# Analysis of a Partial Decorrelator in a Multicell DS-CDMA System

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***Abstract*—For a multicell code-division multiple-access (CDMA) system, we propose a partial decorrelator that decodes a user by suppressing only the in-cell interferers. As a result, each user suffers only from other-cell interference and enhanced receiver noise. By analysis, we show that in random CDMA systems, the partial decorrelator outperforms the conventional receiver, within the operating regime of the conventional receiver. In simulation, we observe that when users have equal received powers at their respective receivers, a multicell system with partial decorrelator receivers yields roughly 1.5 times the capacity of the conventional system.**

***Index Terms*—Code-division multiple access (CDMA), decorrelation, wireless interference suppression.**

## I. INTRODUCTION

ALTHOUGH the *decorrelator* [1] has probably drawn more attention than any other multiuser detector, almost all studies have been for a single-cell code-division multiple-access (CDMA) system. In a CDMA system with multiple cells all using the same frequency carrier, the implementation of a decorrelating detector and its performance are not well understood. In a multicell environment, it is difficult for a base station to form the cross-correlation matrix by acquiring the signatures and timing of all users in other cells. Moreover, the decorrelator exists only when the number of users is less than the processing gain. Thus, it is generally not possible to implement a true decorrelator in a multicell system. For this environment, we propose a partial decorrelator (PD) that decodes a user by decorrelating the in-cell interferers only.

Similar to the current IS-95 system, we adopt a random (R)-CDMA system model in which different bits of a user are transmitted with random signature waveforms. We also assume that the timing offset of a user is fixed throughout its transmission and can be estimated perfectly by the base station. With these assumptions, we compare the PD and the matched filter (MF) receiver for an additive white Gaussian noise (AWGN) asynchronous multicell CDMA system.

When the processing gain is very large and the number of users is less than the processing gain, [2] shows for a single-cell system that under both the conventional receiver and the decorrelator, that expected value of the signal-to-interference ratio (SIR) approaches the ratio of the average signal power to the average total interference power, which we call average SIR. Since the bit error rate (BER) is difficult to analyze, average SIR is used in the analysis as a system performance measure. We verify by simulation that average SIR is a reliable performance measure for comparing the PD and MF receivers.

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## II. SYSTEM MODEL

In our R-CDMA system with $J$ in-cell users and $J_0 - J$ other-cell users, each bit results in a baseband transmission of a sequence of pulses $p[t]$. Each pulse has a duration of one chip period $T_c$. These pulses are sent over an AWGN channel in which the noise $Z(t)$ has power spectral density $N_0/2$. The bit transmission time of a user is $T$ and the processing gain is $L = T/T_c$. To transmit its $k$th bit, user $j$ employs the signature waveform

$$s_{j,k}(t) = \sum_{m=1}^{L} A_{j,k}(m) \frac{1}{\sqrt{L}} p\left[t-(m-1)T_c\right] \qquad (1)$$

where $A_{j,k}(m) \in \{-1,+1\}$ denotes the signature sequence of bit $k$ for user $j$. The energy of the pulse $p[t]$ is normalized so that for all bits $k$ and for every user $j$, $\int_0^T [s_{j,k}(t)]^2\,dt = 1$. Let $\Delta_j$ denote the delay of the $j$th user. In the asynchronous channel, the received signal due to the $j$th user at the desired user's base station is

$$r_j(t) = \sum_{k=-\infty}^{+\infty} \sqrt{E_j} b_{j,k} s_{j,k}\left(t - kT - \Delta_j\right) \qquad (2)$$

where $b_{j,k} \in \{-1,+1\}$ is the $k$th bit and $E_j$ is the received energy of the $j$th user at a desired user's base station. We assume that both users' signature sequences and transmitted bit sequences are independent and identically distributed (i.i.d.) equally likely binary sequences. We wish to decode the bits of user 1, assuming $\Delta_1 = 0$, from the total received signal

