Optimum Signature Sequence Sets for Asynchronous CDMA Systems

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Abstract

In this paper, we characterize the *user capacity*, i.e., the maximum number of supportable users at a common SIR target level for a fixed processing gain, of a single cell symbol *asynchronous* CDMA system. Based on the delay profile of the users, we identify a class of optimum signature sequences that achieve a lower bound on the total squared asynchronous correlation (TSAC) among the users. When the users' signatures achieve this lower bound, the user capacity of a single-cell asynchronous CDMA system becomes the same as that of a single-cell synchronous CDMA system; that is, there is no loss in user capacity due to asynchrony. Further, when the optimum signature sequences are used, the users' received powers are all equal and the M-shot MMSE receiver filters turn out to be scaled matched filters. That is, the maximum user capacity is achieved by observing only one symbol interval of the received signal and using single-user matched filters in that interval. It is a significant open question whether these optimal sequence sets exist for all delay profiles. However, we present iterative and distributed signature adaptation algorithms, which, when executed sequentially by all of the users, appear to converge to these optimum signature sequences.

1 Introduction

Code Division Multiple Access (CDMA) systems are interference limited. Much research has been conducted in the area of multiuser detection [1] to develop techniques to suppress/cancel interference for a fixed (arbitrarily chosen) set of user signatures. Recently, there has been an interest to understand the influence of multiuser detection schemes on the overall system capacity in single-cell CDMA systems. This recent literature can be divided into two general categories: those that assume random signature sequences [2–4], and those that solve for the optimum signature sequences [5–7].

In [2, 3], a random signature sequence analysis is applied to a large system where both the number of users, K, and the processing gain, N, go to infinity, but their ratio $\alpha = K/N$ is fixed and finite. References [2, 3] showed that for this large system with random signature sequences, the signal to inteference ratio (SIR) of all users converged in probability to deterministic quantities, and calculated the *user capacity* of the system. Among all linear receiver filters, much attention is paid to the minimum mean squared error (MMSE) receiver, since it is the optimum linear filter in the sense of maximizing the SIR [8]. Reference [2] showed that the user capacity of the single-cell synchronous CDMA system with MMSE receivers is larger than that with matched filter receivers in the amount of exactly 1 user per dimension. Subsequently, [5] showed that if one can choose the signature sequences of the users carefully, then this gap between the MMSE and the matched filter receivers can be closed. The user capacity of a single-cell synchronous CDMA system using optimum signature sequences is the same for the

MMSE and matched filter receivers. Further, this capacity equals the asymptotic (i.e., large system) user capacity with random signature sequences using MMSE receivers. The signature sequences that maximize the user capacity are Welch bound equality (WBE) sequences [9–11]. In an earlier work [7], WBE sequences were shown to maximize the information theoretic sum capacity of a single-cell synchronous CDMA system with equal received powers. More recently, a generalized version of this problem where users have arbitrary (unequal) received powers was solved in [6].

An iterative and distributed signature sequence adaptation algorithm that converges to a set of optimum signature sequences was given in [12–14]. The algorithm exploits the property of the optimum signature sequences that they minimize the sum of the squares of the cross correlations among the users, called total squared correlation, TSC, in [12–14]. The algorithm converges to a set of WBE sequences if $K > N$, or a a set of orthogonal sequences if $K \leq N$. The algorithm, which was named the *MMSE update* algorithm, was based on the idea that the TSC–minimizing, hence optimum, signature sequences may be obtained if at each iteration one user updates its signature sequence to decrease the TSC of the whole set. In the proposed algorithm, each user updates its signature sequence to be the normalized MMSE receiver filter for that user when the signature sequences of all other users are fixed, hence the name MMSE update.

Recently, [4] generalized the analysis in [2] to a single-cell *asynchronous* CDMA system. Reference [4] showed that under matched filter reception, the user capacity of a single-cell asynchronous CDMA system is the same as the corresponding synchronous system. However, when MMSE reception is employed, there is a loss in user capacity compared to the user capacity in a synchronous system. This gap was shown to diminish as the observation window length is increased.

