The Strong Interference Channel With Common Information

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Abstract

Transmitter cooperation enabled by dedicated links with finite capacities allows for a partial message exchange between encoders. After cooperation, each encoder will know a *common* message partially describing the two original messages, and its own *private* message containing the information that the encoders were not able to exchange. We consider the interference channel with both private and common messages at the encoders. A private message sent at an encoder is intended for a corresponding decoder whereas the common message is to be received at both decoders. We derive conditions under which the capacity region of this channel coincides with the capacity region of the channel in which both private messages are required at both receivers. We show that the obtained conditions and the strong interference conditions determined by Costa and El Gamal for the interference channel with independent messages are satisfied by the same class of interference channels.

1 Introduction

A problem in which encoders partially cooperate in a discrete memoryless channel was proposed by Willems for a multiple access channel (MAC) [1]. To model the transmitter cooperation, two communication links with finite capacities are introduced between the two encoders. The amount of information exchanged between the two transmitters is bounded by the capacities of the communication links. The proposed discrete channel model enables investigation of transmitter cooperation gains. For a Gaussian network with two transmitters and two receivers, improvements in the achievable rates due to node cooperation were demonstrated in [2–6]. In [2], the transmitters fully cooperate by exchanging their intended messages and then jointly encode them using dirty paper coding. More involved cooperation schemes were analyzed in [3–5].

In the discrete memoryless MAC with partially cooperating encoders [1], the outcome of the cooperation is referred to as a *conference*. Willems determined the capacity region of this channel and thus specified the optimum conference. His result was recently extended to a

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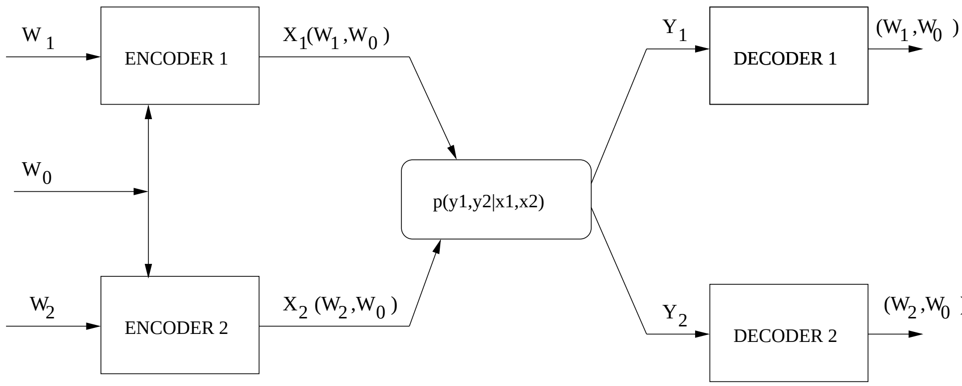

Figure 1: Interference channel with common information.

compound MAC in which two decoders wish to decode messages sent from the encoders [7]. The same form of conference as in [1] was shown to remain optimal.

When cooperating over the links with finite capacities, encoders obtain partial information about each other's messages. This information is referred to as the *common* message as it is known to both encoders after the conference. In addition, each encoder will still have independent information referred to as the *private* message, as this message remains unknown to the other encoder. Both common and private messages are decoded at a single decoder in the case of the MAC [1], or at both receivers in the case of a compound MAC [7].

In this paper, we consider the communication situation in which two encoders each have a private message and a common message they wish to send. Each decoder t is interested in only one private message sent at the corresponding encoder t. Both decoders wish to decode the common message. We refer to this channel as *an interference channel with common information*, denoted $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$. The communication system is shown in Figure 1. Without common information, this channel reduces to the interference channel [8, 9] for which the capacity region is known in the case of *strong interference* [10] satisfying

$$I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2) \tag{1}$$

$$I(X_2; Y_2|X_1) \leq I(X_2; Y_1|X_1) \tag{2}$$

for all product distributions on the inputs X_1 and X_2. The capacity region in this case coincides with the capacity region of the two-sender, two-receiver channel in which both messages are decoded at both receivers, as determined by Ahlswede [11].

