EE 359 - Wireless Communications - Fall 2003

Homework 2: Due Thursday 10/16.

1. (25pts.) The coverage area of a cellular system is the percentage of locations within a cell with received power P_r that exceeds a desired received power threshold P_o. In this problem we investigate the formula for coverage area for a cell with combined path loss and shadowing and show how this coverage area varies with the path loss exponent and the variance of the shadowing. The received signal power under combined path loss and shadowing is given by $P_r(r) = P_t + 10\log_{10} K - 10\gamma\log_{10}(r/r_0) + \psi_{dB}$, where r is the distance from the transmitter, r_0 is the far field distance, and all terms are in dB units except P_r and P_t, which are typically given in dBW (dB relative to a Watt) or dBm (dB relative to a milliwatt). We assume that ψ_{dB} is Gaussian. The average received signal power at distance r is given by the path loss formula alone: $\overline{P_r}(r) = P_t + 10\log_{10} K - 10\gamma\log_{10}(r/r_0)$. (Note that in the reader the P_t is missing from Equations 2.39, 2.40, and the expression for $\overline{P}_R$ just after Equation 2.42: you should correct these omissions in the reader, especially for this problem, by changing the $10\log_{10} K$ is these formulae to $P_t + 10\log_{10} K$).

   (a) Suppose the desired threshold value for your received signal power is P_{min} dBm. Show that the nonoutage probability $p(P_r(r) \geq P_{min})$ as a function of $\overline{P_r}(r)$, P_{min}, and σ_ψ is given by (2.39) in the Reader. Evaluate this formula for $P_{min} = 10$ dBm, $P_t = 30$ dBm, $K = 1$, $\gamma = 2$, $r_0 = 1$m, $\sigma_\psi = 6$ dB, and $r = 10$ m.

   (b) Show using integration by parts that the integral (2.41) in the Reader for C yields

   $$C = Q(a) + \exp\left(\frac{2-2ab}{b^2}\right) Q\left(\frac{2-ab}{b}\right).$$

   where $a = (P_{min} - \overline{P_r}(R))/\sigma_\psi$ and $b = (10\gamma\log e)/\sigma_\psi$.

   (c) Assume $P_{min} = \overline{P_r}(R)$ so $a = 0$. Compute a table of C values for the following parameters: $\gamma = 3, 5$, and $\sigma = 5, 10$ dB. Does C increase or decrease with σ? Why? Does C increase or decrease with γ? Why?

   (d) For $\sigma = 4$ dB compute C for $\gamma = 2, 4, 6$ and $\overline{P_r}(R) - P_{min} = 5$. Comment on how C changes relative to γ when a is no longer zero.

2. (10 pts.) Prove that for X and Y independent zero-mean Gaussian random variables with variance σ^2, the distribution of $Z = \sqrt{X^2 + Y^2}$ is Rayleigh-distributed and the distribution of Z^2 is exponentially-distributed.

3. (25 pts.) Outage probability was defined in Homework 1 to be the probability that signal power falls below a given threshold value P_o. Outage probability is typically applied to signals which vary slowly with time, since once the signal falls below the threshold, it is likely to stay below the threshold for a long time. Since the outage threshold P_o is generally set to the minimum required power for acceptable performance, a slowly-varying channel with received power which falls below P_o basically becomes disconnected. Thus, outage probability is not generally applied to multipath fading channels when the users move at vehicle velocities, since a fade lasts for the duration of just one or two bits. However, pedestrians are generally stationary when they use their wireless devices. Thus, if the pedestrian stops in the middle of a deep multipath fade, his channel is unusable.

   In this problem we compute outage probability in Rayleigh and Rician fading, which is applicable to wireless systems with pedestrian users.

(a) Assume your normalized received signal envelope follows a Rayleigh fading distribution with the average signal power $2\sigma^2 = 20$dB What is the power outage probability relative to the threshold $P_o = 5$dB? How about $P_o = 10$dB?

(b) If your application requires a power outage probability of .01 for the threshold $P_o = 10$dB, what value of the average signal power $2\sigma^2$ is required?

(c) Assume now that your received signal has a LOS component, so its envelope has a Rician distribution. For $2\sigma^2 = 10$dB and $P_o = 10$dB, find the outage probability assuming that the LOS component has the average power $s^2 = 100$. You can either find the value numerically or express it in terms of the Marcum Q Function, defined as

$$Q(a,b) = \int_b^\infty xe^{-(x^2+a^2)/2} I_0(ax)dx.$$

This function is included in most math packages (e.g. Matlab or Mathematica).

4. (20 pts.) In order to improve performance of cellular systems, multiple base stations can receive the signal transmitted from a given mobile unit and combine these multiple signals. This typically increases SNR and reduces the effects of shadowing. Combining of signals received from multiple base stations is called *macrodiversity*, and in this problem we explore the benefits of this technique.

Consider a mobile at the midpoint between two base stations in a cellular network. The received signals (in dBW) from the base stations are given by

$$P_{r,1} = W + Z_1,$$
$$P_{r,2} = W + Z_2,$$

where $Z_{1,2}$ are $\mathcal{N}(0,\sigma^2)$ random variables. We define outage with macrodiversity to be the event that both $P_{r,1}$ and $P_{r,2}$ fall below a threshould T.

(a) Interpret the terms W, Z_1, Z_2 in $P_{r,1}$ and $P_{r,2}$.

(b) If Z_1 and Z_2 are independent, show that the outage probability is given by

$$P_{out} = [Q(\Delta/\sigma)]^2,$$

where $\Delta = W - T$ is the fade margin at the mobile's location.

(c) Now suppose Z_1 and Z_2 are correlated in the following way:

$$Z_1 = a\,Y_1 + b\,Y,$$
$$Z_2 = a\,Y_2 + b\,Y,$$

where Y, Y_1, Y_2 are independent $\mathcal{N}(0,\sigma^2)$ random variables, and a, b are such that $a^2 + b^2 = 1$. Show that

$$P_{out} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{\Delta + by\sigma}{|a|\sigma}\right)\right]^2 e^{-y^2/2} dy.$$

(d) Compare the outage probabilities of (b) and (c) for the special case of $a = b = 1/\sqrt{2}$, $\sigma = 8$ and $\Delta = 5$.

5. (25 points) The goal of this problem is to develop a Rayleigh fading simulator for a mobile communications channel that can be used in future simulations and research. You should use the amplitude distribution and correlation model for Rayleigh fading that was developed in class.

There are several different ways to produce an envelope with the desired characteristics. Each of the recommended class texts show one possible technique, a technique was also discussed in class, or you can develop your own technique. In general a stochastic model is preferable to Jakes' model, which is deterministic and prone to problems when used in actual research, however it is acceptable for this problem to use Jakes' model. To receive full credit for this problem you must do the following:

(a) Develop simulation code to generate a signal with Rayleigh fading amplitude over time. Your sample rate should be at least 1000 samples/sec, the average received envelope should be 0 dB, and your simulation should be parameterized by the Doppler frequency f_D. Matlab is the easiest way to generate this simulation, but any code is fine.

(b) Write a description of your simulation that clearly explains how your code generates the fading envelope using a block diagram and any necessary equations.

(c) Turn in your well-commented code.

(d) Provide plots of received amplitude (dB) vs. time for $f_D = 1, 10, 100$ Hz. over 2 seconds.