Blind Interference Cancellation for the Downlink of CDMA Systems [1]

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Abstract — **In this paper we propose a blind interference cancellation (IC) receiver for asynchronous direct-sequence code-division multiple-access (DS-CDMA) systems using a maximum mean energy (MME) optimization criterion. The simulation results show that this scheme offers performance gains over the well known blind receiver that is based on the minimum mean squared error (MMSE) optimization criterion. Our results show that the blind receiver is particularly effective in the presence of a few strong interferers as may be the case in the downlink of CDMA systems where intracell user transmissions are orthogonal.**

I. Introduction

In DS-CDMA systems, in general, crosscorrelations between signature (spreading) sequences are nonzero. This results in multiple-access interference (MAI) which can disrupt reception of highly attenuated desired user signal. This is known as the near-far effect. To combat this problem several multiuser receivers have been proposed. Several solutions are presented in [1, 2, 3, 4]. These receivers are denoted as centralized because they require knowledge of parameters (signature sequences, amplitudes and timing) for all users in the system. Therefore, they are more suitable for processing at the base station.

For the downlink, it is desirable to devise decentralized receivers. Decentralized receivers exploit the knowledge of the desired user parameters only. They may be further classified into data aided and nondata aided receivers. Data aided adaptive multiuser detection is an approach which does not require a prior knowledge of the interference parameters. But, it requires a training data sequence for every active user. For example, adaptive receivers in [3, 5, 6] are based on the MMSE criterion.

Blind (or nondata aided) multiuser detectors require no training data sequence, but only knowledge of the desired user signature sequence and its timing. The receivers treat MAI and background noise as a random process, whose statistics must be estimated. Majority of blind multiuser detectors are based on estimation of second order statistics of the received signal. In [7], a blind adaptive MMSE multiuser detector is introduced (proven to be equivalent to the minimum output energy (MOE) detector). A subspace approach for blind multiuser detection is presented in [8]; where both the decorrelating and the MMSE detector are obtained blindly. Further, adaptive and blind solutions are analyzed in [9], with an overview in [10].

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The receiver in this paper is based on determining the most (on average) dominant baseband interference components at the output of CDMA system. Accordingly, in Section III, the maximum mean energy (MME) criterion is introduced. It is shown that the MME criterion is strongly related to the Karhunen-Loeve (KL) expansion of the received signal and to the eigendecomposition (ED) of the covariance matrix. In Section IV we present a novel blind receiver. It is based on the MME criterion and requires estimation of the second order statistics. The receiver executes interference cancellation (IC) in a successive manner; starting with most dominant interference component and successively cancelling weaker ones. Therefore it may be viewed as a blind equivalent to the centralized successive interference cancellation (SIC) scheme [4, 11]. Section V presents the simulation results and Section VI contains our conclusion.

II. Background

We now present the asynchronous CDMA system model and briefly review the MMSE criterion. The received baseband signal, $r(t)$, in antipodal K-user asynchronous CDMA additive white Gaussian noise (AWGN) system is

$$r(t) = \sum_{i=-J}^{J} \sum_{k=1}^{K} A_k \, b_k\,[i]\, s_k(t - iT - \tau_k) \; + \; \sigma n(t) \tag{1}$$

where A_k is the received amplitude, $b_k\,[i] \in \{-1, +1\}$ is binary, independent and equiprobable data, $s_k(t)$ is the signature sequence which is assumed to have unit energy, τ_k is relative time offset, all for the k^{th} user. T is the symbol period and $n(t)$ is AWGN with unit power spectral density. $2J + 1$ is the number of data symbols per user per frame.

It is well known that asynchronous system with independent users can be analyzed as synchronous if equivalent synchronous users are introduced, which are effectively additional interferers [12]. Sufficient statistics are obtained by sampling at $2f_0$, where f_0 is the maximum bandwidth of the chip waveforms in the desired user signature sequence [12, 7]. In this paper, we consider the received signal $r(t)$ over only one symbol period, that is synchronous to the desired user ($k = 1$). The discrete representation for the received signal in (1) can be written in vector form as

$$\mathbf{r} = \sum_{k=1}^{L} A_k \, b_k \, \mathbf{s}_k \; + \; \sigma \, \mathbf{n} \tag{2}$$

where the number of the interferers ($L - 1 = 2\,(K - 1)$) is doubled due to equivalent synchronous user analysis. $\mathbf{r}$, $\mathbf{s}_k$ and $\mathbf{n}$ are vectors in $\Re^M$, where M is the number of chips per bit.

