Adaptive Power Control with MMSE Multiuser Detectors*

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Abstract: Power control algorithms assume that the receiver structure is fixed and iteratively update the transmit powers of the users to provide them with an acceptable quality of service while minimizing the total transmitter power. Multiuser detection on the other hand optimizes the receiver structure with the assumption that the users have fixed transmitter powers. In this study, we combine the two approaches and propose an iterative and distributed power control algorithm which iteratively updates the transmitter powers and receiver filter coefficients of the users. We show that the algorithm converges to a minimum power solution for the powers, and an MMSE multiuser detector for the filter coefficients.

1 Introduction

In wireless communication systems, iterative power control is used to provide each user with an acceptable level of communication. In CDMA systems, conventional receivers consist of filters that are matched to the signature sequences of the users. The quality of service is typically defined in terms of the signal to interference ratio (SIR) of the individual users which is only a function of the powers of the users. Distributed power control algorithms [1–5] update the transmitter power levels of the users iteratively so that the power vector converges to a minimum power solution.

Multiuser detection is used to demodulate the signals of the users effectively in a multiple access environment. It was shown in [6] that the optimal multiuser detector has a computational complexity which increases exponentially with the number of active users. Therefore, several suboptimum detectors have been proposed to achieve a performance as close as possible to that of the optimum detector while keeping the complexity low. Examples include the decorrelating detector [7], the decision feedback detector [8], the minimum mean squared error (MMSE) detector [9], multistage detectors [10]. Minimum mean squared error (MMSE) detection [9] is based on the minimization of the expected squared error between the transmitted signal and output of the receiver filter and has the advantage that it can be implemented adaptively. The blind adaptive multiuser detector introduced in [11] converges to the MMSE detector without knowledge of the powers and signature sequences of the interfering users.

Power control theory assumes a fixed receiver structure and optimizes the communication between the base stations and the mobiles by controlling the transmitter powers of the users. Multiuser detection theory, on the other hand, assumes that the transmitter powers of the users are fixed and concentrates on optimizing the receiver structure. Our aim in this study is to combine these two approaches and optimize the communication between the mobiles and the base stations by controlling both the transmitter powers and the receivers of the active users.

In this work, we integrate power control and receiver optimization by adapting the filter coefficients to suppress the interference. The implementation of this approach will require interference measurements at each receiver. We show that the resulting power control algorithm converges to a fixed point power vector where all the users satisfy their SIR-based quality of service requirements and that the linear receiver converges to the MMSE multiuser detector. This fixed point power vector $\bar{\boldsymbol{p}}$ satisfies $\bar{\boldsymbol{p}} \leq \boldsymbol{p}'$ for any power vector $\boldsymbol{p}'$ for which there are filter coefficients that yield acceptable SIR for all users. In [12] a power control algorithm is proposed for a CDMA system with adaptive MMSE receivers. The algorithm in [12] and the algorithm in this paper will converge to the same minimum power solution; however, the algorithm of [12] uses measurements of the minimum mean squared error which requires the knowledge of the information bit transmitted by the users and assumes that an adaptive MMSE receiver will adjust to changes in the transmitter powers.

2 Problem Definition

We consider the uplink of a wireless cellular system with a fixed base station assignment of N users to M base stations. We assume a synchronous CDMA scheme and BPSK modulation in order to simplify the analysis of our algorithm.

We define the chip waveform to be $\psi(t)$, $t \in [0, T_c]$ and 0 elsewhere, where T_c is the chip duration. Thus $\{\psi(t-iT_c), i = 0, \ldots, G-1\}$, where G is the processing gain, is a basis for the signal space. This allows us to represent both the signature

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sequences and the linear receiver filters of the users with G dimensional vectors. We will use $\boldsymbol{s}_i$ and $\boldsymbol{c}_i$ to denote the preassigned unique signature sequence and the linear receiver filter of user i, respectively. In terms of signal vectors, the received signal at the assigned base station of user i can be written as

$$\boldsymbol{r}_i = \sum_{j=1}^{N} \sqrt{p_j}\sqrt{h_{ij}} b_j \boldsymbol{s}_j + \boldsymbol{N} \tag{1}$$

where p_j and b_j are the information bit ($+1$ or -1 with equal probability) and the transmitter power of user j, respectively, h_{ij} is the channel gain of user j to the assigned base station of user i and $\boldsymbol{N}$ is a Gaussian random vector with $E\left[\boldsymbol{N}\boldsymbol{N}^\top\right] = \sigma^2 \boldsymbol{I}$. The receiver filter output of user i at its assigned base station is

$$y_i = \sum_{j=1}^{N} \sqrt{p_j}\sqrt{h_{ij}} (\boldsymbol{c}_i^\top \boldsymbol{s}_j) b_j + n_i \tag{2}$$

where $n_i = \boldsymbol{c}_i^\top \boldsymbol{N}$ is a Gaussian random variable with zero mean and variance $\sigma^2 \boldsymbol{c}_i^\top \boldsymbol{c}_i$. The signal to interference ratio (SIR) of user i can be written as

$$\mathrm{SIR}_i = \frac{p_i h_{ii} (\boldsymbol{c}_i^\top \boldsymbol{s}_i)^2}{\sum_{j\neq i} p_j h_{ij} (\boldsymbol{c}_i^\top \boldsymbol{s}_j)^2 + \sigma^2 (\boldsymbol{c}_i^\top \boldsymbol{c}_i)} \tag{3}$$

