Energy efficient allocation based on service outage constraint

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Abstract

This paper explores an energy efficient, variable rate transmission scheme under a service outage constraint for a block fading channel model. The service outage constraint is motivated by real-time variable-rate applications. In a service outage, the transmission code rate is below a given basic rate and may in fact be zero. We solve the problem of maximizing the energy efficiency, defined as the average rate divided by the average power required to achieve this rate, subject to the service outage probability being sufficiently small. The most efficient average power is allocated as a combination of water filling and channel inversion so that the service outage constraint is satisfied and the corresponding maximum energy efficiency is achieved. Numerical results for the Rayleigh fading channel are given.

1 Introduction

The wireless channel is characterized by its time varying nature. The channel varies with time due to the presence of multipath, user mobility, and changes in the environment. When the current channel state can be estimated and fed back to the transmitter, adaptive transmission can be employed to increase system performance. The system performance criterion is usually application specific, therefore, different classes of applications will result in different adaptive transmission schemes. Wireless applications can be divided into real-time and non real-time services. In order to differentiate these two services, three capacity measures have been defined in the literature: ergodic capacity [1], delay limited capacity [2], and capacity versus outage probability [3, 4]. A comprehensive survey of these concepts can be found in [5].

The delay limited capacity and capacity versus outage probability were developed for fixed-rate real-time applications. But the fixed rate assumption is not essential for many real-time applications. For applications with simultaneous voice and data transmissions, as soon as a basic rate r_o for the voice service has been guaranteed, any excess rate can be used to transmit data service in a best effort fashion. For some video or audio applications, the source rate can be adapted according to the fading channel conditions to provide multiple quality of service levels. Usually in these applications a nonzero basic service rate r_o is required to achieve a minimum acceptable service quality. Motivated by these variable-rate real-time applications, we study variable rate transmission schemes subject to a basic service rate requirement.

Since infinite average power is needed to achieve a given nonzero rate at all times in a Rayleigh fading channel, we set up the basic service rate requirement in an outage sense called service outage constraint. The service is said to be in an outage when the information source rate is smaller than the basic service rate $r_{\rm o}$. The service quality is acceptable as long as the service outage probability is less than ϵ, a parameter indicating the outage tolerance of the application.

The service outage can be distinguished from information outage in [4]. In [4], the information source rate is fixed while the instantaneous mutual information fluctuates with the fading process. When the information source rate is greater than the instantaneous mutual information, reliable communication cannot be achieved and this event is termed information outage. By comparison, in this work, the information source rate can be adapted with the fading process and is always equal to the instantaneous mutual information. However, when the information source rate is less than the basic service rate, the service quality is not acceptable and this event is termed service outage.

The service outage based allocation problem was first discussed in [6]. In [6], we maximize the spectral efficiency measured by the average rate subject to the service outage constraint and an average power constraint. The resulting optimum power allocation was shown to be a combination of water filling and power inversion in the one-block fading channel [6]. The service outage approach can be used to strike a good balance between the average rate and the outage probability.

In this work, we examine the service outage based allocation from an energy efficiency point of view. Since the mobile device is battery-operated, an efficient use of battery energy is of significant interest. Since higher average rate can be achieved at expense of higher average power, the objectives of high average rate and low power consumption are contradictory. We will use an energy efficiency measure to balance these two objectives. A good measure for the energy efficiency is the ratio of average rate and average power in bits per Joule. This measure can be found in, for example, [7, 8, 9]. In this work, the goal is to identify the power allocation that maximizes the energy efficiency under a service outage constraint. We show that the optimum energy efficient power allocation requires assigning the most efficient average power in a most spectrally efficient manner.

2 The Allocation Problem

In this work, we employ the block flat fading Gaussian channel (BF-AWGN) model [3]. In the BF-AWGN channel a block of N symbols experiences the same channel state, which is constant over the whole block, but may vary from block to block. Note that the value of N is related to the product of the coherence time and the coherence bandwidth of the wireless channel. We make the following assumptions:

- *The channel state information is known perfectly at both transmitter and receiver.*

  Within each block we have the time-invariant Gaussian channel

  $$y = \sqrt{h}x + n. \tag{1}$$

  Here x is the channel input, y is the channel output, n is white Gaussian noise with variance σ^2, and h is the channel state. Since the channel state information is known at the transmitter, an adaptive transmission scheme can be employed in the system. Let $p(h)$ denote the power allocation for a channel state h and $r[hp(h)]$ be the capacity of a Gaussian channel with received power $hp(h)$, where

$$r[P] = \frac{1}{2}\log_2\left(1 + \frac{P}{\sigma^2}\right). \tag{2}$$

- *One adaptive codeword spans one fading block and the block size N is sufficiently large for reliable communication.*

  This assumption is reasonable as long as the product of the coherence bandwidth and the block duration is large enough.

