Unquantized and Uncoded Channel State Information Feedback on Wireless Channels

Dragan Samardzija
Bell Labs, Lucent Technologies
791 Holmdel-Keyport Road, Holmdel, NJ 07733, USA
Email: dragan@bell-labs.com

Narayan Mandayam
WINLAB, Rutgers University
73 Brett Road, Piscataway, NJ 08854, USA
Email: narayan@winlab.rutgers.edu

***Abstract*— We propose a channel state information (CSI) feedback scheme based on unquantized and uncoded (UQ-UC) transmission. We consider a system where a mobile terminal obtains the downlink CSI and feeds it back to the base station using an uplink feedback channel. If the downlink channel is an independent Rayleigh fading channel, then the CSI may be viewed as an output of a complex independent identically distributed Gaussian source. Further, if the uplink feedback channel is AWGN, it can be shown that that UQ-UC CSI transmission (that incurs zero delay) is optimal in that it achieves the same minimum mean squared error (MMSE) distortion as a scheme that optimally (in the Shannon sense) quantizes and encodes the CSI while theoretically incurring infinite delay. Since the UQ-UC transmission is suboptimal on correlated wireless channels, we propose a simple linear CSI feedback receiver that can be used to improve the performance of UQ-UC transmission while still retaining the attractive zero-delay feature. We provide bounds on the performance of the UQ-UC CSI feedback. Furthermore, we explore its application and performance in multiple antenna multiuser wireless systems.**

I. Introduction

The tremendous capacity gains due to transmitter optimization in multiple antenna multiuser wireless systems [1]–[5] rely heavily on the availability of the channel state information (CSI) at the transmitter. In such scenarios, aside from the issue of how to estimate the CSI, another interesting question is how to transmit (or feedback) the CSI? A fundamental question that arises is that, is it necessary for reliable CSI feedback to follow the principles outlined by the "digital dogma"? In other words, is it necessary that the CSI be optimally quantized and encoded (in a Shannon theoretic sense) for it to be reliable? Are there ways to mitigate the delay (which is theoretically infinite) that is imposed by such a Shannon theoretic approach?

In this paper we consider a system where a mobile terminal obtains the downlink CSI and feeds it back to the base station using an uplink feedback channel. If the downlink channel is an independent Rayleigh fading channel, then the CSI may be viewed as an output of a complex independent identically distributed (*iid*) Gaussian source. Further, if the uplink feedback channel is AWGN and the downlink CSI is perfectly known at the mobile terminal, it can be shown that unquantized and uncoded (UQ-UC) CSI transmission (that incurs zero delay) is optimal in that it achieves the same minimum mean squared error (MMSE) distortion as a scheme that optimally (in the Shannon sense) quantizes and encodes the CSI while incurring infinite delay. Results on the optimality of unquantized and uncoded transmission have also been discussed in other contexts in [6]–[8]. Since the UQ-UC transmission is suboptimal on correlated wireless channels, we propose a simple linear CSI feedback receiver that can be used in conjunction with the UQ-UC transmission while still retaining the attractive zero-delay feature. Furthermore, we describe an auto regressive (AR) correlated channel model and present the corresponding performance bounds for the UQ-UC CSI feedback scheme. We also explore the performance limits of such schemes in the context of achievable information rates in multiple antenna multiuser wireless systems.

This work is supported in part by the National Science Foundation under Grant No. FMF 0429724.

II. Background

Consider the communication system in Figure 1. The system is used for transmission of unquantized and uncoded outputs (i.e., symbols) of the source. The source is complex, continuous in amplitude and discrete in time (with the symbol period T_{sym}). We assume that the symbols x are zero-mean with unit variance. The average transmit power is P, while the channel introduces additive zero-mean noise n with variance N_0.

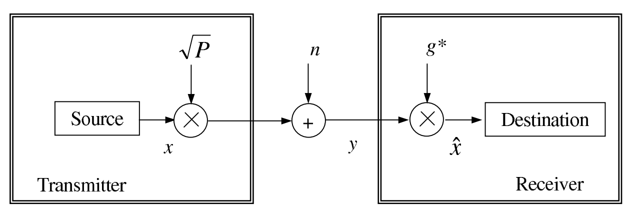

Fig. 1. Unquantized and uncoded transmission that achieves the MMSE distortion of the transmitted signal.

At the receiver, the received signal y is multiplied by the conjugate of g. Consequently, the signal $\hat{x}$ at the destination is

$$\hat{x} = g^* y = g^* \left(\sqrt{P} x + n \right) \tag{1}$$

and $\hat{x}$ is an estimate of the transmitted symbol x. We select the coefficient g to minimize the mean squared error (MSE) between $\hat{x}$ and x. Thus,

$$g = \arg\min \mathrm{E}|\hat{x} - x|^2 = \arg_f \min \mathrm{E}|f^* \left(\sqrt{P} x + n \right) - x|^2. \tag{2}$$

Consequently,

$$g = \frac{\sqrt{P}}{P + N_0} \tag{3}$$

and the corresponding mean squared error is

$$\min\ \mathrm{E}|\hat{x}-x|^2 = \frac{1}{1+\frac{P}{N_0}}. \tag{4}$$

The MSE corresponds to a measure of distortion between the source symbols and estimates at the destination.

