Multiuser Access Detection for CDMA Systems

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Abstract: We consider CDMA systems employing packet switching where multiple users can access the system simultaneously. The problem of detecting the activity of multiple new users in an efficient manner is studied. Under the optimistic assumption of a discrete delay space,, the maximum likelihood receiver that detects the presence of multiple new users, i.e the *optimum multiuser access detector*, has high complexity. Thus, we propose a suboptimum decorrelating receiver which has lower complexity. We then investigate how this simple detector performs under general conditions. The overall recognition of new users by the system is achieved by the decorrelating access detector followed by a bit detector that decodes users' identification information. Performance of the overall access scheme is given and factors effecting the multiuser access capacity of the system are identified.

1 Introduction

Code Division Multiple Access (CDMA) is a promising access scheme for future wireless systems that intend to use packet switching to support a wide range of services with different requirements. Considerable effort has been directed towards establishing efficient protocols for packet switched CDMA networks to date (see [1] and references therein), however all these studies dealt with issues after the timing parameters and the activity status of users are obtained. In reality, future wireless systems may require services where many users arrive and leave the system quickly. Thus, determining which users are active and the parameters associated with these users in an efficient manner become important issues in system design.

In this paper, we consider a packet switched CDMA system where a previously inactive *group* of users attempt to gain access to the system. Their presence must be detected by the system for them to be able to send information. This includes determining which users become active in a given period and the unknown parameters associated with an active user without which the user's information bits cannot be received reliably. The most important of these parameters is the time of arrival (delay) of the user.

Estimation of users' delays in CDMA systems has been an active research area. Timing acquisition work initially concentrated on determining the delay of a single user assuming a Gaussian disturbance, then on determining the delay of a single user in the presence of multi-access interference [2–4]. Finally extensive effort has been directed towards estimation of all users' delays (and amplitudes) jointly [5–8]. These studies assume that each user whose delay is estimated is known to be active. Also, these studies assume a dedicated signature sequence per user suggesting an inherently circuit switched structure. Receivers which can detect one new user's activity at a time along with estimation of its parameters have been constructed in [9]. This work is founded on the assumption that at most one new user attempts to access the system during the detection period which may not be appropriate for packet switched networks.

In this work, we concentrate on detecting *multiple* users' activity along with the delays of each of these users within the context of the model described in Sections 2 and 3. The optimum multiuser access detector under simplistic assumptions is developed and is observed to be highly complex. A decorrelating type suboptimum linear receiver is constructed. The performance of the decorrelating receiver in the absence of these assumptions is investigated. It is observed that the access stage may be potentially a limiting factor in the capacity of a packet switching CDMA system. Factors affecting the multiuser access capacity are discussed.

2 System Model

We consider CDMA systems where multiple users can attempt to access the system at the same time. We assume that the new users have all acquired the base station's pilot signal and are tuned to the downlink paging channel where they can receive broadcast messages. The start time of the access message is broadcasted from the base station along with other necessary access parameters. The delay uncertainty of the new users thus comes from their transmission (propagation) delays relative to the broadcast of the base station. We assume these delays to be less than 1 bit period for each new user (see Figure 1).

We consider a system where new users attempt to access the system through a common signature sequence (access channel). As mentioned in the previous section, this is conceptually different than a conventional CDMA system where users have dedicated signature sequences.

We assume the initial packet that each user sends includes a preamble (a sequence of 1s) that we use to detect the user's activity and estimate its propagation delay followed by the user's identification. If the user's presence is *detected* by the system during the access phase and its identification is decoded correctly, the user receives an acknowledgment to go forward with the information transmission. The model as depicted in Figure 2 suggests a *two-stage* receiver whose initial stage works on the transmitted preambles to detect the activity status of the users and is followed by a detector which will decode the active users' identification information (see Figure 3). In this paper, we will concentrate on how to construct the first stage, i.e. the detection of the presence of the active users. This will be done by detecting the activities (if any) at or around certain delay values determined by the users' propagation delays. The detector for the second stage is an asynchronous multiuser detector and is constructed using the information about the activity status of the users supplied by the first stage, i.e. if users' activity is detected at or around a delay value, a corresponding receiver filter is designed to decode the incoming identification information and/or suppress interference.

