Joint Power and Rate Control in Multiaccess Systems with Multirate Services

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Abstract —

We study joint power and rate control for wireless multiaccess systems providing multirate services in a frequency selective multipath channel environment. We show that the power control framework [1] can be extended to include rate control as well. Using this framework, we prove that a joint power and rate control algorithm converges to optimum assignments of multiaccess resources (time slots for TDMA, spreading codes for CDMA, subcarriers for OFDM etc.) to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient signal to interference plus noise ratio (SINR).

Moreover, in order to provide fairness among users and enforce minimum rate requirements, we introduce minimum cost flow problem formulation. We show how this combinatorial representation can be used to solve multirate resource allocation problem under strict minimum QoS (rate) requirements for all users.

I. Introduction

In wireless multiaccess systems, multiple users share a common communication medium. In TDMA systems, the medium is shared via time slots. In CDMA systems, spreading codes provide users access to the communication medium. Similarly, for an OFDM system, a number of subcarriers provide access to the common medium. While multiple user's information bits are transmitted simultaneously in one way or another, each user has to achieve a level of quality of service (QoS) within system constraints such as total transmit power or bandwidth.

Since wireless resources are scarce and expensive, a careful and efficient allocation of limited resources to users is vital. For example, in CDMA based IS-95 systems, power control is a useful technique to regulate transmit powers of constant bit rate voice users so as to minimize the effect of multiaccess interference (see [3] for a survey on this topic).

On the other hand, current and future wireless networks such as 3G cellular, WLANs or 4G wireless networks, are based on supporting multirate data services such as multimedia applications, internet access etc., in addition to classical voice service. For data service, users may employ multiple time slots or multiple spreading codes, and may receive variable rates. In this case, efficient resource allocation requires optimization and control of multiple parameters simultaneously, such as joint control of transmit power and rate assignments.

In the context of CDMA systems, combined power and rate control algorithms have been studied in [4–6]. Two algorithms have been proposed in [4], one is based on Lagrangian relaxation technique and the other one, called selective power control, is an extension of a fixed rate power control algorithm. On the other hand, the basic idea in [5,6] is to adapt (reduce) the rate when the transmit power required to achieve a target QoS exceeds a threshold. For multirate CDMA systems, uplink throughput maximization problem has been formulated in [7–9]. The focus in these studies is the networks with multiple service classes and the target is to satisfy different QoS requirements while utilizing the system resources in an efficient way.

In [10], we proposed a greedy rate/power scheduling algorithm to maximize the network throughput in the case of a multirate CDMA system. For CDMA systems employing OVSF (orthogonal variable spreading factor) codes, the proposed algorithm finds optimum rate assignments on the binary code tree under constraints on the total transmit power and minimum QoS (rate) requirement of each user. The algorithm achieves maximum total throughput with minimum possible power. We extended the results in [10] to CDMA systems employing multiple codes (MC-CDMA) and the greedy rate scheduling is proven to be optimal in this case as well. The analysis in [10] is based on a simple path loss channel model where the transmit power fades by a single link gain parameter.

In this paper, we extend the power control framework in [1] to jointly determine rate and transmit power level of each user in a multiaccess system in a multipath channel environment. Using the framework in [1], we prove that a standard joint power and rate control algorithm converges to optimum assignments of multiaccess resources (time slots for TDMA, spreading codes for CDMA, subcarriers for OFDM etc.) to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient signal to interference plus noise ratio (SINR).

Moreover, to provide fairness and enforce minimum rate requirements for each user, we introduce minimum cost flow problem formulation. We show how this combinatorial representation can be used to solve multirate resource allocation problem under strict minimum QoS (rate) requirements for all users.

II. Problem Statement

We consider multirate data transmission on the downlink of a single cell multiaccess system. There are K users in the cell. A multiaccess system is represented by a set of unit energy waveforms denoted by $\mathbf{S}(t) = \{\mathbf{s}_1(t), \mathbf{s}_2(t), \ldots, \mathbf{s}_N(t)\}$, where $N \geq K$. Each waveform in $\mathbf{S}(t)$ is zero outside the transmission interval $[0,T]$. For a TDMA system $\{\mathbf{s}_i(t) = \psi(t-(i-1)T/N),\ i=1,\ldots,N\}$ where $\psi(t)$ is a square pulse on the interval $[0,T/N]$. For a CDMA system $\{\mathbf{s}_i(t) = \sum_{j=1}^{N} s_{ij}\psi(t-(j-1)T/N),\ i=1,\ldots,N\}$ where $\psi(t)$ is the chip waveform nonzero in the interval $[0,T/N]$ and $s_{ij} = \int_0^T s_i(t)\psi(t-(j-1)T/N)dt$. In case of an OFDM system, N OFDM tones or subcarriers may be viewed as the waveform set.