For the sake of a completeness, the well known MMSE optimization criterion is briefly repeated here (proven to be equivalent to the MOE criterion [7]). For a vector $\mathbf{d} \in \Re^M$, the mean squared error is $MSE = E\left[(\mathbf{r}^\top \mathbf{d} - b_1)^2\right]$. The linear MMSE detector $\mathbf{c}$ is obtained as

$$\mathbf{c} = \arg\min_{\mathbf{d}} \left(E\left[(\mathbf{r}^\top \mathbf{d} - b_1)^2\right] - \gamma\,(\mathbf{s}_1^\top \mathbf{d} - 1)\right) \tag{3}$$

where the vector $\mathbf{d}$ is constrained to be

$$\mathbf{s}_1^\top \mathbf{d} = 1 \tag{4}$$

The solution of (3) is given as (for user 1) $\mathbf{c} = \mathbf{R}_r^{-1}\mathbf{s_1}$, where $\mathbf{R}_r = E[\mathbf{r}\,\mathbf{r}^\top]$ is the covariance matrix of the input process $\mathbf{r}$ [8]. The matrix $\mathbf{R}_r$ has to be invertible. If an estimate of the covariance matrix $\mathbf{R}_r$ i.e., sample covariance matrix $\hat{\mathbf{R}}_r$, is available, approximation of the optimal MMSE detector is

$$\hat{\mathbf{c}} = \hat{\mathbf{R}}_r^{-1}\mathbf{s_1} \tag{5}$$

which is denoted as blind MMSE (BMMSE) receiver. In this paper, the receiver is used as a reference for performance evaluations.

III. MME Optimization Criterion

Let us define for a M-dimensional vector $\mathbf{u}$, the mean energy (ME) as

$$\mathrm{ME} = E\left[(\mathbf{r}^\top \mathbf{u})^2\right] \tag{6}$$

Let us further constrain the vector $\mathbf{u}$ such that

$$\mathbf{u}^\top \mathbf{u} = 1 \tag{7}$$

We now consider maximization of the ME, with respect to the vector $\mathbf{u}$. The problem can be solved by the method of Lagrange multipliers [13]. Let

$$\psi(\mathbf{u}) = E\left[(\mathbf{r}^\top \mathbf{u})^2\right] - \gamma\,(\mathbf{u}^\top \mathbf{u} - 1) \tag{8}$$

Necessary condition for $\mathbf{v} \in \Re^M$ to maximize (8) is $\nabla(\psi(\mathbf{v})) = 0$, which results in

$$\mathbf{R}_r\,\mathbf{v} = \gamma\,\mathbf{v} \tag{9}$$

It is obvious from (9) that $\mathbf{v}$ and γ are an eigenvector and an eigenvalue of the matrix $\mathbf{R}_r$, respectively. In general, there is a set of eigenvectors and eigenvalues, which are related as

$$\mathbf{R}_r\,\mathbf{V} = \mathbf{V}\,\mathbf{D} \tag{10}$$

where $\mathbf{V}$ is a matrix whose columns are the eigenvectors $(\mathbf{v}_1, \cdots, \mathbf{v}_M)$, and $\mathbf{D}$ is diagonal matrix of the corresponding eigenvalues $(\lambda_1, \cdots, \lambda_M)$.

The constraint (7) only sets the vector $\mathbf{v}$ to have unit energy and it is different from that in (4) which defines energy of the vector $\mathbf{c}$ with respect to the desired user signature sequence $\mathbf{s}_1$. We may note that the MME criterion is more related to signal space, as a whole, unlike the MMSE criterion that is focused on the specific signal component ($\mathbf{s}_1$).