The performance of the first stage is of vital importance to the system since the performance of the second stage detector hinges upon the correctness of the information supplied by the first stage. A false alarm event, the event that the system declares one (or more) user(s) present at a particular delay when there is none, implies waste of resources for the second stage since it may require the detector to try to decode fictitious users and to suppress their actually non-existing interference to other users. A miss event which contributes to the rate at which the system fails to capture a user is also highly undesirable since an active user will not enter the system and its interference will not be cancelled while other users are being decoded during the second stage.

3 Analysis Preliminaries

The problem we will consider in this paper is the detection multiple new users in a Gaussian channel. Users enter the system with the initial packet that contains the access preamble and the identifier, transmit their messages and leave the system before the system announces the next access start time, thus no other connection is present or established during the service of these users.

The total received signal during the first stage of access (the preamble stage) can be expressed as:

$$r(t) = \sum_{i=1}^{N_A} \sqrt{q_i} s_a(t - \tau_i) + n(t) \tag{1}$$

where N_A is the number of active users, q_i and τ_i are the received power and the delay of the user i and $n(t)$ is the zero mean white Gaussian noise with power spectral density σ^2. The accessing signature sequence $s_a(t)$ can be expressed as

$$s_a(t) = \sum_{i=1}^{G} c(i) \frac{1}{\sqrt{G}} p(t - (i-1)T_c) \tag{2}$$

where G is the processing gain, T_c is the chip duration, $c(i) \in \{-1, 1\}$ is the i^{th} chip value, and $p(t)$ is the chip waveform normalized to have unit energy. Throughout the paper we will assume for simplicity that $p(t)$ is rectangular. Thus,

$$p(t) = \begin{cases} \frac{1}{\sqrt{T_c}}, & 0 \le t < T_c \\ 0, & else \end{cases} \tag{3}$$

The received signal is observed from the start of the access message with 1-bit delay, thus for a total of $L - 1$ bits where $L > 1$ is the length of the preamble. The reason of observing the signal with 1-bit delay is to ensure the capture of at least one bit period where all new users are actively sending their access preamble (see Figure 4). Note that during each observed bit interval, the contribution of each active terminal consists of the access signature sequence circularly shifted by that terminal's delay value. This fact will be used extensively in the sequel. Specifically, we will term the contribution of the k^{th} user *the effective signature sequence of user k.*

4 Discrete Timing Delays

To motivate the subsequent development, let us first assume that the time of arrival of each user's access preamble can be one of M possible values between 0 and T_{max} and these M values are uniform and $\Delta = \frac{T_c}{v}$ apart ($M = \lceil \frac{T_{max}}{\Delta} + 1 \rceil$). We can then design a bank of filters each of which is matched to a possible effective signature sequence and filter the received signal in each bit period (starting at $t = T_b = GT_c$) of the preamble stage. Defining

$$\tilde{c}_i = c(k) \quad (k-1)v \le i \le kv, \quad k = 1, \cdots, G \tag{4}$$

$$\tilde{p}(t) = \begin{cases} \frac{1}{\sqrt{T_c}}, & 0 \le t < \Delta \\ 0, & else \end{cases} \tag{5}$$

we can express the j^{th} possible effective signature sequence (j^{th} filter to be used) as:

$$s_j(t) = \sum_{i=1}^{vG} \tilde{c}_{i \oplus (vG - j)} \frac{1}{\sqrt{G}} \tilde{p}(t - (i-1)\Delta) \tag{6}$$

where $\oplus$ denotes the sum modulo vG.

