Discrete Adaptive Transmission for Fading Channels

Lang Lin*, Roy D. Yates, Predrag Spasojevic
WINLAB, Rutgers University
73 Brett Rd., NJ 08854-8060
{llin, ryates, spasojev}@winlab.rutgers.edu

Abstract—**In this work we address optimal adaptive transmission policies in slow varying wireless environments. Continuous rate and power assignments that maximize the average capacity for these channels have been derived previously. Nevertheless, from a practical point of view, use of a finite number of power and rate levels is imperative. Here, we address the mapping from channel states of arbitrary distribution to a discrete set of power level and code rate pairs. Unlike earlier work, our design does not require that the transmitter knows the current channel state; one of L quantization levels is sufficient. For the proposed approach, the limiting case of large L yields Goldsmith's well-known water-filling in time result.**

I. Introduction

In the third generation cellular systems (see [1] and [2]), adaptation on the transmitter side is one of the core technologies leading to power efficient design of high-speed wireless data communication systems. A typical assumption in the design of such systems is that the channel response varies slowly and the seminal work [3] shows that the ergodic capacity of a slowly fading channel can be achieved using an adaptive coding scheme with multiple codebooks each employed for a different channel state.

Optimal adaptive transmission policies [3] require the knowledge of the current channel state at both the receiver and the transmitter. Furthermore, both power and code rate assignments need to adapt continuously to changes in the channel state. Both of these assumptions are widely adopted in further work on information theoretic aspects of communication over fading channels [4,5]. Unfortunately, in practice it is difficult to deploy a system satisfying these two assumptions.

In this paper, we propose a framework to explore the information theoretical capacity, the highest achievable average rate, of finite-level discrete adaptative transmission where the channel state space is quantized to a finite number of intervals such that a transmitter will pick up a pair from a finite set of code rate and power level pairs corresponding to the quantization level of the current channel state and transmit accordingly.

Besides proposing a framework to explore the capacity of finite-level designs, we also explore the capacity-achieving policies. It is demonstrated in this paper that finding an optimal policy which achieves the capacity is an intractable non-convex optimization problem. One way of finding the optimal policy is brute force maximization and its complexity increases exponentially in the number of levels.

In this paper, an convergent iterative algorithm is presented to numerically evaluate the lower bound of the capacity. It is shown that this algorithm produces lower bounds close to the true capacity when the number of levels is small. Moreover, it enables us to see how effective the discrete adaptive transmission is, where brute force approach may not feasible.

*corresponding author

II. System Model

In this paper, we consider the multiplicative flat fading channel model with complex received signal

$$Y = \sqrt{S}X + W, \tag{1}$$

where S is the channel fade state, X is the complex one-dimensional transmitted signal, W is a circularly symmetric additive white Gaussian noise (AWGN) with variance N_0. The fading state S is a real random variable of unit mean with a probability density function $f(s)$, distribution $F(s)$, and domain $\mathcal{S} = \{s | s \geq 0\}$. Following [3], it is assumed that the fading is *slow* and that the channel state is constant during the transmission of a single codeword so that the channel model (1) is sufficient for our purposes.

The proposed adaptive transmission system quantizes any channel state s to one of L levels $v_0 < v_1 < \ldots < v_{L-1}$, where $v_0 = 0$. The L channel state quantization intervals are denoted by $\mathcal{U}_l = [v_l, v_{l+1})$ for $l = 0, \ldots, L-1$, where formally $v_L = \infty$. Note that the set $\{\mathcal{U}_0, \ldots, \mathcal{U}_{L-1}\}$ partitions the channel state space $\mathcal{S}$. When the channel is in state $s \in \mathcal{U}_l$, the transmitter generates codewords of code rate r_l and transmits them at power level p_l. As our channel is an AWGN channel for a fixed channel state s, the maximum mutual information is given by $\log(1 + p_l s/N_0)$. Adopting the convenient notation $R(x) = \log(1 + x/N_0)$, the maximum mutual information is $R(p_l s)$.