In this paper, we investigate the user capacity of an *asynchronous* single-cell CDMA system. Based on the users' delay profile, we identify a class of optimum signature sequences that achieve a lower bound on the total squared asynchronous correlation (TSAC) among the users. When the users' signatures achieve this lower bound, the user capacity of a single-cell asynchronous CDMA system is the same as that of a single-cell synchronous CDMA system, that is, there is no loss in user capacity due to asynchrony. Further, when the optimum signature sequences are used, the M-shot MMSE receiver filters turn out to be scaled matched filters; that is, the maximum user capacity is achieved by observing only one symbol interval of the received signal and using single-user matched filters in that interval. Although it is an open question whether these optimal sequence sets exist for all delay profiles, we propose iterative signature sequence adaptation algorithms which we observe experimentally to converge to optimum signature sequence sets. At each step of the algorithms only one signature sequence from the set is replaced in a way not to increase the TSAC.

2 Problem Statement and Derivations

We consider a single-cell symbol-asynchronous (but chip-synchronous) CDMA system with K users and processing gain of N. The received signal in the nth symbol interval of user k is given as

$$\mathbf{r}_k(n) = \sqrt{p_k} b_k(n) \mathbf{s}_k + \sum_{l \neq k} \sqrt{p_l} \left(b_l(n) T_R^{d_{kl}} \mathbf{s}_l + b_l(n+1) T_L^{d_{kl}} \mathbf{s}_l \right) \tag{1}$$

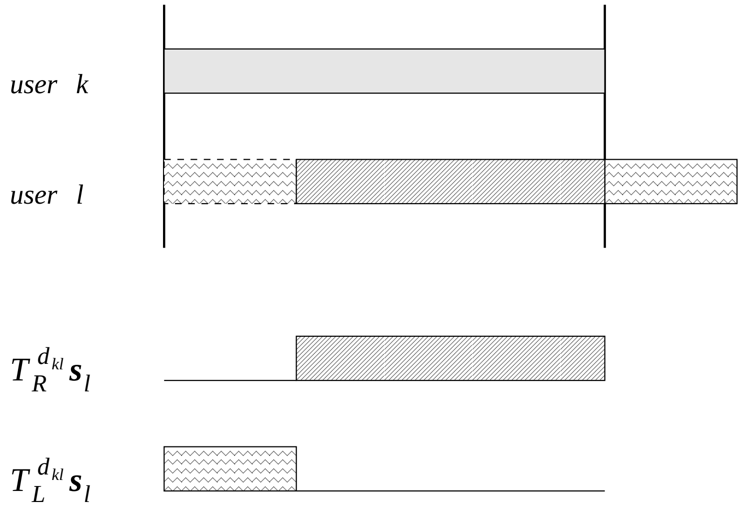

Figure 1: Asynchronous interference calculation.

where p_k, $b_k(n)$ and $\mathbf{s}_k$ are the received power, nth transmitted symbol and signature sequence of user k, respectively. The signature sequences of all users are of unit energy, i.e., $\mathbf{s}_k^\top \mathbf{s}_k = 1$, for all k. For users k and l, d_{kl} represents the relative time delay of user l with respect to the time delay of user k, that is, $d_{kl} = d_l - d_k$, where d_k and d_l are the time delays of users k and l, respectively. Symbols T_R^d and T_L^d denote the operations of shifting, to right and left, respectively, of a vector by d and $N-d$ chips (components). For both operators, the vacated positions in the vector are filled with zeros. That is, for for $d \geq 0$, we use 0^d to denote d consecutive zeros and we define

$$T_R^d \mathbf{x} = [0^d, x_1, \cdots, x_{N-d}] \tag{2}$$

and

$$T_L^d \mathbf{x} = [x_{N-d}, \cdots, x_N, 0^{N-d}] \tag{3}$$

We will use one-shot matched filters as the receivers. In general, for an arbitrary set of signature sequences, single user receivers (matched filters) are suboptimum in multiuser systems, and one-shot detectors are suboptimum in asynchronous systems. Therefore, the one-shot matched filter detector we assume is clearly suboptimum for arbitrarily chosen signature sequences. However, as we will show later, when the optimum signature sequences are used, the optimum linear receivers, i.e., the MMSE receivers, turn out to be scaled matched filters, and there is no loss in capacity due to one-shot filtering.

