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Pilot Assisted Estimation of MIMO Fading Channel Response and Achievable Data Rates

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Abstract. In this paper we analyze the effects of pilot assisted channel estimation error of a frequency-flat time-varying channel on achievable data rates. We consider multiple-input multiple-output (MIMO) wireless systems and emphasize differences with respect to single-input single-output (SISO) systems. The analysis connects results of information theory with practical wireless communication systems. Using a block-fading channel model, where the block length corresponds to the channel coherence time, we consider two pilot based approaches for the estimation. Per transmit antenna, the first approach uses a single pilot symbol per block with different power than data symbols, while the second approach uses more than one pilot symbol per block with same power as data symbols. In the MIMO case, the orthogonality between the pilots assigned to different transmit antennas is assumed. The effects of the estimation error are evaluated in the case of the estimates being available at the receiver only (open loop). The presented analysis is a study of mismatched receiver algorithms in MIMO systems.

1. Introduction

Fading channels are an important element of any wireless propagation environment [9]. Different aspects of fading channels have been studied and publicized. It has been recognized that the inherent temporal and spatial variations of wireless channels impose stringent demand on design of a communication system to allow it approach the data rates that are achievable in, for example, wire-line systems. A number of different solutions exploit variations in wireless channels. For example, a transmitter optimization scheme (using power control), known as the water pouring algorithm, maximizes the capacity for the constrained average transmit power [12] (see also [3]). In addition to power control, recent applications of variable coding rate and modulation formats illustrate a wide range of resource allocation techniques used to exploit and combat effects of fading channels in multiuser wireless systems [24]. An extensive review of the information theoretical aspects of communications in fading channels is given in [1]. Furthermore, modulation and channel coding for fading channels is also being studied (see [1] and references therein).

Multiple-transmit multiple-receive antenna systems represent an implementation of the MIMO concept in wireless communications [6]. This particular multiple

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antenna architecture provides high capacity wireless communications in rich scattering environments. It has been shown that the theoretical capacity (approximately) increases linearly as the number of antennas is increased [6, 7]. This and related results point to the importance of understanding all aspects of MIMO wireless systems. For example, the studies regarding propagation [4, 5, 23], detection [8, 21], space-time coding and implementation aspects [11, 20, 27] are well publicized.

Next generation wireless systems and standards are supposed to operate over wireless channels whose variations are faster and/or further pronounced. For example, using higher carrier frequencies (e.g., 5 GHz for 802.11a) results in smaller scale of spatial variations of the electro-magnetic field. Also, compared to SISO channels, MIMO channels have greater number of parameters that a receiver and/or transmitter has to operate with, consequently pronouncing the channel variations. In addition, there has been a perpetual need for supporting higher mobility within wireless networks. These are just a few motivations for studying the implications of channel variations on achievable data rates in wireless systems.

In this paper we analyze how the estimation error of the channel response affects the performance of a MIMO wireless system. Considering the practical importance of single-input single-output (SISO) systems, we analyze them as a subset of MIMO systems. Considering terminology in literature (see [1] and references therein), the channel response estimate corresponds to *channel state information* (CSI). We assume a frequency-flat time-varying wireless channel with additive white Gaussian noise (AWGN). The above system may also correspond to one subchannel (i.e., carrier) of an OFDM wireless system [25]. We consider two pilot arrangement schemes in this paper. The first scheme uses a single pilot symbol per block with the different power than the data symbol power. The second scheme uses more than one pilot symbol per block, whose power is the same as the data symbol power. For the given pilot schemes, in both cases, *maximum-likelihood* (ML) estimation of the channel response is considered [18]. In the MIMO case, the orthogonality between the pilots assigned to different transmit antennas is assumed. The effects of the estimation error are evaluated in the case of the estimates being available at the receiver only. The presented analysis may be viewed as a study of mismatched receiver algorithms in MIMO systems. The analysis connects results of information theory (see [16, 22] and references therein) with practical wireless communication systems (employing pilot assisted channel estimation) and generalizing it to MIMO systems. We believe that the presented results are directly applicable to current and next generation wireless systems. Furthermore, the results may be used as baseline benchmarks for performance evaluation of more advanced estimation, such as anticipated in future systems.

2. System Model

In the following we present a MIMO communication system that consists on M transmit and N receive antennas (denoted as a $M \times N$ system). At the receiver we assume sampling with the period $T_{smp} = 1/B$, where B is the signal bandwidth, thus preserving the sufficient statistics. The received signal is a spatial vector $\mathbf{y}$