$$R(t) = \sum_{j=1}^{J_0} r_j(t) + Z(t) \qquad (3)$$

The received signal $R(t)$ is passed through a chip MF and sampled at the chip rate. The $(n+d+1)$ bits $\{b_{j,k} | k = -n, \ldots, d\}$ of user 1 will be processed by employing an observation window of duration $[-nT, (d+1)T]$, where $n \geq 0$ is the number of bits into the past, and $d \geq 0$ is the number of bits into the future with respect to bit $b_{1,0}$. Since all users transmit asynchronously, during the observation window $[-nT, (d+1)T]$, an interfering user $j$ transmits $(n+d+2)$ bits $\{b_{j,k} | k = -n-1, \ldots, d\}$. Among the interfering bits $b_{j,k}$, $k = -n-1$ and $k = d$ correspond to partial bits which are truncated at the left boundary $t = -nT$ and the right boundary $t = (d+1)T$, respectively.

There are $M = (n+d+1)L$ chip intervals, and thus, the vector of chip MF output samples in the interval

$[-nT, (d+1)T]$ is given by $\mathbf{R} = [R_1, \dots, R_M]^\top$ where the $m$th chip sample, $R_m$ is

$$R_m = \int_{-nT+(m-1)T_c}^{-nT+mT_c} p\left(t + nT - (m-1)T_c\right) R(t)\, dt. \quad (4)$$

In (4), $m = 1, 2, \dots, M$ and $R_m$ is a function of the parameters of the asynchronous CDMA system. In the observation window $[-nT, (d+1)T]$, an interfering user $j$ transmits $M+1$ chips or partial chips, while user 1 transmits exactly $M$ chips. For notational convenience, we denote by $a_{j,m}$ the $m$th chip of user $j$ in the observation window $[-nT, (d+1)T]$. For user $j \neq 1$, chips $a_{j,1}$ and $a_{j,M+1}$ are truncated at the left and right boundaries of the observation window. Clearly, $\{a_{j,m}\}$ are i.i.d. equally likely $\pm 1/\sqrt{L}$ sequences.

We can write the total received signal as

$$\mathbf{R} = \sum_{j=1}^{J_0} \mathbf{r}_j + \mathbf{Z} \quad (5)$$

where $\mathbf{r}_j = \sum_{k=-n-1}^{d} b_{j,k}\sqrt{E_j}\mathbf{S}_{j,k}$ is the contribution of the $j$th interfering user, and $\mathbf{Z}$ is an $M \times 1$ Gaussian noise vector with cross-correlation matrix $(N_0 T_c/2)\mathbf{I}$. Note that $\mathbf{S}_{j,k}$ is an $M \times 1$ vector that represents an effective chip waveform for bit $k$ of user $j$ over the observation window. For users $j \neq 1$, the effects of asynchronism and the window edge are embedded in $\mathbf{S}_{j,k}$.

Without loss of generality, we can assume $\Delta_j = (e_j + \epsilon_j)T_c$, where $e_j \in \{0, \dots, L-1\}$ and $\epsilon_j \in [0, 1)$. Since the filter is synchronized to user 1, $e_1 = \epsilon_1 = 0$ and $\mathbf{S}_{1,k}$ is simply the chip sequence of user 1, offset by $(n+k)L$ chips from the left edge of the window. For an interferer $j > 1$, $\mathbf{S}_{j,k}$ is a function of the user $j$'s signatures $\{a_{j,m}\}$ along with

$$\delta_j = \int_0^{T_c} p\left(t + (1-\epsilon_j)T_c\right) p(t)\, dt$$

$$\bar{\delta}_j = \int_0^{T_c} p\left(t - \epsilon_j T_c\right) p(t)\, dt$$

which characterize the cross correlation between the chip pulses of the desired user and the offset pulses of user $j$. A detailed description of $\mathbf{S}_{j,k}$ for $j \neq 1$ can be found in [3].