- *The fading process is ergodic.*

  Under this assumption, the time average rate is equal to the expected rate.

Let $f(h)$ denote the probability density function of the channel state h and $F(h)$ denote the corresponding cumulative distribution function. Here, we only consider the case where h is a continuous random variable.[1] The assigned code rate for a channel state h is always equal to the capacity of the Gaussian channel $r[hp(h)]$ with received power $hp(h)$. Thus, the resource allocation problem requires finding only the optimum power allocation $p^*(h)$. Given the basic service rate r_o and the allowable service outage probability ϵ, we wish to maximize the following energy efficiency measure:

$$\eta^* = \max_{p(h)} \frac{E_h\{r[hp(h)]\}}{E_h\{p(h)\}} \tag{3}$$

$$\text{subject to} \quad \Pr\{r[hp(h)] < r_o\} \leq \epsilon \tag{3a}$$

$$p(h) \geq 0\,. \tag{3b}$$

Ergodicity of the fading process implies that the service outage constraint (3a) ensures at least a basic service rate most $(1-\epsilon)$ of the time.

Problem (3) can be equivalently written as follows:

$$\eta^* = \max_{P_{\text{av}}} \frac{1}{P_{\text{av}}} \left\{ \max_{p(h):E_h\{p(h)\}=P_{\text{av}}} E_h\{r[hp(h)]\} \right\} \tag{4}$$

$$\text{subject to} \quad \Pr\{r[hp(h)] < r_o\} \leq \epsilon \tag{4a}$$

$$p(h) \geq 0\,. \tag{4b}$$

Therefore, the energy efficiency problem (3) can be divided into two optimization steps given below.

(I) Fix the average power P_{av} and find the power allocation scheme that maximizes the expected rate under the service outage constraint, as follows:

$$R(P_{\text{av}}) = \max_{p(h)} E_h\{r[hp(h)]\} \tag{5}$$

$$\text{subject to} \quad \Pr\{r[hp(h)] < r_o\} \leq \epsilon \tag{5a}$$

$$E_h\{p(h)\} = P_{\text{av}}\,. \tag{5b}$$

$$p(h) \geq 0\,. \tag{5c}$$

Problem (5) is the spectral efficiency optimization solved by [6]. The main results from [6] are summarized in Section 3.

[1]When h is a discrete random variable, optimum power policies are typically probabilistic.

(II) Find the most energy efficient average power P^*_{av} as follows:

$$\eta^* = \max_{P_{\text{av}}} \frac{R(P_{\text{av}})}{P_{\text{av}}}\,. \tag{6}$$

3 Spectrally efficient power and rate allocation

In this section we summarize main results from [6] which solves the spectrally efficient allocation problem (5). For problem (5) to have a solution, its parameters $(P_{\text{av}}, r_{\text{o}}, \epsilon)$ must satisfy the following feasibility condition:

$$P_{\text{av}} \geq P_{\min} = \int_{h_\epsilon}^{\infty} \frac{\sigma^2(2^{2r_{\text{o}}}-1)}{h} f(h)dh\,, \tag{7}$$

where h_ϵ is the solution to $F(h_\epsilon) = \epsilon$. When Problem (5) is feasible, the corresponding optimum power allocation is given below.

$$p(h, h_0) = \begin{cases} \dfrac{\sigma^2(2^{2r_{\text{o}}}-1)}{h} & h \in \{h \geq h_\epsilon\} \cap \{h < h_0 2^{2r_{\text{o}}}\} \\ \sigma^2\left(\dfrac{1}{h_0} - \dfrac{1}{h}\right)^+ & \text{otherwise} \end{cases}, \tag{8}$$

where h_0 is the solution of $E_h\{p(h, h_0)\} = P_{\text{av}}$.