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{x}(k) + \mathbf{n}(k), \quad \mathbf{y}(k) \in \mathcal{C}^N, \mathbf{x}(k) \in \mathcal{C}^M, \mathbf{n}(k) \in \mathcal{C}^N, \mathbf{H}(k) \in \mathcal{C}^{N\times M} \tag{2.1}$$

where $\mathbf{x}$ is the transmitted vector, $\mathbf{n}$ is AWGN ($\mathrm{E}[\mathbf{n}\mathbf{n}^{\mathrm{H}}] = N_0\, \mathbf{I}_{N\times N}$), and $\mathbf{H}$ is the MIMO channel response matrix, all corresponding to the time instance k. We assign index $m = 1, \cdots, M$ to denote the transmit antennas, and index $n = 1, \cdots, N$ to denote the receive antennas. Thus, $h_{nm}(k)$ is the n-th row and m-th column element of the matrix $\mathbf{H}(k)$. Note that it corresponds to a SISO channel response between the transmit antenna m and the receive antenna n. The n-th component of the received spatial vector $\mathbf{y}(k) = [y_1(k) \cdots y_N(k)]^{\mathrm{T}}$ (i.e., signal at the receive antenna n) is

$$y_n(k) = \sum_{m=1}^{M} h_{nm}(k) g_m(k) + n_n(k) \tag{2.2}$$

where $g_m(k)$ is the transmitted signal from the m-th transmit antenna (i.e., $\mathbf{x}(k) = [g_1(k) \cdots g_M(k)]^{\mathrm{T}}$), and $\mathbf{n}(k) = [n_1(k) \cdots n_N(k)]^{\mathrm{T}}$. To perform estimation of the channel response $\mathbf{H}(k)$, the receiver uses a pilot (training) signal that is a part of the transmitted data. The pilot is sent periodically, every K sample periods. We consider the transmitted signal to be comprised of two parts: one is the data bearing signal and the other is the pilot signal. Within the pilot period consisting of K symbols, L symbols (i.e., signal dimensions) are allocated to the pilot, per transmit antenna. As a common practical solution (see [11,20,27]), we assume that the pilot signals assigned to the different transmit antennas, are mutually orthogonal. For more details on signal design for multiple transmit antenna systems see also [13,14]. Consequently we define a K-dimensional temporal vector $\mathbf{g}_m = [g_m(1) \cdots g_m(K)]^{\mathrm{T}}$, whose k-th component is $g_m(k)$ (in (2.2)), is defined as

$$\mathbf{g}_m = \underbrace{\sum_{i=1}^{K-LM} a_{im}^d d_{im}^d \mathbf{s}_i^d}_{Data} + \underbrace{\sum_{j=1}^{L} a_{jm}^p d_{jm}^p \mathbf{s}_{jm}^p}_{Pilot}. \tag{2.3}$$

In the above the first sum is the information, i.e., data bearing signal and the second corresponds to the pilot signal, corresponding to the transmit antenna m. Superscripts "d" and "p" denote values assigned to the data and pilot, respectively. d_{im}^d is the unit-variance circularly symmetric complex data symbol. The pilot symbols (d_{jm}^p, $j = 1, \cdots, L$) are predefined and known at the receiver. Without loss of generality, we assume that $|d_{jm}^p|^2 = 1$. We also assume that the amplitudes are $a_{im}^d = A$, and $a_{jm}^p = A_P$, and they are known at the receiver. Further, the amplitudes are related as $A_P = \alpha A$. Note that the amplitudes are identical across the transmit antennas (because we assumed that the transmit power is equally distributed across them).

Furthermore, $\mathbf{s}_i^d = [s_i^d(1) \cdots s_i^d(K)]^{\mathrm{T}}$, $(i = 1, \cdots, (K - LM))$ and $\mathbf{s}_{jm}^p = [s_{jm}^p(1) \cdots s_{jm}^p(K)]^{\mathrm{T}}$, $(j = 1, \cdots, L$, and $m = 1, \ldots M)$ are waveforms, denoted as *temporal signatures*. The temporal signatures are mutually orthogonal. For example, $\mathbf{s}_i^d$ (or $\mathbf{s}_{jm}^p$) could be a canonical waveform (i.e., could be a TDMA-like waveform, where $\mathbf{s}_i^d$ (or $\mathbf{s}_{jm}^p$) is the unit-pulse at the time instance i). Alternately, $\mathbf{s}_i^d$ (or $\mathbf{s}_{jm}^p$) could also be a K-dimensional Walsh sequence spanning all K sample intervals (i.e., similar to a CDMA-like waveform [15]). As said earlier, we assume that the pilot signals are orthogonal between the transmit antennas. The indexing and summation limits in (2.3) conform to this assumption, i.e, temporal signatures $\mathbf{s}_{jm}^p (j = 1, \cdots, L)$ are uniquely assigned to the transmit antenna m. In other words, transmit antenna m must not use the temporal signatures that are assigned as pilots to other antennas and assigned to data, which is consequently lowering the achievable data rates (will be revisited in the following sections). Unlike the pilot temporal signatures, the data bearing temporal signatures $\mathbf{s}_i^d$ $(i = 1, \ldots, (K-LM))$ are reused across the transmit antennas, which is an inherent property of MIMO systems, potentially resulting in high achievable data rates.

