# Effect of Node Mobility on Highway Mobile Infostation Networks 

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#### Abstract

In a mobile infostation network, any two nodes communicate when they are in proximity. Under this transmission constraint, any pair of nodes is intermittently connected as mobility shuffles the node locations. In this paper, we evaluate the effect of node mobility on highway mobile infostation networks. Each node enters a highway segment at a Poisson rate with a random speed drawn from a known but arbitrary distribution. Moreover, each node changes speed at each highway segment. Since nodes have different speed, a node may overtake other nodes or be overtaken as time evolves. Using arguments from renewal reward theory, the long run fraction of time an observer node is connected, and the long run average data rate can be derived. In this paper, however, we consider the special case of no speed change in each highway segment. In this case, the performance metrics are functions of the observer node speed. We consider both forward traffic scenarios, in which two nodes moving in the same direction have a transient connection when they are within range from each other, and reverse traffic scenarios in which two nodes travelling in opposite directions are connected transiently when they are in range. For node speed that is uniformly distributed, we reveal that the expected fraction of connection time, or expected number of connections in queuing terminology, is independent of the observer node speed in reverse traffic. In forward traffic, on the other hand, the fraction of connection time increases with observer speed. That is, the network performance improves with node mobility, which is unique to the mobile infostation networking paradigm.


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## 1. INTRODUCTION

In a mobile infostation network, nodes operate on low transmit power. Any two nodes communicate only when they are in proximity and have a very good channel. Under this transmission constraint, any pair of nodes is intermittently connected as mobility shuffles the node locations. The network capacity of mobile infostation networks compares favorably to conventional multihop ad hoc networks. In [5] Gupta and Kumar showed that the per node throughput in a multihop network drops to zero at a rate $O\left(\frac{W}{\sqrt{n \ln n}}\right)$ in the limit of large number of nodes $n$. Thus multihop networks do not scale with large network size. On the other hand, Grossglauser and Tse showed in [3] that the per node throughput of a mobile infostation network is $O(1)$, independent of the number of nodes. This capacity is achieved through a two hop relay strategy.

Assume that each node in the network selects a random destination for unicast. We focus on a source node $i$, which has packets to deliver to a destination node $j$, as shown in Figure 1. As time evolves, node $i$ moves along a random trajectory and eventually runs into nodes 1 and 2 . Although neither nodes 1 nor 2 are the destination of $i, i$ still relays the packets to them, with the expectation that when each of the relay nodes reaches the destination $j$, it will complete the second relay on behalf of node $i$. In steady state, each of the other $n-2$ nodes contains packets generated by node $i$ and destined to node $j$. At any network snapshot, it is almost surely that the nearest neighbor of node $j$ has packets addressed from node $i$ and completes the second relay on the behalf of $i$. That is, the long run per node throughput is constant and is independent of the network size. This capacity improvement comes from the exploitation of node mobility to physically carry the packets around the network, and is independent of the underlying mobility model, as long as the mobility process is ergodic.


Figure 1: Two hop packet relay strategy in a mobile infostation network.

Nevertheless, the order of magnitude improvement in network capacity comes at a cost. End-to-end transmissions incur a random delay that is at the same time scale of the mobility process. Thus, a mobile infostation network is applicable to delay tolerant applications with a heavy bandwidth requirement, say, in a content distribution application where all nodes are subscribers to a movie or news content provider. In this type of applications, a user is not concerned and aware of the movie download schedules. The application typically runs in the background for a few hours or even a few days as a user commutes to different places in his daily routine. This is consistent to the plethora of software applications in ubiquitous computing environments [13], where computing systems become invisible and fade into the background and work for the users. In this case, we can draw a parallel of ubiquitous networking environments since users are not aware of the background networking in the mobile infostation communication paradigm.

On the other hand, there is also a tradeoff between delay and storage in a mobile infostation network. Since a node transmits the same packets to all the relay nodes, there is heavy redundancy in packet transmissions and storage. This may not present a big challenge to researchers, since hardware storage follows the Moore's law quite well and storage capacity is approximately doubled every year. Moreover, a simple time-to-live (TTL) field can also be appended to each packet such that packets can be dropped when the TTL field has expired. This alleviates the storage requirement in individual nodes at the expense of more delay in packet delivery.

Motivated by the dramatic capacity improvement of mobile infostation networks, there are a number of recent papers that examine similar networking architectures that exploit mobility in disseminating data. Whereas [3] focused on unicast, most other papers in the literature focused on multicast. The potential spectrum of applications ranges from biological information acquisition systems used in the habitat monitoring of endangered wildlife species such as whales [12] and zebras [7] on one hand, to mundane movie and news downloading in a content distribution network $[15,16]$ and location specific information services $[9,10]$ on the other hand. Most of the work so far [3, 12, 7, 9, 10] has focused on network scenarios in which nodes cooperate. For some applications such as habitat monitoring of wildlife species, sensor nodes are deployed from a single organization. The cooperation assumption between
nodes is valid. On the other hand, in commercial applications each node in the network is autonomous and may act selfishly. A node is not incentivized to relay other people's packets since it is expending its own bandwidth and energy resources in a transmission. The issue of noncooperation between nodes was examined in [15, 16]. Under the noncooperation assumption, a social contract is defined that is observed by all nodes. Transmissions between two proximate nodes are allowed only when both nodes benefit from a file exchange. It turns out that network throughput is dependent on the size of the content being disseminated. Moreover, in order to minimize its own total download time, a node will exchange for data that it is not interested in per se. That is, nodes are incentivized to cooperate in data dissemination even if nodes are selfish. Nevertheless, simple interference and mobility models are used in [15, 16] to facilitate analysis. Recently, the effect of transmit range on network capacity is examined in [14] using a refined interference model. When nodes in a mobile infostation network are operated at the optimal range that maximizes network capacity, the number of neighbors seen by a node is around 1 . This is in sharp contrast to the well known magic number of 6 to 8 neighbors in conventional multihop ad hoc networks.

Multihop networks and mobile infostation networks are the two extreme instantiations of the capacity-delay tradeoff over many possible networking paradigms. In order to expedite data dissemination in a mobile infostation network, multihop forwarding may also be used occasionally, as in $[9,10]$, if a node has not done so for other nodes for some time. Similarly, node mobility can also be exploited in multihop networks to improve network performance. For instance, in [4] node mobility is exploited to disseminate coordinates of all node locations without incurring any communication overhead. The location information is useful for nodes to make local routing decisions to the destination when geographic routing schemes [6] are used.

In this paper, we examine the effect of node mobility in mobile infostation networks. In [3], mobility provides a mechanism such that numerous instances of excellent channels between different nodes can be exploited. The realization of large network capacity comes from the translation of maximal spatial transmission concurrency in each network snapshot to the long run end-to-end network capacity. The physical implication of mobility in node encounters has been glossed over. In reality, the total connection time of a node over a specific interval depends on the node encounter rate and the connection time in each encounter, both of which depend on the relative mobility of nodes. Although a high node speed results in more node encounters, the connection time in each node encounter also decreases. It is not apparent whether high or low speed results in a larger connection time, and thus, data rate. To this end we propose a new mobility model for highway networks. The highway scenario proves to be interesting despite its mathematical simplicity. Consider the forward traffic scenario, that is traffic travelling at the same direction as the user of interest, or the observer node. The connection time in one node encounter is much larger than that of reverse direction traffic, but the node encounter rate is also much smaller. In the reverse traffic scenario, on the other hand, the connection time in a node encounter is typically small, since nodes are travelling in opposite directions. Nevertheless, node encounter rate is also much higher in the reverse traffic scenario. It is not immediately apparent which traffic type offers the greater fraction of connection time, or number of connections in queueing terminology. Second, the connection time in an encounter depends on the

## forward traffic at rate $\lambda$



Figure 2: Illustration of the highway mobile infostation network model.
transmit range of the nodes. For both forward and reverse traffic, an optimal transmit range exists such that the long run data rate of a node is maximized.

