EE 359 - Wireless Communications - Fall 2003

Homework 4 Solutions

1. (a) Want coherence bandwidth $B_c \leq 10$ MHz.
   So delay spread $T_m \geq 1/B_c 10^{-7}$ sec.

   (b) $\overline{\gamma} = 2$ in this case. Also, $p(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}$. This gives the threshold $\gamma_0 = 0.51$ and capacity $C = 14.8 Mbps$.

   (c) Now $\overline{\gamma} = 1$. This gives for each channel $\gamma_0 = 0.394$ and total capacity $C = 20.56$ Mbps.

   (d) Only the channel with the best instantaneous SNR is kept at each instant and all the average power is sent in it. This is equivalent to having a single channel with SNR $\gamma_{\max} = \max(2\gamma_1, 2\gamma_2)$. Compute the pdf of $\gamma_{\max}$. First compute the cdf

$$\begin{aligned} F_{\gamma_{\max}}(\gamma) &= P(\max(2\gamma_1, 2\gamma_2) < \gamma) \\ &= P(\gamma_1 < \gamma/2, \gamma_2 < \gamma/2) \\ &= P(\gamma_1 < \gamma/2) P(\gamma_2 < \gamma/2) \\ &= [F_{\gamma_1}(\gamma/2)]^2 \end{aligned}$$

   This gives us the pdf

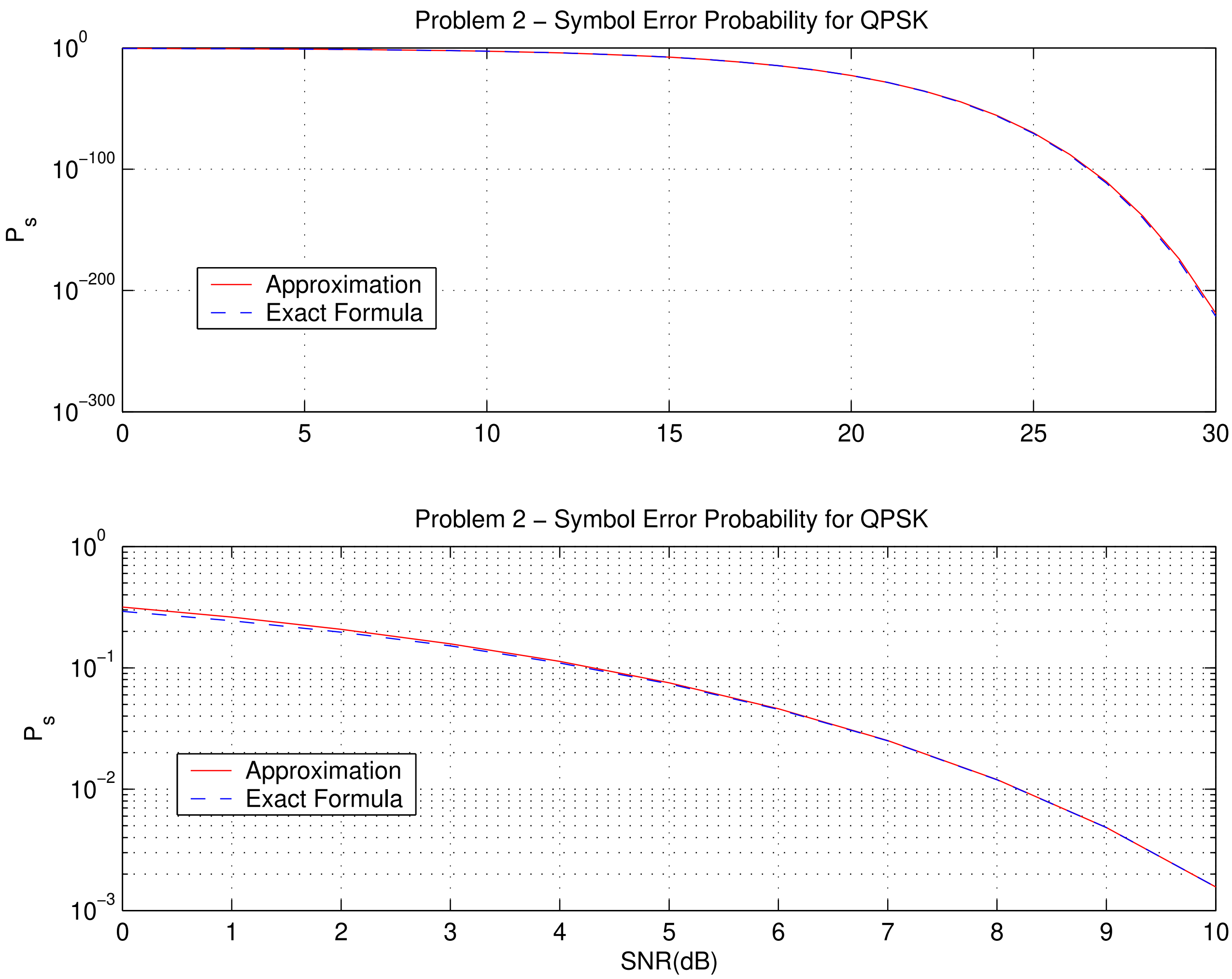
$$\begin{aligned} f_{\gamma_{\max}}(\gamma) &= \frac{\partial}{\partial \gamma} F_{\gamma_{\max}}(\gamma/2) \\ &= F_{\gamma_1}(\gamma/2) f_{\gamma_1}(\gamma/2) \\ &= (1 - e^{-\gamma/2}) e^{-\gamma/2} \end{aligned}$$