Let us also assume that accessing users all arrive at the base with equal received power. Then, given the observation period, the resulting filter outputs that are contained in the vector $\boldsymbol{r}$ constitute a sufficient statistic for determining the number of active users at each possible delay. $\boldsymbol{r}$ can be expressed as:

$$\boldsymbol{r} = \sqrt{q}(L-1)\boldsymbol{\Gamma}\boldsymbol{a} + \boldsymbol{n} \tag{7}$$

where $\boldsymbol{\Gamma}$ is the $M \times M$ circular autocorrelation matrix of the accessing signature sequence, i.e. $\Gamma_{ij} = \int_{T_b} s_i(t) s_j(t) dt$, $a_i \in \{0, 1, ..., N\}$ is the number of active users at i^{th} possible delay[1] and $\boldsymbol{n}$ is the zero-mean filtered Gaussian noise with covariance matrix $(L-1)\sigma^2\boldsymbol{\Gamma}$.

Note that for this simple case, the vector $\boldsymbol{a}$ captures all the information about the time of arrival of the active users.

From Equation (7), the maximum likelihood joint detection of the number of users active at each delay value, i.e. the *optimum multiuser access detection problem*, reduces to the following combinatorial problem:

$$\min_{\boldsymbol{a}\in\{0,1,\ldots,N\}^M} -2\boldsymbol{r}^T\boldsymbol{a} + \sqrt{q}(L-1)\boldsymbol{a}^T\boldsymbol{\Gamma}\boldsymbol{a} \tag{8}$$

The problem is of identical structure to the well-known optimum multi-user symbol detection problem which has been shown to be an NP-hard optimization problem [10]. Following the same approach, it is straight forward to see that one needs to evaluate the above cost function at $(N+1)^M$ points to get the maximum likelihood estimate of $\boldsymbol{a}$. Since this could be of prohibitive complexity for large values of M, we instead suggest a sub-optimum multiuser access receiver that is similar to the decorrelator Lupas and Verdú used for bit detection [11]. The decision statistics then are:

$$\boldsymbol{y} = \frac{1}{(L-1)}\boldsymbol{\Gamma}^{-1}\boldsymbol{r} = \sqrt{q}\boldsymbol{a} + \tilde{\boldsymbol{n}} \tag{9}$$

where the covariance matrix of $\tilde{\boldsymbol{n}}$ is $\frac{\sigma^2}{(L-1)}\boldsymbol{\Gamma}^{-1}$. The value of a_i is then declared in the maximum likelihood sense using y_i.

Note that the existence of the detector depends only on the existence of $\boldsymbol{\Gamma}^{-1}$ and not on the number of potential users. Thus the existence can be guaranteed by using an accessing signature sequence whose possible circular shifts are linearly independent.

It is also important to observe that the detector can be precomputed since the form does not depend on the number of active users. The complexity of constructing the detector is the complexity of inverting a matrix which in general is $O(M^3)$. However, it is easy to show that the circular autocorrelation matrix $\boldsymbol{\Gamma}$ is Toeplitz and the inverse operation can be carried out more efficiently using this observation.

5 Continuous Timing Delays

In the construction of the decorrelating multiuser access receiver we assumed that the delay space of users is discrete which is not a realistic assumption. On the other hand, it was this assumption that led us to the construction of a simple linear detector. Also, it is not realistic to assume that the received power values of the users are known and equal. In this section, we relax these assumptions and investigate how the decorrelating multiuser access detector can be used to detect the accessing users in this case.

[1] We assume at most N users can be active at one delay value and $N_A \leq MN$

We begin with an observation that will be of importance for the sequel. Proof of the observation is omitted due to space limitations. Let us define $s_j^\alpha(t)$ as the circularly shifted version of the basic access signature sequence by $(j+\alpha)\ \Delta$ where $\alpha \in [0,1]$ is the delay mismatch expressed in fraction of one resolution interval of the multiuser access detector (Δ).

Observation 1 *The signal $s_j^\alpha(t)$ filtered with $s_k(t)$ results in*

$$\rho_{jk}(\alpha) = \int_{T_b} s_j^\alpha(t)s_k(t)dt = (1-\alpha)\ \Gamma_{jk} + \alpha\ \Gamma_{(j+1)k} \tag{10}$$

In words, the observation above means that at the output of the matched filters described by Equation (6), the effective signature sequence $s_j^\alpha(t)$ poses as a convex combination of effective signature sequences $s_j(t)$ and $s_{j+1}(t)$ weighted by $1-\alpha$ and α respectively.