To gain more insight into the MME criterion that results in (9) and (10), let us consider the discrete form of the KL expansion of the received vector $\mathbf{r}$ [14]. This expansion allows the M-dimensional stochastic process $\mathbf{r}$ to be represented as a superposition of vectors $\mathbf{x}_i$ from orthonormal basis, scaled by statistically uncorrelated random variables a_i, $(i = 1, \cdots, N)$ as

$$\mathbf{r}_N = \sum_{i=1}^{N} a_i\,\mathbf{x}_i \tag{11}$$

The vectors $\mathbf{x}_i$ are orthonormal ($\mathbf{x}_i^\top \mathbf{x}_j = \delta_{ij}$) and the random variables a_i are defined as $a_i = \mathbf{r}^\top \mathbf{x}_i$. The random variables

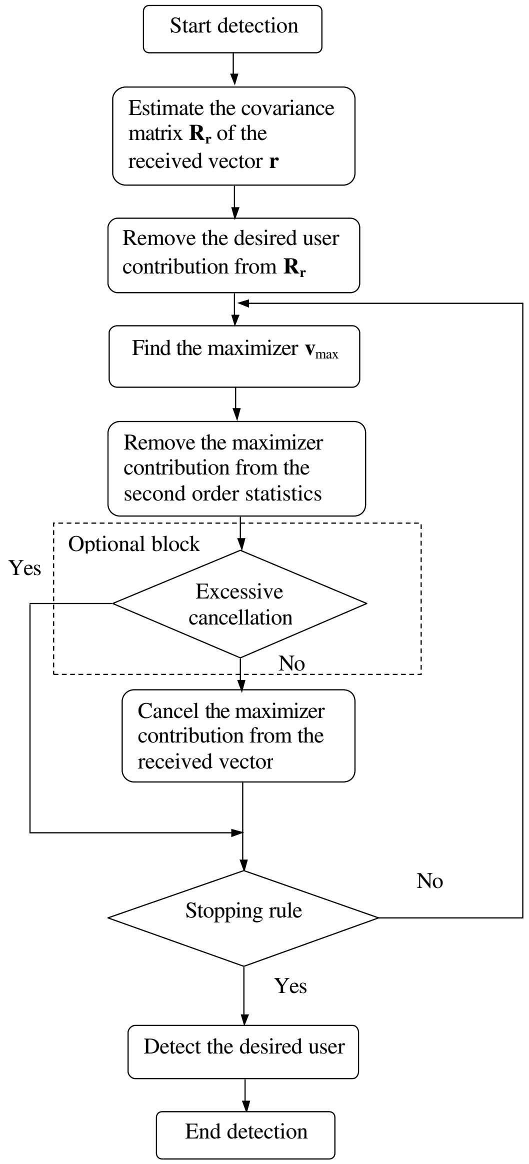

Figure 1: Flow chart illustrating the BIC-MME scheme.

a_i are uncorrelated and with expected energy λ_i ($E\left[a_i\,a_j\right] = \lambda_i\,\delta_{ij}$). The above condition results in

$$E\left[\mathbf{r}\,\mathbf{r}^\top\right]\mathbf{x}_i = \lambda_i\,\mathbf{x}_i,\; i = 1, \cdots, N \tag{12}$$

Further, the following equation

$$E\left[(\mathbf{r} - \mathbf{r}_N)^\top (\mathbf{r} - \mathbf{r}_N)\right] = 0 \tag{13}$$

holds if $E\left[\mathbf{r}\,\mathbf{r}^\top\right]$ is positive semidefinite [15]. In other words $\mathbf{r}_N$ converges to $\mathbf{r}$ in the mean squared sense.

It is obvious that the equations (9) and (12) are identical. Therefore, the vectors $\mathbf{x}_i$ are the column vectors (eigenvectors) of the matrix $\mathbf{V}$ and λ_i are the diagonal elements (eigenvalues) of the matrix $\mathbf{D}$. In the following, $\mathbf{x}_i$ and $\mathbf{v}_i$ $(i = 1, \cdots, N)$ are used interchangeably, and if $\mathbf{R}_r$ is invertible, then $N = M$ [15]. This analogy allows us to make following conclusions: If the matrix $\mathbf{V}$ and $\mathbf{D}$ are obtained from (10), the column vectors in $\mathbf{V}$ are orthonormal basis which span the received