The rest of the paper is organized as follows. In section 2, we describe the system model. Section 3 is devoted to performance analysis for arbitrary speed distribution. The special case of uniform speed distribution is considered in section 4 and numerical results are obtained in section 5. Finally, we discuss the implications of our results in section 6.

## 2. SYSTEM MODEL

We consider a highway network in which fixed infostations are placed regularly at a distance $d$ from each other. We assume that all nodes are subscribers of a content provider, say a movie distribution network. Movies are split into many files and are cached in the infostations at various locations. Besides downloading directly from an infostation, a node participates in data exchanges whenever there is another node in proximity. We assume data exchanges between two proximate nodes always take place without further negotiation. The amount of data exchanged is proportional to the connection time in an encounter and the data transmission rate. It was shown in [16] that in a large network, peer-to-peer node exchanges account for most of the data transmissions. As the network size increases, the importance of fixed infostations in data dissemination dwindles. Thus, in this paper we focus on peer-to-peer connections between proximate mobile nodes in node encounters only. Connections to fixed infostations on the highway are ignored.

In our analysis, we focus on an arbitrary highway segment between infostations $A$ and $B$, as shown in Figure 2. On each highway segment, a node moves at a speed $V$, an iid random variable drawn from a known but arbitrary distribution $G$. Since nodes have different speeds, a node may overtake other nodes or be overtaken as it traverses the highway segment. We make all our observations at a specific node, called the observer node. Two types of traffic are considered here. For forward traffic, nodes are injected into the highway segment at a Poisson rate $\lambda$ from infostation $A$. The Poisson arrival assumption of mobile nodes is valid if the speed of individual nodes is independent and does not interact. That is, we assume there is no delay incurred in a node encounter, in which a platoon of nodes forms behind a node that moves slowly. This
is plausible in a wide highway with multiple lanes and moderate traffic, where nodes overtake others at different lanes. The injected nodes move at the same direction as the observer node. This is called the wide motorway model in [8]. Similarly, for reverse traffic nodes are injected into the highway segment at a Poisson rate $\lambda$ from infostation $B$. The injected nodes move in the opposite direction of the observer node. More generally, a node changes speed as time evolves. We assume each node still moves at a constant speed in a highway segment. Whenever a node traverses a new highway segment, we stipulate that each node selects a new speed from the distribution $G$, independent of the previous speed.

Suppose the observer node moves at a speed $V=v_{0}$ on a highway segment from infostation $A$ to $B$. We denote the time for the node to traverse a highway segment as the cycle duration, given by $T=d / V$, with a corresponding distribution $F . F$ and $G$ are obviously related, given by $\bar{F}(t)=G(d / t)$, where $\bar{F}(t)=1-F(t)$ denotes the complementary distribution function. In this paper, we describe mobility of the observer node in terms of cycle duration rather than node speed for convenience, since the performance metrics are closely related to $t_{0}$.

Given the observer node cycle duration $t_{0}=d / v_{0}$ in a highway segment, we denote $N_{1}\left(t_{0}\right)$ and $N_{2}\left(t_{0}\right)$ as the number of node encounters in forward traffic and reverse traffic scenarios, where a node encounter occurs when two nodes are approaching to within a transmit range $r$ from each other, and the subscripts 1 and 2 denotes a connection with forward and reverse traffic respectively. The connection time in each node encounter is defined as the duration when both nodes are within the transmit range $r$ from each other. Obviously, the connection times in forward traffic $Y_{1}^{i}\left(t_{0}\right)$ and reverse traffic $Y_{2}^{i}\left(t_{0}\right)$ at the $i$-th node encounter are random variables dependent on the relative speed of the nodes and the common transmit range of all nodes $r$. For many speed distributions, two nodes having a similar speed may have a connection time with unbounded mean. However, each node only has a finite amount of data for dissemination to another node. To model this we specify a connection time limit parameter $c$ to limit the actual connection time in a node encounter, given by $B_{1}^{i}\left(t_{0}\right)=\min \left(Y_{1}^{i}\left(t_{0}\right), c\right)$ and $B_{2}^{i}\left(t_{0}\right)=\min \left(Y_{2}^{i}\left(t_{0}\right), c\right)$. We also denote the total connection time of the observer node in a highway segment as $Z_{1}\left(t_{0}\right)$ and $Z_{2}\left(t_{0}\right)$. Obviously,

$$
\begin{align*}
& Z_{1}\left(t_{0}\right)=\sum_{i=1}^{N_{1}\left(t_{0}\right)} B_{1}^{i}\left(t_{0}\right)  \tag{1}\\
& Z_{2}\left(t_{0}\right)=\sum_{i=1}^{N_{2}\left(t_{0}\right)} B_{2}^{i}\left(t_{0}\right) \tag{2}
\end{align*}
$$

When speed changes are incorporated to our mobility model, the long run average fraction of connection time and data rate are the appropriate metrics. It turns out that a simple characterization of these metrics is possible by drawing results from renewal reward theory [11]. Let $M(t), t \geq 0$ be a counting process to denote the number of highway segments traversed by the observer node. At the $n$th highway segment, the observer node selects an iid random speed $V_{n}$ independent of the speed $V_{n-1}$ at the previous highway segment $n-1$. The corresponding cycle durations $T_{n}$ are iid random variables. Since $M(t)$ is a counting process with iid interarrival times, $M(t)$ is a renewal process. Moreover, we denote $R_{n}$ as
the reward earned in the $n$th cycle, or renewal period. If we let

$$
\begin{equation*}
R(t)=\sum_{n=1}^{M(t)} R_{n} \tag{3}
\end{equation*}
$$

then $R(t)$ is the total reward earned by time $t$. Let $E[R]=E\left[R_{n}\right]$ and $E[T]=E\left[T_{n}\right]$, the renewal reward theorem [11] states that if $E[R]<\infty$ and $E[T]<\infty$, then with probability 1 ,

$$
\begin{equation*}
\lim _{t \rightarrow \infty} \frac{R(t)}{t}=\frac{E[R]}{E[T]} \tag{4}
\end{equation*}
$$

That is, the rate of earning reward in the long run is just the ratio of the expected reward in a cycle and the expected cycle duration.