Let us further define I_i, $i = 1, \cdots, M-1$, as the interval $[(i-1)\Delta, i\Delta]$. Using this definition we can restate Equation (1) as:

$$r(t) = \sum_{i=1}^{M-1}\sum_{\tau_k\in I_i}\sqrt{q_k}s_a(t-\tau_k) + n(t) \tag{11}$$

Associated with each active user k, there is a fractional mismatch α_k $(0 \leq \alpha_k < 1)$ such that

$$\alpha_k = \frac{\tau_k}{\Delta} - i_k, \qquad i_k = \lfloor\frac{\tau_k}{\Delta}\rfloor \tag{12}$$

Recall that the received signal is observed starting at time $t = T_b$ and the contribution of user k within each bit of the observed signal is the access signature sequence circularly shifted by τ_k. So, the received signal in the observation window $t \in [T_b, L\ T_b]$ is

$$r(t) = \sum_{i=1}^{M-1}\sum_{\tau_k\in I_i}\sqrt{q_k}s_i^{\alpha_k}(t) + n(t) \tag{13}$$

The output of the j^{th} filter out of M circularly shifted filters can be expressed as:

$$r_j = \sum_{i=1}^{M-1}\sum_{\tau_k\in I_i}\sqrt{q_k}\rho_{ij}(\alpha_k) + n_j \qquad j = 1, \cdots, M \tag{14}$$

Now, using Equation (10), we can express the output vector of the filter bank as

$$\boldsymbol{r} = (L-1)\boldsymbol{\Gamma}\boldsymbol{\beta} + \boldsymbol{n} \tag{15}$$

and the output of the decorrelating multiuser access detector is

$$\boldsymbol{y} = \frac{1}{(L-1)}\boldsymbol{\Gamma}^{-1}\boldsymbol{r} = \boldsymbol{\beta} + \tilde{\boldsymbol{n}} \tag{16}$$

where

$$\beta_i = \begin{cases} \sum_{\tau_k \in I_i} (1-\alpha_k)\sqrt{q_k}, & i = 1 \\ \sum_{\tau_k \in I_{i-1}} \alpha_k \sqrt{q_k} + \sum_{\tau_k \in I_i} (1-\alpha_k)\sqrt{q_k}, & 2 < i < M \\ \sum_{\tau_k \in I_{M-1}} \alpha_k \sqrt{q_k}, & i = M \end{cases} \tag{17}$$

As can be seen from Equation (17), users whose delays τ_k belong to $I_{(i-1)}$ or I_i, i.e. in $[(i-2)\Delta, i\Delta]$ contribute to y_i with their associated weights that depend on their delay mismatch parameters. Users that are outside $[(i-2)\Delta, i\Delta]$, on the other hand, are eliminated by the decorrelating detector.

Since β_i contains contributions from an unknown number of users each of which has an unknown delay offset, it is difficult to solve for the exact number of active users. Instead, we can simply try to detect if there are *any* active users around the i^{th} delay value by designing a simple threshold test. If we ignore the correlations between filter outputs, we can design a Neyman-Pearson type binary hypothesis test for y_i with the hypotheses:

- H_{i1} : Activity detected at the i^{th} filter
- H_{i0} : No activity at the i^{th} filter

The threshold is set for each test so that a fixed false alarm probability performance (α_F) is achieved. Specifically, for y_i we set the threshold Λ_i such that

$$\alpha_F = \int_{\Lambda_i}^{\infty} \frac{1}{\sqrt{2\pi\Gamma_{ii}^{-1}\sigma^2}} e^{-x^2/(2\Gamma_{ii}^{-1}\sigma^2)} dx \tag{18}$$

The threshold Λ_i can be expressed as:

$$\Lambda_i = Q^{-1}(\alpha_F)\sqrt{\Gamma_{ii}^{-1}\sigma^2} \tag{19}$$

where $Q(x)$ is the standard normal complementary CDF. The test is:

$$y_i \underset{H_{0i}}{\overset{H_{1i}}{\gtrless}} \Lambda_i \tag{20}$$

So, for activity to be detected at the output of the i^{th} decorrelator, the output should exceed a factor that depends on the specified false alarm rate times the *enhanced* noise magnitude at that decorrelator. Once activity is detected at the output of the i^{th} decorrelator, the second stage detector uses this information to try and decode the identification information of a user around the corresponding delay value.