Let us now relate the above results to the transmission scheme that applies optimal quantization and channel coding. Based on the Shannon rate distortion theory [9], for a given distortion $D^\star$, the average number of bits per symbol at the output of the optimal quantizer is

$$R = \log_2\left(1+\frac{1-D^\star}{D^\star}\right). \tag{5}$$

Note that the optimal quantizer that achieves the above rate incurs infinite quantization delay. For the AWGN channel, the maximum transmission rate is

$$C = \log_2\left(1+\frac{P}{N_0}\right). \tag{6}$$

As in the case of the optimal quantizer, the optimal channel coding would incur infinite coding delay. Furthermore, optimal matching (in the Shannon sense) of the quantizer and the channel requires that

$$R = C \Rightarrow D^\star = 2^{-C} = \frac{1}{1+\frac{P}{N_0}}. \tag{7}$$

The above distortion is equal to the MSE for the UQ-UC transmission scheme given in (4) (see also [7]). The above result points to the optimality of the UQ-UC scheme (while it incurs zero delay) when the source is *iid* Gaussian and the channel is AWGN.

III. UQ-UC CSI FEEDBACK

Using the above result, we now motivate why UQ-UC transmission schemes can be used for CSI feedback in wireless systems. Consider a communication system that consists of a base station transmitting data over a downlink channel. A mobile terminal receives the data, and transmits the CSI of the downlink channel state h_{dl} over an uplink channel. Let us assume that the mobile terminal estimates the downlink CSI h_{dl} perfectly. If the downlink channel is *iid* Rayleigh, then the CSI is an *iid* complex Gaussian random variable. In this case, if the uplink channel is AWGN and it is independent of the downlink channel, then it follows directly from the earlier discussion that the above UQ-UC scheme is optimal for transmission of the downlink CSI over the uplink channel. In other words, UQ-UC transmission (with zero delay) of the downlink CSI will achieve the same distortion as a scheme that optimally (in the Shannon sense) quantizes and encodes the CSI while incurring infinite delay.

To further distinguish the fact that the UQ-UC CSI feedback transmission does not imply an "analog" communication[1] system, we now illustrate an example of how such a scheme could be applied in the context of a CDMA system. The functional blocks of the mobile terminal in a CDMA system are depicted in Figure 2. Using a pilot-assisted estimation scheme, the mobile terminal obtains an estimate of the downlink channel h_{dl}, denoted as $\bar{h}_{dl}$. The downlink channel estimate $\bar{h}_{dl}$ is the CSI to be transmitted on the uplink channel h_{ul}. The estimate $\bar{h}_{dl}$ modulates (i.e., multiplies) a Walsh code that is specifically allocated as a CSI feedback carrier as shown in Figure 2. The second Walsh code is allocated for the conventional uplink data transmission. For generality, the uplink pilot is also transmitted allowing the base station to obtain an estimate $\bar{h}_{ul}$ of the uplink channel h_{ul}.

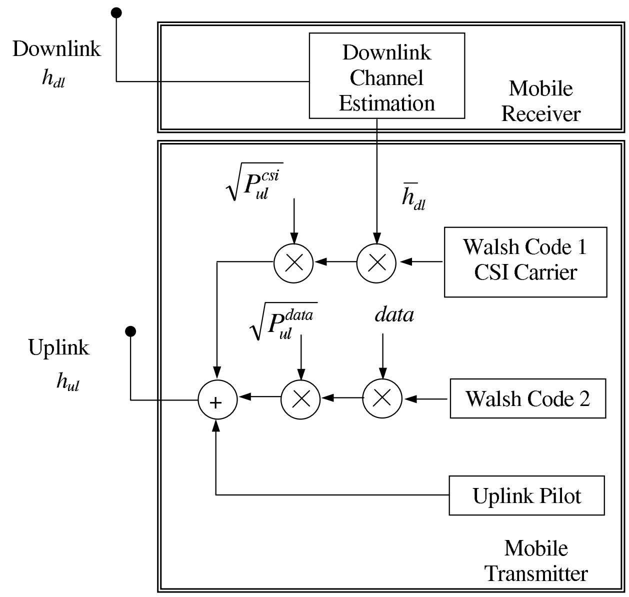

Fig. 2. CDMA mobile terminal that applies the UQ-UC CSI feedback.

In general, the downlink and uplink channel estimation is not perfect, i.e., $\bar{h}_{dl} = h_{dl} + e_{dl}$ and $\bar{h}_{ul} = h_{ul} + e_{ul}$, where e_{dl} and e_{ul} are the channel state estimation errors on the downlink and the uplink, respectively. The estimation errors are modeled as AWGN, which is typical to pilot-assisted channel state estimation schemes (see [10] and the references therein). Consequently, the downlink and uplink estimation errors are distributed as $\mathcal{N}_\mathcal{C}(0, N_{dl}^e)$ and $\mathcal{N}_\mathcal{C}(0, N_{ul}^e)$, respectively, where $\mathcal{N}_\mathcal{C}(0,\sigma^2)$ denotes a complex zero-mean Gaussian random variable distribution with the variance σ^2.

Consider a signal/system model, where at the time instant i, the uplink received signal corresponding to the CSI feedback is

$$y(i) = h_{ul}(i)\sqrt{P_{ul}^{csi}}\,\bar{h}_{dl}(i) + n(i) \tag{8}$$

where $h_{ul}(i)$ is the uplink channel state, P_{ul}^{csi} is the CSI feedback transmit power, $\bar{h}_{dl}(i)$ is the estimate of the downlink channel h_{dl} that is being fed back and $n(i)$ is the AWGN on the uplink with the variance N_0. Using the received signal in (8) and an estimate of $\bar{h}_{ul}(i)$, the CSI feedback receiver at the base station will estimate the transmitted CSI $h_{dl}(i)$. In the following derivations we assume that the uplink and downlink

[1]While we use the term unquantized (UQ) in the UQ-UC nomenclature, it must be pointed out that any practical transmission scheme will require at least some level of coarse quantization.

channel states are mutually independent and correspond to zero-mean and unit-variance complex Gaussian distribution $\mathcal{N}_{\mathcal{C}}(0,1)$.

