Technical Report WINLAB-TR-198

Blind Successive Interference Cancellation for

DS-CDMA Systems

Dragan Samardzija, Narayan Mandayam, Ivan Seskar

Rutgers University

October, 2000

RUTGERS WIRELESS INFORMATION

NETWORK LABORATORY

Rutgers - The State University of New Jersey

73 Brett Road

Piscataway, New Jersey 08854-8060

Phone: (732) 445-5954 FAX: (732) 445-3693

Technical Report WINLAB-TR-198

Blind Successive Interference Cancellation for DS-CDMA Systems

Dragan Samardzija, Narayan Mandayam, Ivan Seskar

WINLAB PROPRIETARY

For one year from the date of this document, distribution limited to WINLAB personnel; members of Rutgers University Administration; and WINLAB sponsors, who will distribute internally when appropriate for their needs.

Copyright ©1999/2000 WINLAB, Piscataway, New Jersey

ALL RIGHTS RESERVED

Blind Successive Interference Cancellation for DS-CDMA Systems

Dragan Samardzija, Narayan Mandayam, Ivan Seskar

RUTGERS WIRELESS INFORMATION

NETWORK LABORATORY

Rutgers - The State University of New Jersey

73 Brett Road

Piscataway, New Jersey 08854-8060

Phone: 732-445-5954 *FAX: 732-445-3693*

ABSTRACT

In this work we propose a blind successive interference cancellation receiver for asynchronous direct-sequence code-division multiple-access (DS-CDMA) systems using a maximum mean energy (MME) optimization criterion. The covariance matrix of the received vector is used in conjunction with the MME criterion to realize a blind successive interference canceler that is referred to as the BIC-MME receiver. Simulation results show that this scheme offers performance gains over the well known blind receiver that is based on the minimum mean squared error (MMSE) optimization criterion. Further, the BIC-MME receiver is particularly effective in the presence of a few strong interferers as may be the case in the downlink of DS-CDMA systems where intracell user transmissions are orthogonal. The receiver is also shown to perform well in the presence of estimation errors of the covariance matrix making it suitable for use in time-varying channels. An iterative implementation that results in reduced complexity is also studied

Table of Contents

1. Introduction

In DS-CDMA systems, in general, crosscorrelations between signature (spreading) sequences are nonzero. This results in multiple-access interference (MAI) which can disrupt reception of highly attenuated desired user signal. This is known as the near-far effect. To combat this problem several multiuser receivers have been proposed (for example, see [1, 2, 3, 4, 5]). These receivers are denoted as centralized because they require knowledge of parameters (signature sequences, amplitudes and timing) for all users in the system. Therefore, they are more suitable for processing at the base station.

For the downlink, it is desirable to devise decentralized receivers. Decentralized receivers exploit the knowledge of the desired user parameters only. The use of short signature sequences simplify the task of multiuser detection and interference cancellation, since a receiver can adaptively learn (estimate) the structure of the MAI [6]. Decentralized receivers may be further classified into data aided and nondata aided receivers. Data aided adaptive multiuser detection is an approach which does not require a prior knowledge of the interference parameters. But, it requires a training data sequence for every active user. For example, adaptive receivers in [3, 7, 8] are based on the MMSE criterion, and the one in [9] is based on minimizing probability of bit-error. More recently, decision feedback detectors using the MMSE criterion have been proposed [10, 11].

Blind (or nondata aided) multiuser detectors require no training data sequence, but only knowledge of the desired user signature sequence and its timing. The receivers treat MAI and background noise as a random process, whose statistics must be estimated. Majority of blind multiuser detectors are based on estimation of second order statistics of the received signal. In [12], a blind adaptive MMSE multiuser detector is introduced (proven to be equivalent to the minimum output energy (MOE) detector). A subspace approach for blind multiuser detection is presented in [13]; where both the decorrelating and the MMSE detector are obtained blindly. Further, adaptive and blind solutions are analyzed in [14], with an overview in [6]. A comprehensive treatment of multiuser detection can be found in [15].

The receiver in this work is based on determining the most (on average) dominant baseband interference components at the output of DS-CDMA system. Preliminary results on this idea was first presented in [16]. In Chapter 3, the maximum mean energy (MME) criterion is introduced. It is shown that the MME criterion is strongly related to the Karhunen-Loéve (KL) expansion of the received signal and to the eigendecomposition (ED) of the covariance matrix. In Chapter 4 we present a novel blind receiver. It is based on the MME criterion and requires estimation of the second order statistics. The receiver executes interference cancellation (IC) in a successive manner; starting with most dominant interference component and successively cancelling weaker ones. Therefore it may be viewed as a blind equivalent to the centralized successive interference cancellation (SIC) scheme [4, 17] and we refer to this receiver as the blind interference cancellation-maximum mean energy (BIC-MME) receiver. In order to reduce the complexity of receiver implementations, we also propose an iterative solution for the MME optimization. Simulation results are presented in Chapter 5.

Time-varying systems are of special interest in future DS-CDMA systems. These variations could be due to either the radio channel or due to the variations in traffic such as anticipated in packet networks [18, 19, 20, 21] that may result in high user activity (on/off) and short transmission periods (burstiness) in the channel. Feasibility of adaptive and blind interference cancellation in these systems is directly impacted by the reliability of required estimates, using limited number of samples. Therefore, in this work, special attention is paid to the analysis of the receiver performance in the case of limited number of samples used for the estimation of the covariance matrix of the input signal. Our simulation results indicate that the BIC-MME receiver outperforms the blind MMSE receiver in all cases, and particularly when the number of samples used for estimation of the covariance matrix is limited. In Chapter 6 we present an interpretation for the above behavior using results from estimation of eigenvectors based on sample covariance matrices. We conclude in Chapter 7.

2. Background

We now present the asynchronous DS-CDMA system model and briefly review the MMSE criterion. The received baseband signal, $r(t)$, in antipodal K-user asynchronous DS-CDMA additive white Gaussian noise (AWGN) system is

$$r(t) = \sum_{i=-J}^{J} \sum_{k=1}^{K} A_k \, b_k \,(i) \, s_k(t - iT - \tau_k) \; + \; \sigma n(t) \tag{2.1}$$

where A_k is the received amplitude, $b_k(i) \in \{-1, +1\}$ is binary, independent and equiprobable data, $s_k(t)$ is the signature sequence which is assumed to have unit energy, τ_k is relative time offset, all for the k^{th} user. T is the symbol period and $n(t)$ is AWGN with unit power spectral density. $2J + 1$ is the number of data symbols per user per frame.

It is well known that asynchronous system with independent users can be analyzed as synchronous if equivalent synchronous users are introduced, which are effectively additional interferers [15]. Sufficient statistics are obtained by sampling at $2f_0$, where f_0 is the maximum bandwidth of the chip waveforms in the desired user signature sequence [15, 12]. In this work we consider the received signal $r(t)$ over only one symbol period that is synchronous to the desired user ($k = 1$). The discrete representation for the received signal in (2.1) can be written in vector form as

$$\mathbf{r} = \sum_{k=1}^{L} A_k \, b_k \, \mathbf{s}_k \; + \; \sigma \, \mathbf{n} \tag{2.2}$$

where the number of the interferers ($L - 1 = 2\,(K - 1)$) is doubled due to equivalent synchronous user analysis. $\mathbf{r}$, $\mathbf{s}_k$ and $\mathbf{n}$ are vectors in $\Re^M$, where M is the number of chips per bit.

For the sake of a completeness, the well known MMSE optimization criterion is briefly repeated here (proven to be equivalent to the MOE criterion [12]). For a vector $\mathbf{d} \in \Re^M$, the mean squared error is $MSE = E\left[(\mathbf{r}^\top \mathbf{d} - b_1)^2\right]$. The linear MMSE detector $\mathbf{c}$ is obtained as

$$\mathbf{c} = \arg\min_{\mathbf{d}} \left(E\left[(\mathbf{r}^\top \mathbf{d} - b_1)^2\right] - \gamma \,(\mathbf{s}_1^\top \mathbf{d} - 1) \right) \tag{2.3}$$

where the vector $\mathbf{d}$ is constrained to be

$$\mathbf{s}_1^\top \mathbf{d} = 1 \tag{2.4}$$

The solution of (2.3) is given as (for user 1) $\mathbf{c} = \mathbf{R}_r^{-1} \mathbf{s_1}$, where $\mathbf{R}_r = E[\mathbf{r}\,\mathbf{r}^\top]$ is the covariance matrix of the input process $\mathbf{r}$ [13]. The matrix $\mathbf{R}_r$ has to be invertible. If an estimate of the covariance matrix $\mathbf{R}_r$ i.e., sample covariance matrix $\hat{\mathbf{R}}_r$, is available, approximation of the optimal MMSE detector is

$$\hat{\mathbf{c}} = \hat{\mathbf{R}}_r^{-1}\mathbf{s_1} \tag{2.5}$$

which is denoted as blind MMSE (BMMSE) receiver. In this work, the above receiver is used as a reference for performance evaluations.

3. MME Optimization Criterion

Let us define for a M-dimensional vector $\mathbf{u}$, the mean energy (ME) as

$$\text{ME} = E\left[(\mathbf{r}^\top \mathbf{u})^2\right] \tag{3.1}$$

Let us further constrain the vector $\mathbf{u}$ such that

$$\mathbf{u}^\top \mathbf{u} = 1 \tag{3.2}$$

We now consider maximization of the ME, with respect to the vector $\mathbf{u}$. The problem can be solved by the method of Lagrange multipliers [22]. Let

$$\psi(\mathbf{u}) = E\left[(\mathbf{r}^\top \mathbf{u})^2\right] - \gamma\left(\mathbf{u}^\top \mathbf{u} - 1\right) \tag{3.3}$$

Necessary condition for $\mathbf{v} \in \Re^M$ to maximize (3.3) is $\nabla(\psi(\mathbf{v})) = 0$, which results in

$$\mathbf{R}_r\,\mathbf{v} = \gamma\,\mathbf{v} \tag{3.4}$$

It is obvious from (3.4) that $\mathbf{v}$ and γ are an eigenvector and an eigenvalue of the matrix $\mathbf{R}_r$, respectively. In general, there is a set of eigenvectors and eigenvalues, which are related as

$$\mathbf{R}_r\,\mathbf{V} = \mathbf{V}\,\mathbf{D} \tag{3.5}$$

where $\mathbf{V}$ is a matrix whose columns are the eigenvectors $(\mathbf{v}_1, \cdots, \mathbf{v}_M)$, and $\mathbf{D}$ is diagonal matrix of the corresponding eigenvalues $(\lambda_1, \cdots, \lambda_M)$.