Our objective is to decode bit $b_{1,0}$ from the total received signal vector $\mathbf{R}$ which is the sum of the desired signal, in-cell interference $\mathbf{R}_i$, other-cell interference $\mathbf{R}_o = \sum_{j=J+1}^{J_0} \mathbf{r}_j$, and Gaussian noise $\mathbf{Z}$. That is

$$\mathbf{R} = b_{1,0}\sqrt{E_1}\mathbf{S}_{1,0} + \mathbf{R}_i + \mathbf{R}_o + \mathbf{Z} \quad (6)$$

where the in-cell interference is

$$\mathbf{R}_i = \sum_{k=-n}^{-1} b_{1,k}\sqrt{E_1}\mathbf{S}_{1,k} + \sum_{k=1}^{d} b_{1,k}\sqrt{E_1}\mathbf{S}_{1,k} + \sum_{j=2}^{J} \mathbf{r}_j. \quad (7)$$

## III. Performance Comparison

From (6), the matched filter output for bit $b_{1,0}$ is

$$R = \mathbf{S}_{1,0}^\top \mathbf{R} = \sqrt{E_1} b_{1,0} + R_i + R_o + Z \quad (8)$$

In (8), $Z$ is a Gaussian random variable with mean zero and variance $\eta^2 = N_0 T_c/2$. The term $R_i = \mathbf{S}_{1,0}^\top \mathbf{R}_i$ denotes the in-cell interference, and $R_o = \mathbf{S}_{1,0}^\top \mathbf{R}_o$ is the other-cell interference. For bit $b_{1,0}$, the average in-cell and other-cell interference power observed at the MF output are

$$\sigma_i^2 = E\left[R_i^2\right] \qquad \sigma_o^2 = E\left[R_o^2\right]. \quad (9)$$

The PD decodes $b_{1,0}$ by projecting $\mathbf{S}_{1,0}$ onto the subspace orthogonal to the in-cell interference $\mathbf{R}_i$ over the observation window $[-nT, (d+1)T]$. Over this window, let $\mathcal{S}_{n,d}$ denote the set of in-cell interfering signatures. Let $\hat{\mathbf{\Phi}}$ be the unit energy PD filter that decodes $b_{1,0}$. [4] finds $\hat{\mathbf{\Phi}}$ by applying Gram–Schmidt orthogonalization on the set of interferers' signatures $\mathcal{S}_{n,d}$. When $\mathbf{S}_{1,0}$ is linearly independent of the signatures in $\mathcal{S}_{n,d}$, the PD filter output will be

$$\hat{R} = \hat{\mathbf{\Phi}}^\top \mathbf{r} = \sqrt{\zeta_{n,d} E_1} b_{1,0} + \hat{R}_o + \hat{Z} \quad (10)$$

where $\hat{Z}$ is a Gaussian random variable with mean zero and variance $\hat{\eta}^2 = N_0 T_c/2$. The term $\hat{R}_o = \hat{\mathbf{\Phi}}^\top \mathbf{R}_o$ denotes the other-cell interference, and $\zeta_{n,d} = (\hat{\mathbf{\Phi}}^\top \mathbf{S}_{1,0})^2$ is the near–far resistance [1] of the PD for decoding the bit $b_{1,0}$ when the other-cell interference $\mathbf{R}_o$ is zero. When $\mathbf{S}_{1,0}$ is a linear combination of signatures in $\mathcal{S}_{n,d}$, the Gram–Schmidt procedure yields $\hat{\mathbf{\Phi}} = 0$, and hence, $\hat{R} = 0$. In this case, the AWGN variance at the PD output will be trivially zero. Otherwise, the AWGN variance $\hat{\eta}^2$ at the PD output will equal $\eta^2$. In either case, we have $\hat{\eta}^2 \leq \eta^2$. We use $\hat{\sigma}_o^2 = E[\hat{R}_o^2]$ to denote the average other-cell interference power observed by user 1 under the PD system. If $\mathrm{SIR}_{\mathrm{MF}}$ and $\mathrm{SIR}_{\mathrm{PD}}$ denote average SIR of user 1 under the MF and PD detectors, respectively, then

$$\mathrm{SIR}_{\mathrm{MF}} = \frac{E_1}{\sigma_i^2 + \sigma_o^2 + \eta^2} \qquad \mathrm{SIR}_{\mathrm{PD}} = \frac{E\left[\zeta_{n,d}\right] E_1}{\hat{\sigma}_o^2 + \hat{\eta}^2}. \quad (11)$$

Note that $\zeta_{n,d}$ is a function of signatures and timing offsets of in-cell users. Since signatures are random, the expectation $E[\zeta_{n,d}]$ has been taken over all in-cell signatures.

