Probability and Stochastic Processes:
A Friendly Introduction for Electrical and Computer Engineers
Edition 2
Roy D. Yates and David J. Goodman

Problem Solutions : Yates and Goodman, 5.7.7 5.7.9 12.5.4 12.5.7 12.5.9 12.10.7 12.11.2 and 12.11.4

Problem 5.7.7 Solution

The difficulty of this problem is overrated since its a pretty simple application of Problem 5.7.6. In particular,

$$\mathbf{Q} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\Bigg|_{\theta=45^\circ} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}. \tag{1}$$

Since $\mathbf{X} = \mathbf{QY}$, we know from Theorem 5.16 that $\mathbf{X}$ is Gaussian with covariance matrix

$$\begin{aligned} \mathbf{C_X} &= \mathbf{QC_YQ'} && (2)\\ &= \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1+\rho & 0 \\ 0 & 1-\rho \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} && (3)\\ &= \frac{1}{2}\begin{bmatrix} 1+\rho & -(1-\rho) \\ 1+\rho & 1-\rho \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} && (4)\\ &= \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}. && (5) \end{aligned}$$

Problem 5.7.9 Solution

(a) If you are familiar with the Gram-Schmidt procedure, the argument is that applying Gram-Schmidt to the rows of $\mathbf{A}$ yields m orthogonal row vectors. It is then possible to augment those vectors with an additional $n-m$ orothogonal vectors. Those orthogonal vectors would be the rows of $\tilde{\mathbf{A}}$.

An alternate argument is that since $\mathbf{A}$ has rank m the nullspace of $\mathbf{A}$, i.e., the set of all vectors $\mathbf{y}$ such that $\mathbf{Ay} = \mathbf{0}$ has dimension $n-m$. We can choose any $n-m$ linearly independent vectors $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{n-m}$ in the nullspace $\mathbf{A}$. We then define $\tilde{\mathbf{A}}'$ to have columns $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_{n-m}$. It follows that $\mathbf{A}\tilde{\mathbf{A}}' = \mathbf{0}$.

(b) To use Theorem 5.16 for the case $m = n$ to show

$$\bar{\mathbf{Y}} = \begin{bmatrix} \mathbf{Y} \\ \hat{\mathbf{Y}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \hat{\mathbf{A}} \end{bmatrix}\mathbf{X}. \tag{1}$$

is a Gaussian random vector requires us to show that

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} \\ \hat{\mathbf{A}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} \\ \tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1} \end{bmatrix} \tag{2}$$

is a rank n matrix. To prove this fact, we will suppose there exists $\mathbf{w}$ such that $\bar{\mathbf{A}}\mathbf{w} = \mathbf{0}$, and then show that $\mathbf{w}$ is a zero vector. Since $\mathbf{A}$ and $\tilde{\mathbf{A}}$ together have n linearly independent rows, we can write the row vector $\mathbf{w}'$ as a linear combination of the rows of $\mathbf{A}$ and $\tilde{\mathbf{A}}$. That is, for some $\mathbf{v}$ and $\tilde{\mathbf{v}}$,

$$\mathbf{w}' = \mathbf{v}\mathbf{t}'\mathbf{A} + \tilde{\mathbf{v}}'\tilde{\mathbf{A}}. \tag{3}$$

The condition $\bar{\mathbf{A}}\mathbf{w} = \mathbf{0}$ implies

$$\begin{bmatrix} \mathbf{A} \\ \tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1} \end{bmatrix} \left(\mathbf{A}'\mathbf{v} + \tilde{\mathbf{A}}'\tilde{\mathbf{v}}'\right) = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \tag{4}$$

This implies

$$\mathbf{A}\mathbf{A}'\mathbf{v} + \mathbf{A}\tilde{\mathbf{A}}'\tilde{\mathbf{v}} = \mathbf{0} \tag{5}$$

$$\tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1}\mathbf{A}\mathbf{v} + \tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}'\tilde{\mathbf{v}} = \mathbf{0} \tag{6}$$