The constraint (3.2) only sets the vector $\mathbf{v}$ to have unit energy and it is different from that in (2.4) which defines energy of the vector $\mathbf{c}$ with respect to the desired user signature sequence $\mathbf{s}_1$. We may note that the MME criterion is more related to signal space, as a whole, unlike the MMSE criterion that is focused on the specific signal component $(\mathbf{s}_1)$.

To gain more insight into the MME criterion that results in (3.4) and (3.5), let us consider the discrete form of the KL expansion of the received vector $\mathbf{r}$ [23]. This expansion allows the M-dimensional stochastic process $\mathbf{r}$ to be represented as a superposition of vectors $\mathbf{x}_i$ from orthonormal basis, scaled by statistically uncorrelated random variables a_i, $(i = 1, \cdots, N)$ as

$$\mathbf{r} = \sum_{i=1}^{N} a_i\,\mathbf{x}_i \tag{3.6}$$

The vectors $\mathbf{x}_i$ are orthonormal ($\mathbf{x}_i^\top \mathbf{x}_j = \delta_{ij}$, where δ_{ij} is the Kronecker delta function) and the random variables a_i are defined as $a_i = \mathbf{r}^\top \mathbf{x}_i$. The random variables a_i are uncorrelated and with expected energy λ_i ($E[a_i\, a_j] = \lambda_i\, \delta_{ij}$). The above condition results in

$$E\left[\mathbf{r}\,\mathbf{r}^\top\right]\mathbf{x}_i = \lambda_i\,\mathbf{x}_i,\; i = 1, \cdots, N \tag{3.7}$$

It is obvious that the equations (3.4) and (3.7) are identical. Therefore, the vectors $\mathbf{x}_i$ are the column vectors (eigenvectors) of the matrix $\mathbf{V}$ and λ_i are the diagonal elements (eigenvalues) of the matrix $\mathbf{D}$. In the following, $\mathbf{x}_i$ and $\mathbf{v}_i$ ($i = 1, \cdots, N$) are used interchangeably, and if $\mathbf{R}_r$ is invertible, then $N = M$ [24]. This analogy allows us to make following conclusions: If the matrix $\mathbf{V}$ and $\mathbf{D}$ are obtained from (3.5), the column vectors in $\mathbf{V}$ are orthonormal basis which span the received signal space in the mean squared sense [23]. The diagonal elements of the matrix $\mathbf{D}$ are the mean energies of the received vector $\mathbf{r}$ along the orthonormal vectors from the basis. Thus, instead of analyzing the actual set of users (vectors) in the received vector $\mathbf{r}$ (as is done in the case of centralized receivers), we are evaluating the corresponding vector space which is characterized by the orthogonal basis and uncorrelated coefficients. From the above we conclude:

Proposition 1 *The eigenvector of $\mathbf{R}_r$ that corresponds to the maximum eigenvalue (λ_{max}) is the vector that maximizes the* ME *(mean energy) in (3.1).*

Proposition 1 follows from the analogy between the MME criterion and the interpretation based on the KL expansion. Let us denote the eigenvector from Proposition 1 as $\mathbf{v}_{max}$ (the maximizer of ME). In addition, we claim:

Proposition 2 *If the contribution of $\mathbf{v}_{max}$ is removed from the matrix $\mathbf{R_r}$, as follows: $\mathbf{R}'_r = \mathbf{R}_r - \lambda_{max}\,\mathbf{v}_{max}\,\mathbf{v}_{max}^\top$, then the eigenvector $\mathbf{v}'_{max}$ that corresponds to the maximum eigenvalue of $\mathbf{R}'_r$ is the same as the eigenvector that corresponds to the second largest eigenvalue of $\mathbf{R}_r$. In addition, the matrix $\mathbf{R}'_r$ is covariance matrix of the vector $\mathbf{r}' = \mathbf{r} - (\mathbf{r}^\top \mathbf{v}_{max})\,\mathbf{v}_{max}$.*

Proposition 2 is a consequence of the spectral theorem [24] and we present a proof in Appendix A. The results in Propositions 1 and 2 form the basis for the blind interference cancellation scheme presented in this work. We now sketch an outline of how the above two results can be exploited to derive a blind successive interference cancellation scheme. Note that the contribution of the desired user can be removed from the covariance matrix $\mathbf{R}_r$ as follows:

$$\mathbf{R}_i = \mathbf{R}_r - A_1^2\,\mathbf{s}_1\,\mathbf{s}_1^\top \tag{3.8}$$

where $\mathbf{R}_i = E[\mathbf{i}\,\mathbf{i}^\top]$ is the interference covariance matrix, with $\mathbf{i} = \sum_{k=2}^{L} A_k\, b_k\, \mathbf{s}_k + \sigma\, \mathbf{n}$. Observe that in the above procedure no knowledge is required of the desired user's bit decision (information). Only the knowledge of the desired signal power A_1^2 is needed. Further, if the MME criterion is now applied on $\mathbf{R}_i$ (i.e., we determine the eigenvector corresponding to the maximum eigenvalue of $\mathbf{R}_i$), then we can capture the most dominant interference (energy) component. The above process can be successively repeated and would result (due to Proposition 2) in successive cancellation of components in the interference subspace, starting from the strongest to the weakest.

4. An Application of the MME Criterion in the Blind IC Receiver

We now present a blind successive interference cancellation scheme where we incorporate the MME criterion and realize the blind interference cancellation-maximum mean energy (BIC-MME) receiver. As depicted in Figures 4.1 and 4.2, the receiver executes the following steps (blocks in Figure 4.1):

1. Estimation of the matrix $\mathbf{R}_r$ according to

$$\hat{\mathbf{R}}_r(i) = \frac{1}{n} \sum_{k=i-n+1}^{i} \mathbf{r}(k)\,\mathbf{r}^\top(k) \tag{4.1}$$

   where $\hat{\mathbf{R}}_r$ is the sample covariance matrix, n is the size of the averaging window (number of samples), and i is time index (will be omitted in the following text). [1]

2. Remove the desired user contribution from $\hat{\mathbf{R}}_r$. If the desired user amplitude (A_1) is known or estimated we can apply (3.8). The result of this step is that $\hat{\mathbf{R}}_i$ contains only the interference components and there is no desired user contribution ($A_1^2\,\mathbf{s}_1\,\mathbf{s}_1^\top$).

   Note that the amplitude A_1 may not be known at the receiver. Therefore in our simulation results (in Chapter 5), we considered the amplitude estimate $\hat{A}_1$ using the outputs of the MF for user 1. Our results indicate that the performance is not very sensitive to the errors in amplitude estimation.

3. Find the maximizer ($\hat{\mathbf{v}}_{max}$) of the ME, i.e., the vector that takes, on average, most of the interference energy. According to Proposition 1, the maximizer is the eigenvector that corresponds to the maximum eigenvalue ($\hat{\lambda}_{max}$) of the matrix $\hat{\mathbf{R}}_i$.

   To find the maximizer it is not necessary to perform the ED in full. An iterative solution can be applied. As an example, we use the power method (PM) [24] to derive an iterative solution for the MME criterion. Starting with an initial guess $\hat{\mathbf{v}}_{max}^0$ which contains some component of $\hat{\mathbf{v}}_{max}$

$$\hat{\mathbf{v}}_{max}^{i+1} = \frac{\hat{\mathbf{R}}_i \hat{\mathbf{v}}_{max}^i}{\left|\hat{\mathbf{R}}_i \hat{\mathbf{v}}_{max}^i\right|} \tag{4.2}$$

[1] Notation: $\hat{z}$ denotes an estimate of z.

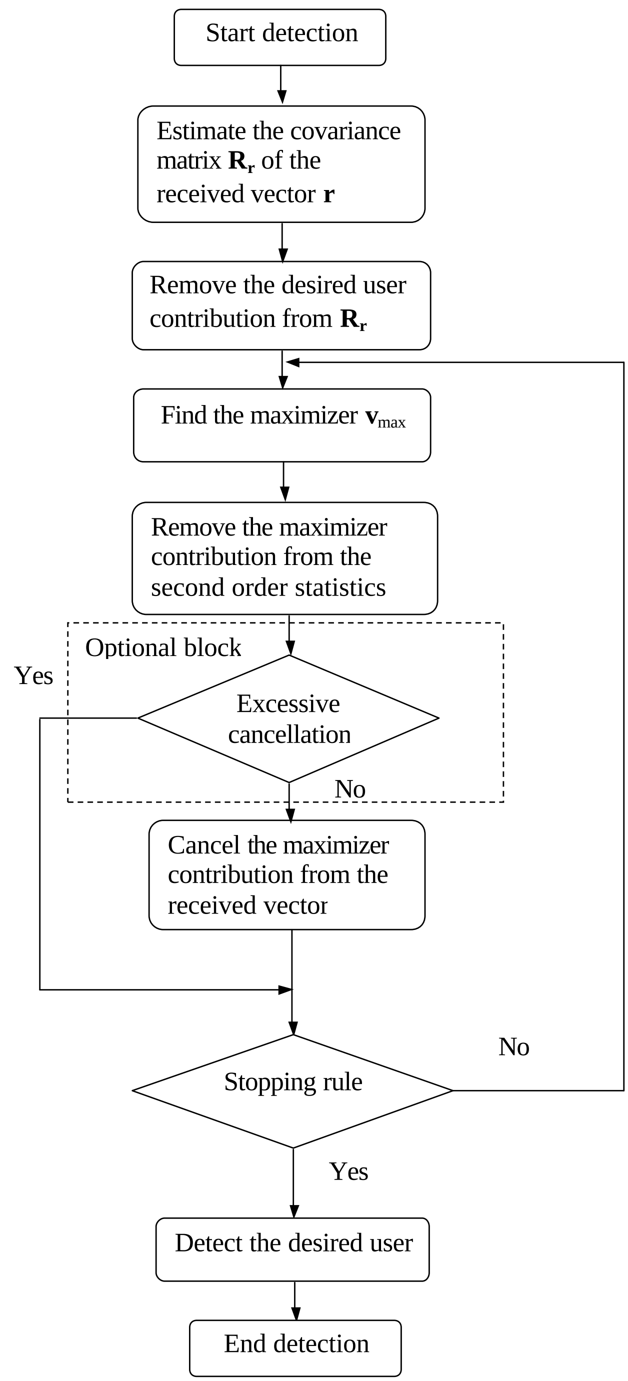


Figure 4.1: Flow chart illustrating the BIC-MME scheme.