The decision statistics for the kth user in the nth symbol interval is $y_k(n) = \mathbf{s}_k^\top \mathbf{r}_k(n)$, where we do assume that the matched filter receiver of each user is perfectly aligned with the symbol interval of the user. The SIR of the kth user is then given by

$$\mathrm{SIR}_k = \frac{p_k (\mathbf{s}_k^\top \mathbf{s}_k)^2}{\sum_{l \neq k} p_l \left\{ (\mathbf{s}_k^\top T_R^{d_{kl}} \mathbf{s}_l)^2 + (\mathbf{s}_k^\top T_L^{d_{kl}} \mathbf{s}_l)^2 \right\} + \sigma^2 (\mathbf{s}_k^\top \mathbf{s}_k)} \tag{4}$$

We define the $K \times K$ matrix $\mathbf{A}$ with the following entries

$$A_{kl} = \begin{cases} (\mathbf{s}_k^\top T_R^{d_{kl}} \mathbf{s}_l)^2 + (\mathbf{s}_k^\top T_L^{d_{kl}} \mathbf{s}_l)^2 & k \neq l \\ 0 & k = l \end{cases} \tag{5}$$

A simple observation that will be important later is that $\mathbf{A}$ is a symmetric non-negative matrix. Since $\mathbf{s}_k^\top \mathbf{s}_k = 1$, the SIR of the kth user in (4) becomes

$$\mathrm{SIR}_k = \frac{p_k}{\sum_{l \neq k} A_{kl} p_l + \sigma^2} \tag{6}$$

The common SIR target β is said to be feasible iff one can find non-negative powers $\{p_k\}_{k=1}^K$ such that $\mathrm{SIR}_k \geq \beta$, for $k = 1, \cdots, K$, i.e.,

$$p_k \geq \beta \left(\sum_{l \neq k} A_{kl} p_l + \sigma^2 \right) \tag{7}$$

which can be written in an equivalent matrix form as

$$\mathbf{p} \geq \beta \left(\mathbf{A}\mathbf{p} + \sigma^2 \mathbf{1} \right) \tag{8}$$

where $\mathbf{1}$ is the vector of all ones. It is well known that if the SIR target β is feasible, then the optimum power vector, i.e., the componentwise smallest feasible power vector, is found by solving (8) with equality:

$$\mathbf{p} = \beta \mathbf{A}\mathbf{p} + \sigma^2 \beta \mathbf{1} \tag{9}$$

That is, if the common SIR target β is feasible, the optimum power vector is given as $\mathbf{p}^* = \sigma^2 \beta (\mathbf{I} - \beta \mathbf{A})^{-1} \mathbf{1}$. The power control problem is feasible iff [15]

$$\beta < \frac{1}{\rho_A} \tag{10}$$

where ρ_A is the largest (also called the Perron-Frobenius) eigenvalue of matrix $\mathbf{A}$. We define the matrix $\mathbf{R} = \mathbf{A} + \mathbf{I}$ so that $\mathbf{R}_{kk} = (\mathbf{s}_k^\top \mathbf{s}_k)^2 = 1$ and $\mathbf{R}$ represents the squared asynchronous cross correlations of the signatures. The Perron-Frobenius eigenvalue of $\mathbf{R}$ satisfies $\rho_R = \rho_A + 1$, and the feasibility condition in (10) can also be expressed as

$$\beta < \frac{1}{\rho_R - 1} \tag{11}$$

That is, for a single cell CDMA system, the range of common achievable SIR values are determined only by the Perron-Frobenius eigenvalue of the squared asynchronous cross correlation matrix $\mathbf{R}$ which depends only on the signature sequences of the users and their relative time delays. For a given signature sequence set $\{\mathbf{s}_k\}_{k=1}^K$ and a set of time delays $\{d_k\}_{k=1}^K$, the supremum of common achievable SIR targets equals $1/(\rho_R - 1)$ Our aim is to choose the signature sequences of the users, for any given set of time delays, such that the common achievable SIR is maximized. Therefore, we seek the signature sequence set that maximizes $1/(\rho_R - 1)$, or, equivalently, minimizes ρ_R.

