# PERFORMANCE EVALUATION OF HIGHWAY MOBILE INFOSTATION NETWORKS 

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#### Abstract

A mobile infostation network stipulates all transmissions to occur when nodes are in proximity. In this paper, we evaluate the effect of mobility on highway mobile infostation networks. Each node enters a highway segment at a Poisson rate with a constant speed drawn from a known but arbitrary distribution. Both forward and reverse traffic are considered. For node speed that is uniformly distributed, the expected fraction of connection time, or expected number of connections in queueing terminology, is independent of observer node speed for reverse traffic, while it increases with observer node speed for forward traffic. We also extend our mobility model such that each node changes speed at each highway segment. The long run fraction of connection time of an observer node is dependent on the ratio of transmit range and connection time limit. Forward traffic connection yields better performance when the ratio is small and vice versa. We also compute the optimal transmit range and the corresponding data rate for both traffic types. We conclude that forward traffic connections yield much higher data rate in most scenarios.


## I. Introduction

In a mobile infostation network, nodes operate on low transmit power. Any two nodes communicate only when they are in proximity and have a very good channel. Under this transmission constraint, any pair of nodes is intermittently connected as mobility shuffles the node locations. The network capacity of mobile infostation networks compares favorably to conventional multihop ad hoc networks. In [3] Gupta and Kumar showed that the per node throughput in a multihop network drops to zero at a rate $O\left(\frac{1}{\sqrt{n \ln n}}\right)$ in the limit of large $n$. Thus multihop networks do not scale with large network size. On the other hand, Grossglauser and Tse showed in [2] that the per node throughput of a mobile infostation network is $O(1)$, independent of the number of nodes. This capacity is achieved through a two hop relay strategy.

Assume that each node in the network selects a random destination for unicast. We focus on a source node $i$, which has packets to deliver to a destination node $j$, as shown in Figure 1. As time evolves, node $i$ moves along a random trajectory and eventually encounters nodes 1 and 2. Although neither nodes 1 nor 2 are the destination of $i, i$ still relays packets to them, with the expectation that when each of these relay nodes reaches the destination $j$, it will complete the second relay on behalf of node $i$. In steady state, each of the other $n-2$ nodes contains packets generated by node $i$ and destined to node $j$. At any


Fig. 1. Two hop packet relay strategy in a mobile infostation network.
network snapshot, it is almost surely that the nearest neighbor of node $j$ has packets addressed from node $i$ and completes the second relay on the behalf of $i$. That is, the long run per node throughput is constant and is independent of the network size. This capacity improvement comes from the exploitation of node mobility to physically carry the packets around the network, and is independent of the underlying mobility model, as long as the mobility process is ergodic.

Motivated by the dramatic capacity improvement of mobile infostation networks, there are a number of recent papers on ad hoc networks that exploit node mobility. Whereas [2] focused on unicast, most other papers focused on multicast. [5], [6] has focused on scenarios in which nodes cooperate. In order to expedite data dissemination, a node also forwards packets for other nodes if it has not done so for some time. The issue of noncooperation between nodes was explored in [9], [10] in the context of a content distribution application. Transmissions between two proximate nodes are allowed only when both nodes benefit from a file exchange. On the other hand, the effect of transmit range on capacity of mobile infostation networks is examined in [8]. A refined interference model is used for analysis.

In this paper, we examine the effect of mobility on mobile infostation networks. In [2], mobility provides a mechanism such that numerous instances of excellent channels between different nodes can be exploited. The realization of large network capacity comes from the translation of maximal spatial transmission concurrency in each network snapshot to the long run end-to-end network capacity. The physical


Fig. 2. Illustration of the highway mobile infostation network model.
implication of mobility in node encounters has been glossed over. In reality, the total connection time of a node over a specific interval depends on the node encounter rate and the connection time in each encounter, both of which depend on the relative mobility of nodes. Although a high node speed results in more node encounters, the connection time in each node encounter also decreases. It is not apparent whether high or low speed results in a larger connection time, and thus, data rate. To this end we propose a new mobility model for highway networks. The highway scenario proves to be interesting despite its mathematical simplicity. First, forward traffic connection time is much larger than that of reverse traffic, but the node encounter rate is also much smaller. It is not apparent which traffic type maximizes the fraction of connection time. Second, the connection time in an encounter depends on the transmit range of the nodes. For both forward and reverse traffic, an optimal transmit range exists such that the long run data rate of a node is maximized.