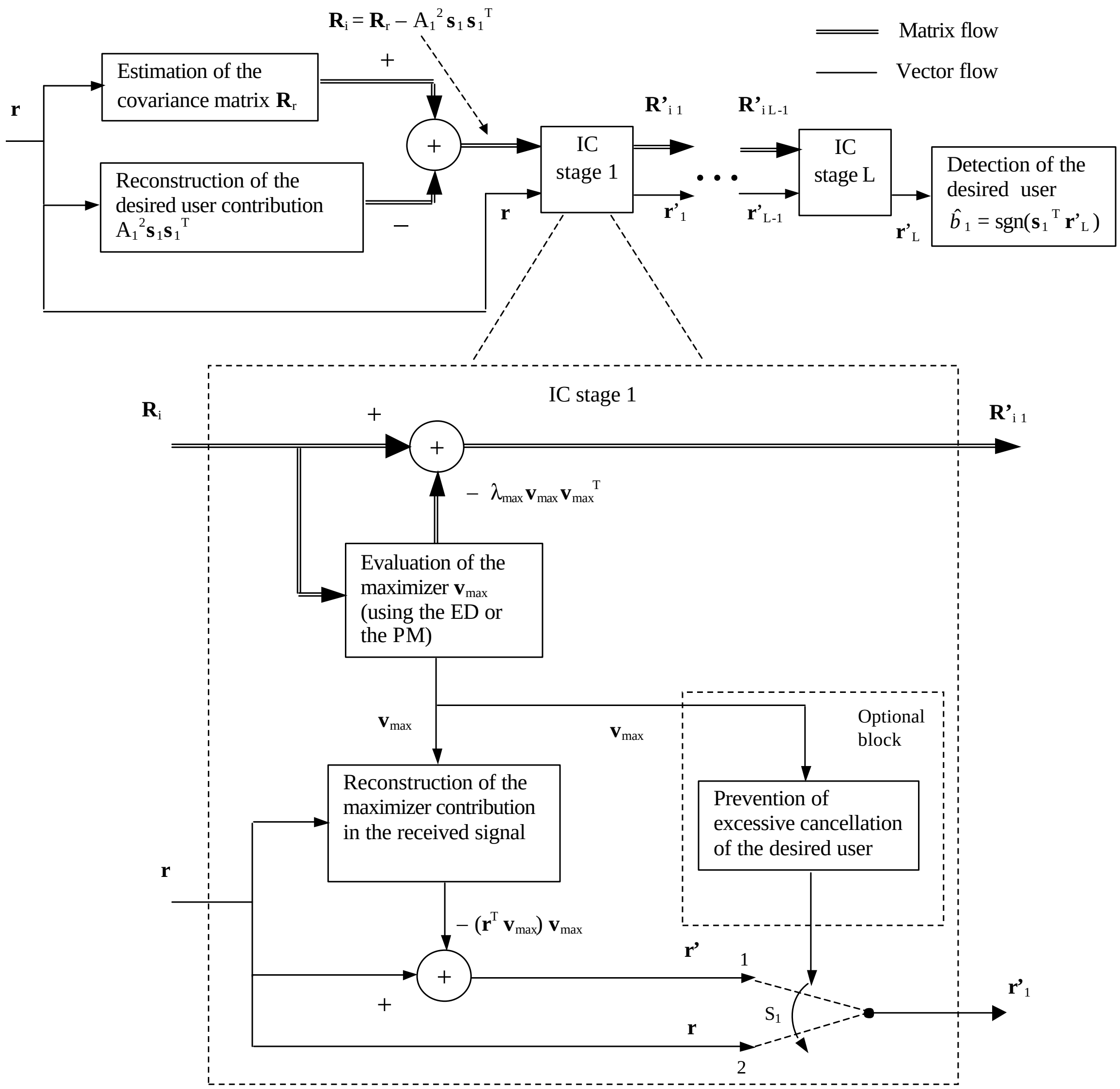


Figure 4.2: Block scheme of the BIC-MME receiver.

where i is iteration step. While other advanced iterative and subspace tracking algorithms are well known [25, 26], this topic is not further analyzed in this work. In the case of the simulations in Chapter 5, the PM is observed to perform well. In Chapter 5, as a stopping rule we have used

$$\left(\hat{\mathbf{v}}_{max}^{i+1}\right)^{\top}\hat{\mathbf{v}}_{max}^{i} \geq T_{PM} \tag{4.3}$$

where T_{PM} is some threshold value.

4. Remove the maximizer contribution from the matrix $\hat{\mathbf{R}}_i$ to yield

$$\hat{\mathbf{R}}_i' = \hat{\mathbf{R}}_i - \hat{\lambda}_{max}\hat{\mathbf{v}}_{max}\hat{\mathbf{v}}_{max}^{\top} \tag{4.4}$$

According to Proposition 2, this step prepares the estimate of the second order statistics ($\hat{\mathbf{R}}_i'$) for evaluation of the maximizer in the next IC stage.

5. To prevent excessive cancellation of the desired user from the input vector $\mathbf{r}$, we introduce an optional block. This block is useful in the case when the crosscorrelation between the desired user signature sequence and the interferer signature sequences is very high. For example, a simple threshold criterion could be applied to determine if cancellation is viable. If

$$\left|(\mathbf{s}_1^{\top}\hat{\mathbf{v}}_{max})\right| > T_C \tag{4.5}$$

where T_C is some threshold value, then step (6) below is skipped, i.e., the IC is not performed (in Figure 4.2, the switch S_1 is in the position 2). If this block is not applied, the switch S_1 is always in the position 1.

6. Cancel the maximizer contribution as

$$\mathbf{r}' = \mathbf{r} - (\mathbf{r}^{\top}\hat{\mathbf{v}}_{max})\,\hat{\mathbf{v}}_{max} \tag{4.6}$$

7. A variety of stopping rules can be defined for the whole procedure. If all significant components of the interference (defined by a specific rule) are cancelled, the detection $\hat{b}_1 = sgn(\mathbf{r}'^{\top}\mathbf{s}_1)$ is performed, otherwise the steps (3) - (7) are repeated, where, for the new IC stage $\mathbf{r}$ and $\mathbf{R}_i$ take the values of $\mathbf{r}'$ and $\mathbf{R}_i'$, respectively. For example, the last IC stage can be the one where the measured (estimated) signal to interference ratio (SIR) is maximum or above a target value. This step can be used to control the trade-off between performance and complexity of the receiver.

5. Simulation Results

We consider a synchronous AWGN DS-CDMA system, using randomly generated signature sequences with processing gain $M = 64$. The users are independent and three cases are analyzed:

1. System with $L = 16$ users, and equal-energy interferers: $A_i^2/A_1^2 = 25$, $i = 2, \cdots, 16$.

2. Lightly loaded system with $L = 4$ users, and very strong equal-energy interferers: $A_i^2/A_1^2 = 400$, $i = 2, \cdots, 4$.

3. System with $L = 16$ users; three strong equal-energy interferers: $A_i^2/A_1^2 = 25$, $i = 2, \cdots, 4$, and twelve interferers with the same energy as the desired user: $A_i^2/A_1^2 = 1$, $i = 5, \cdots, 16$. This scenario may correspond to a system with different transmission powers that accommodate different quality of service (QoS).

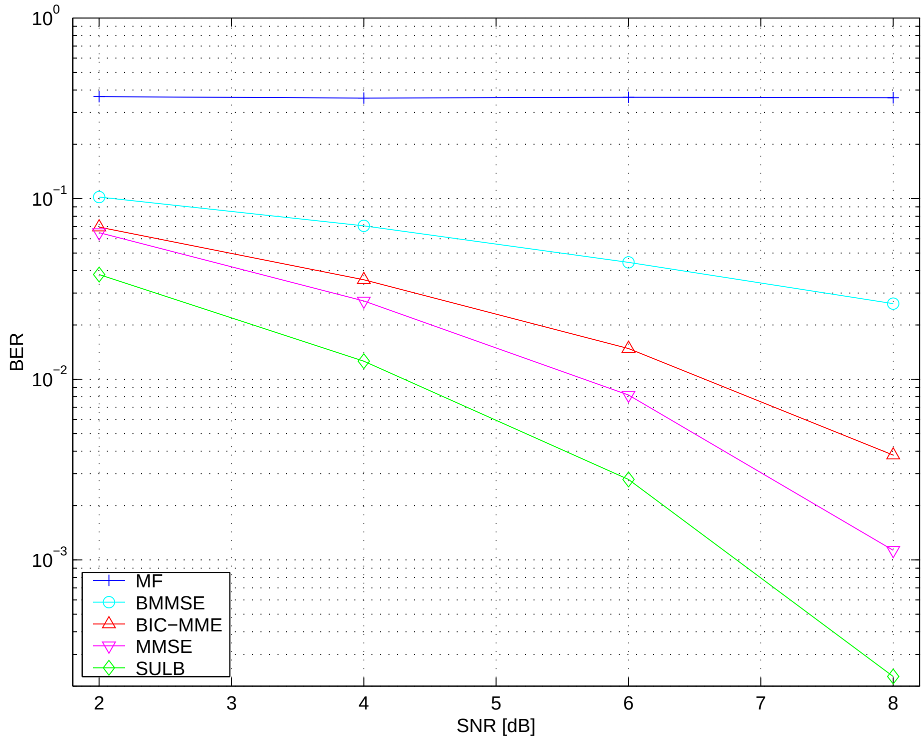
Performance of the conventional matched filter (MF), the centralized MMSE receiver, the BMMSE receiver (detector in (2.5)) and the single user lower bound (SULB) are used as benchmarks for evaluation of the BIC-MME receiver. The centralized MMSE assumes perfect knowledge of all the signature sequences, amplitudes and the variance of the AWGN. The BMMSE and the BIC-MME receiver use the same sample covariance matrix $\hat{\mathbf{R}}_r$. The matrix is estimated according to (4.1). The BIC-MME performs the ED, and the iterative solution (using the PM) is denoted as BIC-MME-PM. Unless stated otherwise, we assume that the amplitude of the desired user is known exactly.