Since $\mathbf{A}\tilde{\mathbf{A}}' = \mathbf{0}$, Equation (5) implies that $\mathbf{A}\mathbf{A}'\mathbf{v} = \mathbf{0}$. Since $\mathbf{A}$ is rank m, $\mathbf{A}\mathbf{A}'$ is an $m \times m$ rank m matrix. It follows that $\mathbf{v} = \mathbf{0}$. We can then conclude from Equation (6) that

$$\tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}'\tilde{\mathbf{v}} = \mathbf{0}. \tag{7}$$

This would imply that $\tilde{\mathbf{v}}'\tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}'\tilde{\mathbf{v}} = 0$. Since $\mathbf{C}_{\mathbf{X}}^{-1}$ is invertible, this would imply that $\tilde{\mathbf{A}}'\tilde{\mathbf{v}} = \mathbf{0}$. Since the rows of $\tilde{\mathbf{A}}$ are linearly independent, it must be that $\tilde{\mathbf{v}} = \mathbf{0}$. Thus $\bar{\mathbf{A}}$ is full rank and $\bar{\mathbf{Y}}$ is a Gaussian random vector.

(c) We note that By Theorem 5.16, the Gaussian vector $\bar{\mathbf{Y}} = \bar{\mathbf{A}}\mathbf{X}$ has covariance matrix

$$\bar{\mathbf{C}} = \bar{\mathbf{A}}\mathbf{C}_{\mathbf{X}}\bar{\mathbf{A}}'. \tag{8}$$

Since $(\mathbf{C}_{\mathbf{X}}^{-1})' = \mathbf{C}_{\mathbf{X}}^{-1}$,

$$\bar{\mathbf{A}}' = \begin{bmatrix} \mathbf{A}' & (\tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1})' \end{bmatrix} = \begin{bmatrix} \mathbf{A}' & \mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}' \end{bmatrix}. \tag{9}$$

Applying this result to Equation (8) yields

$$\bar{\mathbf{C}} = \begin{bmatrix} \mathbf{A} \\ \tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1} \end{bmatrix} \mathbf{C}_{\mathbf{X}} \begin{bmatrix} \mathbf{A}' & \mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}' \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{C}_{\mathbf{X}} \\ \tilde{\mathbf{A}} \end{bmatrix} \begin{bmatrix} \mathbf{A}' & \mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}' \end{bmatrix} = \begin{bmatrix} \mathbf{A}\mathbf{C}_{\mathbf{X}}\mathbf{A}' & \mathbf{A}\tilde{\mathbf{A}}' \\ \tilde{\mathbf{A}}\mathbf{A}' & \tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}' \end{bmatrix}. \tag{10}$$

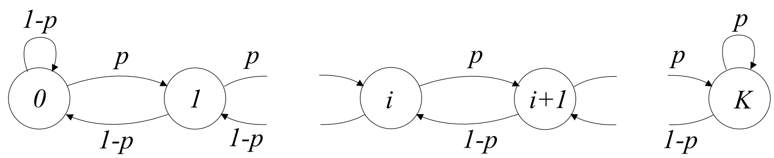
Since $\tilde{\mathbf{A}}\mathbf{A}' = \mathbf{0}$,

$$\bar{\mathbf{C}} = \begin{bmatrix} \mathbf{A}\mathbf{C}_{\mathbf{X}}\mathbf{A}' & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{A}}\mathbf{C}_{\mathbf{X}}^{-1}\tilde{\mathbf{A}}' \end{bmatrix} = \begin{bmatrix} \mathbf{C}_{\mathbf{Y}} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{\hat{\mathbf{Y}}} \end{bmatrix}. \tag{11}$$

We see that $\bar{\mathbf{C}}$ is block diagonal covariance matrix. From the claim of Problem 5.7.8, we can conclude that $\mathbf{Y}$ and $\hat{\mathbf{Y}}$ are independent Gaussian random vectors.