Accordingly, if we define a reward of 1 unit is earned every time the observer node encounters another node, the reward accrued in highway segment $n$ is $R_{n}=N_{1}\left(T_{n}\right)$ for forward traffic and $R_{n}=$ $N_{2}\left(T_{n}\right)$ for reverse traffic. The long run node encounter rate of the observer node is simply

$$
\begin{equation*}
\mathcal{N}_{1}=\lim _{t \rightarrow \infty} \frac{R(t)}{t}=\frac{E\left[N_{1}(T)\right]}{E[T]} \tag{5}
\end{equation*}
$$

in forward traffic scenario and

$$
\begin{equation*}
\mathcal{N}_{2}=\lim _{t \rightarrow \infty} \frac{R(t)}{t}=\frac{E\left[N_{2}(T)\right]}{E[T]} \tag{6}
\end{equation*}
$$

in reverse traffic scenario. Similarly, suppose a reward equivalent to the connection time $B_{1}\left(t_{0}\right)$ is earned each time the observer node encounters another node. Let the observer node mobility at the $n$ th highway segment be $T_{n}=t_{0}$. The accrued reward $R_{n}$ is the sum of the connection times of all node encounters in the highway segment, i.e.

$$
\begin{equation*}
R_{n}=Z_{1}\left(t_{0}\right)=\sum_{i=1}^{N_{1}\left(t_{0}\right)} B_{1}^{i}\left(t_{0}\right) \tag{7}
\end{equation*}
$$

in forward traffic scenarios. In this case, the long run rate of earning reward is given by

$$
\begin{equation*}
\mathcal{Z}_{1}=\lim _{t \rightarrow \infty} \frac{R(t)}{t}=\frac{E\left[Z_{1}(T)\right]}{E[T]} \tag{8}
\end{equation*}
$$

Similarly in reverse traffic scenarios we have

$$
\begin{equation*}
R_{n}=Z_{2}\left(t_{0}\right)=\sum_{i=1}^{N_{2}\left(t_{0}\right)} B_{2}^{i}\left(t_{0}\right) \tag{9}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{Z}_{2}=\lim _{t \rightarrow \infty} \frac{R(t)}{t}=\frac{E\left[Z_{2}(T)\right]}{E[T]} \tag{10}
\end{equation*}
$$

Finally, suppose a reward equivalent to the amount of data sent and received is earned each time the observer node encounters another node. Assuming non-adaptive radios are used, the data rate is the Shannon rate at the transmit range boundary $r$, given by

$$
\begin{equation*}
C(r)=\ln \left(1+1 / r^{4}\right) \tag{11}
\end{equation*}
$$

where we have assumed a path gain exponent of 4 and ignored the effect of mutual interference. Let the cycle duration of the observer node at the $n$-th highway segment be $T_{n}=t_{0}$. The accrued reward $R_{n}$ is the total amount of data transmitted or received by the observer node in the highway segment, denoted as $W_{1}\left(t_{0}\right)$. The average rate of earning reward in the long run should be interpreted
as the long run data rate, given by

$$
\begin{equation*}
\mathcal{W}_{1}=\frac{E\left[W_{1}(T, r)\right]}{E[T]}=C(r) \frac{E\left[Z_{1}(T, r)\right]}{E[T]}=C(r) \mathcal{Z}_{1} \tag{12}
\end{equation*}
$$

in forward traffic scenarios and

$$
\begin{equation*}
\mathcal{W}_{2}=\frac{E\left[W_{2}(T, r)\right]}{E[T]}=C(r) \frac{E\left[Z_{2}(T, r)\right]}{E[T]}=C(r) \mathcal{Z}_{2} \tag{13}
\end{equation*}
$$

in reverse traffic scenarios. We emphasize both connection time $Z$ and the amount of delivered data $W$ are dependent on the transmit range $r$. It is intuitive that $\mathcal{W}_{1}=0$ and $\mathcal{W}_{2}=0$ when the transmit range is either zero or very large. An optimal transmit range $r$ exists for both traffic types such that $\mathcal{W}_{1}$ and $\mathcal{W}_{2}$ are maximized respectively.

In this paper, we consider the special case when each node selects an arbitrary speed upon entrance to the highway. However, each node moves with the same speed in different highway segments. Since the cycle duration $T_{n}$ is still iid, the renewal arguments continues to apply in the constant speed case. Moreover, the long run fraction of connection time $\mathcal{Z}_{1}$ and $\mathcal{Z}_{2}$ simplifies to

$$
\begin{equation*}
\mathcal{Z}_{1}=\eta_{1}\left(t_{0}\right)=\frac{E\left[Z_{1}\left(t_{0}\right)\right]}{t_{0}} \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{Z}_{2}=\eta_{2}\left(t_{0}\right)=\frac{E\left[Z_{2}\left(t_{0}\right)\right]}{t_{0}} \tag{15}
\end{equation*}
$$

which can be interpreted as the expected fraction of connection time $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ as a function of observer node mobility $t_{0}$. In general, since a node can simultaneously maintain more than one connection, $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ can be larger than 1 . In queueing terminology, the observer node is a server and the connection time in a node encounter corresponds to the service time. Although an optimum transmit range exists in both forward and reverse traffic scenarios, we will not pursue this idea further in this paper. Bear in mind that when the transmit range is conditionally given, the fraction of connection time $\mathcal{Z}$ is linearly proportional to the long run data rate $\mathcal{W}$.

## 3. PERFORMANCE ANALYSIS

Consider the forward traffic scenario. Suppose the observer node enters the highway segment at time $s$ and departs at time $s+t_{0}$. We denote an event occurs at time $t \in[0, \infty)$ if a node enters the highway segment at infostation $A$. Since the node travels with random speed $V=d / T$, this node leaves the highway segment at time $t+T$. We define $p_{1}(t)$ as the probability that a forward entrant at time $t$ has an encounter to the observer node at the highway segment. It is straightforward to show that for $t<s$, an encounter occurs if $t+T>s+t_{0}$ when the observer node overtakes the encounter node. That is,

$$
\begin{equation*}
p_{1}(t)=P\left[T+t>s+t_{0}\right]=\bar{F}\left(s+t_{0}-t\right) \tag{16}
\end{equation*}
$$

Similarly, for $s<t<s+t_{0}$, an encounter occurs if $t+T<s+t_{0}$ when the observer node is overtaken by the encounter node. This occurs with probability

$$
\begin{equation*}
p_{1}(t)=P\left[T+t<s+t_{0}\right]=F\left(s+t_{0}-t\right) \tag{17}
\end{equation*}
$$

Finally, for $t>s+t_{0}$, a node encounter will not occur in the highway segment, i.e. $p_{1}(t)=0$. Combining the three cases together,
we have

$$
p_{1}(t)=\left\{\begin{array}{rl}
\bar{F}\left(s+t_{0}-t\right) & t<s  \tag{18}\\
F\left(s+t_{0}-t\right) & s<t<s+t_{0} \\
0 & t>s+t_{0}
\end{array}\right.
$$

Assuming the network has been operated for a long time $s \rightarrow \infty$ before we observe the observer node enters the highway segment. The total number of node encounters is also a Poisson process and in steady state $s \rightarrow \infty$, it is given by

$$
\begin{align*}
\lim _{s \rightarrow \infty} E\left[N_{1}\left(t_{0}\right)\right] & =\lim _{s \rightarrow \infty} \lambda \int_{0}^{\infty} p_{1}(t) d t  \tag{19}\\
& =\lambda\left(\int_{0}^{t_{0}} F(t) d t+\int_{t_{0}}^{\infty} \bar{F}(t) d t\right) \tag{20}
\end{align*}
$$

It can be shown $E\left[N_{1}\left(t_{0}\right)\right]$ attains a global minimum when the observer node cycle duration $t_{0}$ is the median of the distribution $F$ By twice differentiating (20) [11]. This agrees with our intuition that there are few node encounters if the observer node moves at a speed that goes along with the majority.