For the case 1, Figure 5.1(a) depicts bit-error rate (BER) as a function of signal to noise ratio (SNR) (with respect to the desired user). The results are obtained after a total of 15 IC stages, which is where the BER reaches minimum. Additional IC stages result in a deterioration of the performance for this particular example. For $SNR = 8\,dB$, BER versus number of IC stages is presented in Figure 5.1(b). Equivalent results, for the case 2, with a total of 3 IC stages and $SNR = 6\,dB$ are shown in the figures 5.2(a) and 5.2(b), respectively. These results are evaluated for the window size $n = 500$ (the number of the samples used in (4.1)). Note that the performance of the BIC-MME is near-optimum in the case 2. In this lightly loaded system, even in the presence of very strong interferers, a small number of IC stages (3 stages) is sufficient to fully cancel the interference with negligible negative effect on the desired user (just a small fraction of the desired user energy is removed by the IC).

We now study the effect of accuracy of the covariance matrix estimation on the performance of the BIC-MME receiver. Figures 5.3(a) and 5.3(b), correspond to the case 1 (for $SNR = 8dB$) and the case 2 (for $SNR = 6\,dB$), respectively. The above figures depict BER with respect to different window size n. According to the results above, the BIC-MME receiver outperforms the BMMSE receiver. The gain introduced by the BIC-MME, with respect to the BMMSE, increases as the averaging window gets smaller.

Considering the iterative solution (BIC-MME-PM), Figure 5.3(b) shows that the PM successfully replaces the ED (difference in performance between the two schemes is negligible). The case 1 is more affected by the application of the PM (Figure 5.3(a)). This is expected because the number of the IC stages is five times greater than in the case 2 and the computational error accumulated is greater. For the above example, the threshold in (4.3) is $T_{PM} = 0.999$. Regarding the convergence of the PM method, we have observed that the number of iterations, before the criterion in (4.3) is met, has never exceeded 25, and most of the vectors (maximizers) required less than 10 iterations. Further, in our results, no effort has been made to improve the initial guess $\hat{\mathbf{v}}^0_{max}$. It is selected randomly. Therefore, the convergence could be further accelerated if $\hat{\mathbf{v}}^0_{max}$ is improved (see [24]).

We consider the performance of our receiver in the case 3. Figure 5.4 depicts BER with respect to number of IC stages, $SNR = 8\,dB$ and $n = 500$. The same figure presents the performance of the match filter (MF-12) for the system without the strong interferers (only the desired user and the twelve equal-energy interferers, with perfect power control with respect to the desired user). Note, that after 3 IC stages, the BIC-MME completely cancels the strong users (it reaches the MF-12 performance). For this particular case, the minimum BER is reached after 15 IC stages; but, for the sake of lower complexity of the receiver (i.e., smaller number of IC sages), interference cancellation can be stopped in some earlier IC stage at the expense of lower performance (higher BER). Furthermore, the performance of the iterative solution (BIC-MME-PM) seems to follow that of the BIC-MME receiver. This suggests that the low complexity iterative solution cancels the strongest interferers completely, and brings the system into the well studied perfectly power controlled state. This scheme may be applied for the interference cancellation of strong users in a system with different transmission powers that accommodate different QoS, or in a system with a few very strong interferers as may be the case for the downlink where intracell user transmissions are orthogonal.



(a) BER vs. SNR, Case 1, $n = 500$.

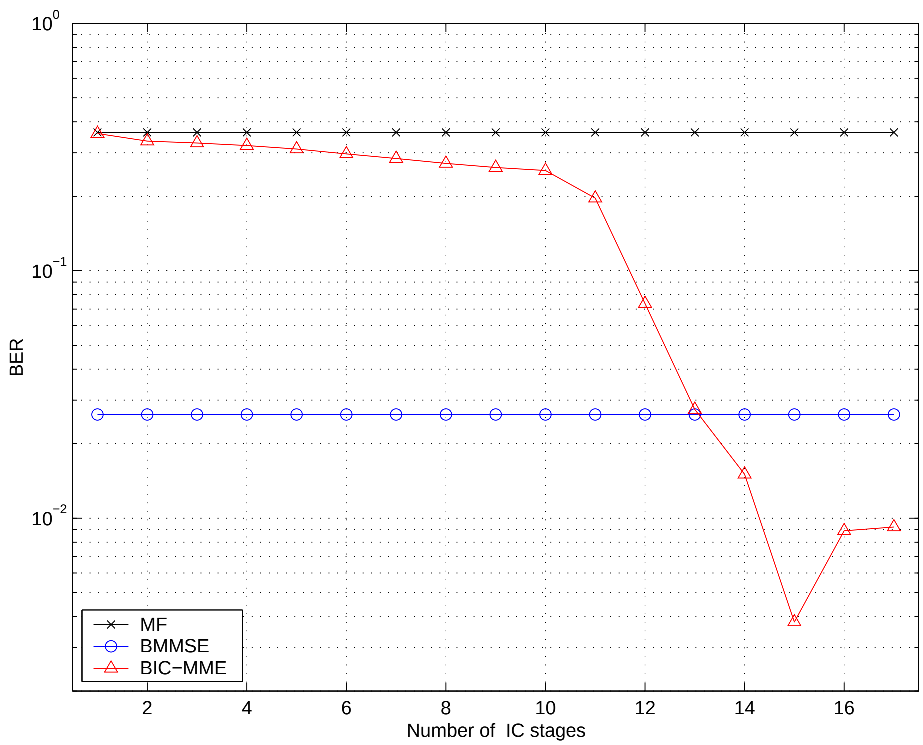
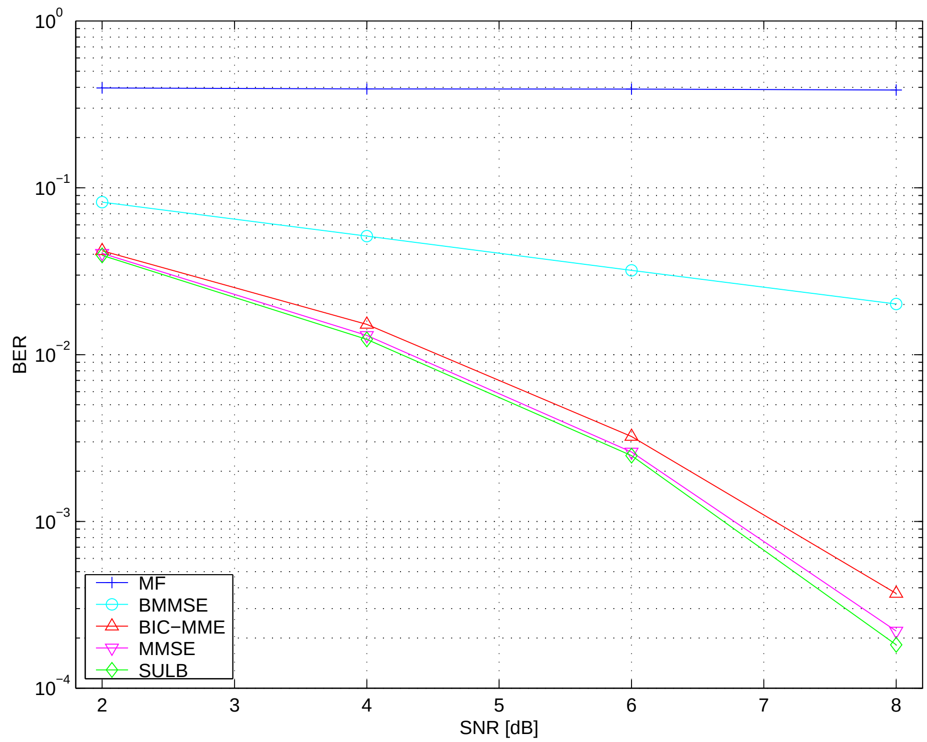


Figure 5.1: (b) BER vs. Number of IC stages, Case 1, $SNR = 8\,dB$, $n = 500$.



(a) BER vs. SNR, Case 2, $n = 500$.

