EE 359 - Wireless Communications - Fall 2003

Homework 3 Solution

1. (a) Outdoor, since delay spread $\approx 10\ \mu$sec.
   Consider that 10 μsec $\Rightarrow$ d = ct = 3km difference between length of first and last path

   (b) Scattering function
   $$S(\tau,\rho) = F_{\Delta t}[A_c(\tau,\Delta t)]$$
   $= \frac{\pi}{W} rect\left(\frac{\pi}{W}\rho\right)$ for $0 \le \tau \le 10\mu sec$

   (c) Avg Delay Spread $= \frac{\int_0^\infty \tau A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau} = 5\mu$sec

   RMS Delay Spread $= \sqrt{\frac{\int_0^\infty (\tau-\mu_{Tm})^2 A_c(\tau)d\tau}{\int_0^\infty A_c(\tau)d\tau}} = 2.89\mu$sec

   Doppler Spread $= \frac{W}{2\pi} = \frac{50}{\pi}$ Hz

   (d) $\beta_u >$ Coherence BW $\Rightarrow$ Freq. Selective Fading $\approx \frac{1}{T_m} = 10^5 \Rightarrow \beta_u >= 10^5$ kHz
   Can also use μ_{Tm} or σ_{Tm} instead of T_m

   (e) Rayleigh fading, since receiver power is evenly distributed relative to delay; no dominant LOS path

   (f) $t_R = \frac{e^{\rho^2}-1}{\rho F_D\sqrt{2\pi}}$ with $\rho = 1$, $f_D = \frac{W}{2\pi} \rightarrow\ t_r = .043$ sec

   (g) Assuming that all fades are of duration t_R, then can send at rate $\frac{2}{t_R} = 46.5$ symbols/sec

2. (a) $T_m \approx .1$msec $= 100\mu$sec
   $B_d \approx .1$Hz

   (b) $B_c \approx \frac{1}{T_m} = 10$kHz
   $\Delta f > 10$kHz for $u_1 \perp u_2$

   (c) $(\Delta t)_c = 10$s

   (d) 3kHz $< B_c \Rightarrow$ Flat
   30kHz $> B_c \Rightarrow$ Freq. Selective

3. (a) Maximize capacity given by
   $$C = \max_{S(\gamma):\int S(\gamma)p(\gamma)d\gamma=\overline{S}} \int_\gamma B\log\left(1+\frac{S(\gamma)\gamma}{\overline{S}}\right)p(\gamma)d\gamma.$$
   Construct the Lagrangian function
   $$\mathcal{L} = \int_\gamma B\log\left(1+\frac{S(\gamma)\gamma}{\overline{S}}\right)p(\gamma)d\gamma - \lambda\int\frac{S(\gamma)}{\overline{S}}p(\gamma)d\gamma$$

Taking derivative with respect to $S(\gamma)$, (refer to discussion section notes) and setting it to zero, we obtain,

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \geq \gamma_0 \\ 0 & \gamma < \gamma_0 \end{cases}$$

Now, the threshold value must satisfy

$$\int_{\gamma_0}^{\infty} \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right) p(\gamma) d\gamma = 1$$

Evaluating this with $p(\gamma) = \frac{1}{10} e^{-\gamma/10}$, we have

$$\begin{aligned} 1 &= \frac{1}{10\gamma_0} \int_{\gamma_0}^{\infty} e^{-\gamma/10} d\gamma - \frac{1}{10} \int_{\gamma_0}^{\infty} \frac{e^{-\gamma/10}}{\gamma} d\gamma & (1) \\ &= \frac{1}{\gamma_0} e^{-\gamma_0/10} - \frac{1}{10} \int_{\frac{\gamma_0}{10}}^{\infty} \frac{e^{-\gamma}}{\gamma} d\gamma & (2) \\ &= \frac{1}{\gamma_0} e^{-\gamma_0/10} - \frac{1}{10} \text{EXPINT}(\gamma_0/10) & (3) \end{aligned}$$

where EXPINT is as defined in matlab. This gives $\gamma_0 = 0.7676$. The power adaptation becomes

$$\frac{S(\gamma)}{\overline{S}} = \begin{cases} \frac{1}{0.7676} - \frac{1}{\gamma} & \gamma \geq 0.7676 \\ 0 & \gamma < 0.7676 \end{cases}$$

(b) Capacity can be computed as

$$C/B = \frac{1}{10} \int_{0.7676}^{\infty} \log\left(\gamma/0.7676\right) e^{-\gamma/10} d\gamma = 2.0649 \ \text{ nats/sec/Hz.}$$

Note that I computed all capacites in nats/sec/Hz. This is because I took the natural log. In order to get the capacity values in bits/sec/Hz, the capacity numbers simply need to be divided by natural log of 2.

(c) AWGN capacity $C/B = \log(1 + 10) = 2.3979$ nats/sec/Hz.

(d) Capacity when only receiver knows γ

$$C/B = \frac{1}{10} \int_{0}^{\infty} \log\left(1 + \gamma\right) e^{-\gamma/10} d\gamma = 2.0150 \ \text{ nats/sec/Hz.}$$

(e) Capacity using channel inversion is ZERO because the channel can not be inverted with finite average power. Threshold for outage probability 0.05 is computed as

$$\frac{1}{10} \int_{\gamma_0}^{\infty} e^{-\gamma/10} d\gamma = 0.95$$

which gives $\gamma_0 = 0.5129$. This gives us the capacity with truncated channel inversion as

$$\begin{aligned} C/B &= \log\left[1 + \frac{1}{\frac{1}{10}\int_{\gamma_0}^{\infty} \frac{1}{\gamma} e^{-\gamma/10} d\gamma}\right] * 0.95 & (4) \\ &= \log\left[1 + \frac{1}{\frac{1}{10}\text{EXPINT}(\gamma_0/10)}\right] * 0.95 & (5) \\ &= 1.5463 \ \text{ nats/s/Hz.} & (6) \end{aligned}$$

(f) Channel Mean=-5 dB = 0.3162. So for perfect channel knowledge at transmitter and receiver we compute $\gamma_0 = 0.22765$ which gives capacity $C/B = 0.36$ nats/sec/Hz.
With AWGN, $C/B = \log(1 + 0.3162) = 0.2748$ nats/sec/Hz.
With channel known only to the receiver $C/B = 0.2510$ nats/sec/Hz.

Capacity with AWGN is always greater than or equal to the capacity when only the reciever knows the channel. This can be shown using Jensen's inequality. However the capacity when the transmitter knows the channel as well and can adapt its power, can be higher than AWGN capacity specially at low SNR. At low SNR, the knowledge of fading helps to use the low SNR more efficiently.

4. (a) If neither transmitter nor receiver knows when the interferer is on, they must transmit assuming worst case, i.e. as if the interferer was on all the time,

$$C = B\log\left(1 + \frac{\overline{S}}{N_0B + \overline{I}}\right) = 10.7Kbps.$$

(b) Suppose we transmit at power S_1 when jammer is off and S_2 when jammer is off,

$$C = B\max\left[\log\left(1 + \frac{S_1}{N_oB}\right)0.75 + \log\left(1 + \frac{S_2}{N_oB + \overline{I}}\right)0.25\right]$$

subject to

$$0.75S_1 + 0.25S_2 = \overline{S}.$$

This gives $S_1 = 12.25mW$, $S_2 = 3.25mW$ and $C = 53.21Kbps$.

(c) The jammer should transmit $-x(t)$ to completely cancel off the signal.