In this paper, we determine the capacity region of interference channels with a common message if

$$I(X_1; Y_1|X_2, U) \leq I(X_1; Y_2|X_2, U) \tag{3}$$

$$I(X_2; Y_2|X_1, U) \leq I(X_2; Y_1|X_1, U) \tag{4}$$

for all joint distributions $p(u, x_1, x_2, y_1, y_2)$ that factor as $p(u)p(x_1|u)p(x_2|u)p(y_1, y_2|x_1, x_2)$. We further show that this class of interference channels is same as those determined by (1) and (2) with independent X_1 and X_2.

2 Channel Model and Statement of Result

The channel consists of finite sets $\mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}_1, \mathcal{Y}_2$ and a conditional probability distribution $p(y_1, y_2|x_1, x_2)$. Symbols $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are channel inputs and $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$

are the corresponding channel outputs. Each encoder t, $t = 1, 2$, wishes to send a private message $W_t \in \{1, \ldots, M_t\}$ to decoder t in N channel uses. In addition, a common message $W_0 \in \{1, \ldots, M_0\}$ needs to be communicated from the encoders to both decoders, as shown in Figure 1. The channel is memoryless and time-invariant in the sense that

$$p(y_{1,n}, y_{2,n}|\mathbf{x}_1^n, \mathbf{x}_2^n, \mathbf{y}_1^{n-1}, \mathbf{y}_2^{n-1}) = p_{Y_1Y_2|X_1X_2}(y_{1,n}, y_{2,n}|x_{1,n}, x_{2,n}) \tag{5}$$

where $\mathbf{x}_t^n = \begin{bmatrix} x_{t,1}, & \ldots, & x_{t,n} \end{bmatrix}$ and where $p_{Y_1Y_2|X_1X_2}(\cdot)$ is the channel probability distribution. We are here following the convention of dropping subscripts of probability distributions if the arguments of the distributions are lower case versions of the corresponding random variables. To simplify notation, we drop the superscript when $n = N$.

Indexes W_0, W_1 and W_2 are independently generated at the beginning of each block of N channel uses. An encoder $t, t = 1, 2$ maps the common message W_0 and the private message W_t into a codeword $\mathbf{x}_t$

$$\mathbf{x}_1 = f_1(W_0, W_1) \tag{6}$$

$$\mathbf{x}_2 = f_2(W_0, W_2). \tag{7}$$

Each decoder t estimates the common message W_0 and the private message W_t based on the received N-sequence

$$(\hat{W}_0, \hat{W}_1) = g_1(\mathbf{Y}_1) \tag{8}$$

$$(\hat{W}_0, \hat{W}_2) = g_2(\mathbf{Y}_2). \tag{9}$$

An (M_0, M_1, M_2, N, P_e) code for the channel consists of two encoding functions f_1, f_2, two decoding functions g_1, g_2 and a maximum error probability

$$P_e \triangleq \max\{P_{e,1}, P_{e,2}\} \tag{10}$$

where

$$P_{e,t} = \sum_{(w_0,w_1,w_2)} \frac{1}{M_0M_1M_2} P\left[g_t(\mathbf{Y}_t) \neq (w_0, w_t)|(w_0, w_1, w_2) \text{ sent}\right], \quad t = 1, 2. \tag{11}$$

A rate triple (R_0, R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_0, M_1, M_2, N, P_e) code such that

$$P_e \leq \epsilon \text{ and } M_i \geq 2^{NR_i} \quad i = 0, 1, 2.$$

The capacity region of the *interference channel with common information* is the closure of the set of all achievable rate triplets (R_0, R_1, R_2).

The next theorem is the main result of this paper. It gives conditions under which the capacity region coincides with the capacity region of the channel in which both private messages are required at both receivers.

Theorem 1 *For an interference channel* $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_1, y_2|x_1, x_2), \mathcal{Y}_1 \times \mathcal{Y}_2)$ *with common information satisfying*

$$I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2) \tag{12}$$

$$I(X_2; Y_2|X_1) \leq I(X_2; Y_1|X_1) \tag{13}$$

for all product distributions on X_1 and X_2, the capacity region $\mathcal{C}$ is given by

$$\begin{aligned}\mathcal{C} = \bigcup\{&(R_0, R_1, R_2) : \\ &0 \le R_1 \le I(X_1; Y_1|X_2, U) &(14)\\ &0 \le R_2 \le I(X_2; Y_2|X_1, U) &(15)\\ &R_1 + R_2 \le \min\{I(X_1, X_2; Y_1|U), I(X_1, X_2; Y_2|U)\} &(16)\\ &0 \le R_0 + R_1 + R_2 \le \min\{I(X_1, X_2; Y_1), I(X_1, X_2; Y_2)\} &(17)\end{aligned}$$