Projecting time signals onto appropriate basis in each multiaccess system, we obtain vector representation of the waveform set, $\mathbf{S} = \{\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_N\}$ where $\mathbf{s}_i \in \Re^N$. To simplify the analysis, we use real valued waveforms, and later real valued channel taps. In each tranmission interval $[0,T]$, base station transmits N waveforms in $\mathbf{S}$. Multirate transmission is provided by assigning multiple waveforms to a user. A waveform $\mathbf{s}_i$ is transmitted with power p_i and is assigned to a user j such that user j can reliably decode (achieves target SINR γ) $\mathbf{s}_i$ using filter $\mathbf{c}_{ij}$.

Each bit is denoted by b_i, thus the base station transmits the signal

$$\mathbf{x} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \sqrt{\mathrm{p_i}}\mathrm{b_i}\mathbf{s}_\mathrm{i} \tag{1}$$

The channel between the base station and user j is modeled as a frequency-selective multipath channel and it is represented by a channel matrix $\mathbf{H}_j \in \Re^{N\times N}$. The base station has full knowledge of each user's channel.

Mobile j receives $\mathbf{r}_j$ which is the channel $\mathbf{H}_j$ distorted version of $\mathbf{x}$

$$\mathbf{r}_j = \sum_{i=1}^{N} \sqrt{p_i} b_i \mathbf{H}_j \mathbf{s}_i \tag{2}$$

There is no predetermined or fixed assignment of waveforms to users and the base station has to decide which waveform should be assigned to which user. This is a crucial point in our problem formulation. Since the channels are frequency-selective, different waveforms get distorted in a different ways by each user's channel. Therefore the SINR of user j decoding bit i will depend on both the waveform $\mathbf{s}_i$ and the channel $\mathbf{H}_j$.

To decode its own bit or bits, mobile j passes the received signal $\mathbf{r}_j$ through a bank of receiver filters, one for each waveform $\mathbf{s}_i$. Denoting the noise power at the output of a filter by σ^2, the signal to noise plus interference ratio γ_i achieved at the output of the filter $\mathbf{c}_{ij}$ is

$$\gamma_i = \frac{p_i(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_i)^2}{\sum_{v\neq i} p_v(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_v)^2 + \sigma^2(\mathbf{c}_{ij}^T\mathbf{c}_{ij})} \tag{3}$$

Our problem is to minimize the total power required to transmit N waveforms to K users where $N \geq K$. For each waveform $\mathbf{s}_i$, we will decide a user j intended to receive $\mathbf{s}_i$, a receiver filter $\mathbf{c}_{ij}$, and a transmit power p_i required to achieve the target SINR γ while user j decodes its transmitted bit on the waveform $\mathbf{s}_i$. Note that the number of waveforms assigned to a user determines the rate assigned to that user.

The optimization problem is as follows

$$\min \quad \sum_{i=1}^{N} p_i \tag{4}$$

$$s.t. \quad \max_j \max_{\mathbf{c}_{ij}} \left(\frac{p_i(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_i)^2}{\sum_{v\neq i} p_v(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_v)^2 + \sigma^2(\mathbf{c}_{ij}^T\mathbf{c}_{ij})} \right) \geq \gamma \tag{5}$$

$$p_i \geq 0,\ \mathbf{c}_{ij} \in \Re^N,\ i=1,\ldots,N$$

The constraint in (5) guarantees that for a given waveform $\mathbf{s}_i$, there is at least one user $j^* \in \{1,\ldots,K\}$ and a receiver filter $\mathbf{c}_{ij^*}$ that can decode $\mathbf{s}_i$ with acceptable quality.

III. Solution

Similar power minimization problems have been well studied in literature [1,2]. Our problem definition adds user selection into the formulation. In each transmission interval, the base station has to determine how many waveforms each user will be assigned to, and accordingly how may bits each user will receive. From this point of view, (4) may be viewed as a joint power and rate control problem.

