EE 359 - Wireless Communications - Fall 2003

Homework 6: Due Thursday 11/20 at noon.

1. (20 pts.) Assume a discrete-time Rayleigh fading channel, where at each time i the transmitter and receiver know $\gamma[i]$, and the distribution of the fading SNR $p(\gamma)$ is exponential with mean $\overline{\gamma} = 10$dB. Compute each part for a target BER of 10^{-3} and for a target BER of 10^{-6}.

   (a) Derive the optimal power and rate adaptation for variable-rate variable-power MQAM over this channel. Assume the rate and power can be varied continuously.

   (b) Compute the average throughput of this channel under this adaptive MQAM scheme.

   (c) Compare your answer in (b) to the capacity of this channel (computed in Homework 3, problem 3b).

2. (40 points) In this problem we explore the behavior of adaptive modulation that is based on different BER bounds, and show that the optimal power adaptation is not necessarily waterfilling depending on the bound. To illustrate this we use the following three BER approximations for MPSK modulation. These bounds assume a received SNR γ due to fading, power control $S(\gamma)/\overline{S}$, and variable rate $M = 2^{k(\gamma)}$. The bounds are tight to within 1 dB for $M > 2$ (you can check this by plotting them if you want to confirm their tightness):

$$\text{Bound 1: } P_b(\gamma) \approx 0.05 \exp\left[\frac{-6\gamma(\frac{S(\gamma)}{\overline{S}})}{2^{1.9k(\gamma)} - 1}\right].$$

$$\text{Bound 2: } P_b(\gamma) \approx 0.2 \exp\left[\frac{-7\gamma(\frac{S(\gamma)}{\overline{S}})}{2^{1.9k(\gamma)} + 1}\right].$$

$$\text{Bound 3: } P_b(\gamma) \approx 0.25 \exp\left[\frac{-8\gamma(\frac{S(\gamma)}{\overline{S}})}{2^{1.94k(\gamma)}}\right].$$

   (a) Show that all three bounds can be expressed in the general form

$$P_b(\gamma) \approx c_1 \exp\left[\frac{-c_2\gamma(\frac{S(\gamma)}{\overline{S}})}{f(k(\gamma))}\right], \quad (1)$$

   where

$$f(k(\gamma)) = 2^{c_3 k(\gamma)} - c_4,$$

   c_1, c_2, and c_3 are positive fixed constants and c_4 is a real constant. Find these constants for each of the three bounds.

   (b) Invert the P_b expression (1) to find the data rate $k(\gamma)$ as a function of the power control $S(\gamma)/\overline{S}$, fixed BER P_b, and constants c_i.

(c) To maximize spectral efficiency we create the Lagrangian

$$J(S(\gamma)) = \int k(\gamma)p(\gamma)d\gamma + \lambda \left[\int S(\gamma)p(\gamma)d\gamma - \overline{S} \right],$$

where $k(\gamma)$ is the expression obtained in part (b). The optimal power and rate adaptation must then satisfy

$$\frac{\partial J}{\partial S(\gamma)} = 0, \tag{2}$$

with the additional constraint that $S(\gamma)$ and $k(\gamma)$ are nonnegative for all γ. Solve (2) for $S(\gamma)$ to show that the optimal power adaptation satisfies

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} -\frac{1}{c_3(\ln 2)\lambda \overline{S}} + \frac{c_4 \ln(P_b/c_1)}{c_2}\frac{1}{\gamma} & S(\gamma) \geq 0, k(\gamma) \geq 0 \\ 0 & \text{else} \end{cases},$$

(d) Find K as a function of the c_is and target P_b such that you can express your answer to part (c) in the form

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma K} & \gamma \geq \gamma_0/K, k(\gamma) \geq 0. \\ 0 & \text{else} \end{cases}, \tag{3}$$

where γ_0 is the optimized cutoff fade depth.

(e) The cutoff fade depth γ_0 must satisfy

$$\int_{\gamma_0/K}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma K} \right) p(\gamma)d\gamma \leq 1.$$

Assuming that γ is exponentially distributed (Rayleigh fading) with $\overline{\gamma} = 30$ dB and the target BER is $P_b = 10^{-7}$, find the cutoff value γ_0 corresponding to the power adaptation for each of the three bounds.

(f) Plot $S(\gamma)/\overline{S}$ and $k(\gamma)$ as a function of γ for Bounds 1, 2, and 3 for γ ranging from 0 to 30 dB. Also state whether the cutoff value below which the channel is not used is based on the power or rate positivity constraint. Which of the three bounds leads to a waterfilling power adaptation? How does the power adaptation for the other bounds differ from this waterfilling at low SNRs? How about at high SNRs?

3. (20 points) Consider a Rayleigh fading channel with an average received SNR of 20 dB, a signal bandwidth of 30 KHz, a Doppler of 80 Hz, and a required BER of 10^{-3}.

(a) Suppose you use adaptive modulation on this channel with constellations restricted to size 0, 2, 4, 16, and 64. Using $\gamma_K^* = .1$ find the fading regions R_j associated with each of these constellations. Also find the average data rate of this restricted adaptive modulation scheme and the average time spent in each region R_j. If the symbol rate is $T_s = B^{-1}$ over approximately how many symbols is each constellation transmitted before a change in constellation size is needed?

(b) For the adaptive modulation scheme with no constellation restriction, suppose the receiver has a perfect estimate of the channel γ while the transmitter has an imperfect estimate $\hat{\gamma}$ due to delay and/or estimation error. If the target BER for the adaptive modulation scheme is 10^{-3} and the ratio $\gamma/\hat{\gamma}$ is uniformly distributed between 0.5 and 1.5, what is the average BER?

4. (20 points) Consider a MIMO system with the following channel matrix:

$$H = \begin{bmatrix} .1 & .3 & .4 \\ .3 & .2 & .2 \\ .1 & .3 & .7 \end{bmatrix} = \begin{bmatrix} -.5196 & -.0252 & -.8541 \\ -.3460 & -.9077 & .2372 \\ -.7812 & .4188 & .4629 \end{bmatrix} \begin{bmatrix} .9719 & 0 & 0 \\ 0 & .2619 & 0 \\ 0 & 0 & .0825 \end{bmatrix} \begin{bmatrix} -.2406 & -.4727 & -.8477 \\ -.8894 & -.2423 & .3876 \\ .3886 & -.8472 & .3621 \end{bmatrix}.$$

Note that H is written in terms of its singular value decomposition (SVD) $H = U\Lambda V$.

(a) Check if $H = U\Lambda V$. You will see that the matrices U, Λ, and V do not have sufficiently large precision so that $U\Lambda V$ is only approximately equal to H. This indicates the sensitivity of the SVD, in particular the matrix Λ, to small errors in the estimate of the channel matrix H.

(b) Based on the singular value decomposition $H = U\Lambda V$, find an equivalent MIMO system consisting of three independent channels. Find the transmit precoding filter and the receiver shaping filter necessary to transform the original system into the equivalent system.

(c) Find the optimal power allocation $P_i, i = 1, 2, 3$ across the three channels found in part (b), and the corresponding total capacity of the equivalent system, assuming $\overline{P}/(N_0 B) = 20$ dB and the system bandwidth $B = 100$ KHz.

(d) Find the SNR when beamforming is used on this MIMO channel with equal weights on each transmit antenna, and compare with the SNR for each of the independent channels obtained via the channel decomposition with the optimal power allocation obtained in part (c).