Problem 12.5.4 Solution

From the problem statement, the Markov chain is



The self-transitions in state 0 and state K guarantee that the Markov chain is aperiodic. Since the chain is also irreducible, we can find the stationary probabilities by solving $\boldsymbol{\pi}' = \boldsymbol{\pi}'\mathbf{P}$; however, in this problem it is simpler to apply Theorem 12.13. In particular, by partitioning the chain between states i and $i+1$, we obtain

$$\pi_i p = \pi_{i+1}(1-p). \tag{1}$$

This implies $\pi_{i+1} = \alpha\pi_i$ where $\alpha = p/(1-p)$. It follows that $\pi_i = \alpha^i \pi_0$. REquiring the stationary probabilities to sum to 1 yields

$$\sum_{i=0}^{K} \pi_i = \pi_0(1 + \alpha + \alpha^2 + \cdots + \alpha^K) = 1. \tag{2}$$

This implies

$$\pi_0 = \frac{1-\alpha^{K+1}}{1-\alpha} \tag{3}$$

Thus, for $i = 0, 1, \ldots, K$,

$$\pi_i = \frac{1-\alpha^{K+1}}{1-\alpha}\alpha^i = \frac{1-\left(\frac{p}{1-p}\right)^{K+1}}{1-\left(\frac{p}{1-p}\right)}\left(\frac{p}{1-p}\right)^i. \tag{4}$$

Problem 12.5.7 Solution

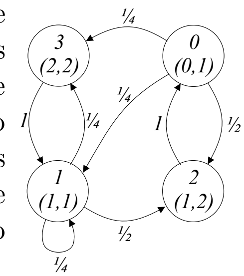
In this case, we will examine the system each minute. For each customer in service, we need to keep track of how soon the customer will depart. For the state of the system, we will use (i, j), the remaining service requirements of the two customers, To reduce the number of states, we will order the requirements so that $i \leq j$. For example, when two new customers start service each requiring two minutes of service, the system state will be $(2, 2)$. Since the system assumes there is always a backlog of cars waiting to enter service, the set of states is

0 $(0, 1)$ One teller is idle, the other teller has a customer requiring one more minute of service

1 $(1, 1)$ Each teller has a customer requiring one more minute of service.

2 $(1, 2)$ One teller has a customer requring one minute of service. The other teller has a customer requiring two minutes of service.

3 $(2,2)$ Each teller has a customer requiring two minutes of service.



The resulting Markov chain is shown on the right. Note that when we departing from either state $(0,1)$ or $(1,1)$ corresponds to both custoemrs finishing service and two new customers entering service. The state transiton probabilities reflect the fact that both customer will have two minute service requirements with probability 1/4, or both customers will hae one minute service requirements with probability 1/4, or one customer will need one minute of service and the other will need two minutes of service with probability 1/2.

Writing the stationary probability equations for states 0, 2, and 3 and adding the constraint $\sum_j \pi_j = 1$ yields the following equations:

$$\pi_0 = \pi_2 \tag{1}$$
$$\pi_2 = (1/2)\pi_0 + (1/2)\pi_1 \tag{2}$$
$$\pi_3 = (1/4)\pi_0 + (1/4)\pi_1 \tag{3}$$
$$1 = \pi_0 + \pi_1 + \pi_2 + \pi_3 \tag{4}$$

Substituting $\pi_2 = \pi_0$ in the second equation yields $\pi_1 = \pi_0$. Substituting that result in the third equation yields $\pi_3 = \pi_0/2$. Making sure the probabilities add up to 1 yields

$$\boldsymbol{\pi} = \begin{bmatrix}\pi_0 & \pi_1 & \pi_2 & \pi_3\end{bmatrix}' = \begin{bmatrix}2/7 & 2/7 & 2/7 & 1/7\end{bmatrix}'. \tag{5}$$

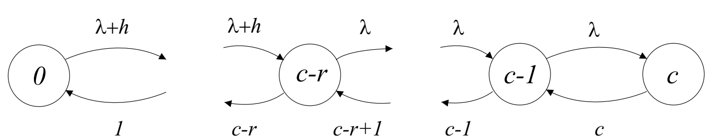
Both tellers are busy unless the system is in state 0. The stationary probability both tellers are busy is $1 - \pi_0 = 5/7$.