We follow a similar analysis to the one in [2]. We rewrite (4) in the form of standard power control problems [1].

$$\min \quad \sum_{i=1}^{N} p_i \tag{6}$$

$$s.t. \quad p_i \geq \min_j \min_{\mathbf{c}_{ij}} \left(\frac{\gamma(\sum_{v\neq i} p_v(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_v)^2 + \sigma^2(\mathbf{c}_{ij}^T\mathbf{c}_{ij}))}{(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_i)^2} \right)$$

$$p_i \geq 0,\ \mathbf{c}_{ij} \in \Re^N,\ i=1,\ldots,N$$

We define

$$\mathbf{p} = [p_1, \ldots, p_N] \tag{7}$$

$$I_i(\mathbf{p}, j, \mathbf{c}_{ij}) = \frac{\gamma(\sum_{v\neq i} p_v(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_v)^2 + \sigma^2(\mathbf{c}_{ij}^T\mathbf{c}_{ij}))}{(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_i)^2} \tag{8}$$

In the context of [1], the interference function $\mathbf{I}(\mathbf{p})$ becomes

$$\mathbf{I}(\mathbf{p}) = [I_1(\mathbf{p}), \ldots, I_N(\mathbf{p})] \tag{9}$$

where

$$I_i(\mathbf{p}) = \min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij}) \tag{10}$$

We propose the following iterative algorithm

$$\mathbf{p(n+1)} \quad = \quad \mathbf{I(p(n))} \tag{11}$$

The framework of [1] tells us that an iterative algorithm in the form of (11) converges to the minimum power solution if the interference function $\mathbf{I}(\mathbf{p})$ is standard [1].

Next, we will show that $\mathbf{I}(\mathbf{p})$ is standard. Therefore, when the algorithm (11)) converges, we obtain 1) optimum matchings $(\mathbf{s}_i, j)$ between waveforms and the users, 2) optimum receiver filter $\mathbf{c}_{ij}$ that user j will use to decode $\mathbf{s}_i$, 3) optimum power assignments $\bar{\mathbf{p}} = [p_1, \ldots, p_N]$ that minimizes the objective function in (4).

Proposition 1: $\mathbf{I}(\mathbf{p}) = [I_1(\mathbf{p}), \ldots, I_N(\mathbf{p})]$ *is a standard interference function.*

Proof: $\mathbf{I}(\mathbf{p})$ is standard if it satisfies *positivity*, *monotonicity* and *scalability* properties, see [1] for details. In the conference proceedings, we skip the proof that $\mathbf{I}(\mathbf{p})$ satisfies all 3 properties.

Proposition 2: The solution of $\min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ *occurs at* j^* *and* $\mathbf{c}_{ij^*}$ *where*

$$\mathbf{A}_{i,k} \quad = \quad \sum_{v\neq i}^{N} p_v(\mathbf{H}_k\mathbf{s}_v)(\mathbf{H}_k\mathbf{s}_v)^T + \sigma^2\mathbf{I} \tag{12}$$

$$j^* \quad = \quad \arg\min_{k\in\{1,\ldots,K\}} ((\mathbf{H}_k\mathbf{s}_i)^T\mathbf{A}_{i,k}^{-1}(\mathbf{H}_k\mathbf{s}_i))^{-1} \tag{13}$$

$$\mathbf{c}_{ij^*} \quad = \quad \frac{\sqrt{p_i}}{1 + p_i(\mathbf{H}_{j^*}\mathbf{s}_i)^T\mathbf{A}_{ij^*}^{-1}(\mathbf{H}_{j^*}\mathbf{s}_i)}\mathbf{A}_{i,j^*}^{-1}\mathbf{H}_{j^*}\mathbf{s}_i \tag{14}$$

Proof: We rewrite (8) as

$$I_i(\mathbf{p}, j, \mathbf{c}_{ij}) = \frac{\gamma\ \mathbf{c}_{ij}^T(\sum_{v\neq i}^N p_v(\mathbf{H}_j\mathbf{s}_v)(\mathbf{H}_j\mathbf{s}_v)^T + \sigma^2\mathbf{I})\mathbf{c}_{ij}}{(\mathbf{c}_{ij}^T\mathbf{H}_j\mathbf{s}_i)^2} \quad (15)$$