Our goal is to compare the capacity of the PD with that of the conventional receiver. In particular, we would like to develop a lower bound on the number of in-cell users, $J$, as a function of system parameters, for which $\mathrm{SIR}_{\mathrm{PD}} \geq \mathrm{SIR}_{\mathrm{MF}}$. To do so, we establish some preliminary results. In [3], we prove that:

*Lemma 1:* For a R-CDMA system, $E[\zeta_{n,d}] \geq E[\zeta_{0,0}] \geq 1 - 2(J-1)/L$.

To characterize the average other-cell interference power, let $\tilde{\mathbf{\Phi}}$ denote an arbitrary receiver filter for bit $b_{1,0}$. For example, $\tilde{\mathbf{\Phi}}$ may represent the MF $\mathbf{S}_{1,0}$, or the partial decorrelator $\hat{\mathbf{\Phi}}$, or perhaps some other linear filter. At the output of filter $\tilde{\mathbf{\Phi}}$, (6) implies the contribution of the other-cell interference is $\tilde{R}_o = \tilde{\mathbf{\Phi}}^\top \mathbf{R}_o = \sum_{j=J+1}^{J_0} \tilde{\mathbf{\Phi}}^\top \mathbf{r}_j$. Averaged over the bits and random signatures of the other-cell interferers, the second moment of the other-cell interference is $\tilde{\sigma}_o^2 = E[\tilde{R}_o^2]$. In Appendix A, we derive the following result for the other-cell interference power.

*Theorem 1:* For a receiver filter $\tilde{\mathbf{\Phi}}$ with unit energy $\|\tilde{\mathbf{\Phi}}\|^2 = 1$

$$\tilde{\sigma}_o^2 = \frac{1}{L} \sum_{j=J+1}^{J_0} \left(\bar{\delta}_j^2 + \delta_j^2\right) E_j + \frac{\tilde{\mathbf{\Phi}}^\top \mathbf{C}_M \tilde{\mathbf{\Phi}}}{L} \sum_{j=J+1}^{J_0} \delta_j \bar{\delta}_j E_j \quad (12)$$

where $\mathbf{C}_M$ is an $M \times M$ matrix whose $(l, m)$th element is 1, if $|l - m| = 1$ and 0, otherwise.

The term $\sum_{j=J+1}^{J_0} \delta_j \bar{\delta}_j E_j$ is a consequence of chip asynchronism. Denoting the $m$th element of $\tilde{\mathbf{\Phi}}$ as $\tilde{\mathbf{\Phi}}(m)$, we have

$$\tilde{\mathbf{\Phi}}^\top \mathbf{C}_M \tilde{\mathbf{\Phi}} = \sum_{m=1}^{M-1} 2\tilde{\mathbf{\Phi}}(m)\tilde{\mathbf{\Phi}}(m+1). \quad (13)$$

For the MF, $\tilde{\mathbf{\Phi}} = \mathbf{S}_{1,0}$. In the R-CDMA system, each chip of $\mathbf{S}_{1,0}$ is chosen independently, and it is straightforward to see