## II. System Model

We consider a highway network in which fixed infostations are placed regularly at a distance $d$ from each other. We assume that all nodes are subscribers of a content provider, say a movie distribution network. Movies are split into many files and are cached in the infostations at various locations. Besides downloading directly from an infostation, a node participates in data exchanges whenever there is another node in proximity. We assume data exchanges between two proximate nodes in range always take place without further negotiation. The amount of data exchanged is proportional to the connection time in an encounter and the data transmission rate. It was shown in [10] that in a large network, peer-to-peer node exchanges account for most of the data transmissions. As the network size increases, the importance of fixed infostations in data dissemination dwindles. Thus, in this paper we focus on peer-to-peer connections between proximate mobile nodes in node encounters only. Connections to fixed infostations on the highway are ignored.

In our analysis, we focus on an arbitrary highway segment between two infostations, say A and B, as shown in Figure 2. Nodes move at a constant speed $V$, an iid random variable drawn from a known but arbitrary distribution $G$. Since nodes
have different speeds, a node may overtake other nodes or being overtaken as it traverses the highway segment. We make all our observations at a specific node, called the observer node. Two types of traffic are considered here. For forward traffic, nodes are injected into the highway segment at a Poisson rate of $\lambda$ from infostation A. Similarly, nodes are injected into the highway at a Poisson rate $\lambda$ from infostation B for reverse traffic. We assume there is no delay incurred in a node encounter. This is plausible in a wide motorway with multiple lanes and moderate traffic, where a node overtakes another at different lanes. This is called the wide motorway model in [4]. More generally, a node changes speed as time evolves. We assume each node still moves at a constant speed in a highway segment. Whenever a node traverses a new highway segment, it selects a new speed from the distribution $G$, independent of the previous speed.

Suppose the observer node moves at a speed $V=v_{0}$ on a highway segment from infostation $A$ to $B$. We denote the time for the node to traverse a highway segment as the cycle duration, given by $T=d / V$, with a corresponding distribution $F . F$ and $G$ are obviously related, given by $\bar{F}(t)=G(d / t)$, where $\bar{F}(t)=1-F(t)$ denotes the complementary distribution function. Given the observer node mobility $t_{0}$, we denote $N_{1}\left(t_{0}\right)$ as the number of node encounters with forward traffic in the duration $t_{0}=d / v_{0}$. The connection time $Y_{1}\left(t_{0}\right)$ in each node encounter is a random variable dependent on the relative speed of the nodes and the common transmit range of all nodes $r$. Two nodes having a similar speed therefore have an unbounded connection time. In reality, however, each node only has a finite amount of data for dissemination to another node. We stipulate a connection time limit parameter $c$ to limit the actual connection time $B_{1}\left(t_{0}\right)$ in a node encounter, given by $B_{1}\left(t_{0}\right)=\min \left(Y_{1}\left(t_{0}\right), c\right)$. We also denote the total connection time of the observer node in a highway segment as $Z_{1}\left(t_{0}\right)$. The expressions $E\left[N_{1}\left(t_{0}\right)\right] / t_{0}$ and $\eta_{1}\left(t_{0}\right)=E\left[Z_{1}\left(t_{0}\right)\right] / t_{0}$ correspond to the expected node encounter rate and the expected fraction of time a node is busy as a function of observer node mobility $t_{0}$. Nevertheless, since a node can simultaneously maintain more than one connection, $\eta_{1}\left(t_{0}\right)$ can be larger than 1. In queueing terminology, the observer node is a server and the connection time in a node encounter corresponds to the service time. $\eta_{1}\left(t_{0}\right)$ can be interpreted as the expected number of connections of the observer node in a cycle. For reverse traffic, the corresponding expected node encounter rate and expected fraction of connection time (expected number of connections) are denoted as $E\left[N_{2}\left(t_{0}\right) / t_{0}\right]$ and $\eta_{2}\left(t_{0}\right)=$ $E\left[Z_{2}\left(t_{0}\right) / t_{0}\right]$.