   With this distribution, the threshold γ_o is found to be $\gamma_0 = 0.643$. The capacity becomes $C = 18.65$ Mbps.

   (e) Subtracting one output from another cancels out the noise and leaves a clean signal. Thus capacity goes to infinity.

2. See figure. The approximation error decreases with SNR because the approximate formula is based on nearest neighbor approximation which becomes more realistic at higher SNR. The nearest neighbor bound over-estimates the error rate because it over-counts the probability that the transmitted signal is mistaken for something other than its nearest neighbors. At high SNR, this is very unlikely and this over-counting becomes negligible.

3. (a) Setup:
   $f_c = 900\text{MHz} \Rightarrow \lambda = \frac{1}{3}\text{m}$
   $P_r = P_t \left[\frac{1}{12\pi d}\right]^2$, $\text{d} = \sqrt{500^2 + 100^2} = 510\text{km}$
   Determine required P_t:
   $P_b = \frac{1}{2} e^{-\gamma_b} \Rightarrow \gamma_b = 13.12$
   $\gamma_b = P_r / N_0 B \rightarrow P_r = \gamma_b N_0 B = 1.312 \times 10^{-11}\text{mW}$
   Thus, $P_t = 4.848$ W using pathloss formula.



Problem 2 – Symbol Error Probability for QPSK
10^{0}
10^{-100}
10^{-200}
10^{-300}
P_s
0
5
10
15
20
25
30
Approximation
Exact Formula
Problem 2 – Symbol Error Probability for QPSK
10^{0}
10^{-1}
10^{-2}
10^{-3}
P_s
0
1
2
3
4
5
6
7
8
9
10
SNR(dB)
Approximation
Exact Formula

(b) From previous part, need $P_0 = 1.312\times10^{-11}$mW $= -108.82$ dBm to meet the bit rate requirement.
$P_{out} = P[P_r < P_o]$
$.1 = P[\Psi < P_o - \overline{P_r}] = \Phi\left(\frac{-108.82dBm - \overline{P_r}}{8}\right)$
$\Phi^{-1}(.1) =$ -1.2816
$\overline{P_r} =$ -98.5 dBm $= 1.413\times10^{-10}$ mW
$P_t = \overline{P_r}\left(\frac{4\pi d}{\lambda}\right)^2 = 52.2$ W $= 47.2$ dBm

4. (a) Law of Cosines:
$c^2 = a^2 + b^2 - 2ab\cos C$ with a,b $= \sqrt{E_s}$, c $= d_{min}$, C $= \Theta = 22.5$
c $= d_{min} = \sqrt{2E_s(1-\cos 22.5)} = .39\sqrt{E_s}$
Can also use formula from reader

(b) $P_s = \alpha_m Q\left(\sqrt{\beta_m \gamma_s}\right) = 2Q\left(\sqrt{\frac{d_{min}{}^2}{2N_o}}\right) = 2Q(\sqrt{.076\gamma_s})$
$\alpha_m = 2$, $\beta_m = .076$