Our aim is to find optimal powers, p_i, and filter coefficients, $\boldsymbol{c}_i$ for $i = 1, \ldots, N$, such that the total transmitter power is minimized while each user i satisfies a quality of service requirement $\mathrm{SIR}_i \geq \gamma_i^*$, where γ_i^*, called the *target* SIR, is the minimum acceptable level of SIR for user i. Therefore, we can state the problem mathematically as

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} p_i \\ \text{s.t.} \quad & p_i \geq \frac{\gamma_i^*}{h_{ii}} \frac{\sum_{j\neq i} p_j h_{ij} (\boldsymbol{c}_i^\top \boldsymbol{s}_j)^2 + \sigma^2 \boldsymbol{c}_i^\top \boldsymbol{c}_i}{(\boldsymbol{c}_i^\top \boldsymbol{s}_i)^2} \\ & p_i \geq 0, \quad \boldsymbol{c}_i \in \mathbf{R}^G \qquad i = 1, \ldots, N \end{aligned} \tag{4}$$

The above problem statement is equivalent to the following one, where an inner optimization is inserted in the constraint set.

$$\begin{aligned} \min_{\boldsymbol{p}} \quad & \sum_{i=1}^{N} p_i \\ \text{s.t.} \quad & p_i \geq \frac{\gamma_i^*}{h_{ii}} \min_{\boldsymbol{c}_i \in \mathbf{R}^G} \frac{\sum_{j\neq i} p_j h_{ij} (\boldsymbol{c}_i^\top \boldsymbol{s}_j)^2 + \sigma^2 (\boldsymbol{c}_i^\top \boldsymbol{c}_i)}{(\boldsymbol{c}_i^\top \boldsymbol{s}_i)^2} && (5) \\ & p_i \geq 0 \qquad i = 1, \ldots, N && (6) \end{aligned}$$

In (6) the outer optimization is defined over the power vector only, whereas the inner optimization problem assumes a fixed power vector and is defined over the filter coefficients of the individual users. Before describing the power control algorithm in the next section, we concentrate on the inner optimization problem for the filter coefficients when the power vector is fixed. Using (3) we observe that the inner optimization problem of (6) can be written as

$$\min_{\boldsymbol{c}_i} p_i h_{ii} \frac{1}{\mathrm{SIR}_i} \tag{7}$$

Note that (7) is equivalent to $\max_{\boldsymbol{c}_i} \mathrm{SIR}_i$ since the power vector therefore p_i is assumed fixed. It was shown in [9] that the MMSE solution for the filter coefficients $\boldsymbol{c}_i$ for a fixed power vector maximizes the SIR. Therefore, the solution of inner optimization problem given in (7) is [9]

$$\boldsymbol{c}_i^* = \sqrt{p_i}(1 + p_i \boldsymbol{s}_i^\top \boldsymbol{A}_i^{-1} \boldsymbol{s}_i)^{-1} \boldsymbol{A}_i^{-1} \boldsymbol{s}_i \tag{8}$$

where $G \times G$ matrix $\boldsymbol{A}_i$ which is a function of the powers of all the users, except the power of user i, is given as

$$\boldsymbol{A}_i = \sum_{j\neq i} p_j h_{ij} \boldsymbol{s}_j \boldsymbol{s}_j^\top + \sigma^2 \boldsymbol{I} \tag{9}$$

3 Power Control Algorithm

When we view (5) as a set of interference constraints on the power vector $\boldsymbol{p}$, we can define a power control algorithm in which each user i iteratively attempts to compensate for the interference. We define

$$\begin{aligned} I_i(\boldsymbol{p}, \boldsymbol{c}_i) &= \frac{\gamma_i^*}{h_{ii}} \frac{\sum_{j\neq i} p_j h_{ij} (\boldsymbol{c}_i^\top \boldsymbol{s}_j)^2 + \sigma^2 (\boldsymbol{c}_i^\top \boldsymbol{c}_i)}{(\boldsymbol{c}_i^\top \boldsymbol{s}_i)^2} && (10) \\ T_i(\boldsymbol{p}) &= \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}, \boldsymbol{c}_i) && (11) \end{aligned}$$

and we propose the power control algorithm

$$\boldsymbol{p}(n+1) = \boldsymbol{T}(\boldsymbol{p}(n)) \tag{12}$$

where

$$\boldsymbol{T}(\boldsymbol{p}) = [T_1(\boldsymbol{p}), \cdots, T_N(\boldsymbol{p})]^\top \tag{13}$$

Each power control iteration (12) includes an optimization of the filter coefficients to maximally suppress the interference. In effect, we choose the filter coefficients to minimize the required transmitter power. This is analogous to integrated power control and base station algorithms [13,14] in which a user's base station assignment is iteratively chosen to minimize the transmitter power. In [5] power control algorithms of the form

$$\boldsymbol{p}(n+1) = \boldsymbol{I}(\boldsymbol{p}(n)) \tag{14}$$

are analyzed for standard interference functions $\boldsymbol{I}(\boldsymbol{p})$. The definition of standard interference functions and the theorem describing the convergence of (14) follow.