When speed change is incorporated to our model, the long run average fraction of connection time and data rate are the appropriate metrics. It turns out that simple characterization of these metrics is possible by drawing results from renewal reward theory [7]. Let $M(t), t \geq 0$ be a counting process to denote the number of highway segments traversed by the observer node. At the $n$-th highway segment, the observer node selects a random speed $V_{n}$ independent of the speed $V_{n-1}$ at the previous highway segment $n-1$. The corresponding cycle durations $T_{n}$ are iid random variables. Since $M(t)$ is a counting process with iid interarrival times, $M(t)$ is a renewal
process. Moreover, we denote $R_{n}$ as the reward earned in the $n$th cycle, or renewal period. If we let

$$
\begin{equation*}
R(t)=\sum_{n=1}^{N(t)} R_{n} \tag{1}
\end{equation*}
$$

then $R(t)$ is the total reward earned by time $t$. Let $E[R]=$ $E\left[R_{n}\right]$ and $E[T]=E\left[T_{n}\right]$, the renewal reward theorem [7] states that if $E[R]<\infty$ and $E[T]<\infty$, then with probability 1,

$$
\begin{equation*}
\lim _{t \rightarrow \infty} \frac{R(t)}{t}=\frac{E[R]}{E[T]} \tag{2}
\end{equation*}
$$

That is, the rate of earning reward in the long run is just the ratio of the expected reward in a cycle and the expected cycle duration. Accordingly, if we define the reward as the number of encounters $N_{1}(T)$ in a highway segment for forward traffic, then the long run node encounter rate of the observer node is simply $\mathcal{N}_{1}=E\left[N_{1}(T)\right] / E[T]$. Similarly, when the reward is defined as the total connection time $Z_{1}(T)$ in a highway segment, $\mathcal{Z}_{1}=E\left[Z_{1}(T)\right] / E[T]$ corresponds to the long run fraction of connection time of the observer node. Last, when the reward is the total amount of delivered data $W_{1}(T)$ in a highway segment, $\mathcal{W}_{1}=E\left[W_{1}(T)\right] / E[T]$ denotes the long run data rate of the observer node. For reverse traffic the long run fraction of connection time $\mathcal{Z}_{2}$ and data rate $\mathcal{W}_{2}$ are defined similarly, with $\mathcal{Z}_{2}=E\left[Z_{2}(T)\right] / E[T]$ and $\mathcal{W}_{2}=E\left[W_{2}(T)\right] / E[T]$.

## III. Performance Analysis

Consider the forward traffic scenario. Suppose the observer node enters the highway segment at time $s$ and departs at time $s+t_{0}$. We denote an event occurs at time $t \in[0, \infty)$ if a node enters the highway segment at infostation $A$. Since the node travels with random speed $V=d / T$, this node leaves the highway segment at time $t+T$. We define $p_{1}(t)$ as the probability that a forward entrant at time $t$ has an encounter to the observer node at the highway segment. It is straightforward to show that

$$
p_{1}(t)=\left\{\begin{array}{rl}
\bar{F}\left(s+t_{0}-t\right) & t<s  \tag{3}\\
F\left(s+t_{0}-t\right) & s<t<s+t_{0} \\
0 & t>s+t_{0}
\end{array}\right.
$$

Assuming the network has been operated for a long time $s \rightarrow$ $\infty$ before we observe the observer node enters the highway segment. The total number of node encounters is also a Poisson process and in steady state $s \rightarrow \infty$, it is given by

$$
\begin{align*}
\lim _{s \rightarrow \infty} E\left[N_{1}\left(t_{0}\right)\right] & =\lim _{s \rightarrow \infty} \lambda \int_{0}^{\infty} p(t) d t \\
& =\lambda\left(\int_{0}^{t_{0}} F(t) d t+\int_{t_{0}}^{\infty} \bar{F}(t) d t\right) \tag{4}
\end{align*}
$$

It can be shown $E\left[N_{1}\left(t_{0}\right)\right]$ attains a global minimum when the observer node cycle duration $t_{0}$ is the median of the distribution $F$ by twice differentiating (4). This agrees with our intuition that there are few node encounters if the observer node moves at a speed that goes along with the majority.

For reverse traffic, we define an event occurs at time $t$ if a node enters the highway segment from infostation $B$. For
an event at time $t$, it is marked with probability $p_{2}(t)$ if there is a node encounter with the observer node at the highway segment, given by

$$
p_{2}(t)=\left\{\begin{array}{rl}
0 & t>s+t_{0}  \tag{5}\\
1 & s<t<s+t_{0} \\
\bar{F}(s-t) & t<s
\end{array}\right.
$$

The total number of node encounters at steady state is

$$
\begin{align*}
\lim _{s \rightarrow \infty} E\left[N_{2}\left(t_{0}\right)\right] & =\lim _{s \rightarrow \infty} \lambda \int_{0}^{\infty} p_{2}(t) d t \\
& =\lambda\left(t_{0}+E[T]\right) \tag{6}
\end{align*}
$$

where $E[T]$ is the expected cycle duration given by

$$
\begin{equation*}
E[T]=\int_{0}^{\infty} \bar{F}(t) d t \tag{7}
\end{equation*}
$$