For a given $\mathbf{p}$ and j, $\mathbf{c}_{ij}$ that minimizes (15) maximizes the left side of (5), which is the SINR achieved at the output of $\mathbf{c}_{ij}$. Therefore $\mathbf{c}_{ij}$ must be the SINR maximizing MMSE filter [2, 11] which is given as

$$\mathbf{c}_{ij} = \frac{\sqrt{p_i}}{1 + p_i(\mathbf{H}_j\mathbf{s}_i)^T\mathbf{A}_{ij}^{-1}(\mathbf{H}_j\mathbf{s}_i)}\mathbf{A}_{ij}^{-1}\mathbf{H}_j\mathbf{s}_i \quad (16)$$

where

$$\mathbf{A}_{ij} = \sum_{v\neq i}^{N} p_v(\mathbf{H}_j\mathbf{s}_v)(\mathbf{H}_j\mathbf{s}_v)^T + \sigma^2\mathbf{I} \quad (17)$$

For a given user j and its MMSE filter $\mathbf{c}_{ij}$ (16), the value of $I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ becomes

$$\gamma\ ((\mathbf{H}_j\mathbf{s}_i)^T\mathbf{A}_{ij}^{-1}\mathbf{H}_j\mathbf{s}_i)^{-1} \quad (18)$$

In this case, $j^* \in \{1, \ldots, K\}$ that minimizes (18) and its corresponding MMSE filter $\mathbf{c}_{ij^*}$ (16) is the solution of $\min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ □.

Comment: Since solving $\min_j \min_{\mathbf{c}_{ij}} I_i(\mathbf{p}, j, \mathbf{c}_{ij})$ for $j^* \in \{1, \ldots, K\}$ and $c_{ij^*} \in \Re^N$ is equivalent to finding the user j^* that decodes $\mathbf{s}_i$ with the largest possible SINR, we conclude that the minimum power solution to the problem (4) is achieved when, at each iteration of the algorithm (11), the base station assigns each signal waveform to the user who can receive that waveform with the best quality (SINR).

We observe in the simulations that although the assignment of waveforms to users may change from iteration to iteration, the set of assignments in the unique minimum total power solution is eventually achieved when the algorithm converges.

IV. Examples

We apply the proposed algorithm (11) on the downlink of a multirate CDMA wireless network. There are 8 users in the cell and the spreading factor (SF) of the system is 16. All 16 orthogonal spreading codes used in the simulations are given in Table 1.

s_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
s_2	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1
s_3	1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1
s_4	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
s_5	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1
s_6	1	1	-1	-1	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1
s_7	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	1	1
s_8	1	1	-1	-1	-1	-1	1	1	-1	-1	1	1	1	1	-1	-1
s_9	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	-1
s_{10}	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
s_{11}	1	-1	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1
s_{12}	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1	1	-1	1	-1
s_{13}	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1	1	-1	-1	1
s_{14}	1	-1	-1	1	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1
s_{15}	1	-1	-1	1	-1	1	1	-1	1	-1	-1	1	-1	1	1	-1
s_{16}	1	-1	-1	1	-1	1	1	-1	-1	1	1	-1	1	-1	-1	1

Table 1: The set of Orthogonal Spreading Codes

Figure 1 shows the locations of the mobiles (stars) and the base station (circle) on a square cell. The x and y coordinates of each mobile location is chosen uniformly on (0-100m).

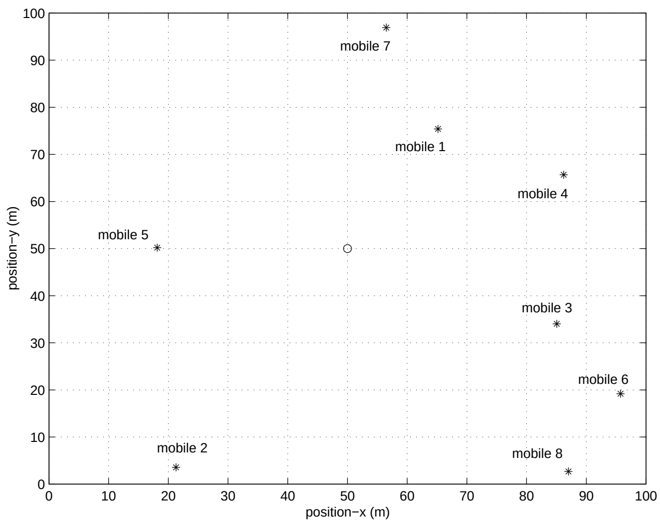

Figure 1: Position of Each Mobile Over the Cell

Between mobile j and the base station, there is a multipath channel $h_j(t)$ which can be modeled as