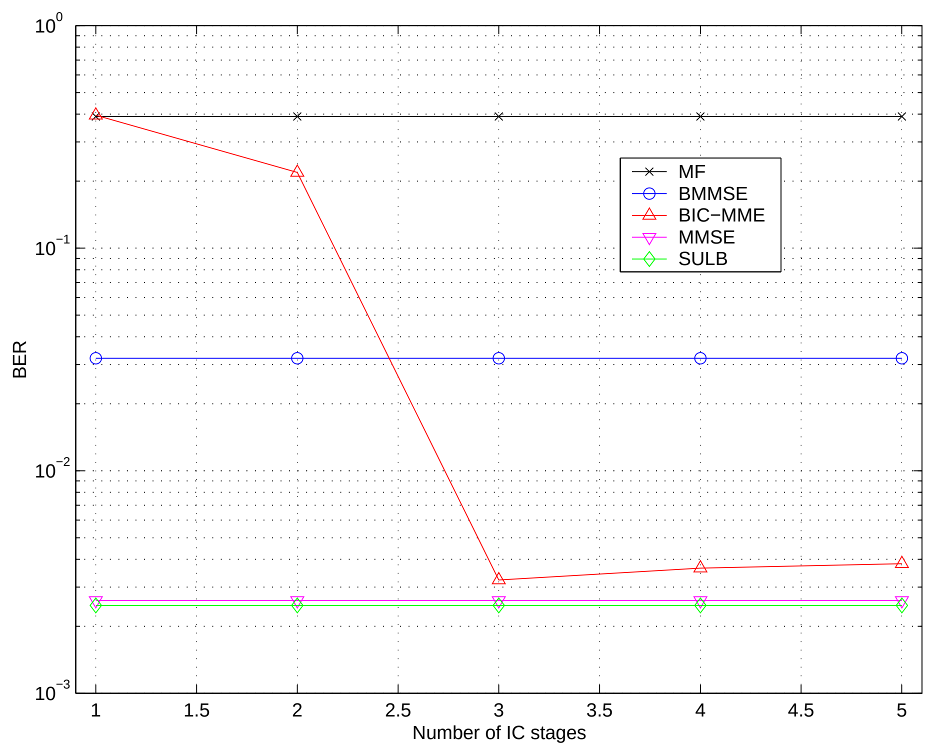
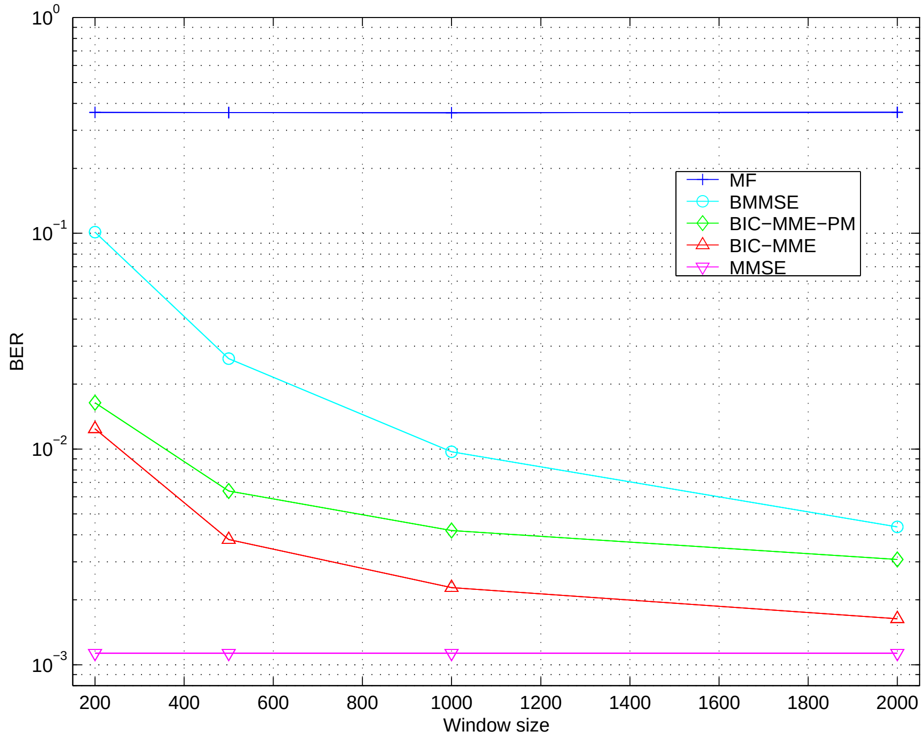


Figure 5.2: (b) BER vs. Number of IC stages, Case 2, $SNR = 6\,dB$, $n = 500$.



(a) BER vs. Window size, Case 1, $SNR = 8\,dB$.

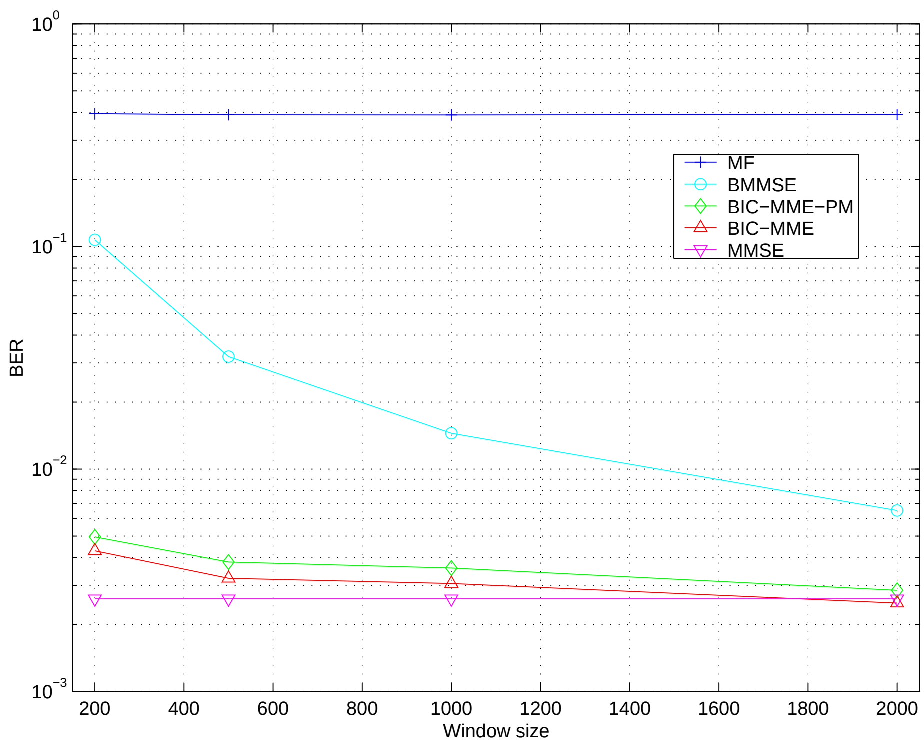


Figure 5.3: (b) BER vs. Window size, Case 2, $SNR = 6\,dB$.

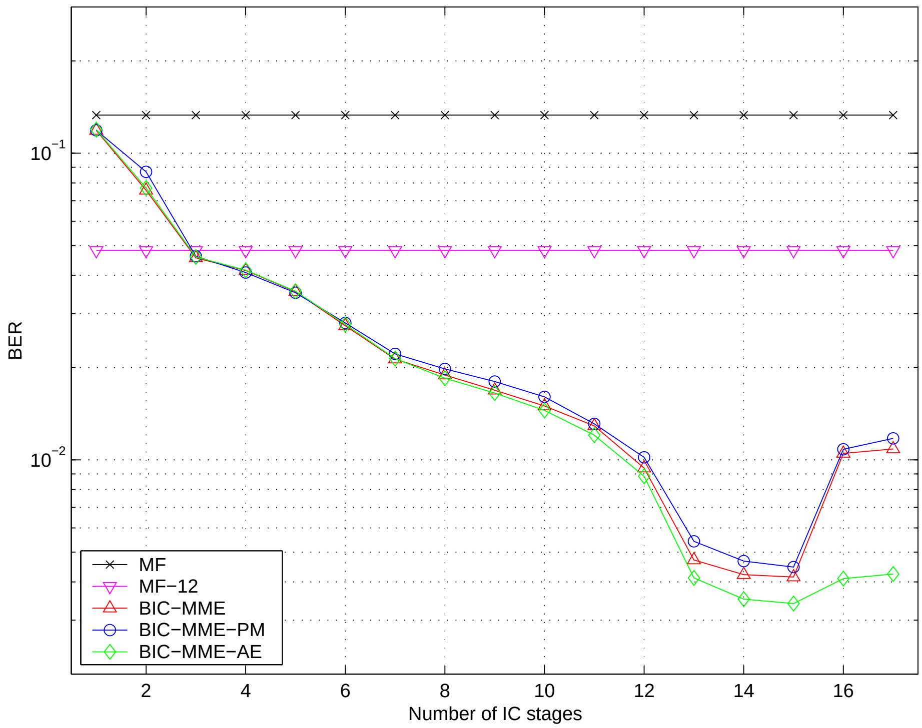


Figure 5.4: BER vs. Number of IC stages, Case 3, $SNR = 8\,dB$, $n = 500$.

We also study performance of the receiver if an estimate of the desired user amplitude is used instead of the perfectly known amplitude. We have tested the performance of the scheme that applies a simple estimation of the amplitude as $\hat{A}_1 = \frac{1}{n} \sum_{i=1}^{n} |\mathbf{r}^T(i)\,\mathbf{s}_1|$. The results, for the case 3, are presented in Figure 5.4, as the BIC-MME-AE. We have not recognized substantial difference in the performance with respect to the solution for the case when the amplitude is known exactly.

6. Interpretation of the BIC-MME Performance

We now present an interpretation of why the BIC-MME performs well in the case of iterative implementation using the PM and in the presence of estimation errors of the covariance matrix. Let us first emphasize the following properties of the BIC-MME scheme:

Property 1 *For the matrix* $\mathbf{R_i}$*, assume that there is a set* $\mathbf{S}$ *of eigenvectors* $\mathbf{v}_i \in \mathbf{S}$ *that correspond to dominant eigenvalues with small absolute difference between them, i.e., the interference has almost the same energy in all directions of the subspace spanned by the true eigenvectors from* $\mathbf{S}$*. The* BIC-MME *scheme tends to cancel the whole subspace, rather than to cancel just a specific eigenvector in* $\mathbf{S}$*. Therefore, there is no need for the vectors to be estimated with high degree of accuracy. It is sufficient that the estimated vectors (*$\hat{\mathbf{v}}_i$*) are orthogonal and fall well into the subspace spanned by the true eigenvectors (*$\mathbf{v}_i \in \mathbf{S}$*) (i.e.,* $\hat{\mathbf{v}}_i$ *has most of its energy confined to the subspace formed by the true eigenvectors from* $\mathbf{S}$*.)*

The following property is the consequence of Proposition 2.

Property 2 *Assuming that the* BIC-MME *scheme executes a total of* Q *IC stages, the total contribution of* Q *eigenvectors of the matrix* $\mathbf{R_i}$ *is cancelled from the input vector* $\mathbf{r}$*. The order in which the vectors are processed (i.e., their contribution is cancelled) does not affect the performance of the scheme (assuming perfect estimation of the vectors).*

6.1 Iterative Implementation Using the PM

We now discuss the performance of the BIC-MME scheme that uses the PM iterative implementation. An attractive feature of such an implementation is that it avoids the need for a full-scale ED of the covariance matrix at every IC stage (see step 3 in Chapter 4). The PM algorithm exhibits very good convergence properties if the absolute difference between dominant eigenvalues are significant [24]. If $\mathbf{v}_{max}$ is one of the vectors that are described in Property 1 ($\mathbf{v}_{max} \in \mathbf{S}$), $\hat{\mathbf{v}}^i_{max}$ (see equation (4.2)) converges very quickly with respect to the vectors that are out of the set, i.e., the estimate lies in the subspace spanned by the set $\mathbf{S}$ after a very small number of iterations. However, once inside the subspace, the convergence is significantly slowed down.

