# EE 359 - Wireless Communications - Fall 2003

## Homework 1 Solutions

1. d = distance between cells with reused freq
   p = transmit power of all the mobiles

$$\left(\frac{S}{I}\right)_{uplink} \geq 20dB$$

   (a) Min. $S/I$ will result when main user is at A and Interferers are at B
   $d_a$ = distance between A and base station #1 = $\sqrt{2}$ km
   $d_a$ = distance between B and base station #1 = $\sqrt{2}$ km

$$\left(\frac{S}{I}\right)_{min} = \frac{P\left[\frac{G\lambda}{4\pi d_A}\right]^2}{2P\left[\frac{G\lambda}{4\pi d_B}\right]^2} = \frac{{d_B}^2}{2{d_A}^2} = \frac{(d_{min}-1)^2}{4} = 100$$

   $\Rightarrow d_{min}$ - 1 = 20km $\Rightarrow d_{min}$ = 21km since integer number of cells should be accommodated in distance d $\Rightarrow d_{min}$ = 22km

   (b)

$$\frac{P_r}{P_u} = k\left[\frac{d_o}{d}\right]^\gamma \Rightarrow \left(\frac{S}{I}\right)_{min} = \frac{Pk\left[\frac{d_o}{d_A}\right]^\gamma}{2Pk\left[\frac{d_o}{d_B}\right]^\gamma} =$$

$$\frac{1}{2}\left[\frac{d_B}{d_A}\right]^\gamma = \frac{1}{2}\left[\frac{d_{min}-1}{\sqrt{2}}\right]^\gamma = \frac{1}{2}\left[\frac{d_{min}-1}{\sqrt{2}}\right]^3 = 100$$

   $\Rightarrow d_{min}$ = 9.27 $\Rightarrow$ with the same argument $\Rightarrow d_{min}$ = 10km

   (c)

$$\left(\frac{S}{I}\right)_{min} = \frac{k\left[\frac{d_o}{d_A}\right]^{\gamma_A}}{2k\left[\frac{d_o}{d_B}\right]^{\gamma_B}} = \frac{{d_B}^{\gamma_B}}{2{d_A}^{\gamma_A}} \times {d_o}^{\gamma_A-\gamma_B} = \frac{(d_{min}-1)^4}{0.04} = 100$$

   $\Rightarrow d_{min}$ = 2.41km $\Rightarrow$ with the same argument $d_{min}$ = 4km

2. optimum no. of data user = d
   optimum no. of voice user = v
   Three different cases:
   Case 1: d=0, v=6
   $\Rightarrow$ revenue = 6×0.8×0.2 = 0.96

Case 2: d=1, v=3
revenue = [prob. of having one data user]×(revenue of having one data user)
+ [prob. of having two data user]×(revenue of having two data user)
+ [prob. of having one voice user]×(revenue of having one voice user)
+ [prob. of having two voice user]×(revenue of having two voice user)
+ [prob. of having three or more voice user]×(revenue in this case)

$$\Rightarrow 0.5^2\binom{2}{1} \times \$1 + 0.5^2 \times \$1 + \binom{6}{1}0.8 \times 0.2^5 \times \$0.2 + \binom{6}{2}0.8^2 \times 0.2^4 \times \$0.4 +$$

$$\left[1 - \binom{6}{1}0.8 \times 0.2^5 \times \$0.2 - \binom{6}{2}0.8^2 \times 0.4^4 \times \$0.4\right] \times \$0.6$$

$$\Rightarrow \$1.35$$

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Case 3: d=2, v=0
revenue = 2×0.5 = \$1

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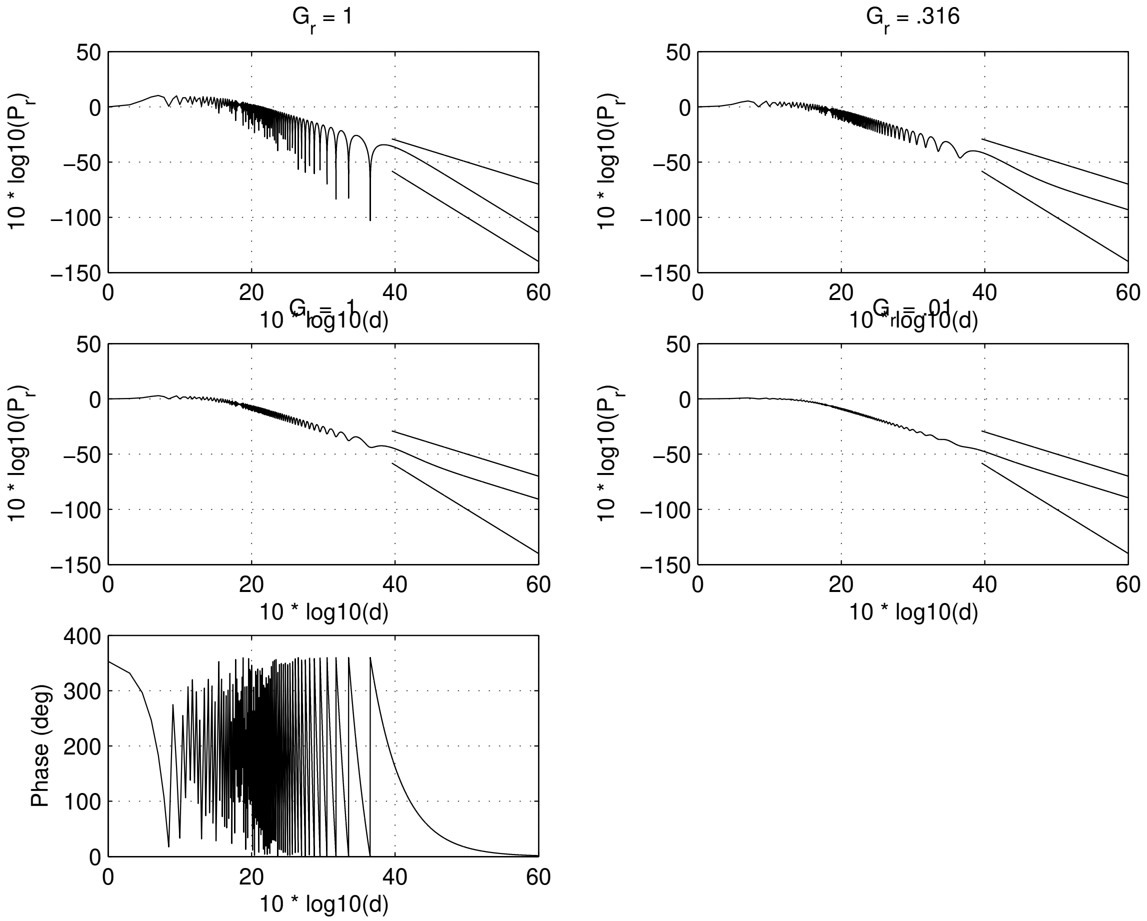
So the best case is case 2, which is to allocate 60kHz to data and 60kHz to voice.