Problem 12.5.9 Solution

Under construction.

Problem 12.10.7 Solution

Since both types of calls have exponential holding times, the number of calls in the system can be used as the system state. The corresponding Markov chain is



When the number of calls, n, is less than $c-r$, we admit either type of call and $q_{n,n+1} = \lambda+h$. When $n \geq c-r$, we block the new calls and we admit only handoff calls so that $q_{n,n+1} = h$. Since the service times are exponential with an average time of 1 minute, the call departure rate in state n is n calls per minute. Theorem 12.24 says that the stationary probabilities p_n satisfy

$$p_n = \begin{cases} \dfrac{\lambda+h}{n} p_{n-1} & n = 1, 2, \ldots, c-r \\ \dfrac{\lambda}{n} p_{n-1} & n = c-r+1, c-r+2, \ldots, c \end{cases} \tag{1}$$

This implies

$$p_n = \begin{cases} \dfrac{(\lambda+h)^n}{n!} p_0 & n = 1, 2, \ldots, c-r \\ \dfrac{(\lambda+h)^{c-r}\lambda^{n-(c-r)}}{n!} p_0 & n = c-r+1, c-r+2, \ldots, c \end{cases} \tag{2}$$

The requirement that $\sum_{n=1}^{c} p_n = 1$ yields

$$p_0 \left[\sum_{n=0}^{c-r} \frac{(\lambda+h)^n}{n!} + (\lambda+h)^{c-r} \sum_{n=c-r+1}^{c} \frac{\lambda^{n-(c-r)}}{n!} \right] = 1 \tag{3}$$

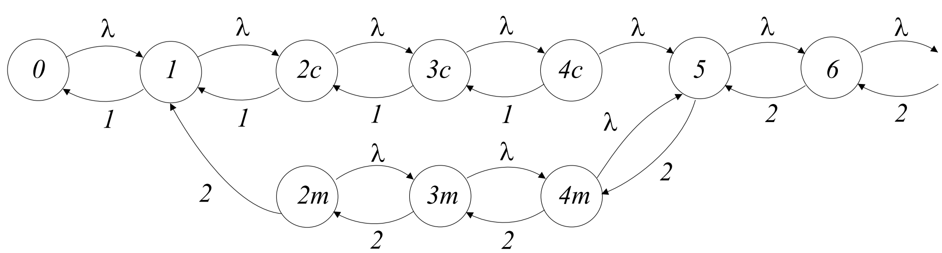
Finally, a handoff call is dropped if and only if a new call finds the system with c calls in progress. The probability that a handoff call is dropped is

$$P[H] = p_c = \frac{(\lambda+h)^{c-r}\lambda^r}{c!} p_0 = \frac{(\lambda+h)^{c-r}\lambda^r/c!}{\sum_{n=0}^{c-r} \frac{(\lambda+h)^n}{n!} + \left(\frac{\lambda+h}{\lambda}\right)^{c-r} \sum_{n=c-r+1}^{c} \frac{\lambda^n}{n!}} \tag{4}$$

Problem 12.11.2 Solution

In this problem, we model the system as a continuous time Markov chain. The clerk and the manager each represent a "server." The state describes the number of customers in the queue and the number of active servers. The Markov chain issomewhat complicated because when the number of customers in the store is 2, 3, or 4, the number of servers may be 1 or may be 2, depending on whether the manager became an active server.

When just the clerk is serving, the service rate is 1 customer per minute. When the manager and clerk are both serving, the rate is 2 customers per minute. Here is the Markov chain:



In states $2c$, $3c$ and $4c$, only the clerk is working. In states $2m$, $3m$ and $4m$, the manager is also working. The state space $\{0, 1, 2c, 3c, 4c, 2m, 3m, 4m, 5, 6, \ldots\}$ is countably infinite. Finding the state probabilities is a little bit complicated because the are enough states that we would like to use MATLAB; however, MATLAB can only handle a finite state space. Fortunately, we can use MATLAB because the state space for states $n \geq 5$ has a simple structure.

