EE 359 - Wireless Communications - Fall 2003

Homework 2 Solutions

1. (a)

$$\begin{aligned} P\left[P_r > P_0\right] &= P\left[P_t + 10\log(K) + \psi_{dB} - 10\gamma\log\left(\frac{r}{r_0}\right) > P_0\right] \\ &= P\left[\psi_{dB} > P_0 - \left(P_t + 10\log(K) - 10\gamma\log\left(\frac{r}{r_0}\right)\right)\right] \\ &= Q\left(\frac{P_0 - \left(P_t + 10\log(K) - 10\gamma\log\left(\frac{r}{r_0}\right)\right)}{\sigma_{\psi dB}}\right) \end{aligned}$$

Plugging in values→ Q(0) = 0.5

(b)

$$C = \frac{2}{R^2}\int_{r=0}^{R} rQ\left(a + b\ln\frac{r}{R}\right)dr$$

To perform integration by parts, we let $du = rdr$ and $v = Q\left(a + b\ln\frac{r}{R}\right)$. Then $u = \frac{1}{2}r^2$ and

$$dv = \frac{\partial}{\partial r}Q\left(a + b\ln\frac{r}{R}\right) = \frac{\partial}{\partial x}Q(x)|_{x=a+b\ln(r/R)}\frac{\partial}{\partial r}\left(a + b\ln\frac{r}{R}\right) = \frac{1}{\sqrt{2\pi}}\exp(-k^2/2)\frac{b}{r}dr. \qquad (1)$$

where $k = a + b\ln\frac{r}{R}$. Then we get

$$\begin{aligned} C &= \frac{2}{R^2}\left[\frac{1}{2}r^2 Q\left(a + b\ln\frac{r}{R}\right)\right]_{r=0}^{R} - \frac{2}{R^2}\int_{r=0}^{R}\frac{1}{2}r^2\frac{1}{\sqrt{2\pi}}\exp(-k^2/2)\frac{b}{r}dr & (2) \\ &= Q(a) - \frac{1}{R^2}\int_{r=0}^{R} r^2\frac{1}{\sqrt{2\pi}}\exp(-k^2/2)\frac{b}{r}dr & (3) \\ &= Q(a) - \frac{1}{R^2}\int_{r=0}^{R}\frac{1}{\sqrt{2\pi}}R^2\exp\left(\frac{2(k-a)}{b}\right)\exp(-k^2/2)\frac{b}{r}dr & (4) \\ &= Q(a) - \int_{k=-\infty}^{a}\frac{1}{\sqrt{2\pi}}\exp\left(\frac{-k^2}{2} + \frac{2k}{b} - \frac{2a}{b}\right)dk & (5) \\ &= Q(a) - \exp\left(\frac{-2a}{b} + \frac{2}{b^2}\right)\int_{k=-\infty}^{a}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(k - \frac{2}{b})^2\right)dk & (6) \\ &= Q(a) - \exp\left(\frac{2-2ab}{b^2}\right)Q\left(\frac{2-ab}{b}\right) & (7) \end{aligned}$$

(c)

$\frac{\gamma}{\sigma}$	2	4	6
5	0.742	0.833	0.877
10	0.651	0.742	0.797

Since $\hat{P}_r(r) \geq P_{min}$ for all $r \leq R$, the probability of non-outage is proportional to $Q\left(\frac{-1}{\sigma}\right)$, and thus decreases as a function of σ. Therefore, C decreases as a function of σ. Since the average power at the boundary of the cell is fixed, C increases with γ, because it forces higher transmit power, hence more received power at $r < R$.

(d)

γ	C
2	0.9667
4	0.9805
6	0.9863

Using the same logic as above, C increases with γ. However, more slowly since the 5dB margin of error smooths the $10\gamma \log\left(\frac{r}{R}\right)$ term.