Let's define $h_m = \sup\{h : f(h) \neq 0\}$. We can see that h_m is the maximum channel state with nonzero probability. In a Rayleigh channel model we have $h_m \to \infty$. It is easy to show that when $h_0 \geq h_m$, the power allocation is the on-off channel inversion as follows:

$$p(h) = \begin{cases} \dfrac{\sigma^2(2^{2r_{\text{o}}}-1)}{h} & h \geq h_\epsilon \\ 0 & h < h_\epsilon \end{cases}, \tag{9}$$

and the corresponding $P_{\text{av}} = P_{\min}$.

Power allocation $p(h, h_0)$ includes a combination of channel inversion and water filling [10, 11]. To obtain a high average rate, we would like to allocate power in the form of water filling allocation [1]; while to meet the service outage constraint (5a), we must assign powers not below the channel inversion allocation to a subset of channel states with probability $1 - \epsilon$. The solution (8) balances these two factors. For a given (r_{o}, ϵ), the optimum power allocation scheme $p(h, h_0)$ belongs to one of the following feasible types depending on the value of h_0:[2]

I When $P_{\text{av}} = P_{\min}$, $p(h, h_0)$ includes no transmission for $h < h_\epsilon$ and channel inversion for $h \geq h_\epsilon$.

II When $P_{\text{av}} > P_{\min}$ but $h_\epsilon \leq h_0$, $p(h, h_0)$ includes no transmission for $h < h_\epsilon$, channel inversion for $h_\epsilon \leq h < h_0 2^{2r_{\text{o}}}$, and water filling for $h \geq h_0 2^{2r_{\text{o}}}$.

III When P_{av} is sufficiently high such that $h_\epsilon 2^{-2r_{\text{o}}} < h_0 < h_\epsilon$, $p(h, h_0)$ includes no transmission for $h < h_0$, water filling for $h_0 \leq h < h_\epsilon$, channel inversion for $h_\epsilon \leq h < h_0 2^{2r_{\text{o}}}$, and water filling for $h \geq h_0 2^{2r_{\text{o}}}$.

[2]In the case of $r_{\text{o}} = 0$ or $\epsilon = 1$, type II and III solutions will degenerate into type IV solution, which is pure water filling allocation.

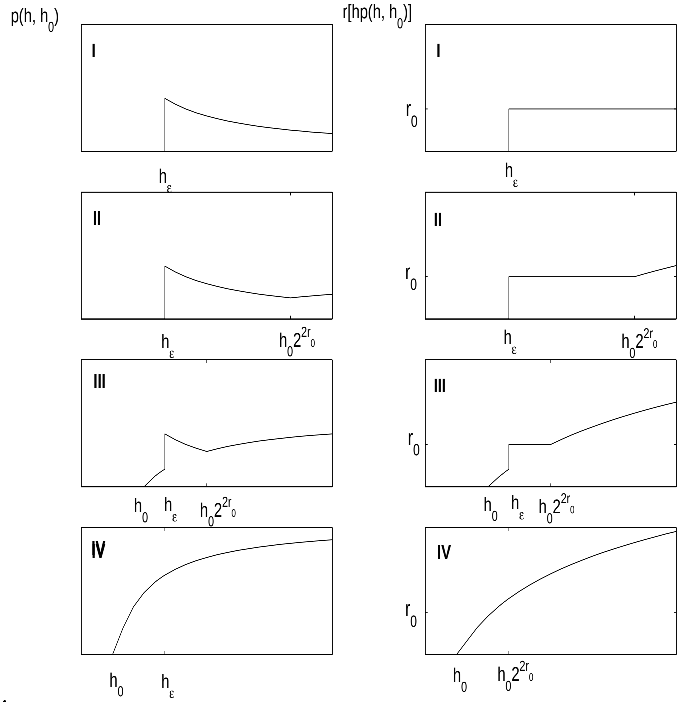

Figure 1: For optimum solution types I-IV, power policies are given on the left and the corresponding rate allocation are on the right.

IV When P_{av} is high enough for $h_0 \leq h_\epsilon 2^{-2r_o}$, $p(h,h_0)$ is just the water filling allocation.

These four types of power allocation schemes as well as the corresponding rate allocations are depicted in Figure 1. We can see that $p(h,h_0)$ gradually changes from the on-off channel inversion scheme [4] to the water filling allocation scheme with decreasing h_0, or equivalently with increasing P_{av}.