The set of quantization levels $\{v_l\}$ and the corresponding set of power and rate assignment pairs $\{(p_l, r_l)\}$ define the adaptive transmission policy. More precisely, the triple of L element vectors

$$(\mathbf{p}, \mathbf{r}, \mathbf{v}) = ([p_0, \ldots, p_{L-1}]^\top, [v_0, \ldots, v_{L-1}]^\top, [r_0, \ldots, r_{L-1}]^\top)$$

specifies the transmission policy. As will be elaborated in the following sections, the transmission policy characterizes a number of communication parameters including achievable average throughput, outage probability, and average power.

We note that given that $s \in \mathcal{U}_l$, the power and rate assignment p_l and r_l, and we effectively have a non-adaptive policy. Under the assumptions on our channel fading model, the fading is non-ergodic during the transmission of one codeword and here, we can use the established (see, e.g., [6]) outage probability definition for non-ergodic channels to define the conditional outage probability

$$\mathcal{P}_{\text{out}}(r_l, p_l|l) = P\left[R(p_l s) < r_l | s \in \mathcal{U}_l\right] = P\left[R(p_l s) < r_l | l\right].$$

In other words, given $s \in \mathcal{U}_l$, $\mathcal{P}_{\text{out}}(r_l, p_l|l)$ is the probability that the channel state $s \in \mathcal{U}_l$ is such that the maximum mutual information $R(p_l s)$ is smaller than the desirable rate r_l and hence, the assigned rate r_l is not achievable using the power level p_l.

Given an L-level policy $(\mathbf{p}, \mathbf{v}, \mathbf{r})$, the *average outage probability* (averaged over channel states $s \in \mathcal{S}$) is

$$\mathcal{P}_{\text{out}}(\mathbf{p}, \mathbf{v}, \mathbf{r}) = \sum_{l=0}^{L-1} P\left[s \in \mathcal{U}_l\right] \mathcal{P}_{\text{out}}(r_l, p_l|l).$$

When $p_l \neq 0$ for all l, the average outage probability is equal to the achievable decoding error probability. Here, we adopt the following convention: when $p_l = 0$ (and consequently $r_l = 0$) for some l, then there is no decoding error and no outage for $s \in \mathcal{U}_l$ since no transmission is attempted, although the average throughput is affected.

Based on the conditional outage probability we define the *average information rate*. Our definition hinges on the assumption that no information is successfully received when an outage occurs. In other words, if the channel state s is such that the assigned rate r_l is not achievable and, therefore, communication is not reliable,

Consequently, given an L-level policy $(\mathbf{p}, \mathbf{v}, \mathbf{r})$, the *average data rate* (throughput) is defined as

$$R_L(\mathbf{p}, \mathbf{v}, \mathbf{r}) = \sum_{l=0}^{L-1} P\left[s \in \mathcal{U}_l\right]\left[1 - \mathcal{P}_{\text{out}}(r_l, p_l|l)\right] r_l \quad (2)$$

Employing the notation $F(s_1, s_2) = F(s_2) - F(s_1) = P\left[s_1 \leq S < s_2\right]$, the average power for the policy $(\mathbf{p}, \mathbf{v}, \mathbf{r})$ is

$$\rho(\mathbf{p}, \mathbf{v}, \mathbf{r}) = \sum_{l=0}^{L-1} F(v_l, v_{l+1}) p_l. \quad (3)$$

For an average power constraint $\overline{p}$, the set of possible L-level transmission policies is

$$\pi_L(\overline{p}) = \left\{(\mathbf{p}, \mathbf{v}, \mathbf{r}) | \, \rho(\mathbf{p}, \mathbf{v}, \mathbf{r}) \leq \overline{p}\right\}. \quad (4)$$

We can now define the *capacity* of the L-level adaptive system as

$$C_L = \max_{(\mathbf{p}, \mathbf{v}, \mathbf{r}) \in \pi_L(\overline{p})} R_L(\mathbf{p}, \mathbf{v}, \mathbf{r}). \quad (5)$$

Finding the optimal L-level policy $(\mathbf{p}^*, \mathbf{v}^*, \mathbf{r}^*)$ that achieves capacity C_L is, in the general case, a non-convex optimization problem.