For reverse traffic, we define an event occurs at time $t$ if a node enters the highway segment from infostation $B$. For an event at time $t$, it is marked with probability $p_{2}(t)$ if there is a node encounter with the observer node at the highway segment. For $t>$ $s+t_{0}$, the reverse entrant node enter the highway segment after the observer node has left, the encounter probability is therefore $p_{2}(t)=0$. For $s<t<s+t_{0}$, the reverse entrant node enters the highway segment after observer node, but before the observer node has left. Thus the encounter probability is $p_{2}(t)=1$. Finally, when $t<s$, a node encounter occurs if the reverse entrant node leave after the observer node arrives at the highway segment. This happens with probability

$$
\begin{equation*}
p_{2}(t)=P[T+t>s]=\bar{F}(s-t) \tag{21}
\end{equation*}
$$

Combining the three cases, we have

$$
p_{2}(t)=\left\{\begin{array}{rl}
0 & t>s+t_{0}  \tag{22}\\
1 & s<t<s+t_{0} \\
\bar{F}(s-t) & t<s
\end{array}\right.
$$

The total number of node encounters in steady state is

$$
\begin{align*}
\lim _{s \rightarrow \infty} E\left[N_{2}\left(t_{0}\right)\right] & =\lim _{s \rightarrow \infty} \lambda \int_{0}^{\infty} p_{2}(t) d t  \tag{23}\\
& =\lambda\left(t_{0}+E[T]\right) \tag{24}
\end{align*}
$$

where $E[T]$ is the expected cycle duration given by

$$
\begin{equation*}
E[T]=\int_{0}^{\infty} \bar{F}(t) d t \tag{25}
\end{equation*}
$$

The long run node encounter rate for both traffic types can be obtained by averaging over the speed distribution. Thus we have

$$
\begin{align*}
E\left[N_{1}(T)\right] & =\int_{0}^{\infty} E\left[N_{1}\left(t_{0}\right)\right] d F\left(t_{0}\right)  \tag{26}\\
& =\lambda \int_{0}^{\infty} \int_{0}^{t_{0}} F(t) d t d F\left(t_{0}\right)  \tag{27}\\
& +\lambda \int_{0}^{\infty} \int_{t_{0}}^{\infty} \bar{F}(t) d t d F\left(t_{0}\right) \tag{28}
\end{align*}
$$

which yields

$$
\begin{equation*}
E\left[N_{1}(T)\right]=2 \lambda \int_{0}^{\infty} \bar{F}(t) F(t) d t \tag{29}
\end{equation*}
$$

upon simplification using integration by parts. Similarly, we have

$$
\begin{align*}
E\left[N_{2}(T)\right] & =\int_{0}^{\infty} E\left[N_{2}\left(t_{0}\right)\right] d F\left(t_{0}\right)  \tag{30}\\
& =2 \lambda E[T] \tag{31}
\end{align*}
$$

(29) and (31) suggest that the expected node encounter rate for reverse traffic is always larger than that for forward traffic, which is obviously true. Moreover, (31) shows that the expected node encounter rate is completely characterized by the traffic intensity $\lambda$ and the first moment of distribution $F$.

To compute the expected connection time in one encounter for forward traffic $E\left[B_{1}\left(t_{0}\right)\right]$, we note that

$$
\begin{align*}
E\left[B_{1}\left(t_{0}\right)\right] & =\int_{0}^{c} P\left[Y_{1}\left(t_{0}\right)>t\right] d t  \tag{32}\\
& =\int_{0}^{c} P\left[\frac{2 r}{\left|v_{0}-V\right|}>t\right] d t  \tag{33}\\
& =\int_{0}^{c} G\left(\frac{2 r}{t}+\frac{d}{t_{0}}\right)-G\left(\frac{d}{t_{0}}-\frac{2 r}{t}\right) d t \tag{34}
\end{align*}
$$

Similarly, in reverse traffic we have

$$
\begin{align*}
E\left[B_{2}\left(t_{0}\right)\right] & =\int_{0}^{c} P\left[Y_{2}\left(t_{0}\right)>t\right] d t  \tag{35}\\
& =\int_{0}^{c} G\left(\frac{2 r}{t}-\frac{d}{t_{0}}\right) d t \tag{36}
\end{align*}
$$

Refer to Figure 2 again, the total connection time for forward traffic is obtained by summing all individual connection time $B_{1}^{i}\left(t_{0}\right)$, $i \in\left[1, N_{1}\left(t_{0}\right)\right]$ over the cycle. In the event that the connection time of the encounter $N_{1}\left(t_{0}\right)$ overshoots the end of the cycle, the observer node undergoes a renewal and selects a new speed. This in turn modifies the connection time $B_{1}^{N_{1}\left(t_{0}\right)}$. Nevertheless, the boundary effect of an overshoot connection time is minimal when either $N_{1}\left(t_{0}\right)$ is large, or when $B_{1}\left(t_{0}\right) \leq c \ll t_{0}=d / v_{0}$. The former assumption is valid when the traffic intensity $\lambda$ is moderate, such that $N_{1}\left(t_{0}\right) \gg 1$. The latter assumption is valid when the distance between fixed infostations $d$ is large, which is likely in an initial deployment of a fixed infostation network. Ignoring the boundary effect of $B_{1}^{N_{1}\left(t_{0}\right)}\left(t_{0}\right)$, we have

$$
\begin{equation*}
Z_{1}\left(t_{0}\right)=\sum_{i=1}^{N_{1}\left(t_{0}\right)} B_{1}^{i}\left(t_{0}\right) \tag{37}
\end{equation*}
$$

It can be shown that $B_{1}^{i}\left(t_{0}\right)$ are iid random variables and $N\left(t_{0}\right)$ is Poisson. However, $N_{1}\left(t_{0}\right)$ and $B_{1}^{i}\left(t_{0}\right)$ are in general not independent. In fact, when node mobility is high, $N_{1}\left(t_{0}\right)$ is large and the corresponding $B_{1}\left(t_{0}\right)$ is small. Thus $Z_{1}\left(t_{0}\right)$ is not a compound Poisson process. Nevertheless, we note that $N_{1}\left(t_{0}\right)$ is a stopping time w.r.t. the sequence $B_{1}^{i}\left(t_{0}\right)$ since the stopping rule $\left\{N_{1}\left(t_{0}\right)=n\right\}$ is completely determined by the information up to time $n$, and is unrelated to $B_{1}^{n+1}\left(t_{0}\right), B_{1}^{n+2}\left(t_{0}\right)$ and so on. Thus, Wald's equality [1] can be applied to (37) to yield

$$
\begin{equation*}
E\left[Z_{1}\left(t_{0}\right)\right]=E\left[N_{1}\left(t_{0}\right)\right] E\left[B_{1}\left(t_{0}\right)\right] \tag{38}
\end{equation*}
$$

Similarly, in reverse traffic we have

$$
\begin{equation*}
E\left[Z_{2}\left(t_{0}\right)\right]=E\left[N_{2}\left(t_{0}\right)\right] E\left[B_{2}\left(t_{0}\right)\right] . \tag{39}
\end{equation*}
$$

The long run fraction of connection time, or number of connections of the observer node for both traffic types can be obtained by conditioning on distribution $F$, given by,

$$
\begin{equation*}
\mathcal{Z}_{1}=\frac{E\left[Z_{1}(T)\right]}{E[T]}=\frac{\int_{0}^{\infty} E\left[Z_{1}\left(t_{0}\right)\right] d F\left(t_{0}\right)}{E[T]} \tag{40}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{Z}_{2}=\frac{E\left[Z_{2}(T)\right]}{E[T]}=\frac{\int_{0}^{\infty} E\left[Z_{2}\left(t_{0}\right)\right] d F\left(t_{0}\right)}{E[T]} \tag{41}
\end{equation*}
$$

Given the transmit range $r, \mathcal{Z}_{1}$ and $\mathcal{Z}_{2}$ are linearly related to the long run average data rate $\mathcal{W}_{1}$ and $\mathcal{W}_{2}$. Since we do not focus on finding an optimal transmit range that maximizes the long run average data rate in this paper, we do not discuss $\mathcal{W}_{1}$ and $\mathcal{W}_{2}$ further.