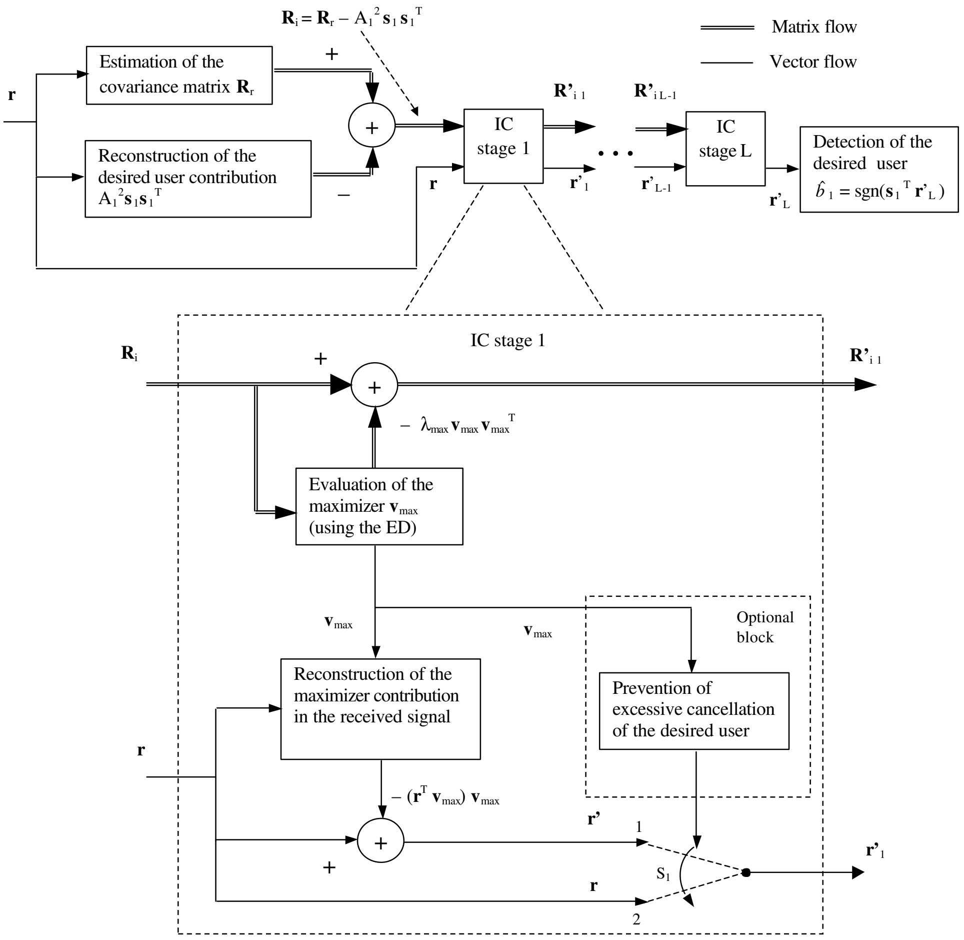

Figure 2: Block scheme of the BIC-MME receiver.

signal space in the mean squared sense. The diagonal elements of the matrix $\mathbf{D}$ are the mean energies of the received vector $\mathbf{r}$ along the orthonormal vectors from the basis. Thus, instead of analyzing the actual set of users (vectors) in the received vector $\mathbf{r}$ (as is done in the case of centralized receivers), we are evaluating the corresponding vector space which is characterized by the orthogonal basis and uncorrelated coefficients. From the given analogy we conclude:

Proposition 1 *The eigenvector of* $\mathbf{R}_r$ *that corresponds to the maximum eigenvalue* (λ_{max}) *is the vector that maximizes the* ME *(mean energy) in (6).*

Let us denote the eigenvector from Proposition 1 as $\mathbf{v}_{max}$ (the maximizer of ME). In addition, we claim:

Proposition 2 *If the contribution of* $\mathbf{v}_{max}$ *is removed from the matrix* $\mathbf{R_r}$, *as follows:* $\mathbf{R}'_r = \mathbf{R}_r - \lambda_{max}\, \mathbf{v}_{max}\, \mathbf{v}_{max}^\top$, *then the eigenvector* $\mathbf{v}'_{max}$ *that corresponds to the maximum eigenvalue of* $\mathbf{R}'_r$ *is the same as the eigenvector that corresponds to the second largest eigenvalue of* $\mathbf{R}_r$.

