EE 359 - Wireless Communications - Fall 2003

Homework 3: Due 10/23.

1. (30 pts) Consider a wideband channel characterized by the autocorrelation function

$$A_c(\tau, \Delta t) = \begin{cases} \operatorname{sinc}(W\Delta t/\pi) & 0 \leq \tau \leq 10\mu\text{sec.} \\ 0 & \text{else} \end{cases},$$

where $W = 100$Hz and $sinc(x) = \sin(\pi x)/(\pi x)$.

   (a) Does this channel correspond to an indoor channel or an outdoor channel and why?

   (b) Sketch the scattering function of this channel.

   (c) Compute the channel's average delay spread, rms delay spread, and Doppler spread.

   (d) Over approximately what range of data rates will a signal transmitted over this channel exhibit frequency-selective fading?

   (e) Would you expect this channel to exhibit Rayleigh or Ricean fading statistics and why?

   (f) Assuming that the channel exhibits Rayleigh fading, what is the average length of time that the signal power is continuously below its average value.

   (g) Assume that your system has error correction coding that can correct two simultaneous errors. Assume also that you always make an error if the received signal power is below its average value, and never make an error if this power is at or above its average value. If the channel is Rayleigh fading then what is the maximum data rate that can be sent over this channel with error-free transmission, making the approximation that the fade duration never exceeds its average value.

2. (20 pts) Let a scattering function $S(\tau, \rho)$ be nonzero over $0 \leq \tau \leq .1$ ms and $-.1 \leq \rho \leq .1$ Hz. Assume that the power of the scattering function is approximately uniform over the range where it is nonzero.

   (a) What are the multipath spread and the doppler spread of the channel?

   (b) For two sinusoidal inputs to the channel $u_1(t) = \sin 2\pi ft$ and $u_2(t) = \sin 2\pi(f + \Delta f)t$, what is the minimum value of Δf for which the channel response to $u_1(t)$ is approximately independent of the channel response to $u_2(t)$.

   (c) For two sinusoidal inputs to the channel $u_1(t) = \sin 2\pi ft$ and $u_2(t) = \sin 2\pi f(t + \Delta t)$, what is the minimum value of Δt for which the channel response to $u_1(t)$ is approximately independent of the channel response to $u_2(t)$.

   (d) Will this channel exhibit flat fading or frequency-selective fading for a typical voice (3KHz) channel? How about for a cellular channel (30KHz)?

3. (30 pts.) Assume a discrete-time Rayleigh fading channel, where at each time i the transmitter and receiver know $\gamma[i]$, and the distribution of the fading SNR $p(\gamma)$ is exponential with mean $\overline{\gamma} = 10$dB.

   (a) Derive the cutoff value γ_0 and the corresponding power adaptation that maximizes capacity for this channel.

   (b) Compute the capacity of this channel using the optimal power and rate adaptation.

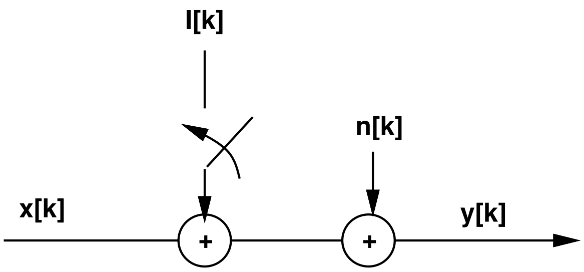
(c) Compare your answer in part (b) with the channel capacity in AWGN with the same average power.

(d) Compare your answer in part (b) with the channel capacity where only the receiver knows $\gamma[i]$.

(e) Compare your answer in part (b) with the channel capacity using channel inversion and truncated channel inversion (with outage probability .05) for the power adaptation.

(f) Repeat parts b, c, and d (i.e. obtain the capacity with perfect transmitter and receiver side information, in AWGN, and with just receiver side information) for the same fading distribution but with mean $\overline{\gamma} = -5$dB. Describe the circumstances under which a fading channel has higher capacity than an AWGN channel with the same average SNR and explain why this behaivor occurs.

4. (20 points) Time-Varying Interference: This problem illustrates the capacity gains that can be obtained from interference estimation, and how a malicious jammer can wreak havoc on your link performance. Consider the following interference channel.



The channel has a combination of AWGN $n[k]$ and interference $I[k]$. We model $I[k]$ as AWGN. The interferer is on (i.e. the switch is down) with probability .25 and off (i.e. the switch is up) with probability .75. The average transmit power is 10 mW, the noise spectral density is 10^{-8} W/Hz, the channel bandwidth B is 10 KHz (receiver noise power is $N_o B$), and the interference power (when on) is 9 mW.

(a) What is the capacity of the channel if neither transmitter nor receiver know when the interferer is on?

(b) What is the capacity of the channel if both transmitter and receiver know when the interferer is on?

(c) Suppose now that the interferer is a malicious jammer with perfect knowledge of $x[k]$ (so the interferer is no longer modeled as AWGN). Assume that neither transmitter nor receiver have knowledge of the jammer behavior. Assume also that the jammer is always on and has an average transmit power of 10 mW. What strategy should the jammer use to minimize the SNR of the received signal?