Using the same approach as given in Section II, the uplink CSI feedbeck receiver w is derived from the following optimization

$$w = \arg_v \min \mathrm{E}_{h_{dl}(i),\, y(i)|\bar{h}_{ul}(i)} |v^* y(i) - h_{dl}(i)|^2 = \frac{s}{u} \quad (9)$$

where

$$\begin{aligned} u &= \mathrm{E}_{y(i)|\bar{h}_{ul}(i)} \left[y(i)\, y(i)^* \right] = \\ &= P_{ul}^{csi} \left(\frac{N_{ul}^e (1+N_{dl}^e)}{1+N_{ul}^e} + \frac{|\bar{h}_{ul}(i)|^2 (1+N_{dl}^e)}{(1+N_{ul}^e)^2} \right) + N_0. \quad (10) \end{aligned}$$

The above result is based on the fact that the conditional distribution $p(h_{ul}(i)|\bar{h}_{ul}(i))$ is a complex Gaussian distribution $\mathcal{N}_{\mathcal{C}}\left(\frac{\bar{h}_{ul}(i)}{1+N_{ul}^e}, \frac{N_{ul}^e}{1+N_{ul}^e} \right)$ and $h_{dl}(i)$ is independent of $\bar{h}_{ul}(i)$. Furthermore,

$$s = \mathrm{E}_{h_{dl}(i),\, y(i)|\bar{h}_{ul}(i)} \left[h_{dl}(i)^*\, y(i) \right] = \sqrt{P_{ul}^{csi}}\ \frac{\bar{h}_{ul}(i)}{1+N_{ul}^e}. \quad (11)$$

The uplink receiver then estimates the downlink CSI $h_{dl}(i)$ as

$$\hat{h}_{dl}(i) = w^* y(i) \quad (12)$$

with the MSE distortion being

$$\begin{aligned} &\mathrm{E}_{h_{dl}(i),\, y(i)|\bar{h}_{ul}(i)} |w^* y(i) - h_{dl}(i)|^2 = 1 - \frac{s\, s^*}{u^*} = \\ &= \frac{\frac{P_{ul}^{csi}}{N_0} \left(\frac{N_{ul}^e(1+N_{dl}^e)}{1+N_{ul}^e} + \frac{N_{dl}^e}{(1+N_{ul}^e)^2} |\bar{h}_{ul}(i)|^2 \right) + 1}{\frac{P_{ul}^{csi}}{N_0} \left(\frac{N_{ul}^e(1+N_{dl}^e)}{1+N_{ul}^e} + \frac{1+N_{dl}^e}{(1+N_{ul}^e)^2} |\bar{h}_{ul}(i)|^2 \right) + 1}. \quad (13) \end{aligned}$$

Note that as the estimation errors approach zero, $N_{dl}^e \to 0$ and $N_{ul}^e \to 0$, the receiver in (9) is identical to the receiver in (3).

IV. UQ-UC CSI Feedback on Correlated Channels

The MSE distortion achieved by the UQ-UC CSI feedback transmission scheme is optimal when the downlink is *iid* Rayleigh and the uplink is AWGN, and further, the uplink and the downlink are also mutually independent with perfect channel estimation of h_{dl} and h_{ul}. In reality, there may the following situations that arise in wireless systems: (1) temporal correlations in the downlink channel, (2) temporal correlations in the uplink channel, and (3) correlations between the uplink and the downlink channels (as is in TDD systems). In each of these cases, it is of interest to quantify the MSE distortion achieved by the UQ-UC CSI feedback. Since, an exact analysis is not readily tractable, we propose to quantify such performance through upper and lower bounds in each of the above scenarios.

A. Performance Bounds

Let us assume that the uplink and downlink channel states are independent (which is typical in FDD wireless systems). Both the uplink and downlink channels are varying in time and are assumed to be ergodic. If the scheme shown in Figure 1 is now applied on the CSI feedback channel, using the result in (13), it follows that the MSE is

$$MSE_{uq-uc}^{ub} = \mathrm{E}_{\bar{h}_{ul}} \left[\frac{\left(\frac{N_{ul}^e(1+N_{dl}^e)}{1+N_{ul}^e} + \frac{N_{dl}^e}{(1+N_{ul}^e)^2} |\bar{h}_{ul}|^2 \right) + \frac{N_0}{P_{ul}^{csi}}}{\left(\frac{N_{ul}^e(1+N_{dl}^e)}{1+N_{ul}^e} + \frac{1+N_{dl}^e}{(1+N_{ul}^e)^2} |\bar{h}_{ul}|^2 \right) + \frac{N_0}{P_{ul}^{csi}}} \right]. \quad (14)$$

Clearly this serves as an upper bound on the MSE achieved by any additional processing that accounts for both the downlink and the uplink CSI feedback channel being correlated channels.

To illustrate an approach to derive a lower bound, consider an Lth order auto regressive (AR) process model for the downlink channel as

$$h_{dl}(i) = \sum_{j=1}^{L} c_j h_{dl}(i-j) + c_0\, n_{dl}(i), \quad (15)$$

where $n_{dl}(i)$ is a complex Gaussian random variable with distribution $\mathcal{N}_{\mathcal{C}}(0,1)$. The coefficients c_j $(j = 0, \cdots, L)$ determine the correlation properties of the channel. $n_{dl}(i)$ is the *innovation sequence* that describes the evolution to successive channel states. This is a quasi-static block-fading channel model where the temporal variations of the channel are characterized by the correlation between successive channel blocks. The above model gives a general framework for describing the correlations in the downlink channel states through the coefficients c_j $(j = 0, \cdots, L)$.