The long run node encounter rate for both traffic types can be obtained by averaging over the speed distribution. Thus

$$
\begin{align*}
E\left[N_{1}(T)\right] & =\int_{0}^{\infty} E\left[N_{1}\left(t_{0}\right)\right] d F\left(t_{0}\right) \\
& =2 \lambda \int_{0}^{\infty} \bar{F}(t) F(t) d t  \tag{8}\\
E\left[N_{2}(T)\right] & =2 \lambda E[T] \tag{9}
\end{align*}
$$

(8) and (9) suggest that the expected node encounter rate for reverse traffic is always larger than that for forward traffic, which is obviously true. Moreover, (9) shows that the expected node encounter rate is completely characterized by the traffic intensity $\lambda$ and the first moment of distribution $F$.

To compute the expected connection time in one encounter for forward traffic $E\left[B_{1}\left(t_{0}\right)\right]$, we note that

$$
\begin{align*}
E\left[B_{1}\left(t_{0}\right)\right] & =\int_{0}^{c} P\left[Y_{1}\left(t_{0}\right)>t\right] d t \\
& =\int_{0}^{c} P\left[\frac{2 r}{\left|v_{0}-V\right|}>t\right] d t \\
& =\int_{0}^{c} G\left(\frac{2 r}{t}+\frac{d}{t_{0}}\right)-G\left(\frac{d}{t_{0}}-\frac{2 r}{t}\right) d t \tag{10}
\end{align*}
$$

Similarly, for reverse traffic we have

$$
\begin{align*}
E\left[B_{2}\left(t_{0}\right)\right] & =\int_{0}^{c} P\left[Y_{2}\left(t_{0}\right)>t\right] d t \\
& =\int_{0}^{c} G\left(\frac{2 r}{t}-\frac{d}{t_{0}}\right) d t \tag{11}
\end{align*}
$$

Refer to Figure 2 again, the total connection time for forward traffic is obtained by summing all individual connection time $B_{1}^{i}\left(t_{0}\right), i \in\left[1, N_{1}\left(t_{0}\right)\right]$ over the cycle. In the event that the connection time of the encounter $N_{1}\left(t_{0}\right)$ overshoots the end of the cycle, the observer node undergoes a renewal and selects a new speed. This in turn modifies the connection time $B_{1}^{N_{1}\left(t_{0}\right)}$. Nevertheless, the boundary effect of an overshoot connection time is minimal when either $N_{1}\left(t_{0}\right)$ is large, or when $B_{1}\left(t_{0}\right) \leq c \ll t_{0}=d / v_{0}$. The former assumption is valid when traffic intensity $\lambda$ is moderate, such that $N_{1}\left(t_{0}\right) \gg 1$. The latter assumption is valid when the distance between fixed infostations $d$ is large, which is likely in an
initial deployment of a fixed infostation network. Ignoring the boundary effect of $B_{1}^{N_{1}\left(t_{0}\right)}\left(t_{0}\right)$, we have

$$
\begin{equation*}
\left.Z_{1}\left(t_{0}\right)\right] \approx \sum_{i=1}^{N_{1}\left(t_{0}\right)} B_{1}^{i}\left(t_{0}\right) \tag{12}
\end{equation*}
$$

It can be shown that $B_{1}^{i}\left(t_{0}\right)$ are iid random variables and $N\left(t_{0}\right)$ is Poisson. However, $N_{1}\left(t_{0}\right)$ and $\left.B_{1}^{( } t_{0}\right)$ are in general not independent. In fact, when node mobility is high, $N_{1}\left(t_{0}\right)$ is large and the corresponding $B_{1}\left(t_{0}\right)$ is small. Thus $Z_{1}\left(t_{0}\right)$ is not a compound Poisson process. Nevertheless, $N_{1}\left(t_{0}\right)$ is a stopping time w.r.t. the sequence $B_{1}^{i}\left(t_{0}\right)$ since the stopping rule $\left\{N_{1}\left(t_{0}\right)=n\right\}$ is completely determined by the information up to time $n$, and is unrelated to $B_{1}^{n+1}\left(t_{0}\right), B_{1}^{n+2}\left(t_{0}\right)$ and so on. Thus, Wald's equality can be applied to (12) to yield

$$
\begin{equation*}
E\left[Z_{1}\left(t_{0}\right)\right]=E\left[N_{1}\left(t_{0}\right)\right] E\left[B_{1}\left(t_{0}\right)\right] \tag{13}
\end{equation*}
$$