According to Property 1, this drawback should not affect the performance of the BIC-MME scheme significantly. This is the reason why the BIC-MME with the iterative PM works well even after a small number of iterations, which is also confirmed by the simulation results in Chapter 5. Furthermore, Property 2 shows that the BIC-MME scheme with the iterative PM solution works well even when the descending order for evaluated eigenvalues is not guaranteed. In other words, if there are a few strong interference vectors in the interference subspace, then the order in which the interference vectors (maximizers) are cancelled does not affect the performance. These are favorable characteristics for low complexity iterative solutions.

6.2 Estimation of Eigenvectors

We now analyze the properties of eigenvector estimates of the sample covariance matrix in (4.1). This analysis is used to explain why the BIC-MME scheme performs well in the presence of eigenvector estimation errors that result when the sample covariance matrix is estimated using small sample sizes. In the following, we will assume that the sample covariance matrix is that of observation vectors that are multivariate Gaussian. Even though this assumption is not true in the presence of MAI, we will justify the following analysis through a numerical validation. Let the subspace spanned by the eigenvectors that correspond to distinct and significant eigenvalues $(\lambda_1, \cdots, \lambda_P)$ be denoted as the signal subspace. The dimension of the signal subspace is P. The noise subspace is spanned by the eigenvectors that correspond to repeated eigenvalues $(\lambda = \lambda_{P+1}, \cdots, \lambda_M)$ with multiplicity $M-P$ [13, 27]. To analyze estimation of the eigenvectors, let us observe the crosscorrelation (projection) $\hat{\mathbf{v}}_i^\top \mathbf{v}_j$ between the estimate of i^{th} eigenvector $(\hat{\mathbf{v}}_i,\ i = 1, \cdots, P)$ and the j^{th} true eigenvector $(\mathbf{v}_j,\ j = 1, \cdots, M)$. These values, $\hat{\mathbf{v}}_i^\top \mathbf{v}_j$, can be used to characterize errors of the eigenvector estimates. When $i \neq j$, it can be shown that $\hat{\mathbf{v}}_i^\top \mathbf{v}_j$ is unbiased, i.e., $E\left[\hat{\mathbf{v}}_i^\top \mathbf{v}_j\right] = 0$. Further, the variance can be approximated as (see Appendix B)

$$E\left[(\hat{\mathbf{v}}_i^\top \mathbf{v}_j)^2\right] \approx \frac{1}{n} \frac{\lambda_i \lambda_j}{(\lambda_i - \lambda_j)^2} \tag{6.1}$$

Note that the second order moment $E\left[(\hat{\mathbf{v}}_i^\top \mathbf{v}_j)^2\right]$ is the mean energy of the eigenvector estimate $\hat{\mathbf{v}}_i$ in the direction of the true eigenvector $\mathbf{v}_j$. We have constrained the eigenvector estimates to be unit energy, i.e., $E\left[(\hat{\mathbf{v}}_i^\top \hat{\mathbf{v}}_i)^2\right] = 1$. From the above an observation follows:

Observation 1 *The estimate $\hat{\mathbf{v}}_i$ of the eigenvector $\mathbf{v}_i$ tends to be clustered (i.e., $\hat{\mathbf{v}}_i$ has most of its energy) within the subspace spanned by the true eigenvector $\mathbf{v}_i$ and eigenvectors that correspond to eigenvalues that are very close to λ_i. Inspection of (6.1) reveals that the above statement is true even when the number of samples is small.*

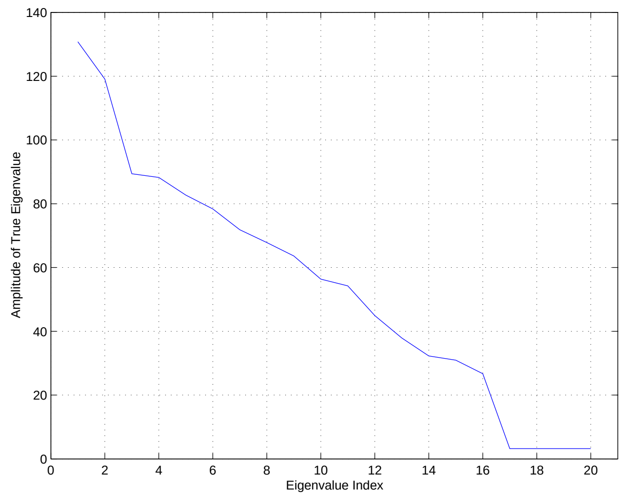
To justify Observation 1 (that is based on analysis using Gaussian statistics), and to validate its claims for a DS-CDMA system with MAI, the following simulation example is presented:

Example 1: We consider a synchronous AWGN DS-CDMA system, using randomly generated signature sequences with $L = 16$ users and processing gain $M = 64$. The signature sequences have happened to be linearly independent in this example; therefore the dimension of the signal subspace is $P = 16$. The users are independent and have the same $SNR = 10\,dB$. The sample covariance matrix is evaluated according to (4.1). The ED is performed for 1000 different sample covariance matrices, and the results are averaged. Because of the identical behavior of the values which correspond to the repeated eigenvalues ($\lambda_i,\ i = 17, \cdots, 64$), the results for $\lambda_i,\ i = 21, \cdots, 64$ are omitted from the following figures.

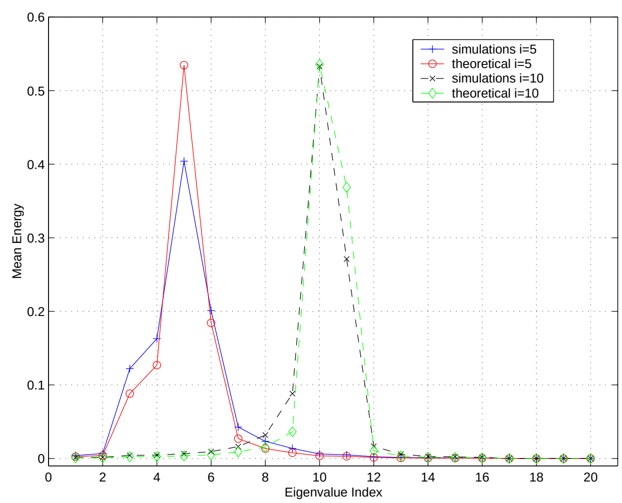
In Figure 6.1(a), a true eigenvalue profile ($\lambda_i,\ i = 1, \cdots, 20$) is presented where the eigenvalues are sorted in descending order. In Figure 6.1(b), we present analytical (see (6.1)) and simulation results for $E\left[(\hat{\mathbf{v}}_i^\top \mathbf{v}_j)^2\right]$ for $i, j = 1, \cdots, M$ (which is the normalized mean energy of the eigenvector estimate $\hat{\mathbf{v}}_i$ in the direction of the true eigenvector $\mathbf{v}_j$). Specifically, we have shown the results for $i = 5$ and $i = 10$, but the results appear to be similar for all $i = 1, \cdots, P$. The abscissa represents the index j arranged in descending order of the eigenvalues. The sample size is $n = 1000$. From the figure, the theoretical results closely resemble the simulations. This confirms the applicability of the above theoretical analysis (based on Gaussian statistics) for the case of DS-CDMA systems. In addition, in Figure 6.1(c) we show the simulation results results for $i = 10$, with different sample size values ($n = 200, 500, 1000$) used for estimating the sample covariance matrix. Both, Figure 6.1(b) and 6.1(c) reveal that the most of the energy of the eigenvector estimate $\hat{\mathbf{v}}_i$ is confined within the space spanned by the true eigenvector $\mathbf{v}_i$ and eigenvectors that correspond to the closest (neighboring) eigenvalues to λ_i even for different sample size n. These results support Observation 1.

Now, let us discuss how the estimation errors of the eigenvectors affect performance of the BIC-MME scheme. According to Property 1, there is no need for the eigenvectors from the set $\mathbf{S}$ to be estimated exactly. Rather, it is sufficient that the eigenvector estimates ($\hat{\mathbf{v}}_i$) are orthogonal and well confined within the subspace spanned by the true eigenvectors ($\mathbf{v}_i \in \mathbf{S}$). Under these conditions, the BIC-MME scheme will successfully cancel the whole subspace spanned by $\mathbf{S}$. Recall that the set $\mathbf{S}$ corresponds to the span of the eigenvectors that have eigenvalues that are very close in amplitude. By Observation 1, even in the case of small sample size n used for the estimation of the sample covariance matrix, the estimates $\{\hat{\mathbf{v}}_i | \mathbf{v}_i \in \mathbf{S}\}$ are clustered within the subspace $\mathbf{S}$. Consequently, the BIC-MME receiver will perform well when small sample sizes are used for the estimation of the covariance matrix. For example, in Figure 6.1(c), it is seen that the estimate $\hat{\mathbf{v}}_i$ for $i = 10$, has most of its energy confined to the subspace formed by $\mathbf{v}_j$, $j = 8, 9, 10, 11, 12$. This is true for all values of n shown, i.e., $n = 200, 500, 1000$.

Unlike the BIC-MME receiver, the BMMSE receiver, in addition to the eigenvector estimates, also requires the eigenvalue estimates [13]. This results in the performance of the BMMSE receiver being more sensitive to the sample size n. Using the analysis in Appendix B and the results in [28] it can be shown that the BMMSE requires greater number of samples for the estimation of the covariance matrix than the BIC-MME receiver to achieve the same performance (see simulation results in Figures 5.3(a) and 5.3(b)). Further investigation of this issue is beyond the scope of this work.



(a) Eigenvalue Profile $SNR = 10\,dB$, $n = 1000$.



(b) The Mean Energy $E\left[(\hat{\mathbf{v}}_i \mathbf{v}_j)^2\right]$: Simulations and Theoretical Results for $i = 5, 10$, $n = 1000$.