We rewrite the received spatial vector in (2.1) as

$$\mathbf{y}(k) = \mathbf{H}(k)(\mathbf{d}(k) + \mathbf{p}(k)) + \mathbf{n}(k), \quad \mathbf{d}(k) \in \mathcal{C}^M, \mathbf{p}(k) \in \mathcal{C}^M \tag{2.4}$$

where $\mathbf{d}(k)$ is the information, i.e., data bearing signal and $\mathbf{p}(k)$ is the pilot portion of the transmitted spatial signal, at the time instance k. The m-th component of the data vector vector $\mathbf{d}(k) = [d_1(k) \cdots d_M(k)]^{\mathrm{T}}$ (i.e., data signal at the transmit antenna m) is

$$d_m(k) = \sum_{i=1}^{K-LM} a_{im}^d d_{im}^d s_i^d(k). \tag{2.5}$$

The m-th component of the pilot vector $\mathbf{p}(k) = [p_1(k) \cdots p_M(k)]^{\mathrm{T}}$ (i.e., pilot signal at the transmit antenna m) is

$$p_m(k) = \sum_{j=1}^{L} a_{jm}^p d_{jm}^p s_{jm}^p(k). \tag{2.6}$$

Let us now describe the assumed properties of the MIMO channel $\mathbf{H}(k)$. The channel coherence time is assumed to be greater or equal to KT_{smp}. This assumption approximates the channel to be constant over at least K samples ($h_{nm}(k) \approx h_{nm}$, for $k = 1, \cdots, K$, for all m and n), i.e., approximately constant during the pilot period. In the literature, channels with the above property are known as block-fading channels [25]. Furthermore, we assume that the elements of $\mathbf{H}$ are independent identically distributed (*iid*) random variables, corresponding to highly scattering channels. In general, the MIMO propagation measurements and modeling have shown that the elements of $\mathbf{H}$ are correlated (i.e., not independent) [4,5,23]. In addition, assuming independence is a common practice because the information about correlation is usually not available at the receiver and/or the correlation is time varying (not stationary) and hard to estimate. Based on the above, the received temporal vector at the receiver n, whose k-th component is $y_n(k)$ (in (2.2)), is

$$\mathbf{r}_n = [y_n(1) \cdots y_n(K)]^{\mathrm{T}} = \sum_{m=1}^{M} h_{nm}\mathbf{g}_m + \mathbf{n}_n, \;\; \mathbf{r}_n \in \mathcal{C}^K \tag{2.7}$$

where $\mathbf{n}_n = [n_n(1) \cdots n_n(K)]^{\mathrm{T}}$ and $\mathrm{E}[\mathbf{n}_n\mathbf{n}_n^{\mathrm{H}}] = N_0\, \mathbf{I}_{K\times K}$.

Note that when applying different number of transmit antennas, the total average transmitted power must remain the same, i.e., conserved. This is a common assumption in MIMO systems [6,7]. Also, the power is equally distributed across the transmit antennas. The average transmit power (from all transmit antennas) is

$$P_{av} = M\frac{\left(\sum_{i=1}^{K-LM}(a_{im}^d)^2 \; + \; \sum_{j=1}^{L}(a_{jm}^p)^2\right)}{K} = M\frac{((K-LM) + L\alpha^2)A^2}{K}. \tag{2.8}$$

Thus

$$A = \sqrt{\frac{K}{((K - LM) + \alpha^2 L)} \frac{P_{av}}{M}}. \tag{2.9}$$

As seen from the above, we assume that the total average transmitted energy (within the pilot period) is the same, but differently distributed between the data bearing portion of the signal and the pilot. Consequently, we observe the performance of the system with respect to the amount of transmitted energy that is allocated to the pilot (percentage wise). This percentage is denoted as μ and is given as

$$\mu = \frac{L\alpha^2}{(K - LM) + L\alpha^2} 100 \ [\%]. \tag{2.10}$$

As said earlier, in this study we consider two different pilot arrangements:

(1) $L = 1$ and $A_P \neq A$. The amplitude is

$$A_1 = \sqrt{\frac{K}{((K - M) + \alpha^2)} \frac{P_{av}}{M}}. \tag{2.11}$$

In the remainder of the paper, the above pilot arrangement is referred to as case 1. For example, in SISO systems the above pilot arrangement is applied in CDMA wireless systems (e.g., IS-95 and WCDMA [15]). In MIMO systems, it is applied in narrowband MIMO implementations described in [11, 20, 27]. Also, it is applied in a wideband MIMO implementation based on 3G WCDMA [2].

(2) $L \geq 1$ and $A_P = A\,(\alpha = 1)$. The amplitude is

$$A_2 = \sqrt{\frac{K}{(K - L(M - 1))} \frac{P_{av}}{M}}. \tag{2.12}$$

In the remainder of the paper, the above pilot arrangement is referred to as case 2. Note that the above pilot arrangement is frequently used in SISO systems, e.g., wire-line modems [19] and some wireless standards (e.g., IS-136 and GSM [25]). To the best to our knowledge, this arrangement is not used in MIMO systems.