We note that it is hard to characterize the dependence of ρ_R on individual signature sequences. If this was not the case, one could devise an algorithm to update the signature sequences of the users in the direction that decreases ρ_R. Instead, our approach is to tie the Perron-Frobenius eigenvalue of $\mathbf{R}$, ρ_R, to another parameter which can be related to the signature sequences in a more direct way. By this approach we will be able to characterize the optimum signature sequences in a closed form expression in addition to being able to devise an iterative and distributed signature sequence update algorithm that will construct progressively better signature sequence sets.

To this end, we start our derivation with the following bounds on the Perron-Frobenius eigenvalue of $\mathbf{R}$ in terms of its row-sums [15]

$$\min_k \sum_{l=1}^{K} R_{kl} \leq \rho_R \leq \max_k \sum_{l=1}^{K} R_{kl} \tag{12}$$

Similar bounds that can be obtained using column-sums of $\mathbf{R}$ are identical to (12) since $\mathbf{R}$ is symmetric. We also have the following bound from a simple application of the Rayleigh quotient [16]

$$\frac{1}{K} \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \leq \rho_R \tag{13}$$

which is equivalent to $(\mathbf{1}^\top \mathbf{R} \mathbf{1})/(\mathbf{1}^\top \mathbf{1}) \leq \rho_R$. Combining (12) and (13) and the fact that the minimum row-sum lower bounds the average of the row-sums yields

$$\min_k \sum_{l=1}^{K} R_{kl} \leq \frac{1}{K} \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \leq \rho_R \leq \max_k \sum_{l=1}^{K} R_{kl} \tag{14}$$

We define the total squared asynchronous correlation (TSAC) as

$$\mathrm{TSAC} = \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \tag{15}$$

Since we want to minimize ρ_R, and since ρ_R is lower bounded by TSAC/K, it is reasonable to try to minimize the TSAC over the space of all possible signature sequences. It is not clear, however, that ρ_R decreases as TSAC decreases. We will show, though, that the signature sequence sets that minimize TSAC are precisely those that minimize ρ_R.

In order to motivate the solution of the asynchronous problem, we will first revisit the synchronous problem which has been recently solved in [5].

3 The Synchronous Problem Revisited

In the synchronous case the (k,l)th component of $\mathbf{R}$ is $R_{kl} = (\mathbf{s}_k^\top \mathbf{s}_l)^2$. The following two theorems guarantee that the signature sequences that minimize the TSC (equivalent of TSAC in synchronous case) are those that minimize ρ_R.

Theorem 1 (Welch [9], Massey [10], Massey-Mittelholzer [11])

$$TSC = \sum_{k=1}^{K} \sum_{l=1}^{K} (\mathbf{s}_k^\top \mathbf{s}_l)^2 \geq \frac{K^2}{N} \tag{16}$$

Theorem 2 (Massey-Mittelholzer [11]) *(The Uniformly-Good Property) If the sequences* $\{\mathbf{s}_k\}_{k=1}^{K}$ *are such that the equality holds in (16) then*

$$\sum_{l=1}^{K} (\mathbf{s}_k^\top \mathbf{s}_l)^2 = \frac{K}{N} \qquad k = 1, \cdots, K \tag{17}$$

Theorem 1, combined with (14) and $R_{kl} = (\mathbf{s}_k^\top \mathbf{s}_l)^2$ for the synchronous case, says that

$$\rho_R \geq \frac{K}{N} \tag{18}$$

Since our aim is to minimize ρ_R, the best we can do is to choose signature sequences so as to achieve (18) with equality. Theorem 2 says that when the signature sequences are chosen such that the TSC is minimized, i.e., the bound on the TSC is satisfied by equality, then all of the row-sums are equal, and they are all equal to K/N. Since the row-sums sandwich ρ_R, (18) is also satisfied with equality, and the lowest possible ρ_R is obtained: $\rho_R = K/N$. Therefore, using (11), in the synchronous case, the bound on the common achievable SIR target is

$$\beta < \frac{1}{K/N - 1} \tag{19}$$

which is equivalent to the user capacity expression

$$\frac{K}{N} < 1 + \frac{1}{\beta} \tag{20}$$

given in [5] where this problem was originally solved.