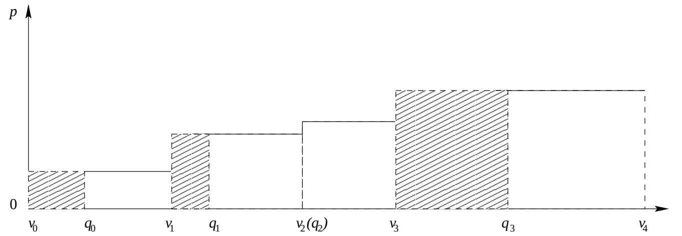

Fig. 1. Illustration of an arbitrary power policy assignment, rate assignment is not explicitly shown. The shaded regions are the channel outage intervals.

III. Properties of Optimal Policies

Coupled with analytical methods, we present sketches of power allocation as a function of channel state serve to illustrate a number of interesting properties of optimal policies. One possible policy is depicted in Fig. 1, where the heights of boxes indicate the power levels assigned to their respective adaptation intervals. Within each $\mathcal{U}_l$, the shaded region indicates states $s \in \mathcal{U}_l$ for which the assigned rate r_l does not permit reliable communication. Note, that any outage interval is contiguous within its respective quantization interval $\mathcal{U}_l$ and would include the left end point v_l of the interval $\mathcal{U}_l$. Thus, there can be at most L outage intervals. In the following, we develop an implicit characterization of the assigned rates for an optimal policy. [1]

Lemma 1: For an optimal policy $(\mathbf{p}^*, \mathbf{v}^*, \mathbf{r}^*)$ we have that

$$r_l^* \in \left[\, R(p_l^* v_l^*), R(p_l^* v_{l+1}^*) \,\right). \quad (6)$$

Since we are only interested in optimal policies, we will, henceforth, assume that any policy of interest $(\mathbf{p}, \mathbf{v}, \mathbf{r})$ also satisfies condition (6). Consequently, for any policy $(\mathbf{p}, \mathbf{v}, \mathbf{r})$ for which (6) holds and any quantization interval $\mathcal{U}_l$, continuity of $R(\cdot)$ implies we can find a channel state $q_l \in \mathcal{U}_l$ such that $r_l = R(p_l q_l)$. This defines a one to one mapping between channel states $\{q_0, \ldots, q_{L-1}\}$ and the respective rate assignments $\{r_0, \ldots, r_{L-1}\}$ for a given power policy $\mathbf{p}$. Thus it will be more convenient to redefine transmission policies of interest as vector triples $(\mathbf{p}, \mathbf{v}, \mathbf{q})$. Fig. 1 is a plot of such a $(\mathbf{p}, \mathbf{v}, \mathbf{q})$ policy and there is no outage in an interval $[v_l, v_{l+1})$ or only a portion of the interval $[v_l, v_{l+1})$ in outage.

Now, we can rewrite the conditional outage probability as follows:

$$\begin{aligned}\mathcal{P}_{\text{out}}(r_l, p_l|l) &= P\left[R(p_l s) < R(p_l q_l) | s \in \mathcal{U}_l\right] \\ &= P\left[s < q_l | s \in \mathcal{U}_l\right]. \end{aligned} \quad (7)$$

Using (7) and (2), the average rate can be expressed in terms of the vector $\mathbf{q}$ as follows:

$$R_L(\mathbf{p}, \mathbf{v}, \mathbf{q}) = \sum_{l=0}^{L-1} F(q_l, v_{l+1}) R(p_l q_l). \quad (8)$$

[1] Due to the length of this paper, we introduce all the lemmas and theorems without any proof except the *Theorem 1.*