3. Estimation of Channel Response

Due to the orthogonality of the pilots and assumption that the elements of $\mathbf{H}$ are *iid*, it can be shown that to obtain the maximum likelihood estimate of $\mathbf{H}$ it is sufficient to estimate h_{nm} (for $m = 1, \cdots, M$, $n = 1, \cdots, N$), independently. This is identical to estimating a SISO channel response between the transmit antenna m and receive antenna n. The estimation is based on averaging the projections of the received signal on $d_{jm}^p \mathbf{s}_{jm}^p$ (for $j = 1, \cdots, L$ and $m = 1, \cdots, M$) as

$$\begin{aligned} \hat{h}_{nm} &= \frac{1}{LA_P} \sum_{j=1}^{L} (d_{jm}^p \mathbf{s}_{jm}^p)^{\mathrm{H}} \mathbf{r}_n \\ &= h_{nm} + \frac{1}{LA_P} \sum_{j=1}^{L} (d_{jm}^p \mathbf{s}_{jm}^p)^{\mathrm{H}} \mathbf{n}_n \end{aligned} \tag{3.1}$$

where $\hat{h}_{nm}$ denotes the estimate of the channel response h_{nm}. It can be shown that for a frequency-flat AWGN channel, given pilot signal [18] and assumed properties of $\mathbf{H}$, (3.1) is the maximum-likelihood estimate of the channel response h_{nm}. The estimation error is

$$n^e_{nm} = \frac{1}{LA_P}\sum_{j=1}^{L}(d^p_{jm}\mathbf{s}^p_{jm})^{\mathrm{H}}\mathbf{n}_n. \tag{3.2}$$

Note that n^e_{nm} corresponds to a white Gaussian random process with distribution $\mathcal{N}(0, N_0/(L\,(\alpha A)^2))$. Thus, the channel matrix estimate $\widehat{\mathbf{H}}$ is

$$\widehat{\mathbf{H}} = \mathbf{H} + \mathbf{H_e} \tag{3.3}$$

where $\mathbf{H}_e$ is the estimation error. It can be shown that each component of the error matrix $\mathbf{H}_e$ is an independent and identically distributed random variable n^e_{nm} given in (3.2) (where n^e_{nm} is the n-th row and m-th column element of $\mathbf{H}_e$).

Having the channel response estimated, the estimate of the transmitted data that is associated with the temporal signature $\mathbf{s}^d_i$ is obtained starting from the following statistics

$$z_{ni} = \frac{1}{A}(\mathbf{s}^d_i)^{\mathrm{H}}\mathbf{r}_n \tag{3.4}$$

where the amplitude A is assumed to be known at the receiver. The above procedure is the matched filtering operation and z_{ni} corresponds to the n-th component of the vector

$$\mathbf{z}_i = [z_{1i}\cdots z_{Ni}]^{\mathrm{T}} = \mathbf{H}\,\mathbf{d}_i + \frac{1}{A}\mathbf{n}_i,\;\; i = 1,\ldots,K-LM \tag{3.5}$$

where the m-th component of $\mathbf{d}_i = [d^d_{i1}\cdots d^d_{iM}]^{\mathrm{T}}$ is d^d_{im} (data transmitted from the antenna m and assigned to the temporal signature $\mathbf{s}^d_i$). Also, $\mathrm{E}[\mathbf{n}_i\mathbf{n}_i^{\mathrm{H}}] = N_0\,\mathbf{I}_{N\times N}$. It can be shown that $\mathbf{z}_i$ is a sufficient statistic for detecting the transmitted data.

As a common practice, the detection procedure assumes that the channel response is perfectly estimated, and that $\widehat{\mathbf{H}}$ corresponds to the true channel response. Let us rewrite the expression in (3.5) as

$$\mathbf{z}_i = (\mathbf{H}+\mathbf{H_e})\,\mathbf{d}_i + \frac{1}{A}\mathbf{n}_i - \mathbf{H_e}\,\mathbf{d}_i = \widehat{\mathbf{H}}\,\mathbf{d}_i + \left(\frac{1}{A}\mathbf{n}_i - \mathbf{H_e}\,\mathbf{d}_i\right). \tag{3.6}$$

The effective noise in the detection procedure (as a spatial vector) is

$$\bar{\mathbf{n}}_i = \left(\frac{1}{A}\mathbf{n}_i - \mathbf{H_e}\,\mathbf{d}_i\right). \tag{3.7}$$

For the given $\widehat{\mathbf{H}}$, the covariance matrix of the effective noise vector is

$$\boldsymbol{\Upsilon} = \boldsymbol{\Upsilon}(A) = \mathrm{E}_{\bar{\mathbf{n}}_i|\widehat{\mathbf{H}}}[\bar{\mathbf{n}}_i\bar{\mathbf{n}}_i^{\mathrm{H}}] = \frac{N_0}{A^2}\mathbf{I} + \mathrm{E}_{\mathbf{H_e}|\widehat{\mathbf{H}}}[\mathbf{H_e}\mathbf{H_e}^{\mathrm{H}}] \tag{3.8}$$

and it is a function of the amplitude A. As said earlier $\mathbf{H_e}$ is a matrix of *iid* Gaussian random variables with distribution $\mathcal{N}(0, N_0/(L\,(\alpha A)^2))$.