Similarly, for reverse traffic we have

$$
\begin{equation*}
E\left[Z_{2}\left(t_{0}\right)\right]=E\left[N_{2}\left(t_{0}\right)\right] E\left[B_{2}\left(t_{0}\right)\right] \tag{14}
\end{equation*}
$$

The long run fraction of connection time, or number of connections of the observer node for both traffic types can be obtained by conditioning on distribution $F$, given by,

$$
\begin{equation*}
\mathcal{Z}_{1}=\frac{E\left[Z_{1}(T)\right]}{E[T]}=\frac{\int_{0}^{\infty} E\left[Z_{1}\left(t_{0}\right)\right] d F\left(t_{0}\right)}{E[T]} \tag{15}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{Z}_{2}=\frac{E\left[Z_{2}(T)\right]}{E[T]}=\frac{\int_{0}^{\infty} E\left[Z_{2}\left(t_{0}\right)\right] d F\left(t_{0}\right)}{E[T]} \tag{16}
\end{equation*}
$$

Finally, we are also interested in the long run data rate for both traffic types. Assuming non-adaptive radios are used, the data rate is the Shannon rate at the transmit range boundary $r$, given by

$$
\begin{equation*}
C(r)=\ln \left(1+1 / r^{4}\right) \tag{17}
\end{equation*}
$$

where we have assumed a path gain exponent of 4 and ignored the effect of mutual interference. We define the long run data rate as

$$
\begin{align*}
& \mathcal{W}_{1}=E\left[W_{1}(T, r)\right]=C(r) E\left[Z_{1}(T, r)\right]  \tag{18}\\
& \mathcal{W}_{2}=E\left[W_{2}(T, r)\right]=C(r) E\left[Z_{2}(T, r)\right] \tag{19}
\end{align*}
$$

where we emphasize both connection time $Z$ and the amount of delivered data $W$ are dependent on the transmit range $r$. Since $W_{1}(r)=0$ and $W_{2}(r)=0$ when the transmit range is zero or very large, an optimal transmit range $r$ exists for both traffic types such that $\mathcal{W}_{1}$ and $\mathcal{W}_{2}$ are maximized respectively.

## IV. Numerical Study

We consider the case when node speed is uniformly distributed according to (20), given by

$$
G(v)=\left\{\begin{array}{rl}
0 & 0 \leq v \leq v_{a}  \tag{20}\\
\frac{v-v_{a}}{v_{b}-v_{a}} & v_{a} \leq v \leq v_{b} \\
1 & v \geq v_{b}
\end{array}\right.
$$

The corresponding distribution of the cycle duration $T=d / V$ is

$$
F(t)=\left\{\begin{array}{rl}
0 & 0 \leq t \leq d / v_{b}  \tag{21}\\
\frac{v_{b}-d / t}{v_{b}-v_{a}} & d / v_{b} \leq t \leq d / v_{a} \\
1 & t \geq d / v_{a}
\end{array}\right.
$$

$E\left[N_{1}(t)\right], E\left[N_{1}(T)\right]$ and $E\left[B_{1}\left(t_{0}\right)\right]$ can be readily computed by evaluating (4),(8), (10) as

$$
\begin{align*}
& E\left[N_{1}\left(t_{0}\right)\right]=\frac{\lambda}{v_{b}-v_{a}}\left(\left(v_{a}+v_{b}\right) t_{0}+d \ln \frac{d^{2}}{t_{0}^{2} e^{2} v_{a} v_{b}}\right)  \tag{22}\\
& E\left[N_{1}(T)\right]=\frac{2 d \lambda}{\left(v_{b}-v_{a}\right)^{2}}\left(\left(v_{a}+v_{b}\right) \ln \frac{v_{b}}{v_{a}}-2\left(v_{b}-v_{a}\right)\right) \tag{23}
\end{align*}
$$