The results in the propositions 1 and 2 form the basis for the blind interference cancellation scheme presented in this paper. We now sketch an outline of how the above two results can be exploited to derive a blind successive interference cancellation scheme. Note that the contribution of the desired user can be removed from the covariance matrix $\mathbf{R}_r$ as follows:

$$\mathbf{R}_i = \mathbf{R}_r - A_1^2\, \mathbf{s}_1\, \mathbf{s}_1^\top \tag{14}$$

where $\mathbf{R}_i = E[\mathbf{i}\,\mathbf{i}^\top]$ is the interference covariance matrix, with $\mathbf{i} = \sum_{k=2}^{L} A_k\, b_k\, \mathbf{s}_k \;+\; \sigma\, \mathbf{n}$. Observe that in the above procedure, no knowledge is required of the desired user's bit decision (information). Only the knowledge of the desired signal power A_1^2 is needed. Further, if the MME criterion is now applied on $\mathbf{R}_i$ (i.e., we determine the eigenvector corresponding to the maximum eigenvalue of $\mathbf{R}_i$), then we can capture the most dominant interference (energy) component. The above process can be successively repeated and would result (due to Proposition 2) in successive cancellation of components in the interference subspace, starting from the strongest to the weakest.

IV. An Application of the MME Criterion in Blind IC Receiver

We now present a blind IC scheme where we incorporate the MME criterion (the scheme is denoted as BIC-MME). As depicted in the figures 1 and 2, the receiver executes the following steps (blocks in Figure 1):

1. Estimation of the matrix $\mathbf{R}_r$ according to

$$\hat{\mathbf{R}}_r(i) = \frac{1}{n} \sum_{k=i-n+1}^{i} \mathbf{r}(k)\, \mathbf{r}^\top(k) \tag{15}$$

where $\hat{\mathbf{R}}_r$ is the sample covariance matrix, n is the size of the averaging window (number of samples), and i is time index (will be omitted in the following text). [2]

2. Remove the desired user contribution from $\hat{\mathbf{R}}_r$. If the desired user amplitude (A_1) is known or estimated we can apply (14). The result of this step is that $\hat{\mathbf{R}}_i$ contains only the interference components and there is no desired user contribution ($A_1^2 \mathbf{s}_1 \mathbf{s}_1^\top$).
3. Find the maximizer ($\hat{\mathbf{v}}_{max}$) of the ME i.e., the vector that takes, on average, most of the interference energy. According to Proposition 1, the maximizer is the eigenvector that corresponds to the maximum eigenvalue ($\hat{\lambda}_{max}$) of the matrix $\hat{\mathbf{R}}_i$.
4. Remove the maximizer contribution from the matrix $\hat{\mathbf{R}}_i$ to yield
$$\hat{\mathbf{R}}_i' = \hat{\mathbf{R}}_i - \hat{\lambda}_{max}\hat{\mathbf{v}}_{max}\hat{\mathbf{v}}_{max}^\top \quad (16)$$
According to Proposition 2, this step prepares the estimate of the second order statistics ($\hat{\mathbf{R}}_i'$) for evaluation of the maximizer in the next IC stage.
5. To prevent excessive cancellation of the desired user from the input vector $\mathbf{r}$, we introduce an optional block. This block is useful in the case when the crosscorrelation between the desired user signature sequence and the interferer signature sequences is very high. A simple threshold criterion is applied to determine if cancellation is viable. For example, if
$$\left|(\mathbf{s}_1^\top \hat{\mathbf{v}}_{max})\right| > T_C \quad (17)$$
where T_C is some threshold value, then step (6) below is skipped, i.e., the IC is not performed (in Figure 2, the switch S_1 is in the position 2). If this block is not applied, the switch S_1 is always in the position 1.
6. Cancel the maximizer contribution as
$$\mathbf{r}' = \mathbf{r} - (\mathbf{r}^\top \hat{\mathbf{v}}_{max})\,\hat{\mathbf{v}}_{max} \quad (18)$$
7. A variety of stopping rules can be defined for the whole procedure. If all significant components of the interference (defined by a specific rule) are cancelled, the detection $\hat{b}_1 = sgn(\mathbf{r}'^\top \mathbf{s}_1)$ is performed, otherwise the steps (3) - (7) are repeated, where, for the new IC stage $\mathbf{r}$ and $\mathbf{R}_i$ take the values of $\mathbf{r}'$ and $\mathbf{R}_i'$, respectively.