4. Estimates Available to Receiver: Open Loop Capacity

Assuming that the channel response estimate is available to the receiver, only, we determine the lower bound for the open loop ergodic capacity as follows.

$$C \geq R = \frac{K - LM}{K} \, \mathrm{E}_{\widehat{\mathbf{H}}} \left[\log_2 \det \left(\mathbf{I}_{M\times M} + \widehat{\mathbf{H}}\widehat{\mathbf{H}}^{\mathrm{H}} \boldsymbol{\Upsilon}^{-1} \right) \right]. \tag{4.1}$$

The term $(K - LM)/K$ is introduced because L temporal signature per each transmit antenna are allocated to the pilot. Also, the random process $\widehat{\mathbf{H}}$ has to be stationary and ergodic (this is a common requirement for fading channel and ergodic capacity [1, 17]). We assume that the channel coding will span across great number of channel blocks (i.e., we use the well known infinite channel coding time horizon, required to achieve error-free data transmission with rates approaching capacity [26]).

In the above expression, equality holds if the effective noise (given in (3.7)) is AWGN with respect to the transmitted signal. If the effective noise is not AWGN, then the above rates represent the worst-case scenario, i.e., the lower bound [10]. In achieving the above rates, the receiver assumes that the effective noise is interference which is independent of the transmitted data with Gaussian distribution and spatial covariance matrix $\boldsymbol{\Upsilon}$. In addition, in the above expression R represents an achievable rate for reliable transmission (error-free) for the specific estimation procedure assumed. Knowing the channel response perfectly or using a better channel estimation scheme (e.g., decision driven schemes) may result in higher achievable rates.

In the following we compare the above result to an information theoretical result presented in [1] (page 2641, expression (3.3.55)). The result is presented for the conventional SISO case, introducing capacity lower bound for mismatched decoding as

$$C \geq R^* = \mathrm{E}_{\hat{h}} \left[\log_2 \left(1 + \frac{\hat{h}^2 P}{\mathrm{E}_{h|\hat{h}}(|h - \hat{h}|^2)P + N_0} \right) \right] \tag{4.2}$$

where h and $\hat{h}$ are the SISO channel response and its estimate, respectively. The above result is general, not specifying the channel response estimation procedure. The bound in (4.1) is an extension of the information theoretical bound in (4.2), capturing the more practical pilot assisted channel response estimation scheme and generalizing it to the MIMO case. Consequently,

Proposition 1. For the SISO case ($M = 1, N = 1$), the rate R in (4.1) and R^* in (4.2), are related as

$$R = \frac{K - L}{K} R^*, \;\; for \;\; P = \frac{K}{(K - L) + \alpha^2 L} P_{av} \tag{4.3}$$

where $\hat{h}$ is obtained using the pilot assisted estimation.

5. Examples and Numerical Results

5.1. SISO Systems. To illustrate the above analysis we start with SISO systems. In the SISO case, all previously defined spatial vectors and related matrices are now single dimensional (e.g., $\mathbf{d}_i$, $\mathbf{H}$, $\widehat{\mathbf{H}}$ and $\boldsymbol{\Upsilon}$ are now scalars d_i, h, $\hat{h}$ and υ, respectively). We assume that $\mathrm{E}[|h|^2] = \Gamma$. We define the average symbol signal-to-noise (SNR) ratio as

$$SNR = 10 \log_{10}\left[\frac{\Gamma P_{av}}{N_0}\right] \tag{5.1}$$

and the effective SNR (in dB) as

$$SNR_e(A) = 10 \log_{10}\left(\frac{\epsilon}{\upsilon}\right) \tag{5.2}$$

where $\epsilon = \mathrm{E}[|d_i\hat{h}|^2]$. In the following, we denote $SNR_{e1} = SNR_e(A_1)$ (where A_1 is defined in (2.11)) and $SNR_{e2} = SNR_e(A_2)$ (where A_2 is defined in (2.12)). In Figure 1, we plot the effective SNR (i.e., SNR_e) as a function of the allocated power to the pilot (equation (2.10)), for different values of SNR (corresponding to the channel background noise). In this example, a pilot period K is 10 symbols and coincides with the coherence time. A frequency-flat Rayleigh fading channel is assumed. The results are shown for the pilot arrangements corresponding to both case 1 and case 2. In the case of the ideal knowledge of the channel response, the effective SNR does correspond to the conventional definition of the SNR (i.e., where the channel background noise is the only noise in the system, as defined in the equation (5.1)).