and $E\left[B_{1}\left(t_{0}\right)\right]=$

$$
\left\{\begin{align*}
\frac{c\left(\frac{d}{t_{0}}-v_{a}\right)+2 r \ln \left[\left(v_{b}-\frac{d}{t_{0}}\right)\left(\frac{c e}{2 r}\right)\right]}{v_{b}-v_{a}} & t_{0} \geq \max \left(\frac{d}{v_{a}+\frac{2 r}{c}}, \frac{d}{v_{b}-\frac{2 r}{c}}\right)  \tag{24}\\
\frac{c\left(v_{b}-\frac{d}{t_{0}}\right)+2 r \ln \left[\left(\frac{d}{t_{0}}-v_{a}\right)\left(\frac{c e}{2 r}\right)\right]}{v_{b}-v_{a}} & t_{0} \leq \min \left(\frac{d}{v_{a}+\frac{2 r}{c}}, \frac{d}{v_{b}-\frac{2 r}{c}}\right) \\
\frac{2 r \ln \left[\left(\frac{c e}{2 r}\right)^{2}\left(v_{b}-\frac{d}{t_{0}}\right)\left(\frac{d}{t_{0}}-v_{a}\right)\right]}{v_{b}-v_{a}} & \frac{d}{v_{b}-\frac{2 r}{c}} \leq t_{0} \leq \frac{d}{v_{a}+\frac{2 r}{c}} \\
c & \frac{d}{v_{a}+\frac{2 r}{c}} \leq t_{0} \leq \frac{d}{v_{b}-\frac{2 r}{c}}
\end{align*}\right.
$$

Recall that $E\left[N_{1}\left(t_{0}\right)\right]$ is minimized when $t_{0}$ is median of $F$, i.e. $F\left(t_{0}\right)=1 / 2$. For uniform distribution, the median is equal to the arithmetic mean. It can be shown that $E\left[N_{1}\left(t_{0}\right)\right.$ is convex with a minimum at $t_{0}=2 d /\left(v_{a}+v_{b}\right)$, or $v_{0}=$ $\left(v_{a}+v_{b}\right) / 2$. Similarly the node encounter rate $E\left[N_{1}\left(t_{0}\right)\right] / t_{0}$ is also convex with a minimum at $t_{0}=d / \sqrt{v_{a} v_{b}} \geq 2 d /\left(v_{a}+v_{b}\right)$, where the inequality follows from the fact that arithmetic mean is greater or equal to the geometric mean. On the other hand, $B\left(t_{0}\right)$ is a concave function with a maximum at $t_{0}=2 d /\left(v_{a}+v_{b}\right)$. Moreover, the connection time $B$ is symmetric about the mean speed, i.e. the connection time is the same when the observer node has speed $v_{0}$ and $v_{b}+v_{a}-v_{0}$.

Similarly by substituting (21) to (6),(9),(11) we have

$$
\begin{align*}
E\left[N_{2}\left(t_{0}\right)\right] & =\lambda\left(t_{0}+\frac{d}{v_{b}-v_{a}} \ln \frac{v_{b}}{v_{a}}\right)  \tag{25}\\
E\left[N_{2}(T)\right] & =\frac{2 \lambda d}{v_{b}-v_{a}} \ln \frac{v_{b}}{v_{a}} \tag{26}
\end{align*}
$$

and $E\left[B_{2}\left(t_{0}\right)\right]=$

$$
\left\{\begin{align*}
\frac{2 r \ln \left(\frac{v_{b}+d / t_{0}}{v_{a}+d / t_{0}}\right)}{v_{b}-v_{a}} & t_{0} \leq \frac{d}{\max \left(v_{a}, 2 r / c-v_{a}\right)}  \tag{27}\\
\frac{2 r \ln \left(\frac{d / t_{0}+v_{b}}{2 r} c e\right)-c\left(d / t_{0}+v_{a}\right)}{v_{b}-v_{a}} & \frac{d}{\min \left(v_{b}, 2 r / c-v_{a}\right)} \leq t_{0} \\
& \leq \frac{d}{\max \left(v_{a}, 2 r / c-v_{b}\right)} \\
& t_{0} \geq \frac{d}{\max \left(v_{b}, 2 r / c-v_{b}\right)}
\end{align*}\right.
$$

For reverse traffic, $E\left[N_{2}\left(t_{0}\right)\right]$ is a decreasing function and $E\left[B_{2}\left(t_{0}\right)\right]$ is an increasing function.

Although both $E\left[N_{1}\left(t_{0}\right)\right]$ and $E\left[B_{1}\left(t_{0}\right)\right]$ are known analytically, the critical points for $\eta_{1}\left(t_{0}\right)=E\left[Z_{1}\left(t_{0}\right)\right] / t_{0}$ cannot be determined analytically since it involves the products of logarithmic functions. We perform numerical experiments to compare the performance of forward and reverse traffic connections at different observer node mobility. The parameters $v_{a}=2, v_{b}=10, d=1000$ are adopted in our numerical study.