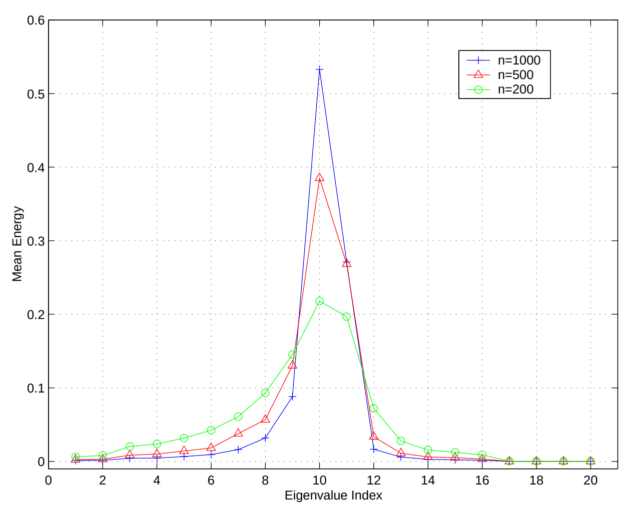


Figure 6.1: (c) The Mean Energy $E\left[(\hat{\mathbf{v}}_i \mathbf{v}_j)^2\right]$ (Simulations), $i = 10$, $n = 200, 500, 1000$.

7. Discussion and Conclusion

We have introduced the MME optimization criterion which is then used to implement a blind IC receiver. The ability of the receiver to exceed the performance of the blind MMSE is confirmed via simulation results. It is seen that this scheme is particularly effective for a system with fewer, very strong interferers and smaller number of samples used for the estimation of the covariance matrix. This may be a very viable solution for implementation on the downlink where transmissions are usually synchronized within a cell such that intracell users are orthogonal and intercell interference may be dominant. However, in the presence of multipath these assumptions do not necessarily hold, but the receiver is still effective since it does not use knowledge of interference parameters. Regarding the use of sample covariance estimates, we have presented an explanation of why the BIC-MME receiver performs well in the presence of estimation errors. A low complexity iterative solution using the power method for eigendecomposition is also studied. The properties of the receiver make it an attractive solution for implementation in time-varying channels as well as packet DS-CDMA systems with bursty traffic. This is an area of further investigation.

A. Appendix

Proof of Proposition 2

According to the spectral theorem [24]

$$\mathbf{R}_r = \sum_{i=1}^{N} \lambda_i \, \mathbf{v}_i \, \mathbf{v}_i^\top \tag{A.1}$$

where the eigenvalues and corresponding eigenvectors are sorted in descending order, i.e., the maximizer for $\mathbf{R}_r$ is $\mathbf{v}_{max} = \mathbf{v}_1$ and $\lambda_{max} = \lambda_1$. Second largest eigenvalue has index 2. Further

$$\mathbf{R}'_r = \mathbf{R}_r - \lambda_{max} \, \mathbf{v}_{max} \, \mathbf{v}_{max}^\top = \sum_{i=1}^{N} \lambda_i \, \mathbf{v}_i \, \mathbf{v}_i^\top - \lambda_1 \, \mathbf{v}_1 \, \mathbf{v}_1^\top = \sum_{i=2}^{N} \lambda_i \, \mathbf{v}_i \, \mathbf{v}_i^\top \tag{A.2}$$

From the above, the largest eigenvalue of the matrix $\mathbf{R}'_r$ has index 2, which is the second largest eigenvalue of the matrix $\mathbf{R}_r$ and $\mathbf{v}'_{max} = \mathbf{v}_2$. This proves the first part of Proposition 2. Consider

$$\mathbf{r}' = \mathbf{r} - (\mathbf{r}^\top \mathbf{v}_{max}) \, \mathbf{v}_{max} = \sum_{i=1}^{N} (\mathbf{r}^\top \mathbf{v}_i) \mathbf{v}_i - (\mathbf{r}^\top \mathbf{v}_1) \mathbf{v}_1 = \sum_{i=2}^{N} (\mathbf{r}^\top \mathbf{v}_i) \mathbf{v}_i \tag{A.3}$$

where the set of equalities follows from the KL expansion (see equation (3.6)). The covariance matrix of $\mathbf{r}'$ is

$$E\left[\mathbf{r}'\mathbf{r}'^\top\right] = E\left[\sum_{i=2}^{N}\sum_{j=2}^{N} \left((\mathbf{r}^\top \mathbf{v}_i)(\mathbf{r}^\top \mathbf{v}_j)\mathbf{v}_i \mathbf{v}_j^\top\right)\right] \tag{A.4}$$

According to the KL expansion, the random variables $a_i = (\mathbf{r}^\top \mathbf{v}_i)$ and $a_j = (\mathbf{r}^\top \mathbf{v}_j)$ for $i \neq j$ and $i, j = 1, \cdots, N$ are uncorrelated and $E\left[a_i \, a_j\right] = \lambda_i \, \delta_{ij}$. Using the above properties in (A.4), we get

$$E\left[\mathbf{r}'\mathbf{r}'^\top\right] = \sum_{i=2}^{N} \lambda_i \, \mathbf{v}_i \, \mathbf{v}_i^\top = \mathbf{R}'_r \tag{A.5}$$

which concludes the proof of Proposition 2.

B. Appendix

In this appendix we present analysis that leads to the approximation in (6.1). Consider the M-dimensional covariance matrix $\mathbf{R} = E\left[\mathbf{r}\,\mathbf{r}^\top\right]$ of the random vector $\mathbf{r}$. Let λ_i and $\mathbf{v}_i$, $(i = 1, \cdots, M)$ be the true eigenvalues and eigenvectors, respectively. Let $\mathbf{V}$ denote a matrix whose columns are the true eigenvectors, sorted in descending order of the corresponding true eigenvalues. The sample covariance matrix is defined as

$$\hat{\mathbf{R}}(i) = \frac{1}{n} \sum_{k=i-n+1}^{i} \mathbf{r}(k)\,\mathbf{r}^\top(k) \tag{B.1}$$

where i and k are time indices (will be omitted in the following), $\hat{\lambda}_i$ and $\hat{\mathbf{v}}_i$ $(i = 1, \cdots, M)$ are the eigenvalue and eigenvector estimates, respectively. Let us perform a similarity transformation as

$$\mathbf{A} = \mathbf{V}^{-1}\,\hat{\mathbf{R}}\,\mathbf{V} = \mathbf{V}^\top\,\hat{\mathbf{R}}\,\mathbf{V} \tag{B.2}$$

In the case of perfect estimate ($\hat{\mathbf{R}} = \mathbf{R}$) the matrix $\mathbf{A}$ is diagonal, with eigenvalues λ_i $(i = 1, \cdots, M)$ on the diagonal. But, in general

$$\mathbf{A} = \begin{bmatrix} \lambda_1 + a_{11} & a_{12} & \cdots & a_{1M} \\ a_{21} & \lambda_2 + a_{22} & \cdots & a_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ a_{M1} & a_{M2} & \cdots & \lambda_M + a_{MM} \end{bmatrix} \tag{B.3}$$

where a_{ij}, $(i,\, j = 1, \cdots, M)$ is the deviation of the sample value. The matrix $\mathbf{A}$ is symmetric $(a_{ij} = a_{ji})$ and for the off-diagonal elements $(i \neq j)$

$$a_{ij} = \frac{1}{n} \sum_{k=i-n+1}^{i} x_i(k) x_j(k) \tag{B.4}$$

and for the diagonal elements

$$a_{ii} = \frac{1}{n} \sum_{k=i-n+1}^{i} x_i(k) x_i(k) \; - \; \lambda_i \tag{B.5}$$

where $x_i(k) = \mathbf{r}^\top(k)\,\mathbf{v}_i$ and $x_j(k) = \mathbf{r}^\top(k)\,\mathbf{v}_j$, where k is time index. We assume that $x_i(k)$ $(i = 1, \cdots, M)$ are observations from a zero-mean normal distribution, and $E[x_i(k)^2\, x_i(l)^2] = E[x_i(k)^2]\, E[x_i(l)^2]$

for $k \neq l$. Under the above assumptions it can be shown that

$$E[a_{ij}^2] = \frac{\lambda_i \lambda_j}{n} \tag{B.6}$$

Further [28],

$$O(a_{ij}) = O\left(\sqrt{E[a_{ij}^2]}\right) = \frac{1}{\sqrt{n}} \equiv O(\delta) \tag{B.7}$$

where we have used $O(\delta)$ to represent $O(a_{ij})$. In order to evaluate the eigenvector estimates, let us introduce a similarity transformation which suppresses the off-diagonal elements of $\mathbf{A}$. Let us denote this new matrix (with off-diagonal elements suppressed) as $\mathbf{B}$. Originally, the idea was presented in [28], and applied in order to analyze the eigenvalue estimates, but here we extend the approach to study characteristics of eigenvector estimates. Specifically, the transformation is

$$\mathbf{B} = \mathbf{C}\,\mathbf{A}\,\mathbf{C}^{-1} \tag{B.8}$$

where the diagonal elements of $\mathbf{B}$ will further approach the eigenvalue estimates (i.e., the eigenvalues of $\hat{\mathbf{R}}$). In general, a similarity transformation leaves the eigenvalues unaltered [24]. In the following, we construct the matrix $\mathbf{C}$ that yields the necessary transformation in (B.8). Let us assume that

$$\mathbf{C} = \mathbf{I} + \mathbf{Y} \tag{B.9}$$

where the entries of $\mathbf{Y}$ are defined as follows

$$\mathbf{Y} = \begin{bmatrix} 0 & y_{12} & \cdots & y_{1M} \\ y_{21} & 0 & \cdots & y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ y_{M1} & y_{M2} & \cdots & 0 \end{bmatrix} \tag{B.10}$$

To simplify the derivations we may assume that

$$\mathbf{Y}^\top = -\mathbf{Y} \Rightarrow y_{ij} = -y_{ji},\ (i,\, j = 1, \cdots, M) \tag{B.11}$$