We observe for $n \geq 5$ that the average rate of transitions from state n to state $n+1$ must equal the average rate of transitions from state $n+1$ to state n, implying

$$\lambda p_n = 2p_{n+1}, \qquad n = 5, 6, \ldots \tag{1}$$

It follows that $p_{n+1} = (\lambda/2)p_n$ and that

$$p_n = \alpha^{n-5} p_5, \qquad n = 5, 6, \ldots, \tag{2}$$

where $\alpha = \lambda < 2 < 1$. The requirement that the stationary probabilities sum to 1 implies

$$1 = p_0 + p_1 + \sum_{j=2}^{4} (p_{jc} + p_{jm}) + \sum_{n=5}^{\infty} p_n \tag{3}$$

$$= p_0 + p_1 + \sum_{j=2}^{4} (p_{jc} + p_{jm}) + p_5 \sum_{n=5}^{\infty} \alpha^{n-5} \tag{4}$$

$$= p_0 + p_1 + \sum_{j=2}^{4} (p_{jc} + p_{jm}) + \frac{p_5}{1-\alpha} \tag{5}$$

This is convenient because for each state $j < 5$, we can solve for the staitonary probabilities. In particular, we use Theorem 12.23 to write $\sum \sum_i r_{ij} p_i = 0$. This leads to a set of matrix equations for the state probability vector

$$\mathbf{p} = \begin{bmatrix} p_0 & p_1 & p_{2c} & p_{3c} & p_{3c} & p_{4c} & p_{2m} & p_{3m} & p_{4m} & p_5 \end{bmatrix}' \tag{6}$$

The rate transition matrix associated with $\mathbf{p}$ is

$$\mathbf{Q} = \left[\begin{array}{ccccccccc} p_0 & p_1 & p_{2c} & p_{3c} & p_{4c} & p_{2m} & p_{3m} & p_{4m} & p_5 \\ \hline 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & \lambda \\ 0 & 2 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & \lambda \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \end{array}\right], \tag{7}$$

where the first row just shows the correspondence of the state probabilities and the matrix columns. For each state i, excepting state 5, the departure rate ν_i from that state equals the sum of entries of the corresponding row of $\mathbf{Q}$. To find the stationary probabilities, our normal procedure is to use Theorem 12.23 and solve $\mathbf{p}'\mathbf{R} = \mathbf{0}$ and $\mathbf{p}'\mathbf{1} = 1$, where $\mathbf{R}$ is the same as $\mathbf{Q}$ except the zero diagonal entries are replaced by $-\nu_i$. The equation $\mathbf{p}'\mathbf{1} = 1$ replaces one column of the set of matrix equations. This is the approach of `cmcstatprob.m`.

In this problem, we follow almost the same procedure. We form the matrix $\mathbf{R}$ by replacing the diagonal entries of $\mathbf{Q}$. However, instead of replacing an arbitrary column with the equation $\mathbf{p}'\mathbf{1} = 1$, we replace the column corresponding to p_5 with the equation

$$p_0 + p_1 + p_{2c} + p_{3c} + p_{4c} + p_{2m} + p_{3m} + p_{4m} + \frac{p_5}{1-\alpha} = 1. \tag{8}$$

That is, we solve

$$\mathbf{p}'\mathbf{R} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}'. \tag{9}$$

where

$$\mathbf{R} = \begin{bmatrix} -\lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1-\lambda & \lambda & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & -1-\lambda & \lambda & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1-\lambda & \lambda & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1-\lambda & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 & -2-\lambda & \lambda & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & -2-\lambda & \lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -2-\lambda & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & \frac{1}{1-\alpha} \end{bmatrix} \tag{10}$$

Given the stationary distribution, we can now find $E[N]$ and $P[W]$.