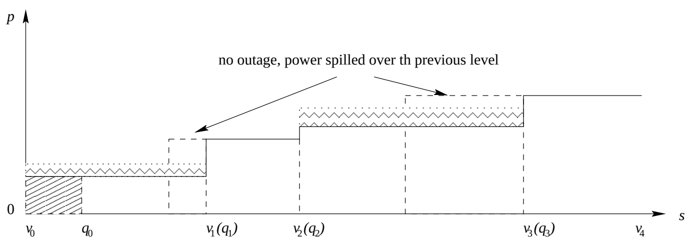

Fig. 2. The policy achieves C_L will have $q_1 = v_1$ and $q_3 = v_3$.

The optimal policy $(\mathbf{p}, \mathbf{v}, \mathbf{q})$ achieves the average capacity

$$C_L = \max_{(\mathbf{p},\mathbf{v},\mathbf{q}) \in \pi_L(\overline{p})} R_L(\mathbf{p}, \mathbf{v}, \mathbf{q}), \tag{9}$$

where the definition of $\pi_L(\overline{p})$ is given by (4). From (8) and Fig. 1, we again note that the outage region does not contribute to overall average rate. Thus, one could intuitively assume that the optimal policy should minimize this region. Solving the non-convex optimization problem (9) is hard. However, further characterization of the optimal solution is helpful in simplifying the search for an optimal policy. The following theorems describe properties of an optimal solution.

Lemma 2: If $R_L(\mathbf{p}, \mathbf{q}, \mathbf{v}) = C_L$,

$$\sum_{l=1}^{L} F(v_l, v_{l+1}) p_l = \overline{p}. \tag{10}$$

In general, the average throughput R_L is a function of both $\mathbf{v}$ and $\mathbf{q}$, nevertheless, Theorem 1 shows that there must be an optimal policy for which $v_l = q_l$ for all $l > 0$.

Theorem 1: If $R_L(\mathbf{p}, \mathbf{v}, \mathbf{q}) = C_L$, then there exists a policy $(\mathbf{p}', \mathbf{v}', \mathbf{q})$ such that $q'_l = v'_l$ for all $l > 0$ and $R_L(\mathbf{p}', \mathbf{v}', \mathbf{q}) = C_L$.

The relevance of Theorem 1 is that it demonstrates that policies of interest are fully determined by the vectors $\mathbf{p}$ and $\mathbf{q}$. For such policies, the average throughput becomes

$$R_L(\mathbf{p}, \mathbf{q}) = \sum_{l=0}^{L-1} F(q_l, q_{l+1}) R(p_l q_l). \tag{11}$$

while the average transmitted power is

$$\rho(\mathbf{p}, \mathbf{q}) = F(0, q_0) p_0 + \sum_{l=0}^{L-1} F(q_l, q_{l+1}) p_l \tag{12}$$

The optimal policy pair $(\mathbf{p}^*, \mathbf{q}^*)$ is that which achieves

$$C_L = \max_{(\mathbf{p},\mathbf{q}) \in \pi_L(\overline{p})} R_L(\mathbf{p}, \mathbf{q}), \tag{13}$$

where the set of L-level policies of interest is now

$$\pi_L(\overline{p}) = \{(\mathbf{p}, \mathbf{q}) | \rho(\mathbf{p}, \mathbf{q}) = \overline{p}\} \tag{14}$$

Although the new optimization problem (13) is much simpler, it remains non-convex and hard to solve in the general case. The next theorem shows that given any adaptation partition, defined now by $\mathbf{q}$, the optimal power assignment is a water-filling policy.

Theorem 2: Given $\mathbf{q}$ and $\overline{p} > 0$, the optimal power allocation is

$$p_l = \begin{cases} \left(\frac{F(q_0, q_1)}{\lambda F(0, q_1)} - \frac{N_0}{q_0} \right)^+ & l = 0 \\ \left(\frac{1}{\lambda} - \frac{N_0}{q_l} \right)^+ & l > 0 \end{cases} \tag{15}$$

where λ is the water-filling level that can be determined from (10) for any given $\overline{p}$ and $\mathbf{q}$.