4 The most energy efficient average power P^*_{av}

In section 2, we have shown that the energy efficiency problem (3) can be solved in two steps. The first step was solved in Section 3. In this section we focus on the second step and on finding the optimum average power P^*_{av}.

For simplicity, we define $P(h_0)$ as the function mapping from h_0 to the corresponding average power P_{av}, that is $P(h_0) = E_h\left\{p(h,h_0)\right\}$. It is easy to verify that $P(h_0)$ is a strictly decreasing function of h_0 when $h_0 \leq h_m$, and $P(h_0) = P_{\min}$ when $h_0 \geq h_m$. Therefore, we can define an inverse function mapping $h_0(P_{\text{av}})$ as follows:

$$h_0(P_{\text{av}}) = \begin{cases} P^{-1}(P_{\text{av}}) & P_{\text{av}} > P_{\min} \\ h_m & P_{\text{av}} = P_{\min} \end{cases} . \tag{10}$$

In this case, we define $R(P_{\text{av}}) = E_h\left\{r[hp(h,h_0(P_{\text{av}}))]\right\}$ as the highest average rate achieved with P_{av}.

Let $\eta(P_{\text{av}})$ denote the achievable energy efficiency at P_{av}, as follows:

$$\eta(P_{\text{av}}) = \frac{R(P_{\text{av}})}{P_{\text{av}}} . \tag{11}$$

In this section we determine P_{av}^* that maximizes $\eta(P_{\rm av})$. We show that $\eta(P_{\rm av})$ is unimodal in the interval $[P_{\rm min}, \infty)$ and, thus, that P_{av}^* is either a stationary point of $\eta(P_{\rm av})$ or an end point of the interval. The computation of P_{av}^* is based on a line search technique that first finds the corresponding h_0^*.

Derivative of $\eta(P_{\rm av})$ can be expressed as follows:

$$\eta'(P_{\rm av}) \quad = \quad \frac{1}{P_{\rm av}}[R'(P_{\rm av}) - \frac{R(P_{\rm av})}{P_{\rm av}}] \quad P_{\rm av} \geq P_{\rm min}\,. \tag{12}$$

In (12), $R(P_{\rm av})$ is an implicit function of $P_{\rm av}$. Its first derivative is given in the following lemma.

Lemma 1 *$R(P_{\rm av})$ is a concave increasing function of $P_{\rm av}$ and its first derivative $R'(P_{\rm av}) = h_0(P_{\rm av})/[2\log(2)]$ for all $P_{\rm av} \geq P_{\rm min}$.*

Lemma 1 follows from the sensitivity theorem in constraint convex optimization problems [12]. The $h_0(P_{\rm av})/[2\log(2)]$ is the Lagrange multiplier associated with the average power $P_{\rm av}$ constraint in the spectral efficiency problem (5). Lagrange multiplier in constraint convex optimization problems determines the slope of the optimum objective value as a function of the constraint parameter. Therefore, the water filling cutoff $h_0(P_{\rm av})$ is a measure of the rate of change of $R(P_{\rm av})$ with the average power $P_{\rm av}$.

Let $\hat{P}_{av}$ denote the solution to $R'(P_{\rm av}) - R(P_{\rm av})/P_{\rm av} = 0$, then $\hat{P}_{av}$ is a stationary point for $\eta'(P_{\rm av})$ if it exists. The following Lemma 2 implies that P_{av}^* is either the stationary point $\hat{P}_{av}$ or the boundary point $P_{\rm min}$.

Lemma 2 *$\eta'(P_{\rm av})$ has the following properties:*

(a) *when $\eta'(P_{\rm min}) > 0$, $\hat{P}_{av}$ exists and is unique.*

(b) *when $\eta'(P_{\rm min}) > 0$, $\eta'(P_{\rm av}) > 0$ for $P_{\rm av} < \hat{P}_{av}$ and $\eta'(P_{\rm av}) < 0$ for $P_{\rm av} > \hat{P}_{av}$.*

(c) *when $\eta'(P_{\rm min}) < 0$, $\eta'(P_{\rm av}) < 0$ for $P_{\rm av} \geq P_{\rm min}$.*

From Lemma 2, we can see that $\eta(P_{\rm av})$ is unimodal in the interval $[P_{\rm min}, \infty)$. When $\eta'(P_{\rm min}) > 0$, $\eta(P_{\rm av})$ achieves it maximum at its stationary point, that is, $P_{av}^* = \hat{P}_{av}$. When $\eta'(P_{\rm min}) < 0$, $\eta(P_{\rm av})$ achieves its maximum at $P_{av}^* = P_{\rm min}$. These two qualitatively different cases are illustrated in Fig 2 and Fig 3, respectively. Combining above results and replacing $R'(P_{\rm av})$ with $h_0(P_{\rm av})/[2\log(2)]$, we obtain the following theorem.