$$h_j(t) = \sum_{p=1}^{L_j} h_{jp}\delta(t - \tau_{jp}) \quad (19)$$

The number of channel taps L_j is chosen uniformly on $\{1, \ldots, 5\}$. The delay of the first path τ_{j1} is set to 0, for all other channel taps, each successive tap is delayed by either 1 or 2 chips, with probability 1/2 each. Therefore the delay spread can be at most 8 chips. Note that $\mathbf{H}_j$ is a lower triangular matrix with all h_{j1}s on the main diagonal and all h_{jp}s on the τ_{jp}th diagonal below the main diagonal where $p \in \{2, \ldots, L_j\}$. From the strongest channel tap to the weakest one, the difference in gain between two successive tap gains is $|A|$ dB where A ˜ N(0,20). Moreover a pathloss exponent of 4 is used and channel gains are scaled to the pathloss gain. The target SINR for each spreading code is 7, and $\sigma^2 = 10^{-12}$.

Figure 2 shows the convergence of the algorithm (11) to the minimum total power solution of the problem (4). When the algorithm converges, mobile 1 receives $\{s_3, s_4, s_7, s_{11}, s_{12}, s_{15}\}$, mobile 4 receives $\{s_5, s_6, s_8, s_{13}\}$, mobile 5 receives $\{s_1, s_2, s_9, s_{10}\}$ and mobile 7 receives $\{s_{14}, s_{16}\}$. The transmit power for each assigned spreading code is $\bar{\mathbf{p}} = 10^-5\times$ [0.0249, 0.0629, 0.0501, 0.0892, 0.1854, 0.1887, 0.1249, 0.2050 0.0247, 0.0634, 0.0548, 0.0654, 0.1940, 0.2202, 0.1014, 0.2202].

To have an insight into how the algorithm assigns the waveforms, we plot the set of all spreading codes in frequency domain in Figure 3. Moreover, Figure 4 shows the channel responses of those users who are assigned at least one spreading code (mobile 1,4,5 and 7) and Figure 5 shows the channels of those users who are not assigned any codes (mobile 2,3,6 and 8).