IV. Optimal Policy Computation

Finding an optimal policy $(\mathbf{p}^*, \mathbf{q}^*)$ which achieves the capacity in (13) is still an intractable non-convex optimization problem. One way of finding the optimal policy is brute force maximization of (13). This approach entails quantization of continuous policy variables $\mathbf{p}$ and $\mathbf{q}$ and can be taken for a small number of quantization levels L since its complexity increases exponentially in L.

Assuming the cdf $F(s)$ is a strictly increasing function of s, instead of optimizing a policy $(\mathbf{p}, \mathbf{q})$, we propose an iterative algorithm which finds the equivalent policy $(\mathbf{p}, \mathbf{a})$, where $a_l = F(q_l)$ for all l based on the water-filling power allocation (Theorem 2) and a quantization optimization process, *water-spilling*. The algorithm is as follows.

1. Choose initial powers $p_l = \bar{p}$ and interval boundaries $a_0 = 1/L$ and $a_l = l/L$ for $l = 1, \ldots, L$. Here, the average rate is zero over $\mathcal{U}_0$ as it is in outage.

2. (water-filling)
Fix $\mathbf{a}$ and find $\mathbf{p}$ based on the water-filling assignment of Theorem 2.

3. (repartitioning)
If $p_1 = 0$ and p_m is the first non-zero power assignment,

$$a_l^* = \begin{cases} a_m/2, & l = 0 \\ a_{l+m-1}, & l = 1, \ldots, L-m \\ a_{L-1} + (1 - a_{L-1}) \frac{l-m}{L-m}, & \text{o.w.} \end{cases}$$

$$p_l^* = \begin{cases} 0, & l = 0 \\ p_{l+m-1}, & l = 1, \ldots, L-m \\ p_{L-1}, & \text{o.w.} \end{cases}$$

Then, let new $a_l = a_l^*$ and $p_l = p_l^*$ for all l.

4. (water-spilling)

– For $l = 1, 2, \ldots, L-1$:
Hold $p_0, \ldots, p_{l-2}, p_l, \ldots, p_{L-1}$ and $a_0, \ldots, a_{l-1}, a_{l+1}, \ldots, a_{L-1}$ constant and vary p_{l-1} and a_l to maximize $R_L(\mathbf{p}, \mathbf{a})$ under the power constraint $\rho(\mathbf{p}, \mathbf{a}) = \bar{p}$.

– Hold $\mathbf{p}$ and $\mathbf{a}$ except a_0, $\max_{a_0} \left[(a_1 - a_0) R(p_0 F^{-1}(a_0)) \right]$.

5. Repeat steps 2, 3, and 4 until convergence occurs where the rate increment after an iteration is less than a preset ϵ. At the point of convergence, the corresponding average rate is the lower bound of C_L.

This algorithm consists of three key operations, water-filling, repartitioning, and water-spilling. The convergence follows from the fact that the average rate R_L is non-decreasing during any step in our algorithm and is upper-bounded by C [3].

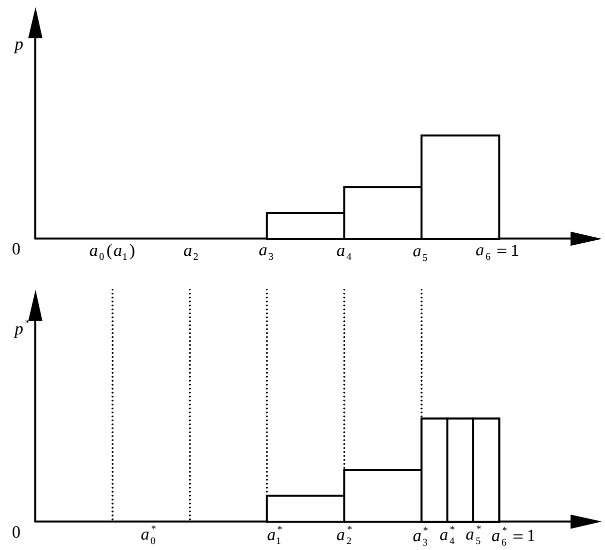

Fig. 3. Illustration of Step 3 of the repartition process for $L = 6$ and $m = 3$.