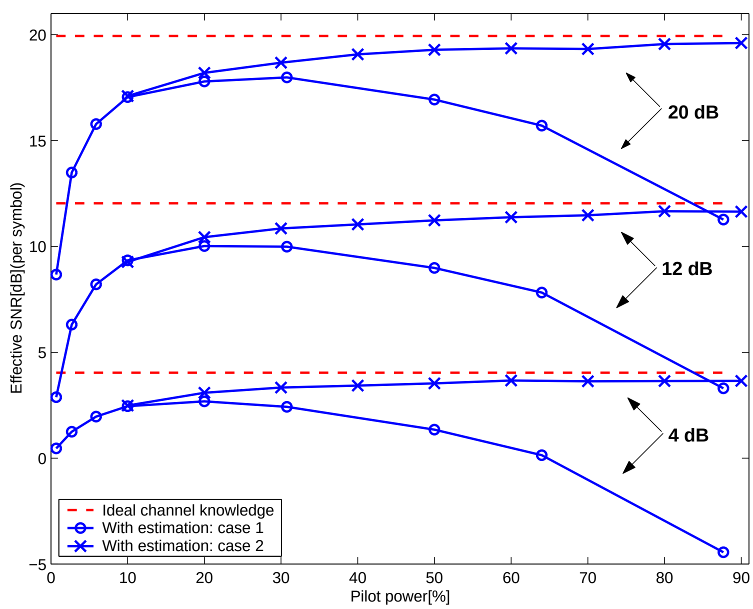

FIGURE 1. Effective SNR vs. power allocated to the pilot, SISO system, $SNR = 4, 12, 20$dB, coherence time $K = 10$ symbols, Rayleigh channel.

Further, in Figure 2, we present the rate R in (4.1) as a function of the power allocated to the pilot (equation (2.10)). The assumptions are identical to ones related to Figure 1. For the ideal knowledge of the channel response we apply the ergodic capacity formula [1]. In Figure 3 and 4, for $SNR = 20$dB, we present the achievable rates vs. power allocated to the pilot for different channel coherence

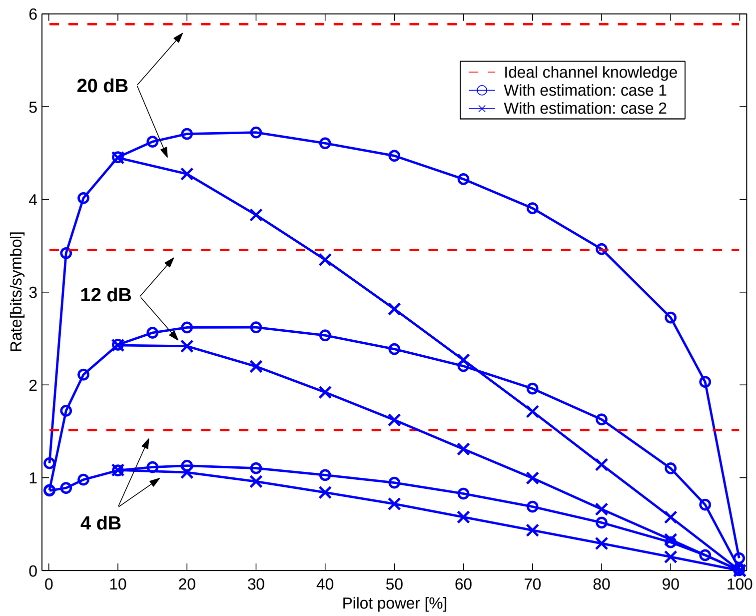

FIGURE 2. Achievable open-loop rates vs. power allocated to the pilot, SISO system, $SNR = 4, 12, 20$dB, coherence time $K = 10$ symbols, Rayleigh channel.

time lengths $K = 10, 20, 40, 100$. In case 1 and case 2, by increasing the channel coherence time length we observe that the maximum rate is reached for lower percentage of the power allocated to the pilot.

Regarding the achievable rates, from the above results we observe that case 1 is less sensitive to the pilot power allocation than case 2 (i.e., in case 2, R is dropping faster if the allocated power is different than the one that results in the maximum value). Further, for the same channel coherence time lengths, case 1 is reaching the maximum rate at a lower power allocated to the pilot than for case 2 (see Figure 3 and 4).

For the given SNR, we define the capacity efficiency ratio η as the ratio between the maximum rate R (with respect to the pilot power) and the ergodic capacity $C_{m\times n}$ in the case of the ideal knowledge of the channel response, i.e.,

$$\eta_{m\times n} = \frac{\max_{\mu} R}{C_{m\times n}}. \tag{5.3}$$

The index m and n correspond to number of transmit and receive antennas, respectively. In Figure 5, we show that the capacity efficiency ratio $\eta_{1\times 1}$ increases as the channel coherence time is getting longer.