We see in the figures that those users who are not assigned

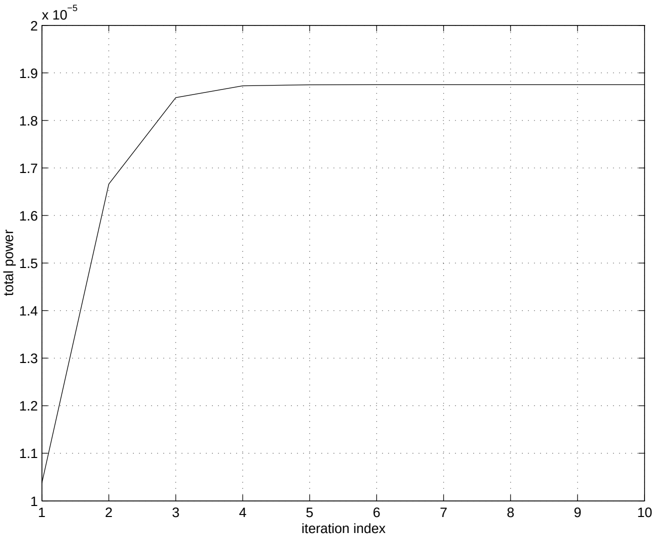

Figure 2: Total Power Convergence

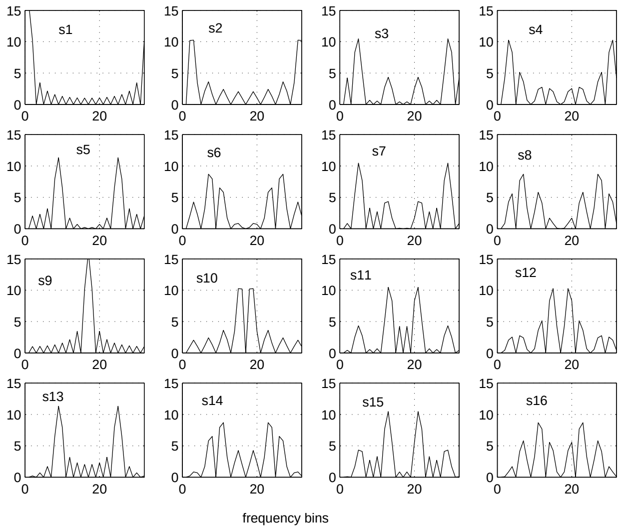

Figure 3: The Spreading Codes in Frequency Domain

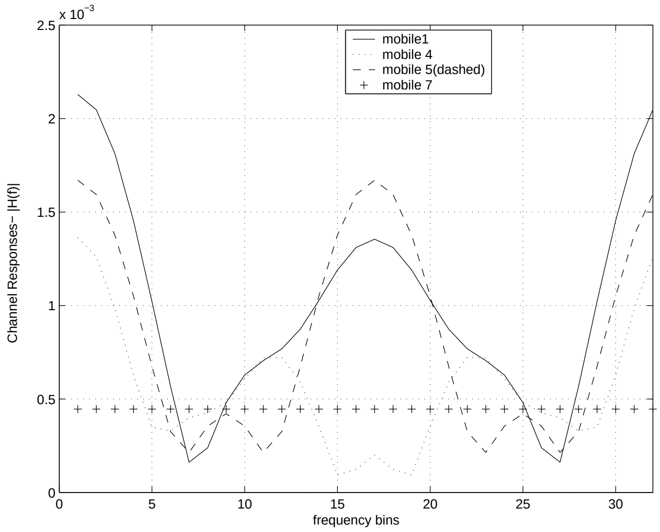

Figure 4: Channel Responses of mobile 1,4,5 and 7

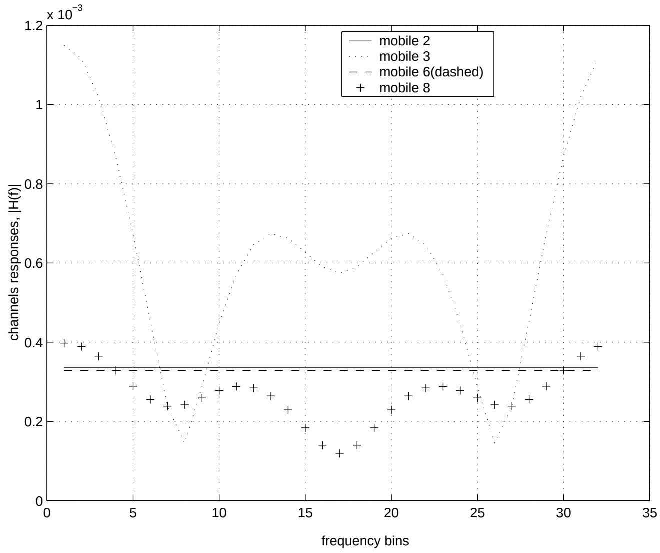

Figure 5: Channel Responses of mobile 2,3,6 and 8

any codes have relatively deeper channel fades compared to those who receive codes. For example, the channel gains of mobile 3,6 and 8 ($\approx< 0.4 \times 10^{-3}$) is almost always below the channel gains of mobile 1, 4 and 5.

On the other hand, those waveforms which are dense in the lowpass frequency range (for example spreading codes 1,2,3 and 4) and those waveforms which are dense in the highpass range (for example spreading codes 9,10,11 and 12) are assigned to mobile 1 and mobile 5 who have the largest channel gains over those frequency bins.

As the example points out, the optimum solution of (4) may result in an unfair assignment of resources to users. In this case, some users with bad channel states might not receive any waveform. On the other hand, some applications such as real-time data requires a minimum level of service without delay. In the next section we will use minimum cost flow problem formulation and analyze the case with strict minimum rate requirements.