Recall that N is the number of customers in the system at a time in the distant future. Defining

$$p_n = p_{nc} + p_{nm}, \qquad n = 2, 3, 4, \tag{11}$$

we can write

$$E[N] = \sum_{n=0}^{\infty} np_n = \sum_{n=0}^{4} np_n + \sum_{n=5}^{\infty} np_5\alpha^{n-5} \tag{12}$$

The substitution $k = n - 5$ yields

$$E[N] = \sum_{n=0}^{4} np_n + p_5 \sum_{k=0}^{\infty} (k+5)\alpha^k \tag{13}$$

$$= \sum_{n=0}^{4} np_n + p_5 \frac{5}{1-\alpha} + p_5 \sum_{k=0}^{\infty} k\alpha^k \tag{14}$$

From Math Fact B.7, we conclude that

$$E[N] = \sum_{n=0}^{4} np_n + p_5 \left(\frac{5}{1-\alpha} + \frac{\alpha}{(1-\alpha)^2} \right) \tag{15}$$

$$= \sum_{n=0}^{4} np_n + p_5 \frac{5-4\alpha}{(1-\alpha)^2} \tag{16}$$

Furthermore, the manager is working unless the system is in state 0, 1, $2c$, $3c$, or $4c$. Thus

$$P[W] = 1 - (p_0 + p_1 + p_{2c} + p_{3c} + p_{4c}). \tag{17}$$

We implement these equations in the following program, alongside the corresponding output.

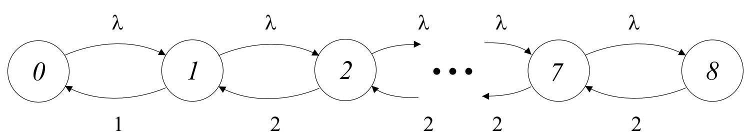
```
function [EN,PW]=clerks(lam);
Q=diag(lam*[1 1 1 1 0 1 1 1],1);
Q=Q+diag([1 1 1 1 0 2 2 2],-1);
Q(6,2)=2; Q(5,9)=lam;
R=Q-diag(sum(Q,2));
n=size(Q,1);
a=lam/2;
R(:,n)=[ones(1,n-1) 1/(1-a)]';
pv=([zeros(1,n-1) 1]*R^(-1));
EN=pv*[0;1;2;3;4;2;3;4; ...
        (5-4*a)/(1-a)^2];
PW=1-sum(pv(1:5));
```

```
>> [en05,pw05]=clerks(0.5)
en05 =
    0.8217
pw05 =
    0.0233
>> [en10,pw10]=clerks(1.0)
en10 =
    2.1111
pw10 =
    0.2222
>> [en15,pw15]=clerks(1.5)
en15 =
    4.5036
pw15 =
    0.5772
>>
```

We see that in going from an arrival rate of 0.5 customers per minute to 1.5 customers per minute, the average number of customers goes from 0.82 to 4.5 customers. Similarly, the probability the manager is working rises from 0.02 to 0.57.

Problem 12.11.4 Solution

This problem is actually very easy. The state of the system is given by X, the number of cars in the system.When $X = 0$, both tellers are idle. When $X = 1$, one teller is busy, however, we do not need to keep track of which teller is busy. When $X = n \geq 2$, both tellers are busy and there are $n - 2$ cars waiting. Here is the Markov chain:



Since this is a birth death process, we could easily solve this problem using analysis. However, as this problem is in the MATLAB section of this chapter, we might as well construct a MATLAB solution:

```
function [p,en]=veryfast2(lambda);
c=2*[0,eye(1,8)]';
r=lambda*[0,eye(1,8)];
Q=toeplitz(c,r);
Q(2,1)=1;
p=cmcstatprob(Q);
en=(0:8)*p;
```

The code solves the stationary distribution and the expected number of cars in the system for an arbitrary arrival rate λ.

Here is the output:

```
>> [p,en]=veryfast2(0.75);
>> p'
ans =
 0.4546 0.3410 0.1279 0.0480 0.0180 0.0067 0.0025 0.0009 0.0004
>> en
en =
    0.8709
>>
```