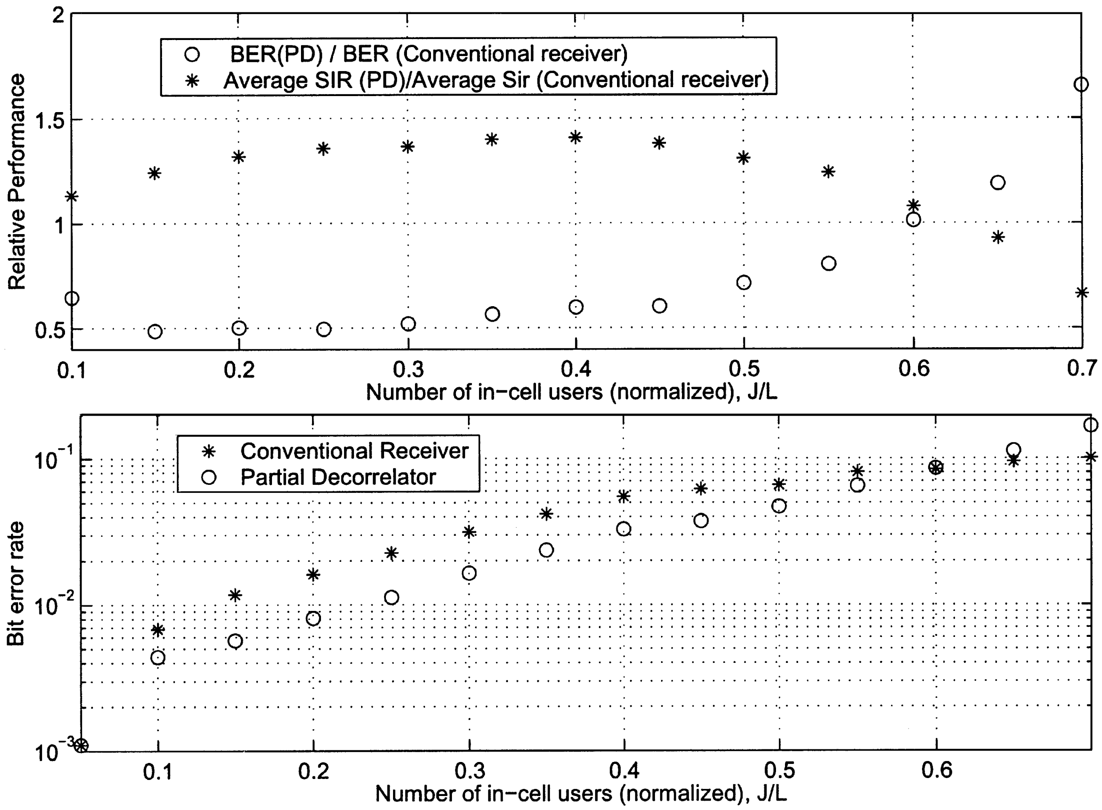


Fig. 1. Upper plot shows the relative performance of the PD as a function of $J/L$. Lower plot shows the BER of user 1 as a function of $J/L$ for the PD and MF systems.

from (13) that $E[\mathbf{S}_{1,0}^{\top}\mathbf{C}_M\mathbf{S}_{1,0}] = 0$. This yields the following corollary.

*Corollary 1:* For the MF $\mathbf{S}_{1,0}$, $\sigma_o^2 = (1/L)\sum_{j=J+1}^{J_0}(\bar{\delta}_j^2 + \delta_j^2)E_j$.

For the decorrelator, a simpler expression than that of *Theorem 1* for the other-cell interference power is not easy to specify. Thus, we develop bounds that apply to any linear filter that is chosen independently of the signatures of the other-cell interferers.

*Theorem 2:* For a receiver filter $\tilde{\mathbf{\Phi}}$ with unit energy $\|\tilde{\mathbf{\Phi}}\|^2 = 1$, the second moment of the other-cell interference, $\tilde{\sigma}_o^2$, satisfies

$$0 \leq \frac{1}{L}\sum_{j=J+1}^{J_0}\left(\delta_j - \bar{\delta}_j\right)^2 E_j \leq \tilde{\sigma}_o^2 \leq \frac{1}{L}\sum_{j=J+1}^{J_0}\left(\delta_j + \bar{\delta}_j\right)^2 E_j. \tag{14}$$

Note that in *Theorem 2*, the upper and lower bounds of $\tilde{\sigma}_o^2$ are maximal and equal for the chip synchronous system with fractional chip offsets $\epsilon_j = 0$. In this case, $\delta_j = 0$, $\bar{\delta}_j = 1$, and $\tilde{\sigma}_o^2 = (1/L)\sum_{j=J+1}^{J_0} E_j$. That is, the chip synchronous system yields higher other-cell interference than the chip asynchronous system when the desired receiver filter is developed by ignoring the other-cell interference. For the chip synchronous system, we observe that $\sigma_o^2 = \tilde{\sigma}_o^2$. When $\mathbf{S}_{1,0}$ is not a linear combination of the in-cell interfering signatures, $\hat{\mathbf{\Phi}}$ has magnitude 1 and $\hat{\sigma}_o^2 = \tilde{\sigma}_o^2$; otherwise, $\hat{\mathbf{\Phi}} = 0$ and $\hat{\sigma}_o^2 = 0$. In either case