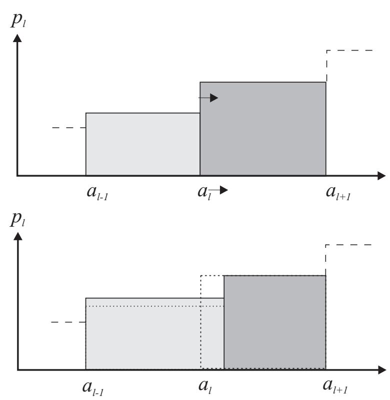

Fig. 4. Illustration of Step 4 of the water-spilling algorithm. The lower plot shows the increase in p_{l-1} as the boundary a_l is increased. That is, as the boundary a_l moves to the right, power from interval $\mathcal{U}_l = [a_l, a_{l+1})$ spills over the boundary to fill the interval $\mathcal{U}_{l-1}$.

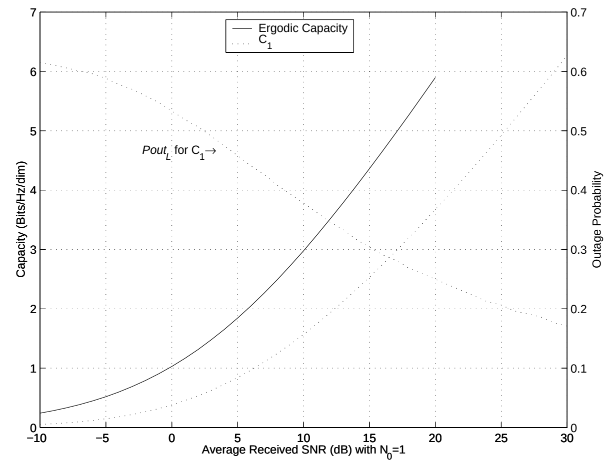

Fig. 5. Capacity C_L vs. outage $\mathcal{P}_{\text{out}}$ with $L = 1$ (no adaptation)

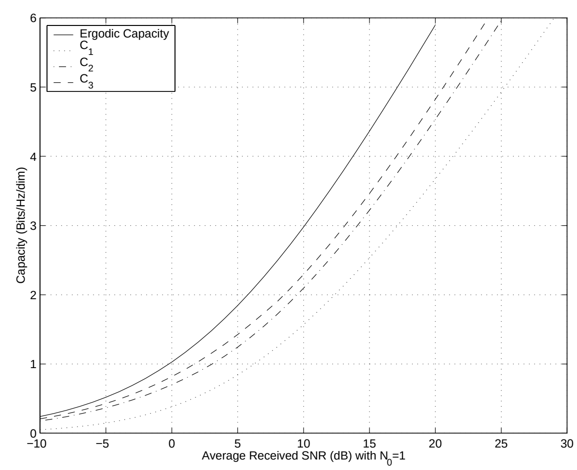

Fig. 6. C_L with $L = 1, 2, 3$.

The repartioning is illustrated in Fig. 3. At this step, partitions with zero power are merged and the $\mathcal{U}_{L-1}$ is split. This is to improve the effectiveness of the following water-spilling process.

The water-spilling process is illustrated in Fig. 4. By expressing the quantization intervals in terms of $\mathbf{a}$, the average power assigned to a given interval is equal to the area of its respective rectangle. With the power constraint $\bar{p}_l$, in Fig. 4, the sum of the areas of the shaded rectangles must remain the same to keep the same average transmitted power. Consequently, when we shift a_l to the right (or, equivalently increase the rate assignment r_l), power spills from interval l to raise the power p_{l-1} so that the shaded area stays constant. Thus we call the above algorithm *water-spilling*.