Theorems 1 and 2 apply to $K > N$ case. When $K \leq N$, the bound in Theorem 1 is loose, the K^2/N bound cannot be achieved, and Theorem 2 loses its applicability. When $K \leq N$, the equivalent of (16) in Theorem 1 is

$$\text{TSC} \geq K \tag{21}$$

In this case, the equivalent of Theorem 2 is the following: if the sequences $\{\mathbf{s}_k\}_{k=1}^K$ are such that the equality holds in (21) then $\sum_l (\mathbf{s}_k^\top \mathbf{s}_l)^2 = 1$ for $k = 1, \cdots, K$. That is, all of the row-sums of $\mathbf{R}$ are equal to 1, and therefore, $\rho_R = 1$. The implication of this result, from (11), is that any (arbitrarily large) common SIR target, β, is feasible with sufficient transmitter power.

4 The Asynchronous Problem

In this section, we derive asynchronous versions of Theorems 1 and 2. The following two theorems guarantee that the signature sequences that minimize the TSAC are those that minimize ρ_R.

Theorem 3 *For any delay profile, the total squared asynchronous correlation satisfies*

$$TSAC = \sum_{k=1}^{K} \sum_{l=1}^{K} R_{kl} \geq \frac{K^2}{N} \tag{22}$$

Theorem 4 *(The Asynchronous Uniformly-Good Property) If the sequences* $\{\mathbf{s}_k\}_{k=1}^K$ *are such that the equality holds in (22) then*

$$\sum_{l=1}^{K} R_{kl} = \frac{K}{N} \qquad k = 1, \cdots, K \tag{23}$$

We will continue our derivation similar to the synchronous case. For this asynchronous CDMA system, Theorem 3 combined with (14) says that

$$\rho_R \geq \frac{K}{N} \tag{24}$$

Similar to the synchronous case, our aim is to minimize ρ_R, and we cannot do better than to choose signature sequences that achieve (24) with equality. Theorem 4 says that when the signature sequences are chosen such that the TSAC is minimized, i.e., the bound on the TSAC is satisfied by equality, then all of the row-sums are equal, and they are all equal to K/N. Since the row-sums sandwich ρ_R, (24) is also satisfied with equality, and the lowest possible ρ_R is obtained: $\rho_R = K/N$. Therefore, using (11), the bound on the common achievable SIR target, in this asynchronous case, is

$$\beta < \frac{1}{K/N - 1} \tag{25}$$

which is the same as the bound (19) found in the synchronous case.

Similar to the synchronous case, Theorems 3 and 4 apply to the $K > N$ case. When $K \leq N$, the bound in Theorem 3 is loose, the K^2/N bound cannot be achieved, and Theorem 4 loses its applicability. When $K \leq N$, the equivalent of (22) in Theorem 3 is

$$\text{TSAC} \geq K \tag{26}$$

The bound is achieved with equality when A_{kl} satisfies $A_{kl} = 0$ for all k and l. In this case, the equivalent of Theorem 4 is the following: if the sequences $\{\mathbf{s}_k\}_{k=1}^K$ are such that the equality holds in (26) then $\sum_l R_{kl} = 1$ for $k = 1, \cdots, K$. That is, all of the row-sums of $\mathbf{R}$ are equal to 1, and therefore, $\rho_R = 1$. The implication of this result, from (11), as in the synchronous case, is that any (arbitrarily large) common SIR target, β, is feasible.