## 4. UNIFORM SPEED DISTRIBUTION

We consider the case when node speed is uniformly distributed according to (42), given by

$$
G(v)=\left\{\begin{array}{rl}
0 & 0 \leq v \leq v_{a}  \tag{42}\\
\frac{v-v_{a}}{v_{b}-v_{a}} & v_{a} \leq v \leq v_{b} . \\
1 & v \geq v_{b}
\end{array}\right.
$$

The corresponding distribution of the cycle duration $T=d / V$ is

$$
F(t)=\left\{\begin{array}{rl}
0 & 0 \leq t \leq d / v_{b}  \tag{43}\\
\frac{v_{b}-d / t}{v_{b}-v_{a}} & d / v_{b} \leq t \leq d / v_{a} \\
1 & t \geq d / v_{a}
\end{array}\right.
$$

Our objective here is twofold. First, we consider the case when each node selects its speed from distribution $F$ and then moves at constant speed at all highway segments. We will examine the effect of observer node mobility $t_{0}$ on its fraction of connection time, or expected number of connections for both forward and reverse traffic scenarios. Second, we incorporate the extended mobility model, where a node selects a new speed at each highway segment. We will examine the long run average number of connections and data rate of a random node in both forward and reverse traffic scenarios.

Substituting (43) into (20), (29), (34), $E\left[N_{1}\left(t_{0}\right)\right], E\left[N_{1}(T)\right]$ and $E\left[B_{1}\left(t_{0}\right)\right]$ can be readily computed as

$$
\begin{align*}
& E\left[N_{1}\left(t_{0}\right)\right]=\frac{\lambda}{v_{b}-v_{a}}\left(\left(v_{a}+v_{b}\right) t_{0}+d \ln \frac{d^{2}}{t_{0}^{2} e^{2} v_{a} v_{b}}\right)  \tag{44}\\
& E\left[N_{1}(T)\right]=\frac{2 d \lambda}{\left(v_{b}-v_{a}\right)^{2}}\left(\left(v_{a}+v_{b}\right) \ln \frac{v_{b}}{v_{a}}-2\left(v_{b}-v_{a}\right)\right), \tag{45}
\end{align*}
$$

and $E\left[B_{1}\left(t_{0}\right)\right]=$

$$
\left\{\begin{array}{cl}
\frac{c\left(\frac{d}{t_{0}}-v_{a}\right)+2 r \ln \left[\left(v_{b}-\frac{d}{t_{0}}\right)\left(\frac{c e}{2 r}\right)\right]}{v_{b}-v_{a}} & t_{0} \geq \max \left(\frac{d}{v_{a}+\frac{2 r}{c}}, \frac{d}{v_{b}-\frac{2 r}{c}}\right)  \tag{46}\\
\frac{c\left(v_{b}-\frac{d}{t_{0}}\right)+2 r \ln \left[\left(\frac{d}{t_{0}}-v_{a}\right)\left(\frac{c e}{2 r}\right)\right]}{v_{b}-v_{a}} & t_{0} \leq \min \left(\frac{d}{v_{a}+\frac{2 r}{c}}, \frac{d}{v_{b}-\frac{2 r}{c}}\right) \\
\frac{2 r \ln \left[\left(\frac{c e}{2 r}\right)^{2}\left(v_{b}-\frac{d}{t_{0}}\right)\left(\frac{d}{t_{0}}-v_{a}\right)\right]}{v_{b}-v_{a}} & \frac{d}{v_{b}-\frac{2 r}{c}} \leq t_{0} \leq \frac{d}{v_{a}+\frac{2 r}{c}} \\
c & \frac{d}{v_{a}+\frac{2 r}{c}} \leq t_{0} \leq \frac{d}{v_{b}-\frac{2 r}{c}}
\end{array}\right.
$$

For forward traffic, given the speed of the observer node $v_{0}$ and


Figure 3: In forward traffic, connection time is truncated when the difference of encounter node speed $V$ and observer node speed $v_{0}$ is less than $2 r / c$, i.e. $\left|V-v_{0}\right| \leq 2 r / c$. The shaded area shows the range of encounter node speed when connection time truncation occurs.
encounter node $V$, the connection time is truncated if

$$
\begin{equation*}
\frac{2 r}{\left|V-v_{0}\right|} \geq c \tag{47}
\end{equation*}
$$

or $\left|V-v_{0}\right| \leq 2 r / c$. That is, the connection time of forward traffic scenarios is truncated when the relative speed of the encounter node and observer node speed is less than $2 r / c$. When the encounter node speed $V$ falls into the shaded area as illustrated in Figure 3, the connection time is truncated. The four cases on the figure correspond to the four cases in (46). Cases 1 and 2 correspond to boundary truncation. When the observer node has a speed $v_{0} \leq v_{a}+2 r / c$ and $v_{0} \geq v_{b}-2 r / c$ respectively, connection time is truncated when the encounter node speed is at the boundary. Case 3 corresponds to partial truncation. Connection time truncation occurs if the difference of encounter node and observer node speed is less than $2 r / c$. For large $r / c$, the shaded area is wide and spans over the interval $\left[v_{a}, v_{b}\right]$. A connection time truncation occurs irrespective of the encounter node speed. This corresponds to case 4 of full truncation. The occurrence of each case is dependent on the ratio $r / c$ and the span of the speed distribution $v_{b}-v_{a}$.