$$\hat{\sigma}_o^2 \leq \tilde{\sigma}_o^2 = \sigma_o^2. \tag{15}$$

Applying $\hat{\eta}^2 \leq \eta^2$, *Lemma 1*, and (15) to (11), we observe that

$$\mathrm{SIR}_{\mathrm{PD}} \geq \frac{\left[1 - \frac{2(J-1)}{L}\right] E_1}{\sigma_o^2 + \eta^2}. \tag{16}$$

We use $F$ to denote the ratio $(\sigma_o^2 + \eta^2)/\sigma_i^2$. Combining (11) and (16) yields the following theorem.

*Theorem 3:* For a chip synchronous R-CDMA system, $\mathrm{SIR}_{\mathrm{MF}} \leq \mathrm{SIR}_{\mathrm{PD}}$ implies $J/L \leq 1/2(1+F)$.

In a conventional multicell direct-sequence (DS)-CDMA system, $\sigma_o^2/\sigma_i^2 \approx 0.55$ [5]. This result relies on the assumption that users have equal received powers at the base station in their own cell, and specifically does not depend on what receiver filters are employed. This suggests that the parameter $F$ is a constant and in an interference-limited system, $F \approx 0.55$. Thus, *Theorem 3* says that for a chip synchronous system, $\mathrm{SIR}_{\mathrm{MF}} \leq \mathrm{SIR}_{\mathrm{PD}}$ implies $J/L \leq 1/[2(1+F)] \approx 0.323$. In this case, when $J/L > 0.323$, we expect the MF will be better. However, for conventional systems, $J/L \approx 0.2$. Hence, within the operating regime of the conventional receiver, the PD should outperform the conventional receiver. Furthermore, note that *Theorem 3* is based on the lower bound on $\mathrm{E}[\zeta_{0,0}]$ in *Lemma 1*. Since for $n, d > 0$, $\mathrm{E}[\zeta_{n,d}] \geq \mathrm{E}[\zeta_{0,0}]$, one could expect the PD with $n,\ d \geq 1$ to outperform the conventional receiver, even when the number of in-cell users per dimension exceeds $1/[2(1+F)]$.

## IV. Empirical Results and Conclusion

To compare the performance of the PD with the conventional receiver, a simulation study was performed with an asynchronous multicell DS-CDMA system of seven contiguous hexagonal cells. The area of each cell was approximately $\pi 1000^2$ m$^2$, which is the area of a circle of radius $r_o = 1000$ m. The cell which was in the middle of seven cells was the cell of interest. It was assumed that a mobile was uniformly distributed within its own cell. This assumption yielded a probability density function $f(r) = 2r/r_0^2$ for the distance of a user from its own base station. The number of in-cell users is 1/7th of the total number of users in the system. We used a path loss exponent $= 4$. The height of the base station was 30 m, so that the uplink channel gain of user $j$ to its own base station, $h_j$, was $1/(r^2 + 30^2)^2$. The system processing gain was 20 and the asynchronous timing offset, $\Delta_j \in [0, T)$, was independently chosen for each user. Perfect power control was assumed, i.e., every user had the same received power at its own base station and its SNR was 9.8 dB, which yields BER $10^{-3}$ in a single-user channel. Using $n = d = 1$, the observation window of the PD covered three bits of the desired user.

In a CDMA system, BER is the performance measure of interest. Since BER is hard to analyze, our analysis employs the average SIR (i.e., the ratio of the average signal power to the average interference power) as the system performance measure. Our simulation results showed that $\text{SIR}_{\text{MF}} \leq \text{SIR}_{\text{PD}}$ is equivalent to $\text{BER}_{\text{MF}} \geq \text{BER}_{\text{PD}}$, where $\text{BER}_{\text{MF}}$ is the BER of user 1 under the conventional system, and $\text{BER}_{\text{PD}}$ is the BER of user 1 under the PD system; see the first plot of Fig. 1.