6 Results

We have simulated a DS/CDMA system with processing gain $G = 31$. The accessing signature sequence is chosen to be an M-sequence, i.e. when $\Delta = T_c$, the correlation between all circularly shifted effective signature sequences are $\Gamma_{i,j} = -1/31, \quad i \neq j$. The access arrivals to the system are generated from a Poisson random variable with mean λ and users which cannot access the system are immediately cleared, i.e. we did not include any retry rule. The received power of each user is assumed to be a log-normal random variable with standard deviation $4dB$. The background noise power σ^2 is unity ($0dB$). Each active user has a propagation delay that is uniform between 0 and 5 chips. We have constructed first stage receivers with 1 and 1/2 chip resolution intervals, i.e. $\Delta = T_c$ and $\Delta = T_c/2$. For $\Delta = T_c$, the first stage has 6 filters matched to the basic signature sequence circularly shifted by the 6 possible delay values $(0, T_c, ..., 5T_c)$ followed by the decorrelator. For $\Delta = T_c/2$, the first stage has 11 filters matching the possible delay values $(0, T_c/2, T_c, ..., 5T_c)$ followed by the decorrelator. We have used $L = 20$ bits of preamble for the first stage. In all experiments, we have set the thresholds for the decorrelator outputs of the first stage such that the false alarm rate of each test is $\alpha_F = 0.01$. The second stage receivers constructed for our experiments are one-shot asynchronous decorrelators and asynchronous matched filters.

We have performed two sets of experiments. First, we simulated the two-stage access receiver assuming a perfect tracking algorithm is inserted before the second stage. That is if an activity detection is made at a delay value by the first stage, a "genie" (the perfect tracking algorithm) would tell the second stage the exact delay values of all active users that are around that delay value. Thus, the second stage asynchronous detectors are designed with this perfect timing knowledge. The performance of this system is a useful upper bound for any two-stage multiuser access detector with a tracking algorithm that may be implemented before decoding the identification information of the active users (the second stage). Second, we have considered a system where no tracking algorithm is employed. That is, if activity is detected at a delay value, the second stage asynchronous detectors would use that delay value to design the receivers (one-shot decorrelators or matched filters) to detect the identification information of the possible active users.

Figure 5 shows the comparison between the system with the perfect tracking algorithm and the system with no tracking algorithm when the average received power of each user is $15dB$. As expected, when active users' delays are estimated perfectly, the one-shot decorrelator outperforms the matched filter in average number of users that can gain access to the system, i.e. *the multiuser access capacity.* In the absence of a tracking algorithm, the second stage receivers suffer from the mismatches between the discrete delay values the first stage supplies and the actual delay values of the active users. We observe that the mismatched one-shot decorrelators do take a great performance hit consistent with the reported sensitivity of the decorrelator to timing errors [12]. In fact, if the users' average received powers are reduced to 1/3 ($10dB$), we see that having a matched filter may yield to slightly better multiuser access capacity (Figure 6).

Figure 7 shows the performance gain to be had by halving the resolution interval in the first stage to $\Delta = T_c/2$.

In this case, the range of timing mismatches are reduced by half and the one-shot decorrelator performance is hurt less. As expected, the performance of the one-shot decorrelator gets better than that of the matched filter as the system load gets heavier, i.e. as the interference level to each user's transmission gets higher.

7 Concluding Remarks

This paper examines a packet switching CDMA system where users access the system through a common signature sequence and need to establish connections to send information. Access detection is achieved by a two-stage receiver. The first stage is a simple multiuser access receiver that uses the preambles to detect the presence of the active users. The receiver constructed to serve this purpose is a linear decorrelator and can eliminate interference due to other accessing users whose propagation delays are sufficiently separated from that of the desired user. The receiver uses a set of decision statistics to detect activity around a set of delay values and supplies its findings to the second stage receiver which will decode the identification information of the active users.

