EE 359 - Wireless Communications - Fall 2003

Homework 4: Due 10/30 at noon.

1. (15 points) Consider a frequency-selective fading channel with total bandwidth $B = 20$ MHz

   (a) For what maximum delay spread on the channel can you divide the frequency-selective fading channel into two subchannels with approximately independent fading on each subband?

   Given two independently fading subchannels from part (a), let the input/output on each subchannel be defined by

$$\begin{aligned} y_1(i) &= \sqrt{\gamma_1(i)}x_1(i) + n_1(i) \\ y_2(i) &= \sqrt{\gamma_2(i)}x_2(i) + n_2(i) \end{aligned}$$

   where during the i^{th} transmission $x_1(i), x_2(i)$ are the inputs to the subchannels, $\gamma_1(i), \gamma_2(i)$ are the i.i.d. exponentially distributed ($\gamma \sim e^{-\gamma}$) random powers (Rayleigh fading) associated with each subchannel, and $n_1(i), n_2(i)$ are i.i.d. zero mean, unit variance Gaussian random variables (AWGN). The channel fades are known to the transmitter and the receiver perfectly. Assume that if the total power is divided equally between the two subchannels then the average received SNR on each subchannel is 0 db.

   (b) Find the channel capacity when only one channel is used with all power allocated to that channel.

   (c) Find capacity when both channels are used and the average power is equally split between the two channels.

   (d) Find the capacity when only the channel with the best instantaneous SNR is used at each instant.

   (e) Show that if $n_1(i) = n_2(i)$ then an infinite capacity can be achieved.

2. (15 points) Plot the symbol error probability P_s for QPSK using the approximation (6.11) and Craig's exact result (6.18) for $0 \leq \gamma_s \leq 30$ dB. Does the error in the approximation (6.11) increase or decrease with γ_s and why?

3. (15 points) This problem illustrates why satellite systems that must compensate for shadow fading are going bankrupt. Consider a LEO satellite system orbiting 500 Km above the earth. Assume the signal follows a free space path loss model with no multipath fading or shadowing. The transmitted signal has a carrier frequency of 900 MHz and a bandwidth of 10 KHz. The handheld receivers have noise spectral density of 10^{-16} (total noise power is N_oB) mW/Hz. Assume nondirectional antennas (0 dB gain) at both the transmitter and receiver. Suppose the satellite must support users in a circular cell on the earth of radius 100 Km at a BER of 10^{-6}.

   (a) For DPSK modulation what transmit power is needed such that all users in the cell meet the 10^{-6} BER target.

   (b) Repeat part (a) assuming that the channel also experiences log normal shadowing with $\sigma = 8$ dB, and that users in a cell must have $P_b = 10^{-6}$ (for each bit) with probability 0.9.

4. (15 points) In this problem we explore the power penalty involved in going to higher level signal modulations, i.e. from BPSK to 16PSK.

   (a) Find the minimum distance between constellation points in 16PSK modulation as a function of signal energy E_s.

   (b) Find α_M and β_M such that the symbol error probability of 16PSK in AWGN is approximately

   $$P_s \approx \alpha_M Q\left(\sqrt{\beta_M \gamma_s}\right).$$

   (c) Using your expression in part (b), find an approximation for the average symbol error probability of 16PSK in Rayleigh fading in terms of $\overline{\gamma}_s$.

   (d) Convert the expressions for average symbol error probability of 16PSK in Rayleigh fading to expressions for average bit error probability assuming Gray coding.

   (e) Find the approximate value of $\overline{\gamma}_b$ required to obtain a BER of 10^{-3} in Rayleigh fading for BPSK and 16PSK. What is the power penalty in going to the higher level signal constellation at this BER?

5. (15 points) The Nakagami distribution for fading signal power was briefly mentioned in lecture and is given by equation (3.36) in the Reader (3.37 for corresponding power). The Nakagami distribution is parameterized by m, which ranges from $m = .5$ to $m = \infty$. The m parameter measures the ratio of LOS signal power to multipath power, so $m = 1$ corresponds to Rayleigh fading, $m = \infty$ corresponds to an AWGN channel with no fading, and $m = .5$ corresponds to fading that is "worse than Rayleigh.". In this problem we explore the impact of the parameter m on the performance of BPSK modulation in Nakagami fading

   Plot the average bit error $\overline{P_b}$ of BPSK modulation in Nakagami fading with average SNR ranging from 0 to 20dB for m parameters $m = 1$ (Rayleigh), $m = 2$, and $m = 4$ (You can integrate the regular Q function against the Nakagami distribution if you want to torture yourself, but the Moment Generating Function technique of Section 6.4.3 leading to Equation 6.40 is much less painful). At an average SNR of 10 dB, what is the difference in average BER?

6. (15 points) Assume a cellular system with log-normal shadowing plus Rayleigh fading. The signal modulation is DPSK. The service provider has determined that it can deal with an outage probability of .01, i.e. 1 in 100 customers are unhappy at any given time. In nonoutage the voice BER requirement is $\overline{P}_b = 10^{-3}$. Assume a noise power spectral density of $N_o = 10^{-16}$ mW/Hz, a signal bandwidth of 30 KHz, a carrier frequency of 900 MHz, free space path loss propagation with nondirectional antennas, and shadowing standard deviation of $\sigma = 6$ dB. Find the maximum cell size that can achieve this performance if the transmit power at the mobiles is limited to 100 mW.

7. (10 points) Consider a wireless channel with an average delay spread of 100 nsec and a doppler spread of 80 Hz. Given the error floors due to doppler and ISI, for DQPSK modulation approximately what range of data rates can be transmitted over this channel with a BER less than 10^{-4}.