5.2. MIMO Systems. In Figure 6, we present the rate R in (4.1) as a function of the power allocated to the pilot (equation (2.10)), for different number of transmit and receive antennas. In this and following cases we observe just the pilot arrangement case 1 (treating case 2 impractical for MIMO systems). We observe the rates for the Rayleigh channel, $SNR = 12$dB and the channel coherence time

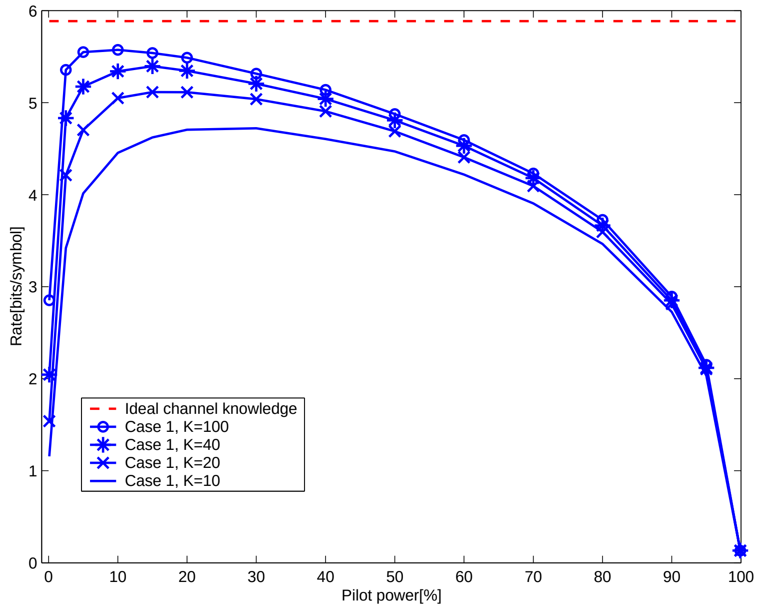

FIGURE 3. Achievable open-loop rates vs. power allocated to the pilot, case 1, SISO system, $SNR = 20$dB, for different channel coherence time lengths $K = 10, 20, 40, 100$ symbols, Rayleigh channel.

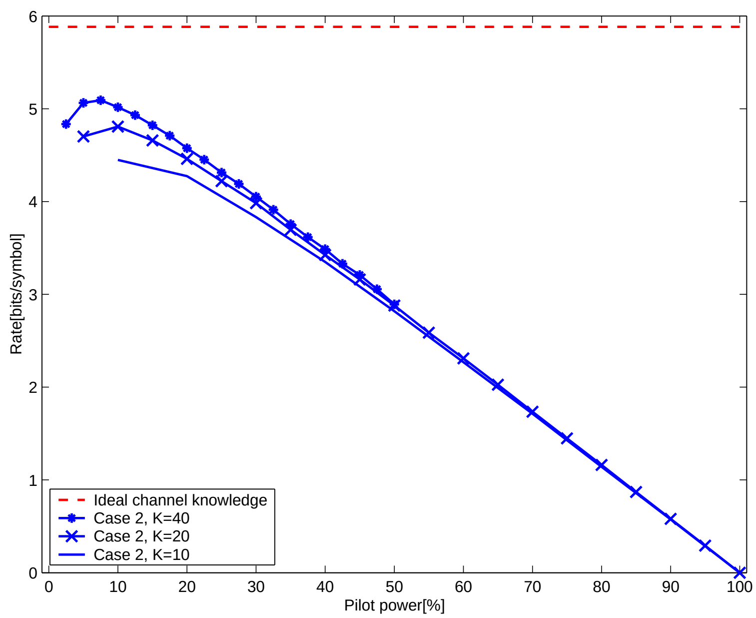

FIGURE 4. Achievable open-loop rates vs. power allocated to the pilot, case 2, SISO system, $SNR = 20$dB, for different channel coherence time lengths $K = 10, 20, 40$ symbols, Rayleigh channel.

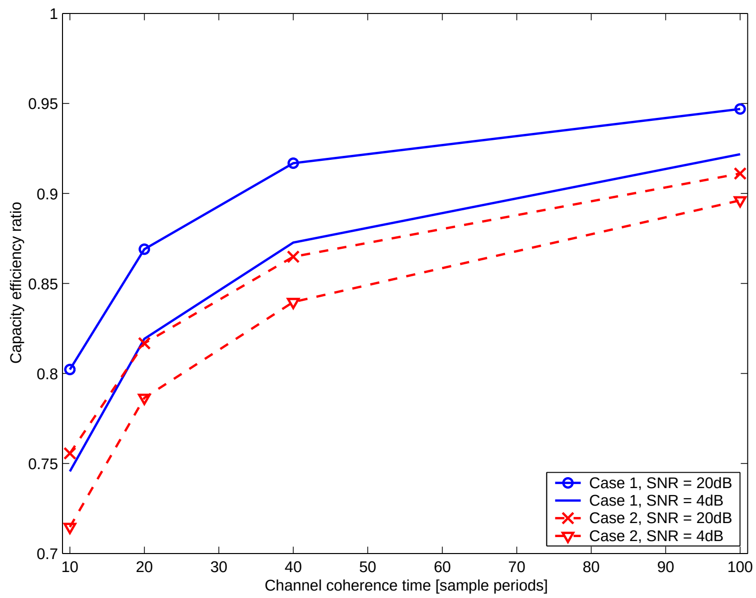

FIGURE 5. Capacity efficiency ratio vs. channel coherence time length ($K = 10, 20, 40, 100$ symbols), SISO system, $SNR = 4, 20$dB, Rayleigh channel.

length $K = 40$. Solid lines correspond to a system with the channel response estimation, and dashed lines to a system with the ideal knowledge of the channel response. Further, in Figure 7 we show the capacity efficiency ratio η for different number of transmit and receive antennas vs. different channel coherence time lengths. We observe that as the number of transmit antenna increases, the sensitivity to the channel response estimation error is more pronounced (while keeping the same number of receive antennas). For example, for the same channel coherence time length, the capacity efficiency ratio of the 4×4 system is lower than in the case of the 3×4 system.