Our experiments showed that the absence of a fine tracking algorithm in the second stage limits the multiuser access capacity of the system. This is due to the sensitivity of the bit detectors used in the second stage to the difference between the exact delays of the active users and the coarse delay estimates supplied by the first stage, i.e. the timing mismatches. In this case, there is not much capacity gain to be had by constructing an asynchronous decorrelator over a matched filter for the second stage.

From Figures 5 and 6, we observe that the no tracking assumption results in a packet CDMA system with very low capacity. That is, with a processing gain of 31, the system would support at most two simultaneous active users. In addition to reducing the first stage delay resolution interval Δ, there may be a number of ways to increase capacity. Two possibilities include developing a frame structure with more than one access period before data transmission, and using the vector $\boldsymbol{y}$ in Equation 16 to estimate the fractional timing offsets α_k. Nevertheless, our observations lead us to believe that the access process can be capacity limiting for a packet switching CDMA system. This is in contrast to earlier literature where capacity of random access CDMA is characterized ignoring the effects of user acquisition.

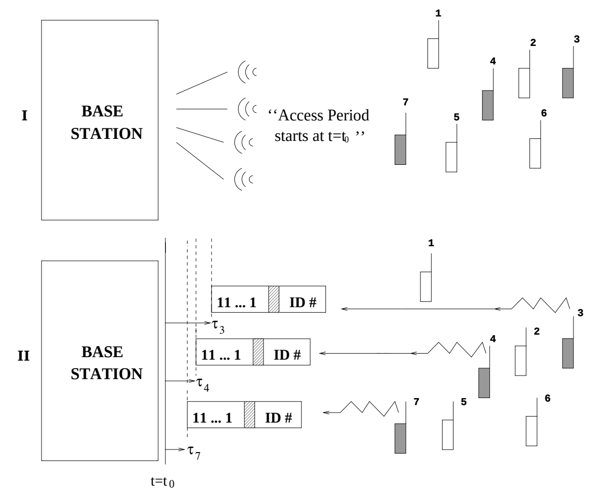

Figure 1: Multiaccess Model for the packet switched CDMA system

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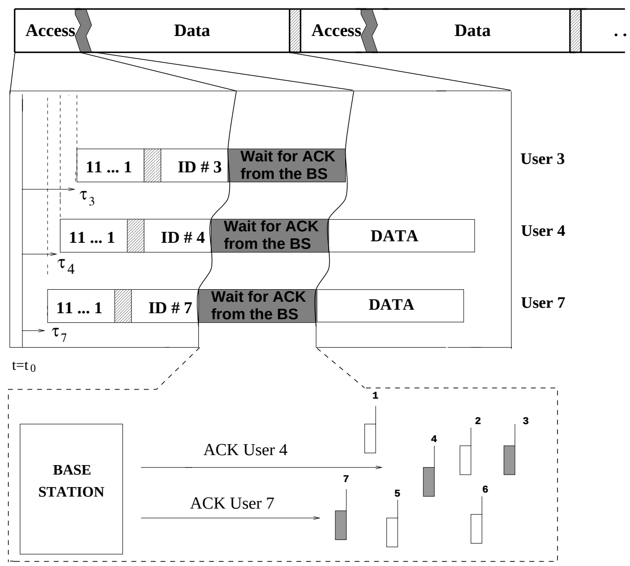