V. Minimum Cost Flow Formulation and Strict Rate Requirements

The definition of minimum cost flow problem is as follows [12, 13]. Let G=(M,A) be a directed network with a cost C_{ij}, capacity u_{ij}, a lowerbound on the arc flow l_{ij} and flow x_{ij} associated with every arc $(i,j) \in A$. Associated with each node $i \in M$, a number $b(i)$ indicates ith node's demand or supply depending on whether $b(i) > 0$ or $b(i) < 0$. In this case the minimum cost flow problem is [12, 13]:

$$\begin{aligned} \min \quad & \sum_{(i,j)\in A} C_{ij}x_{ij} \qquad (20) \\ s.t. \quad & \sum_{\{j:(i,j)\in A\}} x_{ij} - \sum_{\{j:(j,i)\in A\}} x_{ji} = b(i) \text{ for all } i \in M \\ & l_{ij} \le x_{ij} \le u_{ij} \qquad \text{for all } (i,j) \in A \end{aligned}$$

Theorem 1 (Integrality Property, [13]): If all arc capacities

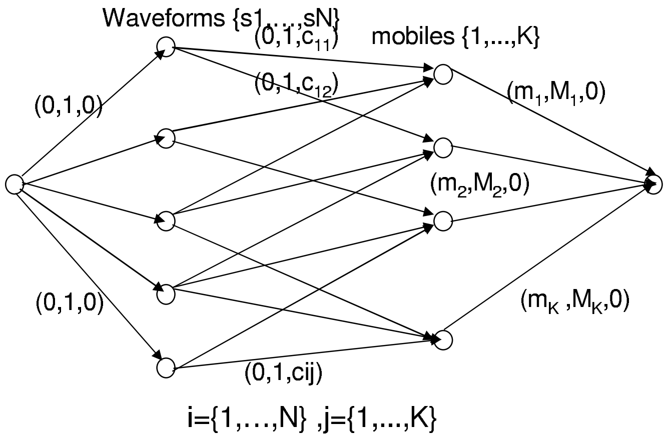

Figure 6: Flow Network

and supplies/demands of nodes are integer, the minimum cost flow problem always has an integer minimum cost flow.

With integrality property, we make sure that all arc flows are integer in the optimal solution [13].

Figure 6 shows how we construct a flow network. The network consists of $N+K+2$ nodes and $N\times K+N+K$ arcs where N denotes the number of available waveforms, K denotes the number of users and 2 additional nodes are the source and the sink nodes. The source node has a supply of N unit flow and the sink node has a demand of N unit flow, all other nodes have zero net flow.

For each arc $(i,j)\in A$, we assign 3 parameters (l_{ij},u_{ij},C_{ij}) where l_{ij} denotes the lower bound on the arc flow, u_{ij} denotes the upper bound on the arc flow (capacity) and C_{ij} denotes the cost associated with each unit flow. For N arcs connecting the source node to N waveforms, we assign $(0,1,0)$. Since the supply of the source node is N unit flow, all of these arcs (and each waveform accordingly) will be included in the minimum cost flow solution with no cost. For K arcs connecting the sink node to K users, we assign $(m_k,M_k,0)$ where m_k and M_k denote the minimum and the maximum number of waveforms user k requires. By this way, we make sure that each user k will be connected to at least m_k and at most M_k waveforms in the optimal solution.

For each $N\times K$ arcs connecting N waveforms to K users, we assign $(0,1,C_{ij})$ where $i\in\{1,\dots,N\}$, $j\in\{1,\dots,K\}$ (although each node has to be labeled with a different integer in a flow network, we use C_{11} indexing to emphasize the assignment of waveform 1 to user 1). In this case if there is a flow on arc (i,j), the waveform $\mathbf{s_i}$ will be assigned to user j. Note that, due to *Theorem 1*, all arc flows in the optimal solution are integers [13].

What is left is to define appropriate cost functions C_{ij} for each $N\times K$ arcs connecting the waveforms to users. The cost of assigning a given waveform $\mathbf{s}_i$ to user j, through channel H_j, may be defined in terms of the transmit power p_{ij} required for user j to decode $\mathbf{s}_i$ reliably. With this cost definition, the minimum cost flow solves the problem of assigning N waveforms to K users with minimum power, under constraints on the number of waveforms each user may receive.

We note that the cost of assigning a given waveform $\mathbf{s}_i$ to user j may be defined in various ways. One may define a cost function which accounts for the delay requirements of each service class or each user's buffer size. Another cost definition might be in terms of the throughput achieved when the waveform $\mathbf{s}_i$ is received by user j, i.e. $log(1+SINR_{ij})$. In this case, the maximum cost flow (which is basically the same problem) solves throughput maximization problem under constraints on the number of waveforms each user may receive.

Here we give a simple OFDM example to present the basic idea of min cost flow problem formulation.