Using an approach outlined in [11], [12] and Appendix, it is possible to approximate the well known Jakes correlated fading model by relating parameters such as carrier frequency and mobile speed to the AR model coefficients. The Jakes model corresponds to a continuous time-varying channel, while the AR model to a quasi-static block-fading channel. To connect these two models, we assume that the channel is constant for a duration of τ seconds (i.e., this duration may be viewed as the channel coherence time) and τ is the absolute time difference between successive channel states $h_{dl}(i)$ and $h_{dl}(i-1)$. Furthermore, the correlation $\mathrm{E}[h_{dl}(i)h_{dl}(i-k)^*] = J_0(2\pi f_d k\tau)$ where f_d is the maximum Doppler frequency (see Appendix). For a more detailed analysis of auto regressive-moving average (ARMA) processes and wireless channel modeling we refer the reader to [13], [14] and the references therein.

Let us assume that the above model and the previous channel states $h_{dl}(i-j)$ $(j = 1, \cdots, L)$ are known at the CSI feedback transmitter and receiver. In addition, in deriving the lower bound, we will assume that the estimation errors $e_{dl} = 0$ and $e_{ul} = 0$ (i.e., perfect channel state estimation). In this idealized case, having only the innovation $n_{dl}(i)$ transmitted over the uplink CSI feedback channel, the receiver can estimate the channel state $h_{dl}(i)$. We will now use arguments similar to that used in deriving (7) to arrive at a lower bound for the MSE of the UQ-UC scheme. Consider the distortion of the innovation sequence

$$D^{in} = \mathrm{E}|\hat{n}_{dl}(i) - n_{dl}(i)|^2, \quad (16)$$

where $\hat{n}_{dl}(i)$ is an estimate of $n_{dl}(i)$. Then the average number of bits per symbol at the output of the optimal quantizer is

$$R^{in} = \log_2\left(1 + \frac{1 - D^{in}}{D^{in}}\right). \tag{17}$$

Furthermore, the ergodic capacity of the uplink channel is

$$\bar{C}_{ul} = \mathrm{E}_{h_{ul}}\left[\log_2\left(1 + \frac{|h_{ul}|^2 P_{ul}^{csi}}{N_0}\right)\right]. \tag{18}$$

Then the optimal matching (in the Shannon sense) of the quantization and channel coding of the innovation $n_{dl}(i)$ results in

$$R^{in} = \bar{C}_{ul}. \tag{19}$$

Hence the MSE

$$D^{in} = \mathrm{E}|\hat{n}_{dl}(i) - n_{dl}(i)|^2 = 2^{-\bar{C}_{ul}}. \tag{20}$$

Thus from equations (15) and (20) it follows that the MSE of $h_{dl}(i)$ is lower bounded as

$$\mathrm{E}|\hat{h}_{dl}(i) - h_{dl}(i)|^2 \geq c_0^2 2^{-\bar{C}_{ul}}. \tag{21}$$

Since this bound is obtained using idealized knowledge of the previous channel states and also a channel coding scheme that achieves the ergodic capacity of the uplink channel, we expect it to be loose.

B. Feedback Receivers for Enhancing UQ-UC CSI Feedback Schemes

While the previous subsection considered the performance limits of the MSE distortion achieved by the UQ-UC CSI feedback transmission, in this subsection we will outline signal processing techniques that could be used to improve the performance of UQ-UC schemes. The specific approach that we propose is to design receivers on the CSI feedback channel that can exploit the channel correlations and thus improve the performance in cases where the UQ-UC CSI feedback transmission is suboptimal. We illustrate such an approach through a design of a linear CSI feedback receiver in the following.

The uplink received signal in (8) is used to form a temporal K-dimensional received vector as

$$\mathbf{y}(i) = [y(i)\, y(i-1) \cdots y(i-K+1)]^{\mathrm{T}}. \tag{22}$$

The uplink receiver then estimates the downlink CSI $h_{dl}(i)$ as

$$\hat{h}_{dl}(i) = \mathbf{w}^{\mathrm{H}}\mathbf{y}(i) \tag{23}$$

where $\mathbf{w}$ is a linear filter that is derived from the following MMSE optimization

$$\mathbf{w} = \arg_{\mathbf{v}} \min\ \mathrm{E}|\mathbf{v}^{\mathrm{H}}\mathbf{y}(i) - h_{dl}(i)|^2. \tag{24}$$

For the given estimates of the uplink channel $\bar{\mathbf{h}}_{ul}(i) = [\bar{h}_{ul}(i)\, \bar{h}_{ul}(i-1) \cdots \bar{h}_{ul}(i-K+1)]^{\mathrm{T}}$ we define the following matrix

$$\mathbf{U} = \mathrm{E}_{\mathbf{y}(i)|\bar{\mathbf{h}}_{ul}(i)}\left[\mathbf{y}(i)\,\mathbf{y}(i)^{\mathrm{H}}\right] \tag{25}$$

and the vector

$$\mathbf{s} = \mathrm{E}_{h_{dl}(i),\,\mathbf{y}(i)|\bar{\mathbf{h}}_{ul}(i)}\left[h_{dl}(i)^*\,\mathbf{y}(i)\right]. \tag{26}$$

It can be shown that the linear MMSE CSI feedback receiver $\mathbf{w}$ is given as

$$\mathbf{w} = \mathbf{U}^{-1}\mathbf{s}. \tag{27}$$

As is evident from the equations (25)-(27), the linear transformation $\mathbf{w}$ takes into account implicitly the following correlations: (1) temporal correlations in the downlink channel, (2) temporal correlations in the uplink channel and (3) the correlations between the uplink and the downlink. In fact, when $K = 1$ and the uplink and the downlink are mutually independent, then the above receiver will achieve the MSE distortion upper bound in equation (14). In all other cases, the performance will be superior, thereby enhancing the performance of the UQ-UC CSI feedback transmission.