When $v_{b}-v_{a}$ is much larger than $r / c$ such that $v_{b}-v_{a} \geq 4 r / c$, a observer node may experience left boundary, right boundary and partial connection time truncations depending on its mobility $t_{0}$. This is usually the case in highway traffic scenarios, where vehicle speed at the fast lane is much larger than that at the slow lane. When $r / c$ is larger, connections are more prone to truncations. In the case $4 r / c \geq v_{b}-v_{a} \geq 2 r / c$, a observer node may experience left boundary, right boundary and full truncation depending on its mobility $t_{0}$. That is, there exists some observer node mobility $t_{0}$ such that connection time is always truncated for all encounter node speed. In a typical mobile infostation network, the transmit range is small such that the ratio $r / c$ is much smaller compared with $v_{b}-v_{a}$. This case may be applicable when highway traffic is slow due to congestion. When $2 r / c \geq v_{b}-v_{a}$, the transmit range is so large such that a truncation always occurs regardless of the speeds of the encounter and observer node. The three regimes are summarized in Table 1 which shows the range of observer node

| Regime | $E\left[B_{1}\left(t_{0}\right)\right]$ | Target Node Mobility $t_{0}$ | $E\left[B_{1}\left(d / v_{a}\right)\right]$ | $E\left[B_{1}\left(d / v_{b}\right)\right]$ |
| :--- | ---: | ---: | ---: | ---: |
| $v_{b}-v_{a} \geq 4 r / c$ | case 1 | $d / v_{a} \geq t_{0} \geq d /\left(v_{a}+2 r / c\right)$ | $\frac{2 r}{v_{b}-v_{a}} \ln \frac{\left(v_{b}-v_{a}\right) c e}{2 r}$ | $\frac{2 r}{v_{b}-v_{a}} \ln \frac{\left(v_{b}-v_{a}\right) c e}{2 r}$ |
|  | case 2 | $d / v_{b} \leq t_{0} \leq d /\left(v_{b}-2 r / c\right)$ |  |  |
| $4 r / c \geq v_{b}-v_{a} \geq 2 r / c$ | case 3 | $d /\left(v_{b}-2 r / c\right) \leq t_{0} \leq d /\left(v_{a}+2 r / c\right)$ |  |  |
|  | case 1 | $d / v_{a} \geq t_{0} \geq d /\left(v_{b}-2 r / c\right)$ | $\frac{2 r}{v_{b}-v_{a}} \ln \frac{\left(v_{b}-v_{a}\right) c e}{2 r}$ | $\frac{2 r}{v_{b}-v_{a}} \ln \frac{\left(v_{b}-v_{a}\right) c e}{2 r}$ |
|  | case 2 | $d / v_{b} \leq t_{0} \leq d /\left(v_{a}+2 r / c\right)$ |  |  |
| $2 r / c \geq v_{b}-v_{a}$ | case 4 | $d /\left(v_{a}+2 r / c\right) \leq t_{0} \leq d /\left(v_{b}-2 r / c\right)$ |  |  |

Table 1: Existence of three regimes for forward traffic scenario.
mobility such that a particular case applies. The connection times for the limiting cases at maximum and minimum observer node speed are also included.

Recall that $E\left[N_{1}\left(t_{0}\right)\right]$ is minimized when $t_{0}$ is the median of $F$, i.e. $F\left(t_{0}\right)=1 / 2$. For uniform distribution, the median is equal to the arithmetic mean. It can be easily verified that $E\left[N_{1}\left(t_{0}\right)\right]$ is convex with a minimum at $t_{0}=2 d /\left(v_{a}+v_{b}\right)$, i.e., when the observer node is at mean speed $v_{0}=\left(v_{a}+v_{b}\right) / 2$. Similarly the node encounter rate $E\left[N_{1}\left(t_{0}\right)\right] / t_{0}$ is also convex with a minimum at $t_{0}=d / \sqrt{v_{a} v_{b}} \geq 2 d /\left(v_{a}+v_{b}\right)$, where the inequality follows from the fact that arithmetic mean is greater than or equal to the geometric mean. On the other hand, $E\left[B_{1}\left(t_{0}\right)\right]$ is concave with a maximum at $t_{0}=2 d /\left(v_{a}+v_{b}\right)$. Moreover, the expected connection time as a function of speed is symmetric about the mean speed. That is, the expected connection time is the same when the observer node has a speed of $v_{0}$ or $v_{b}+v_{a}-v_{0}$. This also explains why the connection times at maximum and minimum observer node speed are equal in Table 1.

In the reverse traffic scenario, we substitute (43) into (24), (31), (36) to obtain

$$
\begin{align*}
E\left[N_{2}\left(t_{0}\right)\right] & =\lambda\left(t_{0}+\frac{d}{v_{b}-v_{a}} \ln \frac{v_{b}}{v_{a}}\right)  \tag{48}\\
E\left[N_{2}(T)\right] & =\frac{2 \lambda d}{v_{b}-v_{a}} \ln \frac{v_{b}}{v_{a}} \tag{49}
\end{align*}
$$

and $E\left[B_{2}\left(t_{0}\right)\right]=$

$$
\left\{\begin{array}{cc}
\frac{2 r \ln \left(\frac{v_{b}+d / t_{0}}{v_{a}+d / t_{0}}\right)}{v_{b}-v_{a}} & t_{0} \leq \frac{d}{\max \left(v_{a}, 2 r / c-v_{a}\right)}  \tag{50}\\
\frac{2 r \ln \left(\frac{d / t_{0}+v_{b}}{2 r} c e\right)-c\left(d / t_{0}+v_{a}\right)}{v_{b}-v_{a}} & \frac{d}{\min \left(v_{b}, 2 r / c-v_{a}\right)} \leq t_{0} \\
c & \leq \frac{d}{\max \left(v_{a}, 2 r / c-v_{b}\right)} \\
& t_{0} \geq \frac{d}{\max \left(v_{b}, 2 r / c-v_{b}\right)}
\end{array} .\right.
$$

Let $V$ and $v_{0}$ be the speed of the encounter node and observer node respectively. A connection is truncated if

$$
\begin{equation*}
\frac{2 r}{V+v_{0}} \geq c, \quad \text { or } \quad V \leq \frac{2 r}{c}-v_{0} \tag{51}
\end{equation*}
$$

That is, given the observer node speed $v_{0}$, a connection time truncation occurs if the encounter node speed $V$ is too low. As illustrated in Figure 4, if the encounter node speed falls into the shaded area, the connection time is truncated. The three depicted cases correspond to a connection with no truncation, partial truncation and full truncation. The expected connection time in one node encounter of the three cases is shown in (50). In case 1 , the shaded area is below $v_{a}$. Thus there is no connection truncation at all encounter node speed. In case 2, there is partial truncation. Connection time is


Figure 4: In reverse traffic, connection time is truncated when the encounter node speed is smaller than $2 r / c-v_{0}$. The shaded area shows the range of encounter node speed when connection time truncation occurs.
truncated if the encounter node speed is smaller than $2 r / c-v_{0}$ and vice versa. In case 3 , a connection time truncation occurs irrespective of the encounter node speed. This is denoted as full truncation. The occurrence of the three cases depends on speed $v_{a}$ and $v_{b}$. Four regimes can be identified and are summarized in Table 2.