Theorem 1 *When $\eta'(P_{\rm min}) > 0$, P_{av}^* is the solution to $R(P_{\rm av})/P_{\rm av} - h_0(P_{\rm av})/[2\log(2)] = 0$. Otherwise, $P_{av}^* = P_{\rm min}$.*

Note that both $h_0(P_{\rm av})$ and $R(P_{\rm av})$ are implicit functions of $P_{\rm av}$, nevertheless, they can be explicitly expressed in terms of $p(h, h_0)$. Therefore, in order to calculate P_{av}^* for the case of $\eta'(P_{\rm min}) > 0$, we first find the optimum h_0^* and then compute P_{av}^* by using the explicit function mapping $P(h_0^*)$. The optimum h_0^* is the solution of the following equation:

$$\frac{E_h\left\{r[hp(h, h_0)]\right\}}{E_h\left\{p(h, h_0)\right\}} - h_0/[2\log(2)] = 0 \quad 0 < h_0 \leq h_m \tag{13}$$

and can be obtained using a line search method.

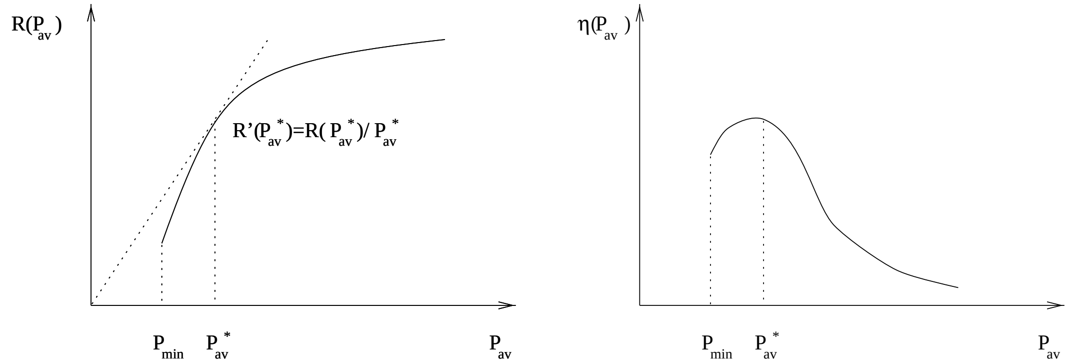

Figure 2: Optimum average rate and the corresponding efficiency when $\eta'(P_{\min}) > 0$.

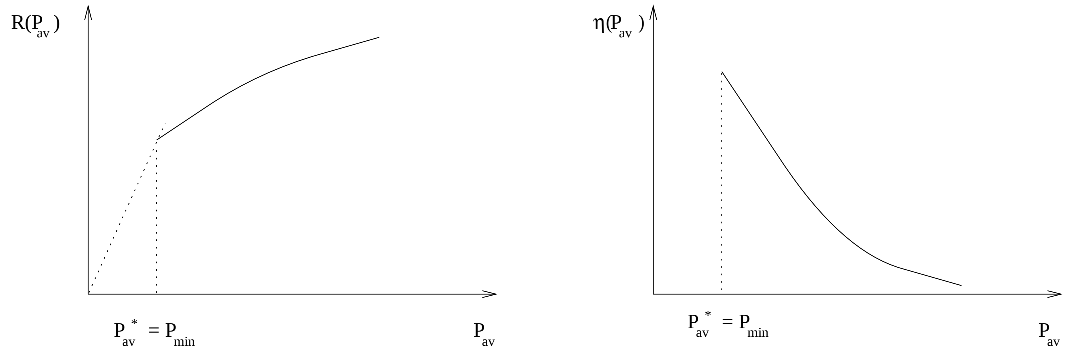

Figure 3: Optimum average rate and the corresponding efficiency when $\eta'(P_{\min}) < 0$.