C. Numerical Results: Distortion Performance

We now present the upper and lower bounds derived in the previous sections for different scenarios corresponding to the uplink and downlink CSI. Specifically we take into account the effect of background noise levels, estimation errors and channel correlation. We characterize the quality of the uplink CSI feedback channel through its SNR given as

$$SNR_{ul}^{csi} = 10\log\frac{P_{ul}^{csi}}{N_0}. \tag{28}$$

In order to quantify the effect of the estimation errors on the UQ-QC scheme, we proceed in the following way. Recall that the uplink channel estimate is given as $\bar{h}_{ul} = h_{ul} + e_{ul}$. We quantify the estimation performance by the following SNR term

$$SNR_{ul}^{e} = 10\log\frac{1}{N_{ul}^{e}}, \tag{29}$$

where N_{ul}^{e} is the variance of e_{ul}. The corresponding quantity that is used to characterized the downlink channel estimation error is

$$SNR_{dl}^{e} = 10\log\frac{1}{N_{dl}^{e}}. \tag{30}$$

First we consider a case when the uplink and downlink channels are mutually independent. The channels correspond to the *iid* Rayleigh block-fading model (i.e., for every time instant independent channel states are instantiated for the uplink and downlink). In Figure 3 we set $SNR_{ul}^{csi} = 20$ dB and present the MSE bounds as functions of SNR_{ul}^{e} and/or SNR_{dl}^{e}. We compare the curves corresponding to the perfect downlink estimation ($SNR_{dl}^{e} = +\infty$) and variable SNR_{ul}^{e} versus the perfect uplink estimation ($SNR_{ul}^{e} = +\infty$) and variable SNR_{dl}^{e} (i.e., the curve with marker $\times$ versus $\bigtriangledown$). From these results we note that the MSE upper bound is more affected by the errors in the uplink than the downlink channel state estimation. In this particular example, for the estimation SNRs exceeding 25 dB, the increase in the distortion due to the imperfect knowledge of the channel states is negligible, as evidenced by the flattening of the MSE upper bound.

We now investigate the MSE distortion for correlated channels. The downlink and uplink channels are modeled as an AR process ($L = 10$) whose coefficients are chosen to correspond

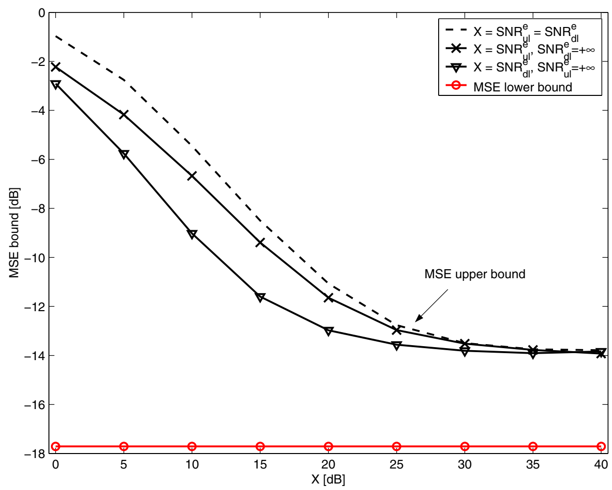

Fig. 3. MSE bounds vs. SNR_{ul}^{e} and/or SNR_{dl}^{e}, for *iid* Rayleigh block-fading on the uplink and downlink and $SNR_{ul}^{csi} = 20$ dB.

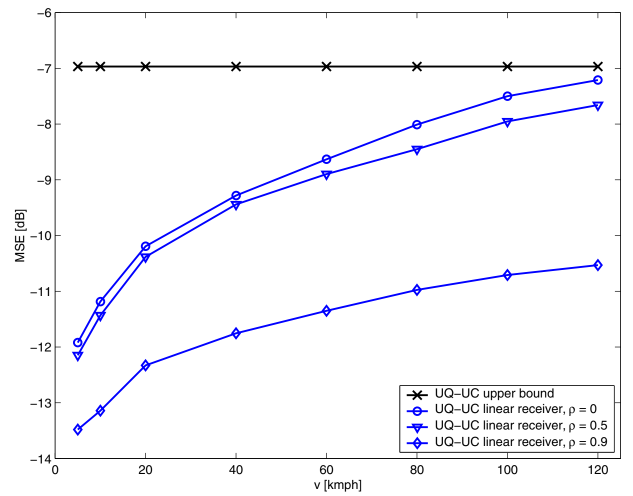

Fig. 4. MSE vs. mobile terminal velocities, $f_c = 2$ GHz and $SNR_{ul}^{csi} = 10$ dB.

to the Jakes model for a carrier frequency of 2 GHz and the coherence time $\tau = 2$ msec (i.e., duration of one channel block). The correlation between the uplink and downlink channel is quantified as

$$\rho = \mathrm{E}\left[h_{dl}(i)\, h_{ul}(i)^{*}\right] \tag{31}$$

where the coefficient $|\rho| \leq 1$. In addition, the uplink has an average $SNR_{ul}^{csi} = 10$ dB and the estimation is perfect ($SNR_{ul}^{e} = +\infty$ and $SNR_{dl}^{e} = +\infty$). In Figure 4 we show the MSE of the UQ-UC scheme with the linear CSI feedback receiver and the MSE upper bound for different mobile terminal velocities. These results show that the linear receiver in combination with the UQ-UC transmission is able to exploit the channel correlations and improve the performance. Note that when the mobile terminal velocities are low the improvement is greater (because the successive channel states are more correlated which is exploited by the linear CSI feedback receiver). Also, the improvement is greater when the uplink and downlink channels are mutually correlated (i.e., for $\rho = 0.9$), as may be the case in TDD systems.