3. (a) 100 cells, 100 users/cell $\Rightarrow$ 10,000 users
   (b) 100 users/cell $\Rightarrow$ 2500 cells required
   $\frac{100km^2}{Area/cell} = 2500$ cells $\Rightarrow \frac{Area}{cell} = .04\ km^2$
   (c) From Rappaport or iteration of formula, we that 100 $\frac{channels}{cell} \Rightarrow 89\ \frac{erlangs}{cell}$ @ $P_b = .02$
   (Note: In the original homework assignment we wrote that the sum in the Erlang B formula begins at $k = 1$ - it should actually begin at $k = 0$. This has been corrected on the posted homework assignment).
   Each subscriber generates $\frac{1}{30}$ of an Erlang of traffic
   Thus, each cell can support 30×89 = 2670 subscribers
   Macrocell: 2670×100 $\Rightarrow$ 267,000 subscribers
   Microcell: 6,675,000 subscribers
   (d) Macrocell: \$50 M
   Microcell: \$1.25 B
   (e) Macrocell: \$13.35 M/month $\Rightarrow$ 3.75 months $\approx$ 4 months to recoop
   Microcell: \$333.75 M/month $\Rightarrow$ 3.75 months $\approx$ 4 months to recoop

4. A program to plot the figures is shown below. The power versus distance curves and a plot of the phase difference between the two paths is shown on the following page. From the plots it can be seen that as $G_r$ (gain of reflected path) is decreased, the asymptotic behavior of $P_r$ tends toward $d^{-2}$ from $d^{-4}$, which makes sense since the effect of reflected path is reduced and it is more like having only a LOS path. Also the variation of power before and around $d_c$ is reduced because the strength of the reflected path decreases as $G_r$ decreases. Also note that the the received power actually increases with distance up to some point. This is because for very small distances (i.e. $d = 1$), the reflected path is approximately two times the LOS path, making the phase difference very small. Since $R = -1$, this causes the two paths to nearly cancel each other out. When the phase difference becomes 180 degrees, the first local maxima is achieved. Additionally, the lengths of both paths are initially dominated by the difference between the antenna heights (which is 35 meters). Thus, the powers of both paths are roughly constant for small values of $d$, and the dominant factor is the phase difference between the paths.

```
clear all;
close all;
ht=50;
hr=15;
f=900e6;
c=3e8;
lambda=c/f;
GR=[1,.316,.1,.01];
Gl=1;
R=-1;
counter=1;
figure(1);
d=[1:1:100000];
l=(d.^2+(ht-hr)^2).^.5;
r=(d.^2+(ht+hr)^2).^.5;
phd=2*pi/lambda*(r-1);
dc=4*ht*hr/lambda;
dnew=[dc:1:100000];
for counter = 1:1:4,
   Gr=GR(counter);
   Vec=Gl./l+R*Gr./r.*exp(phd*sqrt(-1));
   Pr=(lambda/4/pi)^2*(abs(Vec)).^2;
   subplot(2,2,counter);
   plot(10*log10(d),10*log10(Pr)-10*log10(Pr(1)));
   hold on;
   plot(10*log10(dnew),-20*log10(dnew));
   plot(10*log10(dnew),-40*log10(dnew));
end
hold off
```




5. Outage Prob. = Prob. [received $power_{dB} \leqslant Tp_{dB}$]
   Tp = 10dB

   (a)

$$outage prob. = 1 - Q\left(\frac{Tp - \mu_\psi}{\sigma_\psi}\right) = 1 - Q\left(\frac{-5}{8}\right) = Q\left(\frac{5}{8}\right) = 26\%$$

   (b) $\sigma_\psi = 4$dB, outage prob $< 1\% \Rightarrow$

$$Q\left(\frac{Tp - \mu_\psi}{\sigma_\psi}\right) > 99\% \Rightarrow \frac{Tp - \mu_\psi}{\sigma_\psi} < -2.33 \Rightarrow$$

$$\mu_\psi \geq 19.32dB$$

   (c)

$$\sigma_\psi = 12dB, \frac{Tp - \mu_\psi}{\sigma_\psi} < -6.99 \Rightarrow \mu_\psi \geq 37.8dB$$

   (d) For mitigating the effect of shadowing, we can use macroscopic diversity. The idea in macroscopic diversity is to send the message from different base stations to achieve uncorrelated shadowing. In this way the probability of power outage will be less because both base stations are unlikely to experience an outage at the same time, if they are uncorrelated.