6. Conclusion

In this paper we have studied how the estimation error of the frequency-flat time-varying channel response affects the performance of a communication system. We have considered multiple-input multiple-output (MIMO) wireless systems, where we have approximated the channel as a block-fading channel. The presented analysis may be viewed as a study of mismatched receiver algorithms in MIMO systems. We have considered two pilot based schemes for the estimation. The first scheme uses a single pilot symbol per block with the different power than the data symbol power. The second scheme uses more than one pilot symbol per block, whose power is the same as the data symbol power. We have studied the performance in terms of the effective SNR and achievable data rates (i.e., capacity lower bounds). We have presented how the performance depends on the percentage of the total power allocated to the pilot, background noise level and the channel coherence time length. Our results have shown that the first pilot-based approach

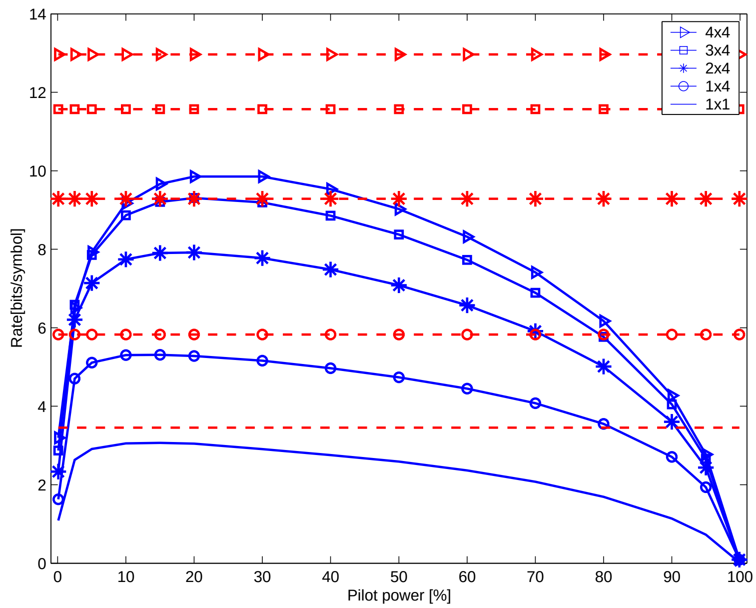

FIGURE 6. Achievable open-loop rates vs. power allocated to the pilot, MIMO system, $SNR = 12$dB, coherence time $K = 40$ symbols, Rayleigh channel, solid line corresponds to a system with the channel response estimation, and dashed line to the case of the ideal channel response knowledge.

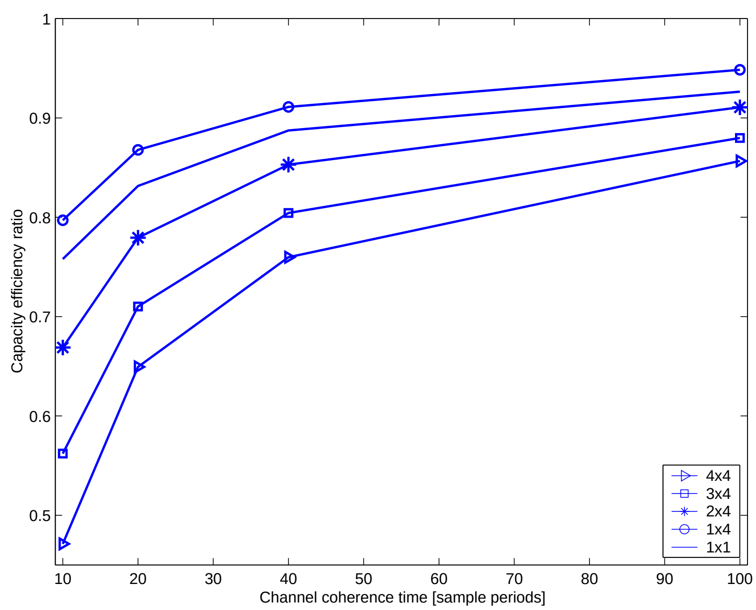

FIGURE 7. Capacity efficiency ratio vs. channel coherence time length ($K = 10, 20, 40, 100$ symbols), MIMO system, $SNR =$ 12dB, Rayleigh channel.

is less sensitive to the fraction of power allocated to the pilot. Furthermore, we have observed that as the number of transmit antennas increase, the sensitivity to the channel response estimation error is more pronounced (while keeping the same number of receive antennas). The effects of the estimation error are evaluated in the case of the estimates being available at the receiver only (open loop), and the future work will address the case when the estimates are fed back to the transmitter (closed loop) allowing water pouring transmitter optimization.

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