V. Simulation Results

We consider a synchronous AWGN CDMA system, using randomly generated signature sequences with processing gain $M = 64$. We assume that the amplitude of the desired user is known exactly in the results presented here. The users are independent and two cases are analyzed:

1. System with $L = 16$ users, and equal-energy interferers: $A_i^2/A_1^2 = 25,\ i = 2, \cdots, 16$.
2. Lightly loaded system with $L = 4$ users, and very strong equal-energy interferers: $A_i^2/A_1^2 = 400,\ i = 2, \cdots, 4$.

Performance of the conventional matched filter (MF), the centralized MMSE receiver, the BMMSE receiver (detector in (5)) and the single user lower bound (SULB) are used as benchmarks for evaluation of the BIC-MME receiver. The BMMSE and the BIC-MME receiver use the same sample covariance matrix $\hat{\mathbf{R}}_r$. The matrix is estimated according to (15).

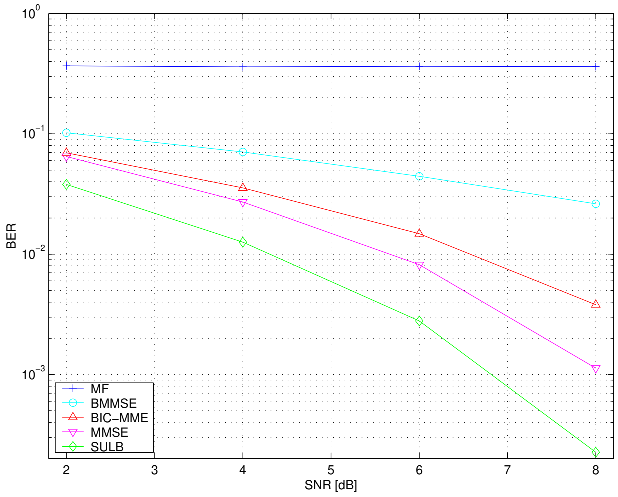

Figure 3: BER vs. SNR, Case 1, $n = 500$.

For the case 1, Figure 3 depicts bit-error rate (BER) as a function of signal to noise ratio (SNR) (with respect to the desired user). The results are obtained after a total of 15 IC stages, which is where the BER reaches minimum. Additional IC stages result in a deterioration of the performance for this particular example. For $SNR = 8\,dB$, BER versus number of the IC stages is presented in Figure 4. Equivalent results, for the case 2, with a total of 3 IC stages and $SNR = 6\,dB$ are shown in the figures 5 and 6, respectively. These results are evaluated for the window size $n = 500$ (the number of the samples used in (15)).

Note that the performance of the BIC-MME is near-optimum in the case 2. In this lightly loaded system, even in the presence of very strong interferers, a small number of the IC stages (3) is sufficient to fully cancel the interference with negligible negative effect on the desired user (just a small fraction of the desired user energy is removed by the IC).

We now study the effect of the accuracy of the covariance matrix estimation on the performance of the BIC-MME receiver. Figure 7 corresponds to the case 2 (for $SNR = 6\,dB$). The figure depicts BER with respect to different window size n.

According to the results above, the BIC-MME receiver outperforms the BMMSE receiver. The gain introduced by the BIC-MME, with respect to the BMMSE, increases as the averaging window gets smaller.

VI. Conclusion

We have introduced the MME optimization criterion which is then used to implement a blind IC receiver. The ability of the receiver to exceed the performance of the blind MMSE is confirmed via the simulation results. It is seen that this scheme is particularly effective for a system with fewer, very strong interferers and smaller number of samples used for the estimation of the covariance matrix. This may be a very viable solution for implementation on the downlink. A more detailed

[2] Notation: $\hat{z}$ denotes an estimate of z.