In the first regime, the minimum speed $v_{a}$ is large such that $v_{a} \geq r / c$. Suppose the encounter node moves at speed $V$ and the observer node moves at speed $v_{0}$, the corresponding connection time is

$$
\begin{equation*}
\frac{2 r}{V+v_{0}} \leq \frac{2 r}{2 v_{a}} \leq c \tag{52}
\end{equation*}
$$

Thus, there is no connection time truncation at all observer and encounter node speeds. In a highway environment, the minimum node speed $v_{a}$ is typically much larger than $r / c$. Thus we expect there is no connection time truncation in reverse traffic scenarios. In the second regime, $v_{a} \leq r / c$ and $v_{a}+v_{b} \geq 2 r / c$. When there is traffic congestion on the highway, it is possible that the minimum speed is small and satisfies $v_{a} \leq r / c$. On the other hand, congestion may be local and occurs only in one or two lanes. The fast lanes may experience no congestion such that $v_{a}+v_{b} \geq 2 r / c$ is satisfied. In this scenario, a observer node undergoes no connection time truncation if it has high mobility such that $d / v_{b} \leq$ $t_{0} \leq d /\left(2 r / c-v_{a}\right)$. On the other hand, if the observer node has low mobility such that $d /\left(2 r / c-v_{a}\right) \leq t_{0} \leq d / v_{a}$, a connection time truncation occurs when the encounter node speed is smaller than $2 r / c-v_{0}$. In the third regime, $v_{a} \leq r / c$ and $v_{a}+v_{b} \leq$ $2 r / c$. When the maximum speed $v_{b}$ is also small, a observer node will undergo partial connection time truncation at high mobility

| Regime | $E\left[B_{2}\left(t_{0}\right)\right]$ | Target Node Mobility $t_{0}$ | $E\left[B_{2}\left(d / v_{a}\right)\right]$ | $E\left[B_{1}\left(d / v_{b}\right)\right]$ |
| :--- | ---: | ---: | ---: | ---: |
| $r / c \leq v_{a}$ | case 1 | $d / v_{b} \leq t_{0} \leq d / v_{a}$ | $\frac{2 r}{v_{b}-v_{a}} \ln \frac{v_{a}+v_{b}}{2 v_{a}}$ | $\frac{2 r}{v_{b}-v_{a}} \ln \frac{2 v_{b}}{v_{a}+v_{b}}$ |
| $v_{a} \leq r / c$ | case 1 | $d / v_{b} \leq t_{0} \leq d /\left(2 r / c-v_{a}\right)$ | $\frac{2 r \ln \frac{\left(v_{a}+v_{v}\right)}{2 r}-2 v_{a}}{v_{b}-v_{a}}-2 c v_{a}$ | $\frac{2 r}{v_{b}-v_{a}} \ln \frac{2 v_{b}}{v_{a}+v_{b}}$ |
| $v_{a}+v_{b} \geq 2 r / c$ | case 2 | $d /\left(2 r / c-v_{a}\right) \leq t_{0} \leq d / v_{a}$ |  |  |
| $v_{a} \leq r / c$ | case 2 | $d / v_{b} \leq t_{0} \leq d /\left(2 r / c-v_{b}\right)$ |  | c |
| $v_{a}+v_{b} \leq 2 r / c$ | case 3 | $d /\left(2 r / c-v_{b}\right) \leq t_{0} \leq d / v_{a}$ |  |  |
| $v_{b} \leq r / c$ | case 3 | $d / v_{b} \leq t_{0} \leq d / v_{a}-c\left(v_{a}+v_{b}\right)$ |  |  |
| $v_{b}-v_{a}$ | c |  |  |  |

Table 2: Existence of four regimes for reverse traffic scenario.
when $d / v_{b} \leq t_{0} \leq d /\left(2 r / c-v_{b}\right)$. Again, connection time truncation occurs when the encounter node speed is smaller than $2 r / c-v_{0}$. When the observer node has low mobility such that $d /\left(2 r / c-v_{b}\right) \leq t_{0} \leq d / v_{a}$, full truncation always occurs irrespective of the encounter node speed. In the fourth regime, the maximum speed $v_{b}$ is small such that $v_{b} \leq r / c$. Even if both the observer node and the encounter node move at maximum speed, the corresponding connection time is $2 r / 2 v_{b} \geq c$. In practice, a mobile infostation network has a small transmit range $r$ and a moderate large connection time limit $c$. It is unlikely that the last two regimes are of importance in reverse traffic scenarios. In the usual highway traffic scenarios, it is reasonable to assume that the first regime holds most of the time. We will therefore perform our numerical experiments for the first regime only.

Although both node encounter rates $E\left[N_{1}\left(t_{0}\right)\right] / t_{0}, E\left[N_{2}\left(t_{0}\right)\right] / t_{0}$ and connection times $E\left[B_{1}\left(t_{0}\right)\right], E\left[B_{2}\left(t_{0}\right)\right]$ are known analytically, the critical points for $\eta_{1}\left(t_{0}\right)=E\left[Z_{1}\left(t_{0}\right)\right] / t_{0}$ and $\eta_{2}\left(t_{0}\right)=$ $E\left[Z_{2}\left(t_{0}\right)\right] / t_{0}$ cannot be determined analytically as both involves the products of logarithmic functions in $t_{0}$. Thus it is impossible to examine the variations of $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ as as function of observer node mobility without employing numerical studies, as we do in the next section. Nevertheless, it is instructive to compare the values of $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ at limiting cases of maximum and minimum observer node speed. Specifically, we compute the ratios $\eta_{1}\left(d / v_{b}\right) / \eta_{1}\left(d / v_{a}\right)$ and $\eta_{2}\left(d / v_{b}\right) / \eta_{2}\left(d / v_{a}\right)$.

Consider the forward traffic scenario. Recall in Table 2 that the connection time at minimum and maximum speed is the same at all regimes by symmetry. The ratio $\eta_{1}\left(d / v_{b}\right) / \eta_{1}\left(d / v_{a}\right)$ therefore depends on node encounter rate only. Thus for all the three regimes in the forward traffic scenario, this ratio is given by

$$
\begin{equation*}
\frac{\eta_{1}\left(d / v_{b}\right)}{\eta_{1}\left(d / v_{a}\right)}=\frac{v_{b} E\left[N_{1}\left(d / v_{b}\right)\right]}{v_{a} E\left[N_{2}\left(d / v_{a}\right)\right]}=\frac{\left(\frac{v_{b}}{v_{a}}\right) \ln \left(\frac{v_{b}}{v_{a}}\right)-\left(\frac{v_{b}}{v_{a}}\right)+1}{\left(\frac{v_{b}}{v_{a}}\right)-1-\ln \left(\frac{v_{b}}{v_{a}}\right)} \tag{53}
\end{equation*}
$$

It is noteworthy that (53) is completely determined by the ratio $v_{b} / v_{a}$ and is independent of the transmit range $r$ and connection time limit $c$. With reference to Figure 5(a), we observe that the ratio $\eta_{1}\left(d / v_{b}\right) / \eta_{1}\left(d / v_{a}\right)$ is always larger than 1 for all choices of $v_{a}$ and $v_{b}$. In particular, when the difference $v_{b}-v_{a}$ is large, say $v_{a}=2$ and $v_{b}=30$, the ratio is as large as 2.25 . That is, the fraction of connection time, or the average number of connections of the observer node is more than double when observer node mobility is high.