Figure 2: Communication Model for the packet switched CDMA system

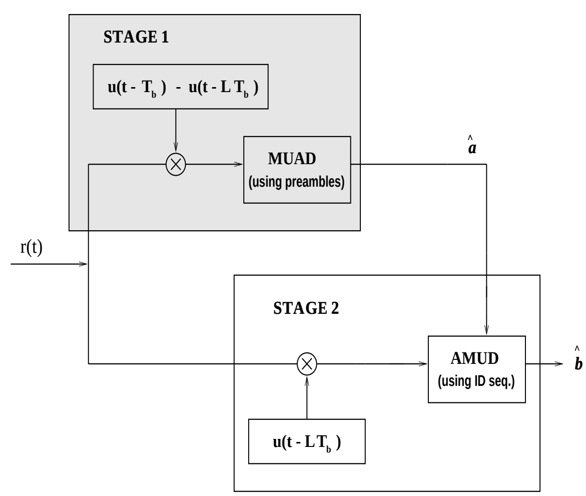

Figure 3: Two-stage detector for the packet switched CDMA system. MUAD: Multiuser Access Detector, AMUD: Asynchronous Multiuser Detector, $u(.)$ is the step function $(u(t) = 1, \quad t > 0)$

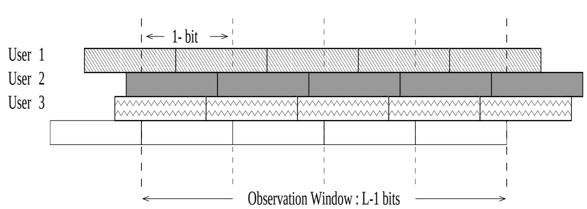

Figure 4: Observation of the access preamble by the multiuser access receiver

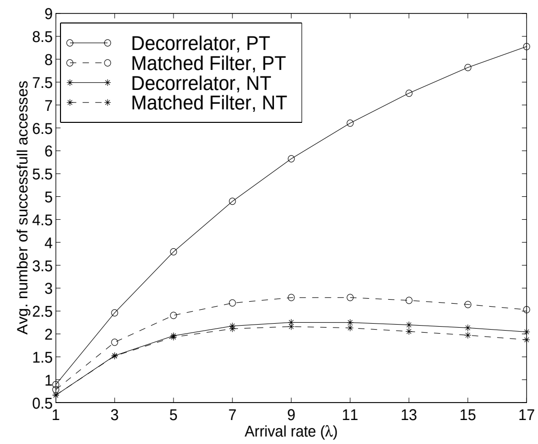

Figure 5: Perfect Tracking (PT) and No Tracking (NT), average received power for each user $= 15dB$

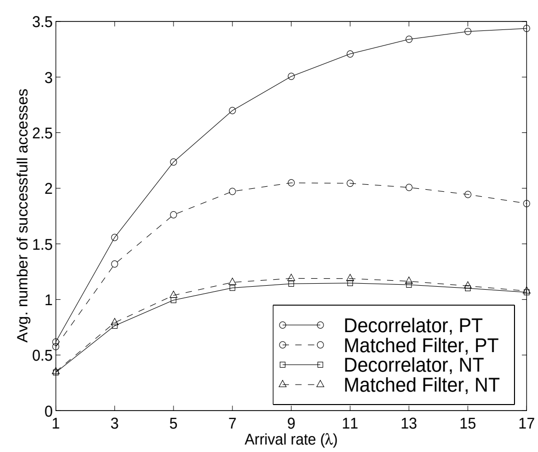

Figure 6: Perfect Tracking (PT) and No Tracking (NT), average received power for each user $= 10dB$

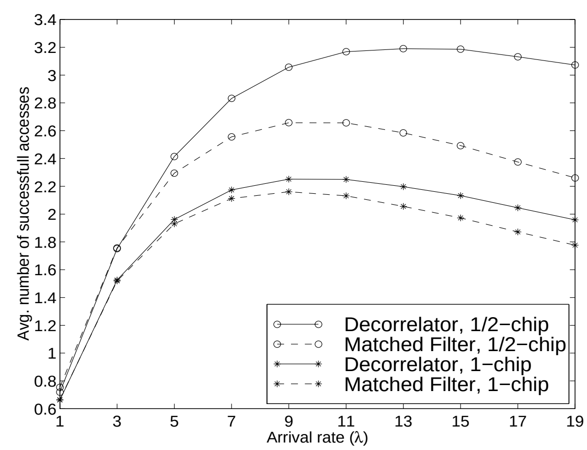

Figure 7: 1-chip resolution versus 1/2-chip resolution, average received power for each user $= 15dB$