V. UQ-UC CSI Feedback for Transmitter Optimization in Multiple Antenna Multiuser Systems

The discussion thus far has focused on performance limits and enhancements from the point of view of the MSE distortion achieved due to the UQ-UC CSI feedback transmission. A more direct performance issue that needs to be considered is the overall capacity of a system that actually uses the CSI feedback information. We will consider the UQ-UC CSI feedback in a multiple antenna multiuser system. As an example, consider the system shown in Figure 5, where there are M transmit antennas at the base station and N single-antenna mobile terminals. In the above model, x_n is the information bearing signal intended for mobile terminal n and y_n is the received signal at the corresponding terminal (for $n = 1, \cdots, N$). The received vector $\mathbf{y} = [y_1, \cdots, y_N]^{\mathrm{T}}$ is

$$\begin{aligned} &\mathbf{y} = \mathbf{H}\mathbf{S}\mathbf{x} + \mathbf{n}, \\ &\mathbf{y} \in \mathcal{C}^{N}, \mathbf{x} \in \mathcal{C}^{N}, \mathbf{n} \in \mathcal{C}^{N}, \mathbf{S} \in \mathcal{C}^{M\times N}, \mathbf{H} \in \mathcal{C}^{N\times M} \end{aligned} \tag{32}$$

where $\mathbf{x} = [x_1, \cdots, x_N]^{\mathrm{T}}$ is the transmitted vector ($\mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{H}}] = P_{dl}\, \mathbf{I}_{N\times N}$), $\mathbf{n}$ is AWGN ($\mathrm{E}[\mathbf{n}\mathbf{n}^{\mathrm{H}}] = N_0\, \mathbf{I}_{N\times N}$), $\mathbf{H}$ is the MIMO channel state matrix, and $\mathbf{S}$ is a transformation (spatial pre-filtering) performed at the transmitter. Note that the vectors $\mathbf{x}$ and $\mathbf{y}$ have the same dimensionality. Further, h_{nm} is the nth row and mth column element of the matrix $\mathbf{H}$ corresponding to a channel between mobile terminal n and transmit antenna m.

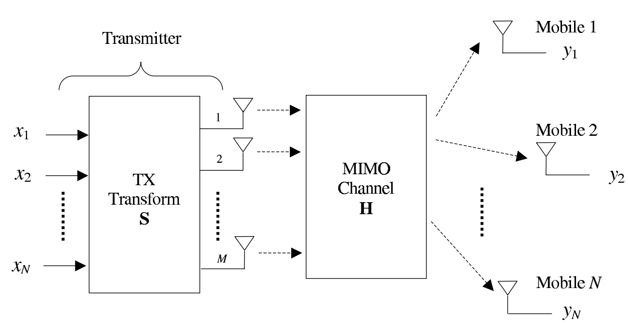

Fig. 5. System model consisting of M transmit antennas and N mobile terminals.

Application of the spatial pre-filtering results in the composite MIMO channel $\mathbf{G}$ given as

$$\mathbf{G} = \mathbf{H}\mathbf{S}, \qquad \mathbf{G} \in \mathcal{C}^{N\times N} \tag{33}$$

where g_{nm} is the nth row and mth column element of the composite MIMO channel state matrix $\mathbf{G}$. The signal received at the nth mobile terminal is

$$y_n = \underbrace{g_{nn} x_n}_{\text{Desired signal for user } n} + \underbrace{\sum_{i=1, i\neq n}^{N} g_{ni} x_i}_{\text{Interference}} + n_n. \tag{34}$$

In the above representation, the interference is the signal that is intended for other mobile terminals than terminal n. As said earlier, the matrix $\mathbf{S}$ is a spatial pre-filter at the transmitter. It is determined based on optimization criteria that we address later in the text and has to satisfy the following constraint

$$\text{trace}\left(\mathbf{S}\mathbf{S}^{\mathrm{H}}\right) \leq N \tag{35}$$

which keeps the average transmit power conserved. We represent the matrix $\mathbf{S}$ as

$$\mathbf{S} = \mathbf{A}\mathbf{P}, \quad \mathbf{A} \in \mathcal{C}^{M\times N}, \mathbf{P} \in \mathcal{C}^{N\times N} \tag{36}$$

where $\mathbf{A}$ is a linear transformation and $\mathbf{P}$ is a diagonal matrix. $\mathbf{P}$ is determined such that the transmit power remains conserved. We study the zero-forcing (ZF) spatial pre-filtering scheme where $\mathbf{A}$ is represented by

$$\mathbf{A} = \mathbf{H}^{\mathrm{H}}(\mathbf{H}\mathbf{H}^{\mathrm{H}})^{-1}. \tag{37}$$