where the union is over joint distributions $p(u, x_1, x_2, y_1, y_2)$ that factor as

$$p(u)p(x_1|u)p(x_2|u)p(y_1, y_2|x_1, x_2). \tag{18}$$

3 The MAC With Common Information and Achievability

The interference channel with common information is closely related to a discrete channel model in which private and common messages are transmitted to a single receiver, referred to as the MAC with common information [12]. The capacity region of this channel, $\mathcal{C}_{MAC}$, was shown in [12] and [13] to be

$$\begin{aligned}\mathcal{C}_{MAC} = \bigcup\{&(R_0, R_1, R_2) : \\ &0 \le R_1 \le I(X_1; Y|X_2, U)\\ &0 \le R_2 \le I(X_2; Y|X_1, U)\\ &R_1 + R_2 \le I(X_1, X_2; Y|U)\\ &0 \le R_0 + R_1 + R_2 \le I(X_1, X_2; Y)\} &(19)\end{aligned}$$

where the union is over all $p(u, x_1, x_2, y)$ that factor as $p(u)p(x_1|u)p(x_2|u)p(y|x_1, x_2)$. (In [13] the convex hull operation used in [12] was shown to be unnecessary).

In this paper we analyze a channel model with two receivers. When each receiver wishes to decode both private messages and the common message, the considered channel becomes a compound MAC with common information. This channel defines two MAC channels with common information $(\mathcal{X}_1 \times \mathcal{X}_2, p(y_t|x_1, x_2), \mathcal{Y}_t)$, one for each receiver t where

$$p(y_1|x_1, x_2) = \sum_{y_2 \in \mathcal{Y}_2} p(y_1, y_2|x_1, x_2) \tag{20}$$

and

$$p(y_2|x_1, x_2) = \sum_{y_1 \in \mathcal{Y}_1} p(y_1, y_2|x_1, x_2). \tag{21}$$

As described in [7, Section IV], the encoding and decoding strategy proposed by Willems in [13] can be adopted for the compound MAC with common information to guarantee the achievability of rates

$$\mathcal{C}_{\text{CMAC}} = \bigcup \{\mathcal{R}_{\text{MAC1}} \cap \mathcal{R}_{\text{MAC2}}\} \tag{22}$$

where $\mathcal{R}_{\text{MACt}}$, $t = 1, 2$ satisfies the bounds (19) with Y replaced by Y_t, and the union is over all $p(u)p(x_1|u)p(x_2|u)p(y_1, y_2|x_1, x_2)$. We remark that under the conditions (12) and (13), the regions (17) and (22) are the same.

Consider next the strong interference channel with common information. The achievability of the rates of Theorem 1 in the case in which both messages are required at the receivers guarantees that these rates are also achieved when a weaker constraint of decoding of a single message is imposed at the receivers. Hence the proof of achievability in Theorem 1 is immediate. We next prove the converse.

4 Converse

Consider a code (M_0, M_1, M_2, N, P_e) for the interference channel with common information. Applying Fano's inequality results in

$$H(W_0, W_1|\mathbf{Y}_1) \leq P_{e1} \log(M_0 M_1 - 1) + h(P_{e1}) \triangleq N\delta_{1,N} \tag{23}$$

$$H(W_0, W_2|\mathbf{Y}_2) \leq P_{e2} \log(M_0 M_2 - 1) + h(P_{e2}) \triangleq N\delta_{2,N}. \tag{24}$$

where $\delta_{t,N} \to 0$ as $P_{et} \to 0$ (or as $P_e \to 0$). It follows that

$$H(W_0, W_1|\mathbf{Y}_1) = H(W_0|\mathbf{Y}_1) + H(W_1|\mathbf{Y}_1, W_0) \leq N\delta_{1,N} \tag{25}$$

$$H(W_0, W_2|\mathbf{Y}_2) = H(W_0|\mathbf{Y}_2) + H(W_2|\mathbf{Y}_2, W_0) \leq N\delta_{2,N}. \tag{26}$$