In reverse traffic scenarios, we consider the first two regimes, namely $v_{a} \geq r / c$ and $\left\{v_{a} \leq r / c, v_{a}+v_{b} \geq 2 r / c\right\}$, since these regimes are most likely to happen in realistic scenarios. Here, the


Figure 5: (a) Ratio of the average number of connections at maximum speed and minimum speed for forward traffic. (b) Ratio of the average number of connections at maximum speed and minimum speed for reverse traffic ( $v_{a} \geq r / c$ ). (c) Ratio of the average number of connections at maximum speed and minimum speed for reverse traffic ( $v_{a} \leq r / c$ and $v_{a}+v_{b} \geq$ $2 r / c$ ).
node encounter rate is increasing with node speed and connection time is decreasing with node speed. For $v_{a} \geq r / c$, we have

$$
\begin{equation*}
\frac{\eta_{2}\left(d / v_{b}\right)}{\eta_{2}\left(d / v_{a}\right)}=\frac{\left[\left(\frac{v_{b}}{v_{a}}\right)+\left(\frac{v_{b}}{v_{a}}\right) \ln \left(\frac{v_{b}}{v_{a}}\right)-1\right] \ln \left(\frac{2}{1+\frac{v_{b}}{v_{a}}}\right)}{\left[\left(\frac{v_{b}}{v_{a}}\right)-1+\ln \left(\frac{v_{b}}{v_{a}}\right)\right] \ln \left(\frac{1+\frac{v_{b}}{v_{a}}}{2}\right)} \tag{54}
\end{equation*}
$$

Although the connection time at $t_{0}=d / v_{a}$ and $t_{0}=d / v_{b}$ is different, it turns out that the ratio $\eta_{2}\left(d / v_{b}\right) / \eta_{1}\left(d / v_{a}\right)$ is also independent of transmit range $r$ and dependent on the ratio $v_{b} / v_{a}$ only. In this regime, no connections are truncated. The expected connection times $E\left[Z_{1}\left(t_{0}\right)\right]$ and $E\left[Z_{2}\left(t_{0}\right)\right]$ are linear to the transmit



Figure 7: Expected number of connections $\eta\left(t_{0}\right)$ versus node mobility $t_{0}=d / v_{0}$ for different transmit range $r$ and connection time limit $c$. (a) $r=1, c=1$ (b) $r=2, c=1$ (c) $r=0.5, c=1$ (d) $r=1, c=10$.

100 by two orders of magnitude. With reference to Figure 7, the expected fraction of connection time, or expected number of connections $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ are plotted together versus $t_{0}$ in the range $d / v_{b}=100$ to $d / v_{a}=500$. At mean speed $v_{0}=6$, the corresponding $t_{0}$ is 166.67 unit. Consider scenario 1 for $r=1, c=1$. For forward traffic, $\eta_{1}\left(t_{0}\right)$ attains a global maximum of 0.6 when $t_{0}$ is minimum. $\eta_{1}\left(t_{0}\right)$ decreases steadily as $t_{0}$ increases and hits the minimum of 0.3 at $t_{0}=267.73$. Beyond that, there is a slight increase of $\eta_{1}\left(t_{0}\right)$ when $t_{0}$ is increased further. Similar trends are observed for other scenarios in Figure 7(b),(c),(d). Nevertheless, a slight dip of $\eta_{1}\left(t_{0}\right)$ occurs at low mobility $\left(t_{0} \approx 500\right)$ for Figure $7(\mathrm{~d})$. Although there are slightly more encounters at low mobility, there is a steeper decrease in connection time. Thus $\eta_{1}\left(t_{0}\right)$ is not convex in general. In the particular case of $v_{0}=v_{a}=0$, the observer node is stationary. The expected fraction of connection time for forward and reverse traffic should be arbitrarily close. That is, the two curves should coincide when $t_{0}$ is arbitrarily large. In our example, the observer node moves slowly when $v_{0}=v_{a}=2$. The dip in Figure 7(d) is consistent to our intuition that the fraction of connection time for forward and reverse traffic are close when the observer node has low mobility.

In contrast to forward traffic, the expected fraction of connection time, or expected number of connections $\eta_{2}\left(t_{0}\right)$ is almost constant at all observer node speed in reverse traffic scenarios. The relative value of $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ depends on the ratio of transmit range to connection time limit $r / c$. When $r / c$ is large (Figure 7(b)), it is likely that the connection time for forward traffic is truncated. Thus $\eta_{1}\left(t_{0}\right)$ is consistently smaller than $\eta_{2}\left(t_{0}\right)$ except for very high ob-
server node speed. When $r / c$ is small (Figure 7(c),(d)), the connection time of each node encounter is large. In fact, if there is no connection time limit, the expected connection time for forward traffic is unbounded. The large connection time at large $c$ stipulates that $\eta_{1}\left(t_{0}\right)>\eta_{2}\left(t_{0}\right)$ at all node speed. Incidentally, when $r / c=1$ (Figure 7(a)), $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ intersects at $t_{0}=162.7$, which is close to the cycle duration at mean speed $d / E[V]=166.67$. Thus, if a observer node moves at a constant speed $v_{0}$ less than the mean speed $E[V]$, reverse traffic connections are more preferable. Similarly, forward traffic connections are more preferable if a node moves at a constant speed $v_{0} \geq E[V]$ in this particular example.

Our results show that the data rate of forward traffic connections and reverse traffic connections is dependent on $c$. The value of $c$, in turn, is closely related to the correlation of the contents between two nodes. If nodes have highly correlated contents, any two arbitrary nodes may want to exchange only a few files with each other, effectively modeled by a small $c$. It is more efficient to maintain reverse traffic connections and exchange files with more nodes, as in the case ( $r=2, c=1$ ) shown in Figure 7(b). In a content distribution application, this is an appropriate strategy when most nodes get most of the files already. Similarly, when new content is disseminated, nodes have few files in common and can be modeled by a large $c$. In this case, a node should maintain forward traffic connections to exploit the long expected connection time as warranted by the uniform speed distribution, as in the case ( $r=1, c=10$ ) shown in Figure 7(d).

## 6. DISCUSSIONS

In [3], it was shown that mobility increases the capacity of a mobile infostation network. Capacity gain arises from the realization of the maximal spatial transmission concurrency in each network snapshot. Mobility comes into the picture by shuffling node locations, creating numerous instances when excellent channels between different nodes can be exploited (multiuser diversity). As a result of mobility, the sum capacity of each network snapshot translates to the long run end-to-end network throughput. It is noteworthy that in this networking paradigm, end-to-end capacity does not depend on node mobility per se. Node mobility, however, do impact the delay performance. The delay of a transiting packet is directly related to the time scale of the mobility process.

In this paper we have focused on the physical implications of mobility. The fraction of connection time, or number of connections of a observer node over an interval, is determined by the rate of node encounters and the connection time of each encounter, both of which are obviously related to node mobility. It turns out that in reverse traffic scenarios, the expected number of connections is really independent of node mobility. In forward traffic scenarios, however, the expected number of connections (and thus the data rate) increases as mobility increases. Numerical results show that the expected fraction of connection time, or expected number of connections at high node mobility can be much greater than that at low mobility. In particular, in the case when node speed is uniformly distributed between 2 to 30 units, the fraction of connection time is improved by more than a factor of 2 when the observer node increases its speed from minimum to the maximum. Thus, mobility not only provides a mechanism for the exploitation of multiuser diversity. The increase of the fraction of connection time and data rate is a physical consequence of node mobility. Incidentally, this also provides an incentive for network nodes to be mobile. If a particular mobile user wants to enjoy a higher throughput, or mini-
mizes the downloading time of the files he is interested in, he is motivated to become more mobile and roam around the network. This mobile user in turn helps the network to disseminate data more efficiently, such that the end-to-end delay performance of other users are improved.

It is well known that mobility degrades network performance in many wireless paradigms such as cellular networks and multihop networks. In multihop networks, for instance, extraneous overhead is needed for route maintenance to cope with link failures in node mobility. On the other hand, the fraction of connection time in a fixed infostation model [2] is constant regardless of node mobility. We have shown in this paper that the fraction of connection time, and data rate increases with node mobility in a mobile infostation network. Thus the mobile infostation network paradigm is superior to multihop networks and fixed infostation networks in its robustness to node mobility.

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