We will next show that the optimum received powers of the users are equal. First, we set the received power of each user k to be

$$p_k = p = \frac{\sigma^2}{1 + 1/\beta - K/N} \qquad k = 1, \cdots, K \tag{27}$$

and then we show that this selection guarantees that all users have SIRs equal to β when the signature sequences of the users are chosen to satisfy (22) with equality in Theorem 3. Note that $p > 0$ as long as K, N and β satisfy the user capacity inequality (25). From (6), the SIR of the kth user is

$$\text{SIR}_k = \frac{p}{p(\sum_{l \neq k} A_{kl}) + \sigma^2} = \frac{p}{p(\sum_l R_{kl} - 1) + \sigma^2} \tag{28}$$

If the signature sequences satisfy (22) with equality in Theorem 3, then using (23) in Theorem 4, we can write (28) as

$$\text{SIR}_k = \frac{p}{p(K/N - 1) + \sigma^2} \tag{29}$$

It is now straightforward to show that when (27) is inserted into (29), we have $\text{SIR}_k = \beta$.

As stated earlier, in general, using the matched filter as we have done in this section for the asynchronous case and in the previous section in the treatment of the synchronous case is suboptimum. However, an important observation in the original solution for the optimum signature sequences in the synchronous case in [5] was that once the optimum signature sequences were chosen, the MMSE receiver filters reduce to scaled matched filters. As stated in the following theorem, when we choose the optimum signature sequences in the asynchronous case, i.e., the signature sequences that minimize the TSAC, then the M-shot MMSE receiver filters, for $M \geq 1$, corresponding to those signature sequences reduce to be scaled matched filters.

Theorem 5 *If the sequences* $\{\mathbf{s}_k\}_{k=1}^{K}$ *are such that the equality holds in (22) then the* M*-shot MMSE filters for all users reduce to corresponding scaled matched filters.*

5 TSAC Reduction: Iterative Algorithms

Following the closed-form expressions for the signature sequence sets maximizing the information theoretic sum capacity [6, 7] and user capacity [5], references [12–14] introduced the iterative adaptation of signature sequences for synchronous CDMA systems. Since the optimum signature sequences minimize the TSC in the synchronous case, the algorithm presented in [12–14] was designed to decrease (more precisely, not to increase) the TSC at each iteration of the algorithm. Since we have shown in the previous section that the optimum signature sequences minimize the TSAC in the asynchronous case, we will design algorithms which decrease the TSAC at each iteration. To this end, we will first separate the terms that depend on the signature sequence of the kth user in the TSAC. Using the TSAC definition (15) and the definition of $\mathbf{R}$ as $\mathbf{R} = \mathbf{A} + \mathbf{I}$, along with the definition of $\mathbf{A}$ in (5), we can express the TSAC as

$$\text{TSAC} = (\mathbf{s}_k^\top \mathbf{s}_k)^2 + 2\mathbf{s}_k^\top \left(\sum_{l \neq k} \bar{\mathbf{s}}_{kl} \bar{\mathbf{s}}_{kl}^\top + \tilde{\mathbf{s}}_{kl} \tilde{\mathbf{s}}_{kl}^\top \right) \mathbf{s}_k + \gamma_k \tag{30}$$

where we used notation $\bar{\mathbf{s}}_{kl} = T_R^{d_{kl}} \mathbf{s}_l$ and $\tilde{\mathbf{s}}_{kl} = T_L^{d_{kl}} \mathbf{s}_l$, to represent the left and right signatures of the lth asynchronous user with respect to the kth user. We also used the fact that $\mathbf{A}$ is symmetric in deriving (30) from the definition of TSAC in (15). In (30),

$$\gamma_k = \sum_{i \neq k} \sum_{j \neq k} R_{kl} \tag{31}$$

denotes the squared asynchronous correlation terms that do not depend on $\mathbf{s}_k$. Let us define $\mathbf{B}_k$ as

$$\mathbf{B}_k = \sum_{l \neq k} \left(\bar{\mathbf{s}}_{kl} \bar{\mathbf{s}}_{kl}^\top + \tilde{\mathbf{s}}_{kl} \tilde{\mathbf{s}}_{kl}^\top \right) \tag{32}$$