Since conditioning cannot increase entropy, from (26) it follows that

$$H(W_2|\mathbf{Y}_2, W_0, W_1) \leq H(W_2|\mathbf{Y}_2, W_0) \leq N\delta_{2,N}. \tag{27}$$

To prove the converse, we will use the data processing inequality for the following Markov chains:

Lemma 1 *The following form Markov chains for the interference channel with a common message:*

$$W_1 \to (\mathbf{X}_1, W_0, W_2) \to \mathbf{Y}_1 \tag{28}$$
$$W_2 \to (\mathbf{X}_2, W_0, W_1) \to \mathbf{Y}_2 \tag{29}$$
$$(W_0, W_t) \to (\mathbf{X}_t, W_0) \to \mathbf{Y}_t \tag{30}$$

for $t = 1, 2$.

Proof:

The result follows easily by the problem definition and is omitted. □

We will need the data processing inequality in the following form:

Lemma 2 *For a Markov chain* $W \to (U, X) \to Y$

$$I(W; Y|U) \leq I(X; Y|U). \tag{31}$$

Proof:

We have

$$\begin{aligned} H(Y|U, X) &=^{(a)} H(Y|W, U, X) \\ &\leq^{(b)} H(Y|W, U) \end{aligned} \tag{32}$$

where (a) holds because of the Markov property and (b) since conditioning cannot increase entropy. Subtracting both sides from $H(Y|U)$ gives the desired result. □

Applying Lemma 2 to the Markov Chains (29)- (30) and using (30) yields,

$$I(W_2; \mathbf{Y}_2|W_0, W_1) \leq I(\mathbf{X}_2; \mathbf{Y}_2|W_0, W_1) \tag{33}$$
$$I(W_t; \mathbf{Y}_t|W_0) \leq I(\mathbf{X}_t; \mathbf{Y}_t|W_0) \tag{34}$$
$$I(W_0, W_1; \mathbf{Y}_1) \leq I(W_0, \mathbf{X}_1; \mathbf{Y}_1). \tag{35}$$

We first consider the bound (17) at the decoder 1. We have

$$\begin{aligned} N(R_0+&R_1+R_2) \\ &= H(W_0)+H(W_1)+H(W_2) \\ &=^{(a)} H(W_0,W_1)+H(W_2|W_0,W_1) \\ &= I(W_0,W_1;\mathbf{Y}_1)+I(W_2;\mathbf{Y}_2|W_0,W_1)+H(W_0,W_1|\mathbf{Y}_1)+H(W_2|\mathbf{Y}_2,W_0,W_1) \\ &\leq^{(b)} I(W_0,\mathbf{X}_1;\mathbf{Y}_1)+I(\mathbf{X}_2;\mathbf{Y}_2|W_0,W_1)+H(W_0,W_1|\mathbf{Y}_1)+H(W_2|\mathbf{Y}_2,W_0,W_1) \\ &\leq^{(c)} I(W_0,\mathbf{X}_1;\mathbf{Y}_1)+I(\mathbf{X}_2;\mathbf{Y}_2|W_0,W_1)+N\delta_{1,N}+N\delta_{2,N} \\ &=^{(d)} I(W_0,\mathbf{X}_1;\mathbf{Y}_1)+I(\mathbf{X}_2;\mathbf{Y}_2|W_0,W_1,\mathbf{X}_1(W_0,W_1))+N\delta_{1,N}+N\delta_{2,N} \\ &=^{(e)} I(W_0,\mathbf{X}_1;\mathbf{Y}_1)+I(\mathbf{X}_2;\mathbf{Y}_2|W_0,\mathbf{X}_1)+N\delta_{1,N}+N\delta_{2,N} \end{aligned} \tag{36}$$

where (a) follows from the independence of W_0, W_1, W_2; (b) from (33) and (35); (c) from (25) and (27); (d) and (e) from (6). If

$$I(\mathbf{X}_2;\mathbf{Y}_2|W_0,\mathbf{X}_1) \leq I(\mathbf{X}_2;\mathbf{Y}_1|W_0,\mathbf{X}_1) \tag{37}$$

then it follows from (36) that

$$\begin{aligned} N(R_0+R_1+R_2) &\leq I(W_0,\mathbf{X}_1;\mathbf{Y}_1)+I(\mathbf{X}_2;\mathbf{Y}_1|\mathbf{X}_1,W_0)+N\delta_{1,N}+N\delta_{2,N} \\ &= I(W_0,\mathbf{X}_1,\mathbf{X}_2;\mathbf{Y}_1)+N\delta_{1,N}+N\delta_{2,N} \\ &= I(\mathbf{X}_1,\mathbf{X}_2;\mathbf{Y}_1)+N\delta_{1,N}+N\delta_{2,N} \\ &\leq \sum_{n=1}^{N} I(X_{1n},X_{2n};Y_{1n})+N\delta_{1,N}+N\delta_{2,N}. \end{aligned} \tag{38}$$