It can be proved that if the fading cdf $F(s)$ satisfies

$$\frac{2}{1 - F(s)} + \frac{F''(s)}{[F'(s)]^2} \geq 0, \; s \in \mathcal{S} \tag{16}$$

the objective function in the water-spilling step will be convex.

V. Numerical results

In this section, we present numerical results of C_L and lower bounds which obtained by brute force and the proposed iterative algorithm, respectively. We compare the C_L and lower bounds with the ergodic capacity C for Rayleigh fading. The most important point is that C_L of small L are very close to C.

For a Rayleigh fading channel, the cdf is

$$F(s) = 1 - e^{-s} \qquad s \in \mathcal{S}. \tag{17}$$

In this case, (16) holds and the convexity condition holds.

We numerically evaluate C_L and $\mathcal{P}_{\text{out}}$ with $L \leq 10$. In Fig. 5, it is shown for a non-adaptive system with $L = 1$

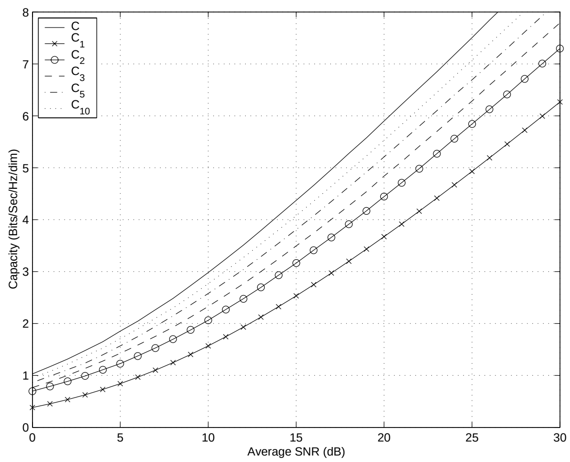

Fig. 7. C_L with $L \leq 10$.

that there is a 6 to 7 dB gap between the ergodic capacity curve and the optimal non-ergodic throughput. The outage probability corresponding to this optimal solution ranges from 17% to 62%. It confirms there is a significant penalty on the capacity and a high outage probability when using a non-adaptive communication system in a fading environment. In particular, when the received power is low, the outage probability is extremely high which implies that the optimal q_0 is quite large. Hence, most of time, the channel is poor and the channel state $s < q_0$.

Fig. 6 shows that by applying an $L = 2$ level adaptive transmission policy, the required SNR for the same capacity can be reduced by approximately 3 dB in comparison with using the non-adaptive polity. In other words, a 2-level adaptive system can eliminate about half of the SNR gap between the capacity curves of the ergodic case and the non-adaptive case. Furthermore, increasing L from 2 to 3 yields another 1 dB reduction in the SNR requirement. Note, C_2 and C_3 are obtained from exhaustive search.

As exhaustive search becomes prohibitive for increasing L, we use the heuristic iterative algorithm to lower bound C_L for large L. In Fig. 7, we present lower bounds of C_L for $L \leq 10$. There is no perceptible difference between C_L and the lower bounds produced by the heuristic algorithm for $L < 3$. Moreover, with these lower bounds, we note that $L = 10$ levels, the capacity of our discrete design could be within 1 dB from the ergodic capacity when SNR is less than 20 dB.

VI. Conclusion

In this work, we optimize the design of an adaptive transmission system with a discrete set of power levels and code rates for wireless fading environments. We show that our design procedure yields results close to the well-known water-filling result [3].