2. Use CDF strategy.

$$F_z(z) = P[x^2+y^2 \leq z^2] = \int\int_{x^2+y^2 \leq z^2} \frac{1}{2\pi\sigma^2} e^{\frac{-(x^2+y)^2}{2\sigma^2}} dxdy = \int_0^{2\pi}\int_0^{z} \frac{1}{2\pi\sigma^2} e^{\frac{-r^2}{2\sigma^2}} rdrd\theta = 1 - e^{\frac{-z^2}{2\sigma^2}} \, (z \geq 0)$$

$$\frac{df_z(z)}{dz} = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}} \rightarrow Rayleigh$$

For Power:

$$F_{z^2}(z) = P[Z \leq \sqrt{z}] = 1 - e^{\frac{-z^2}{2\sigma^2}}$$

$$f_z(z) = \frac{1}{2\sigma^2} e^{\frac{-z}{2\sigma^2}} \rightarrow Exponential$$

3. (a) $P_{outage} = F_{z^2}(P_0) = 1 - e^{\frac{-P_0}{\Omega_p}}$ where $\Omega_p = 20dB$

$P_{outage} = \begin{matrix} 0.0311 & P_0 = 5dB \\ 0.0951 & P_0 = 10dB \end{matrix}$

(b)

$$P_{outage} = 0.01 = 1 - e^{\frac{-P_0}{\Omega_p}} \rightarrow \Omega_p = 995 \approx 30dB$$

(c)

$$P_{outage} = P[P_r < P_0]$$

$$P\left[\sqrt{P_r} < \sqrt{P_0}\right], \texttt{ where} \sqrt{P_r} is\ Ricean$$

$$= \int_{0}^{\sqrt{P_0}} \frac{x}{\sigma^2} exp(-\frac{x^2+s^2}{2\sigma^2}) I_0(\frac{x}{\sigma^2}s)dx$$

Numerically integrate over this equations, or:

$$y \doteq \frac{x}{\sigma} \rightarrow P_{outage}{=}1{-} \int_{\frac{\sqrt{P_0}}{\sigma}}^{\infty} yexp(-\frac{y^2{+}(\frac{s}{\sigma})^2)}{2}) I_0(y\frac{s}{\sigma}dx$$

$$= 1 - Q_m\left(\frac{s}{\sigma}, \frac{\sqrt{P_0}}{\sigma}\right)$$

$$= 1 - Q_m\left(\frac{10}{\sqrt{5}}, \frac{\sqrt{10}}{\sqrt{5}}\right) = 0.057\%$$

4. (a) W = average received power
   Z_i = Shadowing over link i
   P_{ri} = Received power in dBW, which is Gaussian with mean W, variance σ^2

   (b)

$$P_{outage} = P\left[P_{r,1} < T \cap P_{r,2} < T\right] = P\left[P_{r,1} < T\right] P[P_{r,2} < T] \ since \ Z_1, Z_2 \ independent$$

$$= \left[Q\left(\frac{W-T}{\sigma}\right)\right]^2 = \left[Q\left(\frac{\triangle}{\sigma}\right)\right]^2$$

   (c)

$$P_{out} = \int_{-\infty}^{\infty} P\left[P_{r,1} \leq T \ , \ P_{r,2} < T | Y=y\right] f_y\left(y\right) dy$$

$$P_{r,1}|Y=y \ \tilde{} \ N\left(W+by{,}a^2\sigma^2\right), \ and \ \left[P_{r,1}|Y=y\right] \perp \left[P_{r,2}|Y=y\right]$$

$$P_{outage}{=} \int_{-\infty}^{\infty} \left[Q\left(\frac{W+by-T}{a\sigma}\right)\right]^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}dy}$$

let $\frac{y}{\sigma}= u$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{W-T+bu\sigma}{a\sigma}\right)\right]^2 e^{\frac{-u^2}{2}} du = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{\triangle+by\sigma}{a\sigma}\right)\right]^2 e\frac{-y^2}{2}dy$$