Applying Willems' notation [13, Section 3]

$$U_n = W_0 \qquad n=1,\ldots,N \tag{39}$$

to condition (37) yields

$$I(\mathbf{X}_2;\mathbf{Y}_2|\mathbf{X}_1,\mathbf{U}) \leq I(\mathbf{X}_2;\mathbf{Y}_1|\mathbf{X}_1,\mathbf{U}) \tag{40}$$

We can now proceed as in [10], [14] to show that the condition (40) reduces to a per letter condition

$$I(X_2;Y_2|X_1,U) \leq I(X_2;Y_1|X_1,U) \tag{41}$$

to be satisfied for every distribution of the form (18). It can then be shown that (41), and thus condition (40), is equivalent to the strong interference condition (2). In the following, we present a more direct proof.

Theorem 2 *Condition*

$$I(\mathbf{X}_2;\mathbf{Y}_2|\mathbf{X}_1,\mathbf{U}) \leq I(\mathbf{X}_2;\mathbf{Y}_1|\mathbf{X}_1,\mathbf{U}) \tag{42}$$

is satisfied for all input distributions of the form

$$p(\mathbf{u},\mathbf{x}_1,\mathbf{x}_2) = p(\mathbf{x}_1|\mathbf{u})p(\mathbf{x}_2|\mathbf{u})p(\mathbf{u}) \tag{43}$$

if and only if the vector version of the strong interference condition (2)

$$I(\mathbf{X}_2;\mathbf{Y}_2|\mathbf{X}_1) \leq I(\mathbf{X}_2;\mathbf{Y}_1|\mathbf{X}_1) \tag{44}$$

is satisfied for all $p(\mathbf{x}_1,\mathbf{x}_2) = p(\mathbf{x}_1)p(\mathbf{x}_2)$.

Proof.

To show that (42) implies (44), we observe that since (42) holds for all input distributions of the form (43), it must also hold for $\mathbf{U}$ independent from $\mathbf{X}_1, \mathbf{X}_2$. By choosing such distribution on $(\mathbf{U}, \mathbf{X}_1, \mathbf{X}_2)$ we obtain (44).

To prove the other direction, we write the mutual information in (42) as

$$I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, \mathbf{U}) = E_{\mathbf{U}}[I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, \mathbf{U} = \mathbf{u})] \tag{45}$$

where

$$I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, \mathbf{U} = \mathbf{u}) = \sum_{\mathbf{x}_1} \sum_{\mathbf{x}_2} \sum_{\mathbf{y}_1} p(\mathbf{y}_1 | \mathbf{x}_1, \mathbf{x}_2, \mathbf{u}) p(\mathbf{x}_1, \mathbf{x}_2 | \mathbf{u}) \log \frac{p(\mathbf{y}_1 | \mathbf{x}_1, \mathbf{x}_2, \mathbf{u})}{p(\mathbf{y}_1 | \mathbf{x}_1, \mathbf{u})}. \tag{46}$$

We next observe that Markovity $(W_0, \mathbf{Y}_1^{n-1}, \mathbf{X}_1^{n-1}, \mathbf{X}_2^{n-1}) \to (X_{1,n}, X_{2,n}) \to (Y_{1,n}, Y_{2,n})$ and (39) imply

$$(\mathbf{U}, \mathbf{Y}_1^{n-1}, \mathbf{X}_1^{n-1}, \mathbf{X}_2^{n-1}) \to (X_{1,n}, X_{2,n}) \to (Y_{1,n}, Y_{2,n}) \tag{47}$$

and therefore

$$p(\mathbf{y}_1 | \mathbf{x}_1, \mathbf{x}_2, \mathbf{u}) = p(\mathbf{y}_1 | \mathbf{x}_1, \mathbf{x}_2) \tag{48}$$