As can be seen, the above linear transformation is zeroing the interference between the signals dedicated to different mobile terminals, i.e., $\mathbf{H}\mathbf{A} = \mathbf{I}_{N\times N}$. The x_n's are assumed to be circularly symmetric complex random variables each having Gaussian distribution $\mathcal{N}_{\mathcal{C}}(0, P_{dl})$. Consequently, the maximum achievable data rate (capacity) for mobile terminal n is

$$R_n^{\mathrm{ZF}} = \log_2\left(1 + \frac{P_{dl}|p_{nn}|^2}{N_0}\right) \tag{38}$$

where p_{nn} is the nth diagonal element of the matrix $\mathbf{P}$ defined in (36). In this study we apply a suboptimal, yet a simple solution

$$\mathbf{P} = \sqrt{\frac{N}{\text{trace}\left(\mathbf{A}\mathbf{A}^{\mathrm{H}}\right)}}\,\mathbf{I}_{N\times N} \tag{39}$$

that guarantees the constraint in (35).

To perform the above spatial pre-filtering, the base station obtains CSI corresponding to each downlink channel state h_{nm}. The CSI is obtained from each mobile terminal using the UQ-UC CSI feedback. In other words, at time instant i, terminal n ($n = 1, \cdots, N$) is transmitting the corresponding CSI $h_{nm}(i)$ ($m = 1, \cdots, M$) via the uplink CSI feedback channel. Relating to the analysis in the previous sections, each $h_{nm}(i)$ corresponds to a different $h_{dl}(i)$. Instead of the ideal channel state $h_{nm}(i)$, the spatial pre-filter applies the estimate $\hat{h}_{nm}(i)$ obtained from the uplink CSI feedback receiver. Therefore at the base station instead of the true $\mathbf{H}$, in the expressions (37) and (39), $\hat{\mathbf{H}}$ is applied whose entries are $\hat{h}_{nm}(i)$ ($m = 1, \cdots M$ and $n = 1, \cdots, N$). Consequently, the maximum achievable data rate for mobile terminal n is

$$\hat{R}_n^{\mathrm{ZF}} = \log_2\left(1 + \frac{P_{dl}|\hat{g}_{nn}|^2}{P_{dl}\sum_{i=1, i\neq n}^{N}|\hat{g}_{ni}|^2 + N_0}\right). \tag{40}$$

where $\hat{g}_{nm}$ is the nth row and mth column element of the composite MIMO channel state matrix

$$\mathbf{G} = \mathbf{H}\hat{\mathbf{A}}\hat{\mathbf{P}} \tag{41}$$

with

$$\hat{\mathbf{A}} = \hat{\mathbf{H}}^{\mathrm{H}}(\hat{\mathbf{H}}\hat{\mathbf{H}}^{\mathrm{H}})^{-1} \text{ and } \hat{\mathbf{P}} = \sqrt{\frac{N}{\text{trace}\left(\hat{\mathbf{A}}\hat{\mathbf{A}}^{\mathrm{H}}\right)}}\mathbf{I}_{N\times N}. \tag{42}$$

Note that $\hat{\mathbf{A}}\hat{\mathbf{P}}$ forms a spatial pre-filter. It is mismatched because it applies $\hat{\mathbf{H}}$ instead of the true $\mathbf{H}$.

In Figure 6 we present downlink sum data rates where $SNR_{dl} = 10$ dB, and $M = 3$ and $N = 3$. The rates are presented as functions of the mobile terminal velocity using the approximate Jakes model for a carrier frequency 2 GHz and the coherence time $\tau = 2$ msec and spatially uncorrelated channels. The uplink CSI feedback channel is with the average $SNR_{ul}^{csi} = 10$ dB, and it is independent of the downlink. In addition, we present the rates for instantaneous ideal channel knowledge and a delayed ideal channel knowledge (2 msec delay) which may correspond to a practical feedback scheme that quantizes and encodes the CSI. For example, in 3G WCDMA HSDPA system 2 msec corresponds to the duration of a radio packet which may be used to transmit quantized and encoded CSI, incurring the minimum delay of 2 msec. We note that under the UQ-UC CSI feedback with the linear receiver, the performance is better for channels with higher correlations (i.e., lower mobile terminal velocities). For the moderate and higher velocities, the UQ-UC CSI feedback scheme is outperforming the case of the delayed ideal channel knowledge. Note that in the above example we assume that the estimation is perfect ($SNR_{ul}^{e} = +\infty$ and $SNR_{dl}^{e} = +\infty$).

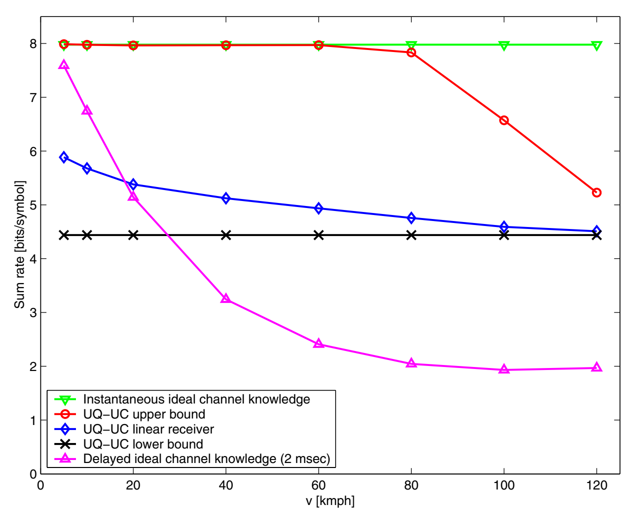

Fig. 6. Average downlink sum data rate vs. mobile terminal velocity, $f_c = 2$ GHz, $M = 3$, $N = 3$, spatially uncorrelated, $SNR_{dl} = 10$ dB and $SNR_{ul}^{csi} = 10$ dB.