5 Discussion

Before we discuss the energy efficiency under the service outage constraint for flat fading channels, we first examine the energy efficiency for AWGN channels and that for flat fading channels without the service outage constraint. Several numerical results given at the end of this section illustrate the analysis given in this work as applied to a Rayleigh fading channel.

The energy efficiency versus the spectral efficiency in AWGN channels is studied in [13]. Since maximizing energy efficiency in bits/Joule is equivalent to minimizing the bit energy $\mathcal{E}_{\rm b}$ in Joule/bit, $\mathcal{E}_{\rm b}/N_0$ is used as the energy efficiency measure in [13]. $\mathcal{E}_{\rm b}/N_0$ approaches its lower limit when the information rate approaches zero. In this case, the corresponding power goes to zero for finite bandwidth systems. Therefore, in AWGN channels the energy efficiency and spectral efficiency are two competing objectives. To achieve a high energy efficiency, the system should transmit at a low rate, resulting in a low spectral efficiency.

The energy efficiency for the ergodic capacity problem in flat fading channels can be regarded as a special case of problem (3) with $\epsilon = 1$ or $r_{\rm o} = 0$. In this case, the corresponding optimum power allocation is always the water filling allocation for any $P_{\rm av} \geq 0$. It can be shown that the energy efficiency $\eta(P_{\rm av})$ decreases with $P_{\rm av}$ and achieves its maximum value at $P_{\min} = 0$. In the same manner as for the AWGN channel, a high energy efficiency for fading channels without a service outage constraint requires a low average power and results in a low average rate, hence, a low spectral efficiency.

For flat fading channels with a service outage constraint, the optimum average power is not necessarily the minimum required average power $P_{\min}$. The reason is as follows:

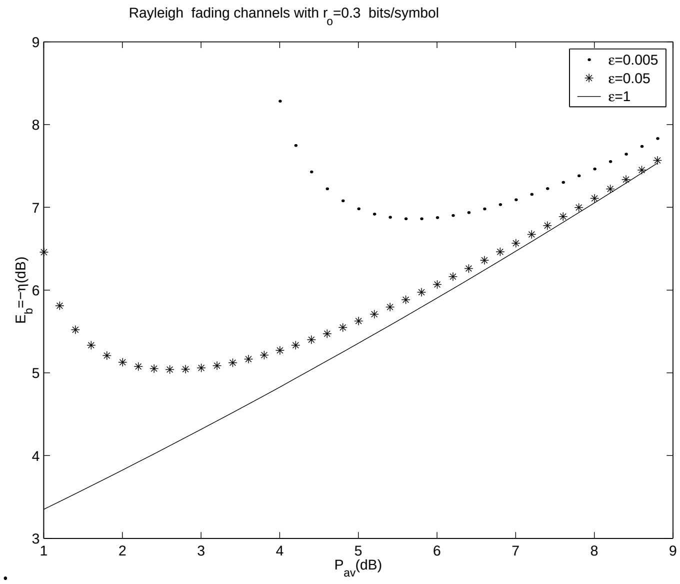

Figure 4: $\mathcal{E}_\mathrm{b}$ versus average power in Rayleigh fading channel

under the service outage constraint the optimum power allocation is a combination of channel inversion and water filling allocations. With an increase in the average power, the optimum power allocation includes a larger water filling component and a smaller channel inversion component. Since the water filling allocation achieves the highest average rate without the service outage constraint, it is more energy efficient than the channel inversion allocation. On the other hand, the energy efficiency for pure water filling allocation decreases with an increase in the average power. Therefore, the energy efficiency may increase with P_av initially when the first factor is dominant, and then decrease with P_av when the second factor is dominant. Relative to P_min, the optimum average power P_av^* achieves both higher energy efficiency and higher spectral efficiency.