(c) $\overline{P_e} = \int\limits_0^\infty P_s(\gamma_s) f(\gamma_s) d\gamma_s$
$= \int\limits_0^\infty \alpha_m Q(\sqrt{\beta_m\gamma_s}) f(\gamma_s) d\gamma_s$
Using alternative Q form
$= \frac{\alpha_m}{\pi} \int\limits_0^{\frac{\pi}{2}} \left(1 + \frac{g\overline{\gamma_s}}{(\sin\phi)^2}\right)^{-1} d\phi$ with g $= \frac{\beta_m}{2}$
$= \frac{\alpha_m}{2}\left[1 - \sqrt{\frac{g\overline{\gamma_s}}{1+g\overline{\gamma_s}}}\right] = 1 - \sqrt{\frac{.038\overline{\gamma_s}}{1+.038\overline{\gamma_s}}} = \frac{1}{.076\overline{\gamma_s}}$, where we have used an integral table to evaluate the integral

(d) $P_d = \frac{P_s}{4}$

(e) BPSK: $\overline{P}_b = \frac{1}{4\overline{\gamma}_b} = 10^{-3}$, $\Rightarrow \overline{\gamma_b} = 250$, 16PSK: From above get $\overline{\gamma_s} = 3289.5$
Penalty $= \frac{3289.5}{250} = 11.2$dB
Also will accept $\gamma_b(16PSK) = 822 \Rightarrow = 5.2dB$