VI. Conclusion

In this paper we have considered a system where a mobile terminal obtains the downlink CSI and feeds it back to the base station using an uplink feedback channel. If the downlink channel is an independent Rayleigh fading channel and the uplink feedback channel is AWGN, we have shown that unquantized and uncoded CSI transmission (that incurs zero delay) is optimal in that it achieves the same minimum mean squared error distortion as a scheme that optimally quantizes and encodes the CSI while incurring infinite delay. We have proposed a simple linear CSI feedback receiver that exploits the channel correlations while still retaining the attractive zero-delay feature. Furthermore, we described the AR correlated channel model and presented the corresponding performance bounds for the UQ-UC CSI feedback scheme. We explored the performance limits of the scheme in the context of downlink multiple antenna, multiuser transmitter optimization. We showed that the UQ-UC scheme can a provide reliable and fast feedback of CSI even in the case of high terminal mobility.

Appendix

In this appendix we show how for the given correlation between the downlink channel states, the correlated channel states are generated and the coefficients c_0 to c_L of the AR model in (15) are determined. The correlation between the downlink channel states is given as

$$\phi(k) = \mathrm{E}[h_{dl}(i)h_{dl}(i-k)^*] \text{ for } |k| \leq L \quad (43)$$

where $\phi(-k) = \phi(k)^*$, and for $|k| > L$, $\phi(k) = 0$. As said earlier, we assume that $\phi(0) = 1$. The corresponding correlation matrix is $\mathbf{R} = \mathrm{E}[\mathbf{h}_{dl}(i)\mathbf{h}_{dl}(i)^{\mathrm{H}}]$ where $\mathbf{h}_{dl}(i) = [h_{dl}(i)\, h_{dl}(i-1) \cdots h_{dl}(i-L)]^{\mathrm{T}}$. Considering that the matrix $\mathbf{R}$ can be decomposed as $\mathbf{R} = \mathbf{Q}\mathbf{Q}^{\mathrm{H}}$, the correlated channel states $h_{dl}(i), \cdots, h_{dl}(i-L)$ are obtained from the following operation

$$\mathbf{h}_{dl}(i) = \mathbf{Q}\,\mathbf{n} \quad (44)$$

where $\mathbf{n}$ is a random, $L+1$-dimensional, zero-mean vector with the correlation matrix $\mathrm{E}[\mathbf{n}\mathbf{n}^{\mathrm{H}}] = \mathbf{I}$.

Further, based on the AR model in (15) we form a set of $L+1$ linear equations

$$\phi(0) = \sum_{j=1}^{L} c_j \phi(-j) + c_0^2 \quad (45)$$

and

$$\phi(k) = \sum_{j=1}^{L} c_j \phi(k-j) \quad k = 1, \cdots, L. \quad (46)$$

Let us define the following matrix

$$\mathbf{\Phi} = \begin{bmatrix} 1 & \phi(1)^* & \phi(2)^* & \cdots & \phi(L)^* \\ 0 & \phi(0) & \phi(1)^* & \cdots & \phi(L-1)^* \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & \phi(L-1) & \phi(L-2) & \cdots & \phi(0) \end{bmatrix} \quad (47)$$

and vectors

$$\mathbf{c} = [c_0^2\, c_1\, \cdots\, c_L]^{\mathrm{T}} \quad (48)$$

and

$$\mathbf{f} = [\phi(0)\, \phi(1)\, \cdots\, \phi(L)]^{\mathrm{T}}. \quad (49)$$

The above system of linear equations can be rewritten as

$$\mathbf{f} = \mathbf{\Phi}\mathbf{c}. \quad (50)$$

The least squares solution of the above linear equation is

$$\tilde{\mathbf{c}} = [\tilde{c}_0^2\, \tilde{c}_1\, \cdots\, \tilde{c}_L]^{\mathrm{T}} = (\mathbf{\Phi}^{\mathrm{H}}\, \mathbf{\Phi})^{-1}\, \mathbf{\Phi}^{\mathrm{H}}\, \mathbf{f}. \quad (51)$$

From the above we directly adopt the solutions for the coefficients $c_i = \tilde{c}_i$ for $i = 1, \cdots, L$. Let us now determine the coefficient c_0. From the model in (15), the innovation term is

$$c_0\, n_{dl}(i) = h_{dl}(i) - \sum_{j=1}^{L} c_j h_{dl}(i-j) = \mathbf{z}^{\mathrm{H}}\mathbf{h}_{dl}(i) \quad (52)$$

where $\mathbf{z} = [1\ \ -c_1^* \cdots -c_L^*]^{\mathrm{T}}$. In order to guarantee that the innovation is unit-variance, while maintaining the correlation $\mathbf{R}$, the coefficient c_0 is selected as

$$c_0 = \sqrt{\mathbf{z}^{\mathrm{H}}\mathbf{R}\mathbf{z}}. \quad (53)$$

To approximate the Jakes model using the finite length AR model in (15) we select elements of the vector $\mathbf{f}$ as

$$\phi(k) = J_0(2\pi f_d k\tau), \ \ k = 0, \cdots, L \quad (54)$$

where f_d is the maximum Doppler frequency and τ is the time difference between successive channel states $h_{dl}(i)$ and $h_{dl}(i-1)$. Satisfying the Nyquist sampling rate, the period τ should be such that $\tau < 1/(2f_d)$.

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