(d) Let a = b = $\frac{1}{\sqrt{2}}$, $\sigma = 8$, $\triangle = 5$. With independent fading we get

$$P_{out} = \left[Q\left(\frac{5}{8}\right)\right]^2 = 0.0708.$$

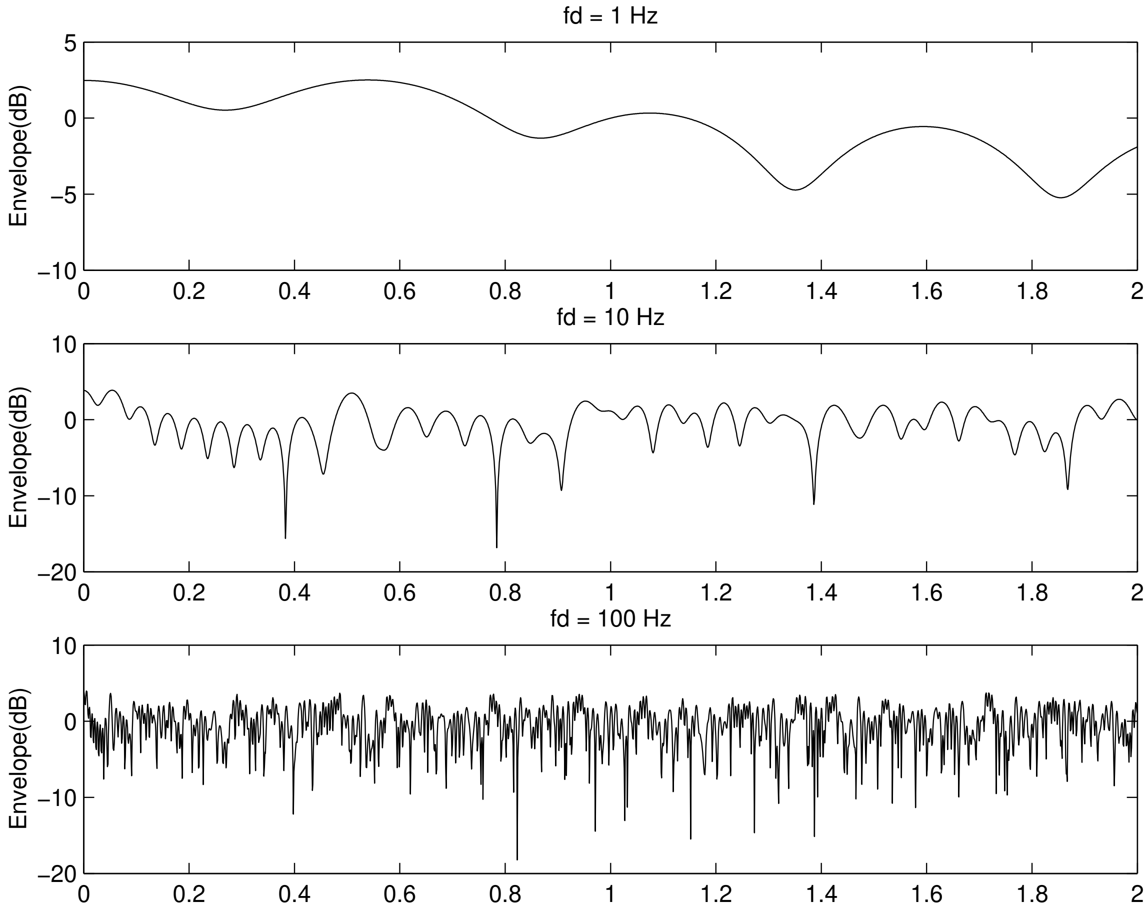
With correlated fading we get $P_{out} = 0.1316$.
Conclusion : Independent shadowing is prefarable for diversity.

5. There are many acceptable techniques for this problem. Sample code for both the stochastic technique(preferred) and the Jake's technique are included.