5.
```
%Script used to plot the average probability of bit error for BPSK modulation in
%Nakagami fading m = 1, 2, 4.
%Initializations
avg_SNR = [0:0.1:20];
gamma_b_bar = 10.^(avg_SNR/10);
m = [1 2 4];
line = ['-k', '-r', '-b']

for i = 1:size(m,2)
   for j = 1:size(gamma_b_bar, 2)
        Pb_bar(i,j) = (1/pi)*quad8('nakag_MGF',0,pi/2,[],[],gamma_b_bar(j),m(i),1);
   end
   figure(1);
   semilogy(avg_SNR, Pb_bar(i,:), line(i));
   hold on;
end

xlabel('Average SNR ( gamma_b ) in dB');
ylabel('Average bit error probability ( P_b ) ');
title('Plots of P_b for BPSK modulation in Nagakami fading for m = 1, 2, 4');
legend('m = 1', 'm = 2', 'm = 4');
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```
function out = nakag_MGF(phi, gamma_b_bar, m, g);
%This function calculates the m-Nakagami MGF function for the specified values of phi.
%phi can be a vector. Gamma_b_bar is the average SNR per bit, m is the Nakagami parameter
%and g is given by Pb(gamma_b) = aQ(sqrt(2*g*gamma_b)).

out = (1 + gamma_b_bar./(m*(sin(phi).^2)) ).^(-m);
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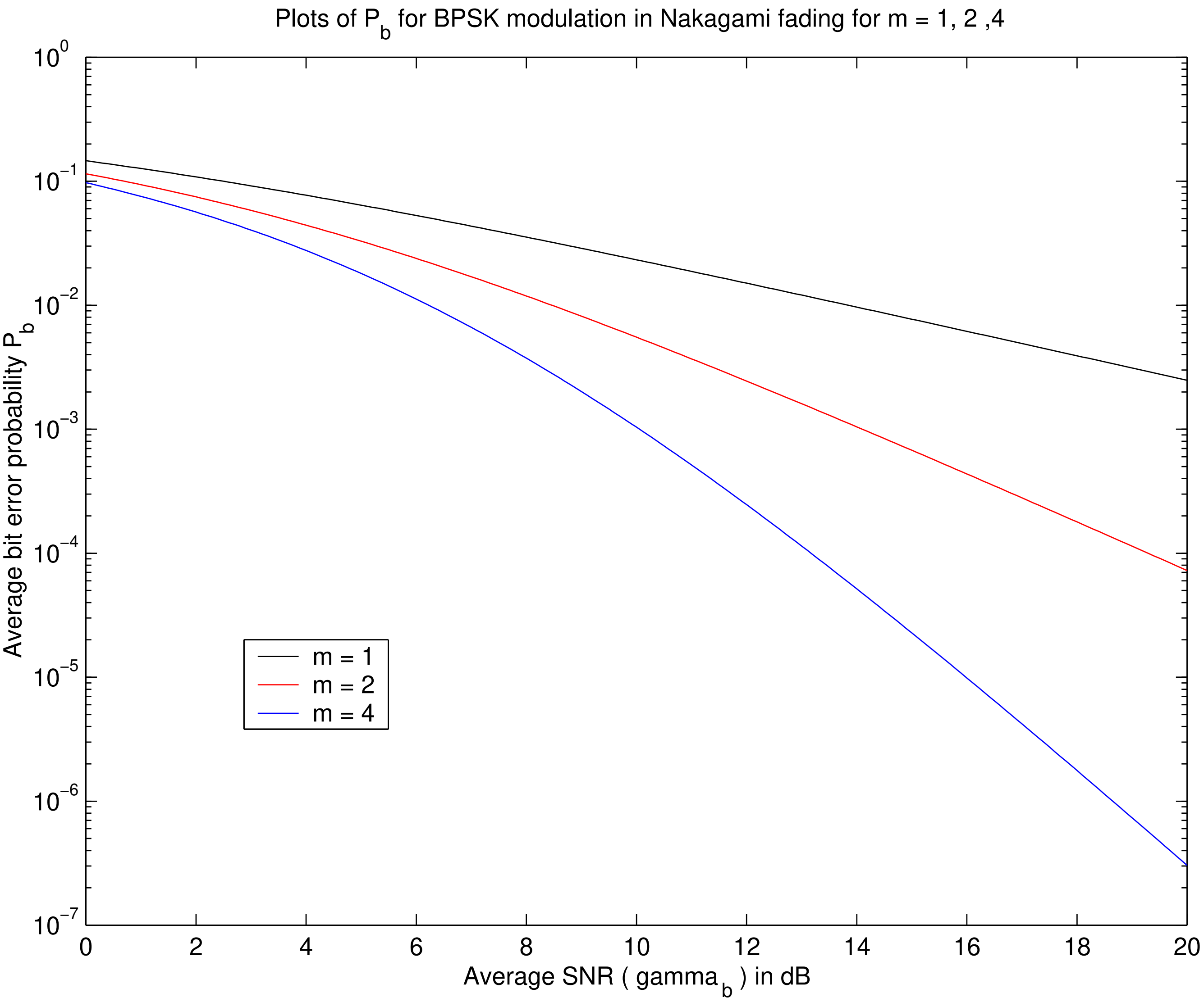

SNR = 10dB

M	BER
1	2.33×10^{-2}
2	5.53×10^{-3}
4	1.03×10^{-3}

6. For DPSK in Rayleigh fading, $\overline{P_b} = \frac{1}{2\overline{\gamma_b}} \Rightarrow \overline{\gamma_b} = 500$
   $N_oB = 3 \times 10^{-12} mW \Rightarrow P_{target} = \overline{\gamma}_b N_0 B = 1.5 \times 10^{-9} mW$ = -88.24 dBm

   Now, consider shadowing:
   $P_{out} = P[P_r < P_{target}] = P[\Psi < P_{target} - \overline{P_r}] = \Phi\left(\frac{P_{target} - \overline{P_r}}{\sigma}\right)$
   $\Rightarrow \Phi^{-1}(.01) = 2.327 = \frac{P_{target} - \overline{P_r}}{\sigma}$
   $\overline{P_r} = -74.28$ dBm $= 3.73 \times 10^{-8}$ mW $= P_t\left(\frac{\lambda}{4\pi d}\right)^2$
   $\Rightarrow$ d = 1372.4 m

7. $\mu_t = 100$ nsec will determine ISI. As long as $T_s \gg \mu_T$, ISI will be negligible. Let $T_s = 5\ \mu_T$ (values of 2-10 μ_T are fine). Then $R \leq \frac{2bits}{symbol}\frac{1}{T_s}\frac{symbols}{sec} = 4$Mbps

$B_d \approx f_d$ will determine lower bound due to irreducible error floor. Using Fig 5.5 for DQPSK, $f_D T_s \leq 0.002 \rightarrow T_s \leq 2.5 \times 10^{-5}$ sec. Thus $R \geq 80$ Kbps

80Kbps $\leq$ R $\leq$ 4Mbps