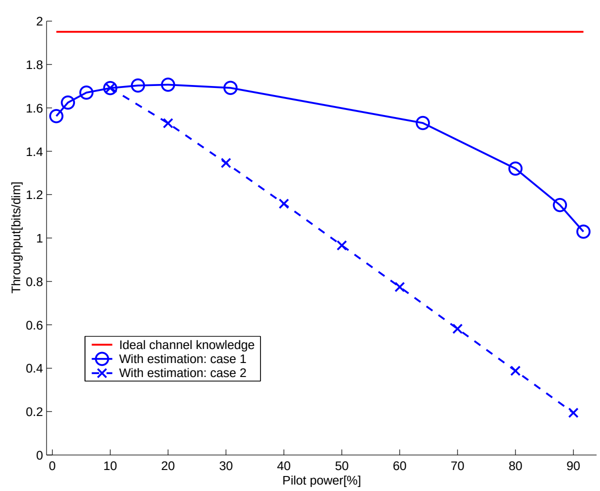

Figure 7: Throughput vs. power allocated to the pilot, QPSK modulation, $N = 10$, $SNR = 4$dB, Rayleigh channel.

V Conclusion

In this paper we have studied how the estimation error of the frequency-flat time-varying channel response affects the performance of a communication system. We have approximated the channel as a block-fading channel. We have considered two pilot based schemes for the estimation. The first scheme (case 1) uses a single pilot symbol per block with variable power, while the second scheme (case 2) uses more than one symbol per block with constant power per symbol. We have studied the performance in terms of the effective SNR, system capacity, symbol-error-rate and throughput for various QAM schemes. We have presented how the performance depends on percentage of the total power allocated to the pilot, background noise level and the channel coherence time length. Our results have shown that the first pilot-based approach is less sensitive to the fraction of power allocated to the pilot. Further, both schemes performed quite well under optimum power allocation between the pilot and information bearing signal, more so in the case of long channel coherence times.

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