Fig. 3. Expected number of connections $\eta\left(t_{0}\right)$ versus node mobility $t_{0}=$ $d / v_{0}$ for different transmit range $r$ and connection time limit $c$. (a) $r=$ $1, c=1$ (b) $r=2, c=1$ (c) $r=0.5, c=1$ (d) $r=1, c=10$

With reference to Figure 3, the expected number of connections $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ are plotted together versus $t_{0}$ in the range $d / v_{b}=100$ to $d / v_{a}=500$. At mean speed $v_{0}=6$, the corresponding $t_{0}$ is 166.67 unit. Consider scenario 1 for $r=$ $1, c=1$. For forward traffic, $\eta_{1}\left(t_{0}\right)$ attains a global maximum of 0.6 when $t_{0}$ is minimum. $\eta_{1}\left(t_{0}\right)$ decreases steadily as $t_{0}$ increases and hits the minimum of 0.3 at $t_{0}=267.73$. Beyond that, there is a slight increase of $\eta_{1}\left(t_{0}\right)$ when $t_{0}$ is increased further. Similar trends are observed for other scenarios in Figure 3(b),(c),(d). Nevertheless, a slight dip of $\eta_{1}\left(t_{0}\right)$ occurs at low mobility $\left(t_{0} \approx 500\right)$ for Figure 3(d). Although there are slightly more encounters at low mobility, there is a steeper decrease in connection time. Thus $\eta_{1}\left(t_{0}\right)$ is not convex in general. In the particular case of $v_{0}=v_{a}=0$, the observer node is stationary. The expected fraction of connection time for forward and reverse traffic should be arbitrarily close. That is, the two curves should coincide when $t_{0}$ is arbitrarily large. In our example, the observer node moves slowly when $v_{0}=v_{a}=2$. The dip in Figure 3(d) is consistent to our intuition that the fraction of connection time for forward and reverse traffic are close when the observer node has low mobility.

In contrast to forward traffic, the expected fraction of connection time $\eta_{2}\left(t_{0}\right)$ is almost constant at all observer node speed for reverse traffic. The relative value of $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ depends on the ratio of transmit range and connection time limit $r / c$. When $r / c$ is large (Figure 3(b)), it is likely that the connection time for forward traffic is truncated. Thus $\eta_{1}\left(t_{0}\right)$ is consistently smaller than $\eta_{2}\left(t_{0}\right)$ except for very high observer node speed. When $r / c$ is small (Figure 3(c),(d)), the connection time of each node encounter is large. In fact, if there is no connection time limit, the expected connection


Fig. 4. Long run average number of connections $\mathcal{Z}$ versus transmit range $r$ for connection time limit $c=0.5,1,2$.
time for forward traffic is unbounded. The large connection time at large $c$ stipulates that $\eta_{1}\left(t_{0}\right)>\eta_{2}\left(t_{0}\right)$ at all node speed. Incidentally, when $r / c=1$ (Figure 3(a)), $\eta_{1}\left(t_{0}\right)$ and $\eta_{2}\left(t_{0}\right)$ intersects at $t_{0}=162.7$, which is close to the cycle duration at mean speed $d / E[V]=166.67$. Thus, if an observer node moves at a constant speed $v_{0}$ less than the mean speed $E[V]$, reverse traffic connections are more preferable. Similarly, forward traffic connections are more preferable if a node moves at a constant speed $v_{0} \geq E[V]$ in this particular example.

When nodes move with random speed in different highway segments, the long run fraction of connection time or average number of connections $\mathcal{Z}_{1}(r, c)$ and $\mathcal{Z}_{2}(r, c)$ are relevant and dependent on the transmit range $r$ and connection time limit $c$. In practice, $c$ is typically long enough such that the connection time for reverse traffic $Y_{2}$ is not truncated. This is satisfied when $\max \left(Y_{2}\right)=r / v_{a} \leq c$, or $r / c \leq 2$. Thus $\mathcal{Z}_{2}(r)$ is independent of $c$ for the cases of our interest. In Figure 4 the long run average number of connections for both forward and reverse traffic is plotted for $c=0.5,1,2$. Both $\mathcal{Z}_{1}$ and $\mathcal{Z}_{2}$ are increasing functions of the transmit range. This is obvious since as $c$ increases, the connection time $B_{1}$ and $B_{2}$ also increase. We also observe that when the long run average number of connections for both traffic types are the same, $r \approx c$ holds. For $r / c>1$ the network nodes have a large transmit range relative to $c$, reverse traffic connections are more preferable due to the truncated connection time for forward traffic. Similarly, forward traffic connections are more preferable when $r / c<1$.