   Jakes: Summing of appropriate sine waves

```
%Jake's Method
close all;
clear all;
%choose N=30
N=30;
M=0.5*(N/2-1);
Wn(M)=0;
beta(M)=0;
%We choose 1000 samples/sec
ritemp(M,2001)=0;
rqtemp(M,2001)=0;
rialpha(1,2001)=0;
fm=[1 10 100];
Wm=2*pi*fm;
for i=1:3
    for n=1:1:M
        for t=0:0.001:2
            %Wn(i)=Wm*cos(2*pi*i/N)
            Wn(n)=Wm(i)*cos(2*pi*n/N);
            %beta(i)=pi*i/M
            beta(n)=pi*n/M;
            %ritemp(i,20001)=2*cos(beta(i))*cos(Wn(i)*t)
            %rqtemp(i,20001)=2*sin(beta(i))*cos(Wn(i)*t)
            ritemp(n,1000*t+1)=2*cos(beta(n))*cos(Wn(n)*t);
            rqtemp(n,1000*t+1)=2*sin(beta(n))*cos(Wn(n)*t);
            %Because we choose alpha=0,we get sin(alpha)=0 and cos(alpha)=1
            %rialpha=(cos(Wm*t)/sqrt(2))*2*cos(alpha)=2*cos(Wm*t)/sqrt(2)
            %rqalpha=(cos(Wm*t)/sqrt(2))*2*sin(alpha)=0
            rialpha(1,1000*t+1)=2*cos(Wm(i)*t)/sqrt(2);
        end
    end
    %summarize ritemp(i) and rialpha
    ri=sum(ritemp)+rialpha;
    %summarize rqtemp(i)
    rq=sum(rqtemp);
    %r=sqrt(ri^2+rq^2)
    r=sqrt(ri.^2+rq.^2);
    %find the envelope average
```

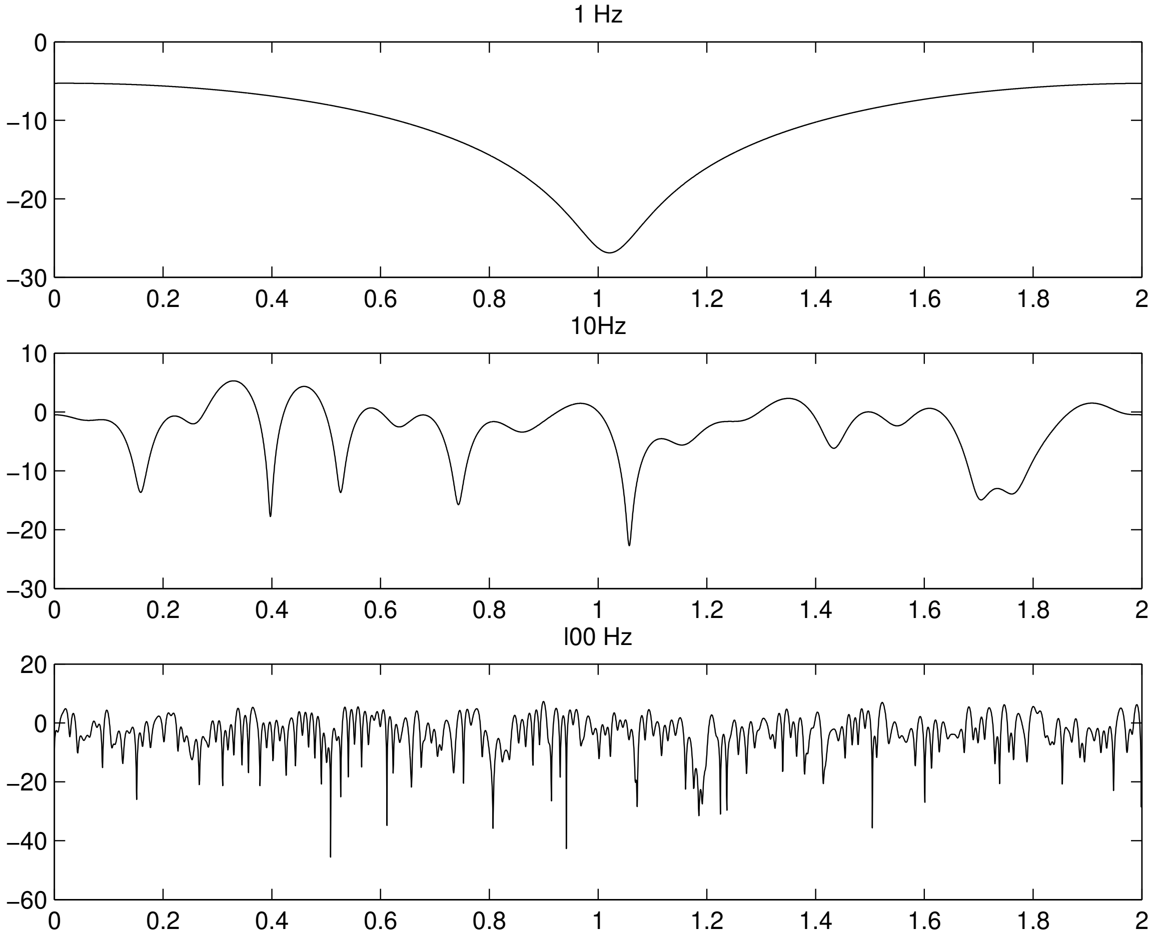
```
    mean=sum(r)/2001;
    subplot(3,1,i);
    time=0:0.001:2;
    %plot the figure and shift the envelope average to 0dB
    plot(time,(10*log10(r)-10*log10(mean)));
    titlename=['fd = ' int2str(fm(i)) ' Hz'];
    title(titlename);
    xlabel('time(second)');
    ylabel('Envelope(dB)');
end
```