Appendix

Proof: Theorem 1 Suppose $v_{l+1} < q_{l+1}$ for some $0 \leq l < L-1$. We will construct a new policy $(\mathbf{p}', \mathbf{v}', \mathbf{q}) \in \pi_L(\overline{p})$ such that $v'_{l+1} = q_{l+1}$ and $R_L(\mathbf{p}', \mathbf{v}', \mathbf{q}) \geq C_L$. If there is more than one l such that $v_{l+1} < q_{l+1}$, we can repeat the same construction for each value of l. Let

$$\hat{R} = \sum_{i \neq l} F(q_i, v_{i+1}) R(q_i p_i) \tag{18}$$

denote the contributions to R_L from channel states $s \notin \mathcal{U}_l$. Since $R(0) = 0$, we can write

$$\begin{aligned} & R_L(\mathbf{p}, \mathbf{v}, \mathbf{q}) \\ = \; & \hat{R} + F(q_l, v_{l+1}) R(q_l p_l) + F(v_{l+1}, q_{l+1}) R(0) \\ = \; & \hat{R} + F(q_l, q_{l+1}) [\alpha R(q_l p_l) + (1-\alpha) R(0)] \end{aligned}$$

where $\alpha = F(q_l, v_{l+1}) / F(q_l, q_{l+1})$ satisfies $0 \leq \alpha \leq 1$. Since $R(\cdot)$ is a concave function, we observe that

$$R_L(\mathbf{p}, \mathbf{v}, \mathbf{q}) \leq \hat{R} + F(q_l, q_{l+1}) R(q_l \alpha p_l) \tag{19}$$

The right side of (19) can be achieved by a policy in which we use power p_l for channel states $s \in [v_l, q_l)$ and power αp_l when $s \in [q_l, q_{l+1})$. By defining $\beta = F(v_l, q_l)/F(v_l, q_{l+1})$, the average power over the states $s \in [v_l, q_{l+1})$ is

$$\hat{p} = \beta p_l + (1-\beta)\alpha p_l \geq \alpha p_l$$

Since $\hat{p} \geq \alpha p_l$, we observe that (19) and the fact that $R(\cdot)$ is an increasing function imply

$$R'_L = \hat{R} + F(q_l, q_{l+1}) R(q_l \hat{p}) \geq R_L(\mathbf{p}, \mathbf{v}, \mathbf{q})$$

Furthermore, the transmission policy $(\mathbf{p}', \mathbf{v}', \mathbf{q})$ with

$$v'_i = \begin{cases} q_{l+1} & i = l+1 \\ v_i & i \neq l+1 \end{cases}$$

$$p'_i = \begin{cases} \hat{p} & i = l \\ p_i & i \neq l \end{cases}$$

achieves the average rate $R_L(\mathbf{p}', \mathbf{v}', \mathbf{q}) = R'_L$ while meeting the average power constraint $\rho(\mathbf{p}', \mathbf{v}') \leq \overline{p}$. □

References

[1] F. Adachi, M. Sawahashi, and H. Suda, "Wideband DS-CDMA for next-generation mobile communications systems," *IEEE Commun. Mag.*, vol. 36, no. 9, pp. 56–69, Sep. 1998.

[2] K. Balachandran, R. Ejzak, S. Nanda, S. Vitebskiy, and S. Seth, "GPRS-136: high-rate packet data service for North American TDMA digital cellular systems," *IEEE Personal Commun.*, vol. 6, no. 3, pp. 34–47, Jun. 1999.

[3] A. Goldsmith and P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Info. Theory*, vol. 43, no. 6, pp. 1986–1992, Nov. 1997.

[4] D. Tse and S. Hanly, "Multi-access fading channels: Part I: Polymatroid structure, optimal resource allocation and throughput capacities," *IEEE Trans. Info. Theory*, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.

[5] G. Caire, G. Taricco, and E. Biglieri, "Optimum power control over fading channels," *IEEE Trans. Info. Theory*, vol. 45, no. 5, pp. 1468–1489, Jul. 1999.

[6] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Trans. Telecomm.*, vol. 10, no. 6, pp. 585–595, Nov.-Dec. 1999.