Therefore the TSAC in (30) can be written as

$$\text{TSAC} = (\mathbf{s}_k^\top \mathbf{s}_k)^2 + 2\mathbf{s}_k^\top \mathbf{B}_k \mathbf{s}_k + \gamma_k \tag{33}$$

In order to minimize the TSAC, we are looking for updates of the signature sequence of the kth user from $\mathbf{s}_k$ to some $\mathbf{c}_k$ that is guaranteed to decrease (not to increase) the TSAC. Let us denote the TSAC after the $\mathbf{s}_k \rightarrow \mathbf{c}_k$ update as $\overline{\text{TSAC}}$. Then,

$$\overline{\text{TSAC}} = (\mathbf{c}_k^\top \mathbf{c}_k)^2 + 2\mathbf{c}_k^\top \mathbf{B}_k \mathbf{c}_k + \gamma_k \tag{34}$$

Restricting the new (updated) signature sequence of the kth user to be of unit energy as well, i.e., $\mathbf{c}_k^\top \mathbf{c}_k = 1$, we note that $\overline{\text{TSAC}} \leq \text{TSAC}$ iff

$$\mathbf{c}_k^\top \mathbf{B}_k \mathbf{c}_k \leq \mathbf{s}_k^\top \mathbf{B}_k \mathbf{s}_k \tag{35}$$

Although there are many possible $\mathbf{s}_k \to \mathbf{c}_k$ updates that would guarantee that (35) holds, we will propose two of them here. The two similar updates used in the synchronous CDMA context were given in [12–14] and in [17]. We call the first update the *asynchronous MMSE update* which is given as

$$\mathbf{c}_k = \frac{\left(\mathbf{B}_k + a^2 \mathbf{I}_N\right)^{-1} \mathbf{s}_k}{\left[\mathbf{s}_k^\top \left(\mathbf{B}_k + a^2 \mathbf{I}_N\right)^{-2} \mathbf{s}_k\right]^{1/2}} \tag{36}$$

and we call the second update the *asynchronous eigen update* which is defined as the normalized version of the eigenvector of $\mathbf{B}_k$ corresponding to its smallest eigenvalue. The proof that the asynchronous eigen update decreases the TSAC follows from the Rayleigh quotient applied to the matrix $\mathbf{B}_k$ [16]. The proof that the asynchronous MMSE update decreases the TSAC can be carried out in a very similar fashion to the proof that the MMSE update decreases the TSC in [12–14].

6 Further Remarks

Due to space limitations, we were unable to provide proofs of our theorems in this paper. The derivations of the bounds in the proofs of Theorems 3 and 4 describe the conditions that the signature sequences must satisfy in order for them to achieve the lower bound on the TSAC. In a synchronous CDMA system, the optimum (WBE) sequences must satisfy $\mathbf{S}\mathbf{S}^\top = (K/N)\mathbf{I}_N$, where $\mathbf{S}$ is a matrix containing the signature sequences of the users in its columns. That is, the optimum signature sequences must satisfy three sets of conditions: (1) each column of $\mathbf{S}$ must have unit length, (2) rows of $\mathbf{S}$ must be orthogonal to each other, and (3) each row of $\mathbf{S}$ must have equal length (a length of $\sqrt{K/N}$). These conditions were identified in [10,11] in re-deriving the Welch's bound, but the existence of sequences satisfying these conditions was first addressed in [5]. Similar to [10, 11] in the synchronous case, currently, in the asynchronous case, we have derived the conditions that the optimum signature sequences should satisfy, however, we have not addressed the issue of existence of such sequences. Similarly, we have proved that the optimum signature sequences should achieve a lower bound on TSAC and provided two algorithms that monotonically decrease TSAC; however we have not proved that the algorithms proposed here are guaranteed to converge to the desired lower bound starting from an arbitrary set. We can report however that, through a large number of numerical experiments with randomly generated initial signature sequence sets, we have observed that the TSAC reduction algorithms we have proposed here have always converged to signature sequences with TSAC $= K^2/N$. That is, we have observed not only the existence of such sequence sets, but also the convergence of the proposed algorithms to these sets.

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