Further, let us assume that

$$O(y_{ij}) \leq O(\delta)\ \ (i,\, j = 1, \cdots, M) \tag{B.12}$$

Any element b_{ij} of the matrix $\mathbf{B}$ is

$$b_{ij} = \sum_{l=1}^{M} \left\{ \left[\lambda_l c_{il} + \sum_{k=1}^{M} c_{ik} a_{kl} \right] \left[c_{jl} - \sum_{k=1,\, k\neq l,\, k\neq j}^{M} c_{lk} c_{jk} + \sum_{m=1,\, m\neq l}^{M} c_{lm} \sum_{k=1,\, k\neq l,\, k\neq j}^{M} c_{lk} c_{jk} + \cdots \right] \right\} \tag{B.13}$$

where $c_{ii} = 1$ and $c_{ij} = y_{ij} \Rightarrow c_{ij} = -c_{ji}$. Under the assumptions in (B.9)-(B.12), all off-diagonal elements b_{ij} $(i \neq j)$ can be further simplified as

$$b_{ij} = \lambda_j c_{ij} + \lambda_i c_{ji} + a_{ji} \ + \ O(\delta^2) \tag{B.14}$$

Now, in the case of $\lambda_i \neq \lambda_j$, we can suppress the off-diagonal elements b_{ij} in (B.14) as follows. Choose

$$y_{ij} = \frac{a_{ij}}{\lambda_i - \lambda_j} \tag{B.15}$$

This implies $O(b_{ij}) = O(\delta^2)$. Note, that to satisfy the assumption in (B.12), it follows from (B.15) that we require

$$O(a_{ij}) < O(|\lambda_i - \lambda_j|) \tag{B.16}$$

or, in other words, we require that the sampling errors a_{ij} (which are of order $\frac{1}{\sqrt{n}}$) be smaller than the distances $|\lambda_i - \lambda_j|$ between the corresponding eigenvalues. In the case of $\lambda_i = \lambda_j$ (as would happen in noise subspace), we set

$$y_{ij} = 0 \tag{B.17}$$

This results in $O(b_{ij}) = O(\delta)$, which implies that the off-diagonal elements in $\mathbf{A}$ that correspond to the noise subspace are left unaltered by the transformation in (B.8).

Having the off-diagonal elements suppressed, the diagonal elements of the matrix $\mathbf{B}$ approach the eigenvalue estimate, i.e., eigenvalues of $\hat{\mathbf{R}}$. We now study the eigenvector estimates. Note that

$$\mathbf{C}^{-1} = (\mathbf{I} \ + \ \mathbf{Y})^{-1} = (\mathbf{I} \ - \ \mathbf{Y} + \ \mathbf{Y}^2 \ - \mathbf{Y}^3 \cdots) \tag{B.18}$$

Let us approximate

$$\mathbf{C}^{-1} \approx (\mathbf{I} \ - \ \mathbf{Y}) \Rightarrow \mathbf{C}^{-1} = \mathbf{C}^{\top} \tag{B.19}$$

This approximation is justified by (B.18) and (B.12). Then the transformations in (B.2) and (B.8) can be written as

$$\mathbf{B} = \mathbf{C}\,\mathbf{V}^{\top}\,\hat{\mathbf{R}}\,\mathbf{V}\,\mathbf{C}^{\top} \tag{B.20}$$

Based on the above, the matrix $\mathbf{B}$ can be approximated as a diagonal matrix. From (B.19), the matrix $\mathbf{V}\,\mathbf{C}^{\top}$ is orthogonal. Therefore, according to the spectral theorem [24], the columns of the matrix $\mathbf{V}\,\mathbf{C}^{\top}$ in (B.20) are approximately the eigenvectors of the sample covariance matrix $\hat{\mathbf{R}}$. Thus

$$\tilde{\mathbf{V}} \approx (\mathbf{V}\,\mathbf{C}^{\top}) \Rightarrow \mathbf{C}^{\top} \approx (\mathbf{V}^{\top}\tilde{\mathbf{V}}) \tag{B.21}$$

where $\tilde{\mathbf{V}}$ is the matrix whose columns are the eigenvectors ($\hat{\mathbf{v}}_i, i = 1, \cdots, M$) of $\hat{\mathbf{R}}$. To characterize the estimation errors of the eigenvector estimates we observe crosscorrelations (projections)

$\hat{\mathbf{v}}_i^\top \mathbf{v}_j$, $i \neq j$, $(i, j = 1, \cdots, M)$. Now, we study the first and second moments of the above projections. From (B.21) it follows that

$$c_{ij} \approx (\hat{\mathbf{v}}_i^\top \mathbf{v}_j) \tag{B.22}$$

For $i \neq j$ and $\lambda_i \neq \lambda_j$

$$E\left[(\hat{\mathbf{v}}_i^\top \mathbf{v}_j)\right] \approx E[c_{ij}] = 0 \tag{B.23}$$

and

$$E\left[(\hat{\mathbf{v}}_i^\top \mathbf{v}_j)^2\right] \approx E[c_{ij}^2] = \frac{1}{n} \frac{\lambda_i \lambda_j}{(\lambda_i - \lambda_j)^2} \tag{B.24}$$

which is the required result in (6.1).

References

[1] S. Verdú, "Minimum Probability of Error for Asynchronous Gaussian Multiple-access Channels," *IEEE Transactions on Information Theory*, vol. 32, pp. 85–96, January 1986.

[2] R. Lupas and S. Verdú, "Linear Multiuser Detectors for Synchronous Code-Division Multiple-Access Channels," *IEEE Transactions on Information Theory*, vol. 35, pp. 123–136, January 1989.

[3] U. Madhow and M. Honig, "MMSE Interference Suppression for Direct-Sequence Spread-Spectrum CDMA," *IEEE Transactions on Communications*, vol. 42, pp. 3178–3188, December 1994.

[4] P. Patel and J. Holtzman, "Analysis of a Simple Successive Interference Cancellation Scheme in DS/CDMA Systems," *IEEE JSAC, Special Issue on CDMA*, vol. 12, pp. 796–807, June 1994.

[5] A. Duel-Hallen, "A Family of Multiuser Decision-Feedback Detectors for Asynchronous Code-Division Multiple-Access Channels," *IEEE Transactions on Communications*, vol. 43, pp. 421–434, Feb./March/April 1995.

[6] U. Madhow, "Blind Adaptive Interference Suppression for Direct-Sequence CDMA," *IEEE Proceedings, Special Issue on Blind Identification and Equalization*, pp. 2049–2069, October 1998.

[7] S. Miller, "An Adaptive Direct-Sequence Code-Division Multiple-Access Receiver for Multiuser Interference Rejection," *IEEE Transactions on Communications*, vol. 43, pp. 1746–1755, February/March/April 1995.

[8] P. Rapajic and B. Vucetic, "Adaptive Receiver Structure for Asynchronous CDMA Systems," *IEEE JSAC*, vol. 12, pp. 685 – 697, May 1994.

[9] N. Mandayam and B. Aazhang, "Gradient Estimation for Sensitivity Analysis and Adaptive Multiuser Interference Rejection in Code Division Multiple Access Systems," *IEEE Transactions on Communications*, vol. 45, pp. 848–858, July 1997.

[10] G. Woodward, R. Ratasuk, and M. Honig, "Multistage Multiuser Decision Feedback Detection for DS-CDMA," *International Conference on Communications*, vol. 1, pp. 68–72, June 1999. Vancouver.

[11] R. Ratasuk, G. Woodward, and M. Honig, "Adaptive Multiuser Decision Feedback for Asynchronous Cellular DS-CDMA," *37th Allerton Conference*, 1999.

[12] M. Honig, U. Madhow, and S. Verdú, "Blind Adaptive Multiuser Detection," *IEEE Transactions on Information Theory*, vol. 41, pp. 944–960, July 1995.

[13] X. Wang and V. Poor, "Blind Multiuser Detection: A Subspace Approach," *IEEE Transactions on Information Theory*, vol. 44, pp. 677–690, March 1998.

[14] S. Ulukus and R. Yates, "A Blind Adaptive Decorrelating Detector for CDMA Systems," *IEEE JSAC*, vol. 16, pp. 1530–1541, October 1998.

[15] S. Verdú, *Multiuser Detection.* Cambridge University Press, 1998.

[16] D. Samardzija, N. Mandayam, and I. Seskar, "Blind Interference Cancellation for the Downlink of CDMA Systems," *Conference on Information Sciences and Systems*, vol. 2, pp. TP3.17–TP3.22, March 2000. Princeton University.

[17] I. Seskar, K. Pedersen, T. Kolding, and J. Holtzman, "Implementation Aspects for Successive Interference Cancellation in DS/CDMA Systems," *Wireless Networks*, no. 4, pp. 447–452, 1998.

[18] T. Ojanpera and R. Prasad, "An Overview of Air Interface Multiple Access for IMT-2000/UMTS," *IEEE Communication Magazine*, vol. 36, pp. 82 – 95, September 1998.

[19] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-CDMA for Next-Generation Mobile Communications Systems," *IEEE Communication Magazine*, vol. 36, pp. 56 – 69, September 1998.

[20] R. Pichna and Q. Wang, "A Medium Access Control Protocols for a Cellular Packet CDMA Carrying Multirate Traffic," *IEEE JSAC*, vol. 14, pp. 1728 – 1736, December 1996.

[21] I. Akyildiz and et al., "Medium Access Control Protocols for Multimedia Traffic in Wireless Networks," *IEEE Network*, pp. 39 – 47, July/August 1999.

[22] D. Bertsekas, *Constrained Optimization and Lagrange Multiplier Methods*. Academic Press, 1982.

[23] V. Poor, *An Introduction to Signal Detection and Estimation*. Springer-Verlag, second ed., 1994.

[24] G. Strang, *Linear Algebra and its Applications*. Harcourt Brace Jovanovich, third ed., 1988.

[25] P. Comon and G. Golub, "Tracking a Few Extreme Singular Values and Vectors in Signal Processing," *Proceedings of IEEE*, pp. 1327 – 1343, August 1990.

[26] B. Yang, "Projection Approximation Subspace Tracking," *IEEE Transactions on Signal Processing*, vol. 43, pp. 95 – 107, January 1995.

[27] D. Ramakrishna, N. Mandayam, and R. Yates, "Subspace Based Estimation of the Signal to Interference Ratio for CDMA Cellular Systems," *Accepted for IEEE Transactions on Vehicular Technologies*, 2000.

[28] D. N. Lawley, "Test of Significance for the Latent Roots of Covariance and Correlation Matrix," *Biometrika*, vol. 43, pp. 128–136, 1956.