In the following, we apply the analysis given in this paper to Rayleigh fading channels. For the Rayleigh fading channel with normalized mean, we have

$$f(h) = \begin{cases} e^{-h} & h \geq 0 \\ 0 & \text{otherwise} \end{cases} . \tag{14}$$

For an active service outage constraint, that is, when $r_\mathrm{o} > 0$ and $\epsilon < 1$, we have $P_\mathrm{min} > 0$ and $R(P_\mathrm{min}) = r_\mathrm{o}(1-\epsilon)$. It is easy to show that $\eta'(P_\mathrm{min}) > 0$ for Rayleigh fading channel as follows:

$$\eta'(P_\mathrm{min}) = \frac{1}{P_\mathrm{min}}\left(R'(P_\mathrm{min}) - \frac{R(P_\mathrm{min})}{P_\mathrm{min}}\right) \tag{15}$$

$$= \frac{1}{P_\mathrm{min}}\left(\frac{h_m}{2\log(2)} - \frac{R(P_\mathrm{min})}{P_\mathrm{min}}\right). \tag{16}$$

Note that $h_0(P_\mathrm{min}) = h_m \to \infty$, therefore $\eta'(P_\mathrm{min}) > 0$. Consequently, the optimum $P_\mathrm{av}^* > P_\mathrm{min}$ and can be obtained as a solution to $R(P_\mathrm{av})/P_\mathrm{av} - h_0(P_\mathrm{av})/[2\log(2)] = 0$.

We define $\mathcal{E}_\mathrm{b}(P_\mathrm{av}) = 1/\eta(P_\mathrm{av}) = P_\mathrm{av}/R(P_\mathrm{av})$ as the minimum required average energy per average information bit when the average power P_av is used. $\mathcal{E}_\mathrm{b}(P_\mathrm{av})$ in dB is plotted in Figure 4 for Rayleigh fading channels. We consider the following three service outage probabilities $\epsilon = 0.005$, $\epsilon = 0.05$, and $\epsilon = 1$ for fixed basic service rate

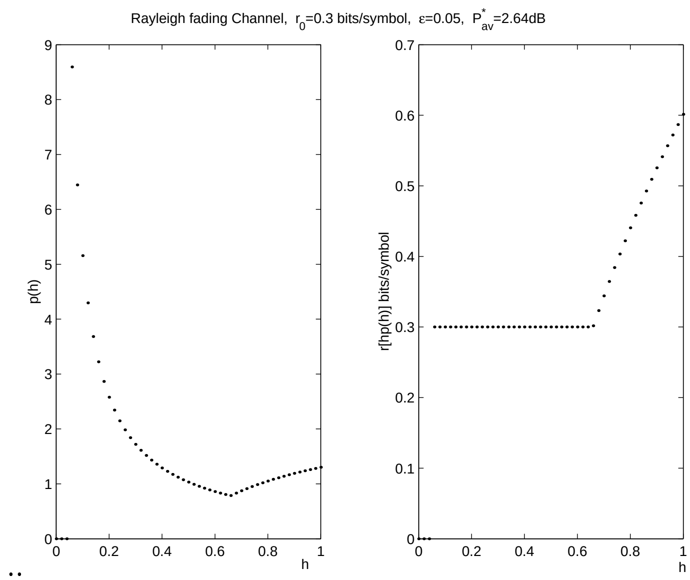

Figure 5: At the optimum average power $P_{\text{av}}^* = 2.64$ dB, the power allocation is given on the left and corresponding rate allocation is on the right

$r_{\text{o}} = 0.2$ bits/symbol. In the case of $\epsilon = 1$ there is no service outage constraint and the ergodic capacity is achieved for an given P_{av}. In this case, required $\mathcal{E}_{\text{b}}(P_{\text{av}})$ increases with P_{av} and the highest power efficiency is achieved at zero power. When $\epsilon = 0.005$, the optimum average power $P_{\text{av}}^* = 5.7$ dB is 1.83 dB above the corresponding $P_{\min} = 3.87$ dB, and it saves 2.26 dB in required bit energy compared to the case when $P_{\min}$ is used. When $\epsilon = 0.05$, the optimum average power $P_{\text{av}}^* = 2.64$ dB is 1.64 dB above the corresponding $P_{\min} = 1$ dB, and it saves 1.4 dB in bit energy compared to $P_{\min}$. Therefore, significant energy saving can be achieved by operating at the optimum average power.

The power and rate allocation for the optimum average power $P_{\text{av}}^* = 2.64dB$, $\epsilon = 0.05$, and $r_{\text{o}} = 0.3$ bits/symbol is plotted in Figure 5. We can see that the optimum energy efficient power allocation is a combination of water filling and channel inversion.

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