Whereas the long run fraction of connection time is an increasing function of the transmit range, there exists an optimal range such that the long run data rate is maximized. With reference to Figure 5, the data rate for both traffic types are plotted versus transmit range for the cases $c=0.25,0.5,1,2$. The optimal transmit range corresponding to the cases $c=$ $0.25,0.5,1,2$ is $r=0.15285,0.19022,0.22187,0.24722$ for forward traffic. For reverse traffic, the optimal range is $r=$ 0.37713 independent of $c$. The smaller optimal range for forward traffic connections is intuitively plausible. Forward traffic enjoys longer connection time. A short range is favored such that a high channel rate can be realized. When the connection time limit $c$ is small (Figure 5(a)), it is likely the connection time of forward traffic is truncated. Reverse traffic enjoyed much higher encounter rate that contributes


Fig. 5. Long run data rate $\mathcal{W}$ versus transmit range $r$ for different connection time limit $c$ (a) $\mathrm{c}=0.25$, (b) $\mathrm{c}=0.5$, (c) $\mathrm{c}=1$, (d) $\mathrm{c}=2$.
to the total connection time. Thus reverse traffic outperforms forward traffic at large transmit range. The long run data rate for forward and reverse traffic at optimal range are roughly the same. However, the discrepancy of the optimized data rate increases as $c$ increases. At $c=1,2$ (Figure 5(c)(d)), the optimized data rate of forward traffic is more than twice the reverse traffic. Thus, forward traffic connections yields much higher data rate. The result of Figure 5 can be compared to Figure 4. Figure 4 shows that for $r / c>1$, reverse traffic connections are more preferable due to the increased long run fraction of connection time. However, the achievable channel rate also decreases rapidly at large transmit range. Thus the extraneous data rate at $r / c>1$ is negligible as shown in Figure 5(b),(c). In most scenarios, forward traffic connections yield much higher data rate. The optimized range for forward traffic is also considerably smaller than that in reverse traffic. Thus it is also energy efficient to maintain forward traffic connections.

Our results show that the data rate of forward traffic connections and reverse traffic connections is dependent on $c$. The value of $c$, in turn, is closely related to the correlation of the contents between two nodes. If nodes have highly correlated contents, any two arbitrary nodes may want to exchange only a few files with each other, effectively modeled by a small $c$. It is more efficient to maintain reverse traffic connections and exchange files with more nodes. In a content distribution application, this is an appropriate strategy when most nodes get most files already. Similarly, when new content is disseminated, nodes have few files in common and should maintain forward traffic connections to exploit the long expected connection time as warranted by the uniform speed distribution.

## V. Discussions

In [2], it was shown that mobility increases the capacity of a mobile infostation network. Capacity gain arises from the realization of the maximal spatial transmission concurrency in each network snapshot. Mobility comes into the picture by shuffling node locations, creating numerous instances when excellent channels between different nodes can be exploited (multiuser diversity). As a result of mobility, the sum capacity of each network snapshot translates to the long run end-toend network throughput. It is noteworthy that in this networking paradigm, end-to-end capacity does not depend on node mobility per se. Node mobility, however, do impact the delay performance. The delay of a transiting packet is directly related to the time scale of the mobility process.

In this paper we have focused on the physical implications of mobility. It turns out that for reverse traffic, the expected fraction of connection time or number of connections is really independent of node mobility. For forward traffic, however, the expected number of connections increases as mobility increases. Numerical results show that the expected number of connections at high node mobility can be much greater. Thus, mobility not only provides a mechanism for the exploitation of multiuser diversity. The increase of the fraction of connection time and data rate is a physical consequence of node mobility.

It is well known that mobility degrades network performance in many wireless paradigms such as cellular networks and multihop networks. In multihop networks, for instance, extraneous overhead is needed for route maintenance to cope with link failures in node mobility. On the other hand, the fraction of connection time in a fixed infostation model [1] is constant regardless of node mobility. We have shown in this paper that the total connection time, and data rate increases with node mobility in a mobile infostation network. Thus the mobile infostation network paradigm is superior to multihop networks and fixed infostation networks in its robustness to node mobility.

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