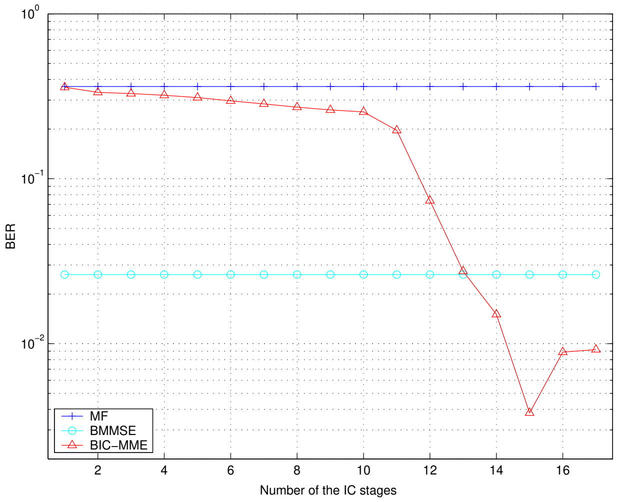

Figure 4: BER vs. Number of the IC stages, Case 1, $SNR = 8\,dB$, $n = 500$.

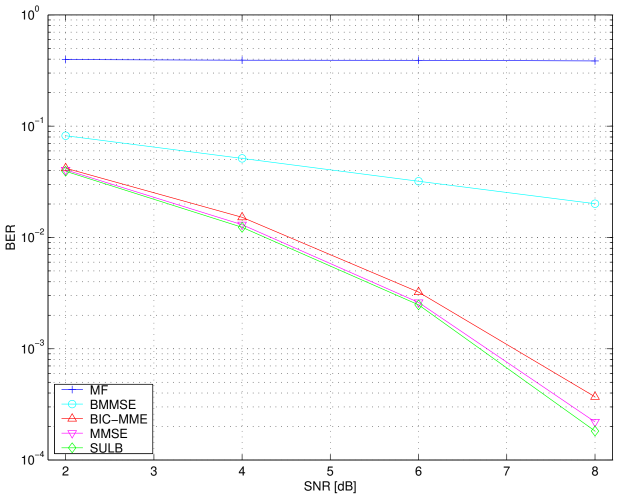

Figure 5: BER vs. SNR, Case 2, $n = 500$.

analysis of the BIC-MME scheme is presented in [16], including a detailed explanation of why the receiver performs well in the presence of estimation errors. Low complexity iterative solutions are also presented in [16].

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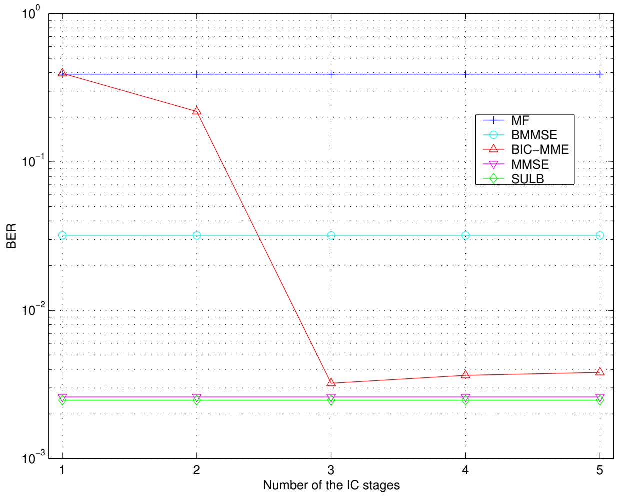

Figure 6: BER vs. Number of the IC stages, Case 2, $SNR = 6\,dB$, $n = 500$.

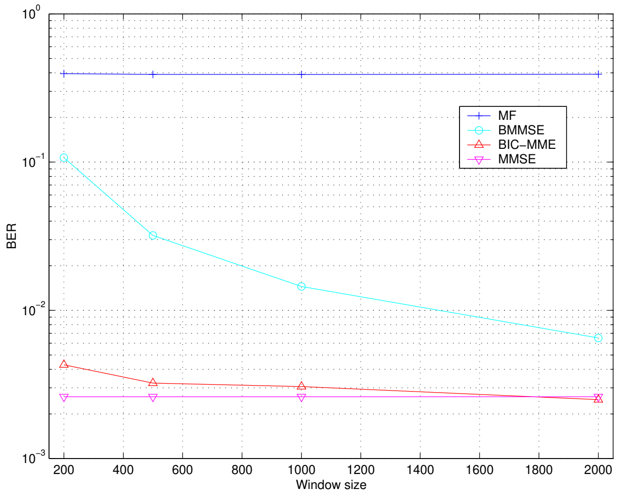

Figure 7: BER vs. Window size, Case 2, $SNR = 6\,dB$.

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