Furthermore, the same reasoning as in [13, $3.4.11$] shows that (6), (7) and (39) imply $\mathbf{X}_1 \to \mathbf{U} \to \mathbf{X}_2$, that is

$$p(\mathbf{x}_1, \mathbf{x}_2 | \mathbf{u}) = p(\mathbf{x}_1 | \mathbf{u}) p(\mathbf{x}_2 | \mathbf{u}). \tag{49}$$

In particular, (just to convince me/you, but will be omitted in the paper)

$$\begin{aligned} p(\mathbf{x}_1, \mathbf{x}_2 | \mathbf{u}) &=^{(a)} p(\mathbf{x}_1, \mathbf{x}_2 | w_0) \\ &=^{(b)} \sum_{w_1} \sum_{w_2} p(w_1) p(w_2) p(\mathbf{x}_1 | w_1, w_2, w_0) p(\mathbf{x}_2 | \mathbf{x}_1, w_1, w_2, w_0) \\ &=^{(c)} \sum_{w_1} \sum_{w_2} p(w_1) p(w_2) p(\mathbf{x}_1 | w_1, w_0) p(\mathbf{x}_2 | w_2, w_0) \\ &= \sum_{w_1} p(w_1) p(\mathbf{x}_1 | w_1, w_0) \sum_{w_2} p(w_2) p(\mathbf{x}_2 | w_2, w_0) \\ &= p(\mathbf{x}_1 | \mathbf{u}) p(\mathbf{x}_2 | \mathbf{u}) \end{aligned} \tag{50}$$

where (a) follows from (39); (b) follows since W_1 and W_2 are independent; (c) from (6) and (7).

Equations (48) and (49) imply that the mutual information (46) equals $I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1)$ for the conditional input distribution $p(\mathbf{x}_1, \mathbf{x}_2 | \mathbf{u}) = p(\mathbf{x}_1 | \mathbf{u}) p(\mathbf{x}_2 | \mathbf{u})$. Since (44) holds for any input distribution on independent inputs, it follows that for all $\mathbf{u}$

$$I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1, \mathbf{U} = \mathbf{u}) \leq I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, \mathbf{U} = \mathbf{u}). \tag{51}$$

Therefore, every element in the average (45) is greater that the corresponding element in $I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1, \mathbf{U})$ and claim (42) follows. □

Equivalently, conditions $I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1, \mathbf{U}) \leq I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1, \mathbf{U})$ satisfied for all distributions as in (43) can be shown to be equivalent to $I(\mathbf{X}_2; \mathbf{Y}_2 | \mathbf{X}_1) \leq I(\mathbf{X}_2; \mathbf{Y}_1 | \mathbf{X}_1)$ satisfied for all independent inputs.

To prove that the bound (16) at the decoder 1 is valid, we consider

$$
\begin{aligned}
N(R_1{+}R_2)& \\
&= H(W_1) + H(W_2) \\
&=^{(a)} H(W_1|W_0) + H(W_2|W_0, W_1) \\
&= I(W_1; \mathbf{Y}_1|W_0) + I(W_2; \mathbf{Y}_2|W_0, W_1) + H(W_1|\mathbf{Y}_1, W_0) + H(W_2|\mathbf{Y}_2, W_0, W_1) \\
&\leq^{(b)} I(\mathbf{X}_1; \mathbf{Y}_1|W_0) + I(\mathbf{X}_2; \mathbf{Y}_2|W_0, W_1) + H(W_1|\mathbf{Y}_1, W_0) + H(W_2|\mathbf{Y}_2, W_0, W_1) \\
&\leq^{(c)} I(\mathbf{X}_1; \mathbf{Y}_1|W_0) + I(\mathbf{X}_2; \mathbf{Y}_2|W_0, W_1) + N\delta_{1,N} + N\delta_{2,N} \\
&=^{(d)} I(\mathbf{X}_1; \mathbf{Y}_1|W_0) + I(\mathbf{X}_2; \mathbf{Y}_2|W_0, W_1, \mathbf{X}_1(W_0, W_1)) + N\delta_{1,N} + N\delta_{2,N} \\
&=^{(e)} I(\mathbf{X}_1; \mathbf{Y}_1|\mathbf{U}) + I(\mathbf{X}_2; \mathbf{Y}_2|\mathbf{X}_1, \mathbf{U}) + N\delta_{1,N} + N\delta_{2,N}.
\end{aligned}
\tag{52}
$$