Example- OFDM Carrier Assignment Problem

We consider an OFDM system with $N = 16$ subcarriers, and there are $K = 8$ mobiles to be served. The location and channel of each mobile are the same as the previous example in Section IV. Mobile 1,2 and 3 require at least 1 OFDM subcarrier, mobile 4,5,6 require at least 2 carriers, mobile 7 and 8 are delay tolerant and may or may not receive any subcarrier. In this case we set $m_k = 1$ for $k\in\{1,\dots,3\}$, $m_k = 2$ for $k\in\{4,\dots,6\}$ and $m_k = 0$ for $k\in\{7,8\}$. We set $M_k = 16$ for $k\in\{1,\dots,16\}$.

Such an OFDM system can be represented as a set of N independent Gaussian channels [14].

$$y_{ij} = h_{ij}x_i + n_i \qquad i\in\{1,\dots,N\} \quad j\in\{1,\dots,K\} \tag{21}$$

where the subcarrier i (equivalently $\mathbf{s_i}$) is assigned to a user j, n_i denotes the Gaussian noise density, x_i denotes the transmitted symbol and y_{ij} denotes the received signal. The channel coefficient h_{ij} is obtained by N point DFT of jth user's channel response $h_j(t)$ (19), i.e. $[h_{1j},\dots,h_{Nj}]^T = DFT_N(h_j(t))$ [14]. Each $\mathbf{s_i}$ is transmitted to a user j with power p_{ij} and there is a target SNR γ_{ij} that needs to be achieved by user j to decode the symbol x_i reliably.

We can express the transmit power p_{ij} in terms of γ_{ij}, σ^2 and h_{ij} as

$$p_{ij} = \frac{\gamma_{ij}\sigma^2}{|h_{ij}|^2} \tag{22}$$

In this case we choose the cost function as $C_{ij} = p_{ij}$.

Minimum cost flow problems are well known combinatorial problems and there are numerous algorithms proposed in literatue [12,13]. For our example, we used the source code [15] which uses a network simplex algorithm. In the experiment, the SNR target is assumed to be 7 and $\sigma^2 = 10^{-12}$ as before.

The assignment of subcarriers to users is as follows. User 1 receives $\{s_2,s_3,s_7,s_{11},s_{15},s_{16}\}$, user 2 receives $\{s_{14}\}$, user 3 receives $\{s_1\}$, user 4 receives $\{s_5,s_6,s_{12}\}$, user 5 receives $\{s_8,s_9,s_{10}\}$, user 6 receives $\{s_4,s_{13}\}$ and user 7 and 8 are not assigned any OFDM carriers. The total power required for this set of assignments is 3×10^{-4}. Figure 4 and Figure 5 shows the consistency between the assignment of subcarriers and user channels clearly. Note that 2 successive frequency bins represent an OFDM carrier in the figures.

In case there is no minimum number of subcarrier requirements ($m_k = 0$ for all users), user 1 receives $\{s_1,s_2,s_3,s_{11},s_{15},s_{16}\}$, user 2 receives no codes, user 3 receives no codes, user 4 receives $\{s_5,s_6,s_{12},s_{13}\}$, user 5 receives $\{s_8,s_9,s_{10}\}$, user 6 receives no codes, user 7 receives $\{s_4,s_{14}\}$ and user 8 receives no codes. The total power required for this set of assignments is 2.05×10^{-4}. Observe that each carrier is assigned to the user with the best channel gain over the given frequency band in this case.

VI. Conclusion and Future Work

In this paper, we examined joint power and rate control problem for wireless multiaccess systems providing multirate services in a frequency selective multipath channel environment. We proposed an iterative algorithm that converges to optimum assignments of multiaccess resources (time slots for TDMA, spreading codes for CDMA, subcarriers for OFDM etc.) to users, and to optimum transmit power levels such that the total transmit power is minimized while each transmitted bit can be decoded with sufficient signal to interference plus noise ratio (SINR).

We also presented a combinatorial analysis and introduced the minimum cost flow problem formulation. We showed how this combinatorial representation can be used to solve multirate resource allocation problem under strict minimum QoS requirements for all users. The minimum cost flow is a fairly general problem formulation in which one can define different cost functions to model different optimization objectives. We currently investigate how to apply this formulation into various places. Our current focus is the iterative version of min cost flow problem for CDMA systems.

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