Stochastic: Usually two guassian R.V.'s are filtered by the Doppler Spectrum and summed. Can also just do a Rayleigh distribution with an adequate LPF, although the above technique is prefered.

```
function [Ts, z_dB] = rayleigh_fading(f_D, t, f_s)
%
% function [Ts, z_dB] = rayleigh_fading(f_D, t, f_s)
% generates a Rayleigh fading signal for given Doppler frequency f_D,
% during the time perios [0, t], with sampling frequency f_s >= 1000Hz.
%
% Input(s)
%   -- f_D  : [Hz] [1x1 double] Doppler frequency
%   -- t    : simulation time interval length, time interval [0,t]
%   -- f_s  : [Hz] sampling frequency, set to 1000 if smaller.
% Output(s)
%   -- Ts   : [Sec][1xN double] time instances for the Rayleigh signal
%   -- z_dB : [dB] [1xN double] Rayleigh fading signal
%

% Required parameters
if f_s < 1000;
  f_s = 1000;               % [Hz]  Minumum required sampling rate
end;
N = ceil( t*f_s );          % Number of samples
```

```
% Ts contains the time instances at which z_dB is specified
Ts = linspace(0,t,N);

if mod( N, 2) == 1
  N = N+1;                    % Use even number of samples in calculation
end;
f = linspace(-f_s,f_s,N);     % [Hz]  Frequency samples used in calculation

% Generate complex Gaussian samples with line spectra in frequency domain
% Inphase :
Gfi_p = randn(2,N/2);
CGfi_p = Gfi_p(1,:)+i*Gfi_p(2,:);
CGfi = [ conj(fliplr(CGfi_p)) CGfi_p ];

% Quadrature :
Gfq_p = randn(2,N/2);
CGfq_p = Gfq_p(1,:)+i*Gfq_p(2,:);
CGfq = [ conj(fliplr(CGfq_p)) CGfq_p ];

% Generate fading spectrum, this is used to shape the Gaussian line spectra
omega_p = 1;   % this makes sure that the average received envelop can be 0dB
S_r = omega_p/4/pi./(f_D*sqrt(1-(f/f_D).^2));

% Take care of samples outside the Doppler frequency range, let them be 0
idx1 = find(f>f_D);
idx2 = find(f<-f_D);
S_r(idx1) = 0;
S_r(idx2) = 0;

% Generate r_I(t) and r_Q(t) using inverse FFT:
r_I = N*ifft(CGfi.*sqrt(S_r));
r_Q = -i*N*ifft(CGfq.*sqrt(S_r));

% Finally, generate the Rayleigh distributed signal envelope
z = sqrt(abs(r_I).^2+abs(r_Q).^2);
z_dB = 20*log10(z);

% Return correct number of points
z_dB = z_dB(1:length(Ts));
```


1 Hz
0
−10
−20
−30
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
10Hz
10
0
−10
−20
−30
l00 Hz
20
0
−20
−40
−60