where again (a) follows from the independence of W_0, W_1, W_2; (b) from (33) and (34); (c) from (25) and (27); (d) from (6); (e) from (6) and (39). Again, if (37) holds, then (52) becomes

$$
\begin{aligned}
R_1 + R_2 &\leq I(\mathbf{X}_1; \mathbf{Y}_1|\mathbf{U}) + I(\mathbf{X}_2; \mathbf{Y}_1|\mathbf{X}_1, \mathbf{U}) + N\delta_{1,N} + N\delta_{2,N} \\
&= I(\mathbf{X}_1, \mathbf{X}_2; \mathbf{Y}_1|\mathbf{U}) + N\delta_{1,N} + N\delta_{2,N} \\
&\leq \sum_{n=1}^{N} I(X_{1n}, X_{2n}; Y_{1n}|U_n) + N\delta_{1,N} + N\delta_{2,N}.
\end{aligned}
\tag{53}
$$

The same approach can be used to show that the bounds (16) and (17) are satisfied at decoder 2 under a condition equivalent to (40)

$$I(\mathbf{X}_1; \mathbf{Y}_1|\mathbf{X}_2, \mathbf{U}) \leq I(\mathbf{X}_1; \mathbf{Y}_2|\mathbf{X}_2, \mathbf{U}) \tag{54}$$

which, due to Theorem 2, reduces to

$$I(\mathbf{X}_1; \mathbf{Y}_1|\mathbf{X}_2) \leq I(\mathbf{X}_1; \mathbf{Y}_2|\mathbf{X}_2). \tag{55}$$

Finally, the bounds (14) and (15) are the single user upper bounds and hence have to be satisfied. □

Combining Theorem 2 and Lemma in [10] we conclude that the conditions (40) and (54) reduce to the strong interference conditions (1) and (2) respectively, thus proving Theorem 1.

5 Gaussian Channel

We next consider the Gaussian interference channel in the standard form [9, 15]

$$y_{1i} = x_{1i} + h_{12}x_{2i} + z_{1i} \tag{56}$$

$$y_{2i} = h_{21}x_{1i} + x_{2i} + z_{2i} \tag{57}$$

where the Z_t are independent, zero-mean, unit-variance Gaussian random variables. The code definition is the same as that given in Section 2 with the addition of the power constraints

$$\frac{1}{N}\sum_{i=1}^{N} x_{ti}^2 \leq P_t, \qquad t = 1, 2. \tag{58}$$

From the maximum-entropy theorem [16, Thm. 9.6.5] it follows that Gaussian inputs are optimal. We have the following result.

Corollary 1 *When the strong interference conditions $h_{12}^2 \geq 1$, $h_{21}^2 \geq 1$ are satisfied, the capacity region of the Gaussian strong interference channel with common information is given by*

$$\mathcal{R} = \bigcup \{(R_1, R_2) :$$
$$0 \leq R_1 \leq C(\bar{a}P_1) \tag{59}$$
$$0 \leq R_2 \leq C(\bar{b}P_2) \tag{60}$$
$$R_1 + R_2 \leq \min_{j\in\{1,2\}} C\left(h_{j1}^2\bar{a}P_1 + h_{j2}^2\bar{b}P_2\right) \tag{61}$$
$$0 \leq R_0 + R_1 + R_2 \leq \min_j C\left(h_{j1}^2P_1 + h_{j2}^2P_2 + 2h_{j1}h_{j2}\sqrt{aP_1bP_2}\right)\} \tag{62}$$

where the union is over all a, b, for $0 \leq a \leq 1, 0 \leq b \leq 1$, $\bar{a} = 1 - a$, $\bar{b} = 1 - b$, and $h_{11} = h_{22} = 1$.

6 Discussion

Communication systems with encoders that have to send both private and common information naturally arise in the case when encoders can partially cooperate as in [1,7]. After such cooperation, the common information consists of two indexes each partially describing one of the two original messages. The assumption of our model that the entire common message is decoded simplifies the problem. However, a receiver interested in a message from only one encoder, as is the case in the interference channel, will be interested in only a *part* of the common message. Understanding such communication problems appears to be much more challenging and is the subject of our future work.

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