The BER requirement of a conventional system is approximately $10^{-2}$, which is obtained at $J/L \approx 0.2$. Our simulation result also agreed with this previous observation; see the second plot of Fig. 1. Here, we also found that the PD's performance at $J/L = 0.3$ is the same as the performance of the conventional receiver at $J/L = 0.2$. This result suggests that the PD yields $50\%$ capacity gain over the conventional receiver.

In particular, [5] notes that if the in-cell interference can be completely cancelled, then the capacity improvement over the conventional receiver would be approximately $(1+F)/F$, and the factor $(1+F)/F$ can be considered as an upper bound on the capacity gain of any multiuser detection scheme. For $F = 0.55$, this upper bound is 2.8. We have observed that the partial decorrelator achieves roughly half of this potential capacity enhancement. The capacity of any multiuser receiver, including the conventional receiver, will degrade under imperfect timing. [6] shows that decorrelators outperform adaptive minimum mean-square error (MMSE) receivers when timing offset errors are less than a chip.

## Appendix

### Proofs

### A. Proof: Theorem 1

Squaring $\tilde{\mathbf{\Phi}}^\top \mathbf{R}_o$, we obtain

$$\tilde{R}_o^2 = \tilde{\mathbf{\Phi}}^\top \mathbf{R}_o \mathbf{R}_o^\top \tilde{\mathbf{\Phi}} = \sum_{j=J+1}^{J_0} \sum_{k=J+1}^{J_0} \tilde{\mathbf{\Phi}}^\top \mathbf{r}_j \mathbf{r}_k^\top \tilde{\mathbf{\Phi}}. \tag{17}$$

Combining $\mathbf{r}_j = \sum_{k=-n-1}^{d} b_{j,k} \sqrt{E_j} \mathbf{S}_{j,k}$ with (17), we write

$$\tilde{R}_o^2 = \sum_{j=J+1}^{J_0} \sum_{j'=J+1}^{J_0} \sum_{l=-n-1}^{d} \sum_{k=-n-1}^{d} b_{j,l} b_{j',k} \sqrt{E_j} \sqrt{E_{j'}} \tilde{\mathbf{\Phi}}^\top \mathbf{S}_{j,l} \mathbf{S}_{j',k}^\top \tilde{\mathbf{\Phi}} \tag{18}$$

Since the transmitted bits are an i.i.d. equally likely $\pm 1$ sequence, taking the expectation with respect to transmitted bits on both sides of the above equation, and then taking the expectation with respect to signatures $\mathbf{S}_{j,k}$, we prove the desired result. □

### B. Proof: Theorem 2

First, we employ the following lower and upper bounds:

$$\begin{aligned} -\tilde{\mathbf{\Phi}}^2(m) - \tilde{\mathbf{\Phi}}^2(m+1) &\leq 2\tilde{\mathbf{\Phi}}(m)\tilde{\mathbf{\Phi}}(m+1) \\ &\leq \tilde{\mathbf{\Phi}}^2(m) + \tilde{\mathbf{\Phi}}^2(m+1) \end{aligned} \tag{19}$$

on (13). Second, we use $\|\tilde{\mathbf{\Phi}}\|^2 = \sum_{m=1}^{M} \tilde{\mathbf{\Phi}}^2(m) = 1$ to obtain

$$-1 - \sum_{m=2}^{M-1} \tilde{\mathbf{\Phi}}^2(m) \leq \tilde{\mathbf{\Phi}}^\top C_M \tilde{\mathbf{\Phi}} \leq 1 + \sum_{m=2}^{M-1} \tilde{\mathbf{\Phi}}^2(m). \tag{20}$$

Since $\tilde{\mathbf{\Phi}}^2(1) \geq 0$ and $\tilde{\mathbf{\Phi}}^2(M) \geq 0$, from inequalities (20), we get $-2 \leq \tilde{\mathbf{\Phi}}^\top C_M \tilde{\mathbf{\Phi}} \leq 2$. For $\tilde{\sigma}_o^2$ given in *Theorem 1*, we apply these inequalities to complete the proof. □

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