Definition 1 $\boldsymbol{I}(\boldsymbol{p})$ *is a standard interference function if for all* $\boldsymbol{p} \geq \boldsymbol{0}$ *the following properties are satisfied.*

- *Positivity:* $\boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{0}$
- *Monotonicity: If* $\boldsymbol{p} \geq \boldsymbol{p}'$ *then* $\boldsymbol{I}(\boldsymbol{p}) \geq \boldsymbol{I}(\boldsymbol{p}')$
- *Scalability: For all* $\alpha > 1$, $\alpha \boldsymbol{I}(\boldsymbol{p}) > \boldsymbol{I}(\alpha \boldsymbol{p})$

Theorem 1 *If $\boldsymbol{I}(\boldsymbol{p})$ is a standard interference function, then given that there exists $\mathbf{p}' \geq \boldsymbol{I}(\mathbf{p}')$, for any initial power vector $\boldsymbol{p}(0)$, the sequence $\boldsymbol{p}(n) = \boldsymbol{I}(\boldsymbol{p}(n-1))$ converges to a unique fixed point $\bar{\boldsymbol{p}}$ such that $\bar{\boldsymbol{p}} \leq \mathbf{p}'$ for any $\mathbf{p}' \geq \boldsymbol{I}(\mathbf{p}')$.*

The condition that there exists $\boldsymbol{p}' \geq \boldsymbol{I}(\boldsymbol{p}')$ is simply a requirement that a feasible power vector exists. The fixed point $\bar{\boldsymbol{p}}$ is a minimum power solution in that $\bar{\boldsymbol{p}} \leq \boldsymbol{p}'$ for any feasible power vector $\boldsymbol{p}'$. Thus, we prove the convergence of the power control algorithm (12) by proving that the transformation $\boldsymbol{T}(\boldsymbol{p})$ is standard.

Theorem 2 *$\boldsymbol{T}(\boldsymbol{p})$ is a standard interference function.*

Proof: Theorem 2 From (10), for any fixed $\boldsymbol{c}_i$ we have $I_i(\boldsymbol{p}, \boldsymbol{c}_i) > 0$. Therefore, $T_i(\boldsymbol{p}) = \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}, \boldsymbol{c}_i) > 0$ and $\boldsymbol{T}(\boldsymbol{p})$ is positive. To prove monotonicity, we note for any fixed $\boldsymbol{c}_i$ that $\boldsymbol{p} \geq \boldsymbol{p}'$ implies $I_i(\boldsymbol{p}, \boldsymbol{c}_i) \geq I_i(\boldsymbol{p}', \boldsymbol{c}_i)$. If the minimum of $I_i(\boldsymbol{p}, \boldsymbol{c}_i)$ is achieved at $\boldsymbol{c}_i^*$, then,

$$\begin{aligned} T_i(\boldsymbol{p}) &= \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}, \boldsymbol{c}_i) && (15) \\ &= I_i(\boldsymbol{p}, \boldsymbol{c}_i^*) && (16) \\ &\geq I_i(\boldsymbol{p}', \boldsymbol{c}_i^*) && (17) \\ &\geq \min_{\boldsymbol{c}_i} I_i(\boldsymbol{p}', \boldsymbol{c}_i) = T_i(\boldsymbol{p}') && (18) \end{aligned}$$

For scalability, we note that for any fixed $\boldsymbol{c}_i$ and $\alpha > 1$ we have $\alpha I_i(\boldsymbol{p}, \boldsymbol{c}_i) > I_i(\alpha \boldsymbol{p}, \boldsymbol{c}_i)$. Assuming again that the minimum of $I_i(\boldsymbol{p}, \boldsymbol{c}_i)$ is achieved at $\boldsymbol{c}_i^*$, we have

$$\begin{aligned} \alpha T_i(\boldsymbol{p}) &= \min_{\boldsymbol{c}_i} \alpha I_i(\boldsymbol{p}, \boldsymbol{c}_i) && (19) \\ &= \alpha I_i(\boldsymbol{p}, \boldsymbol{c}_i^*) && (20) \\ &> I_i(\alpha \boldsymbol{p}, \boldsymbol{c}_i^*) && (21) \\ &\geq \min_{\boldsymbol{c}_i} I_i(\alpha \boldsymbol{p}, \boldsymbol{c}_i) = T_i(\alpha \boldsymbol{p}) && (22) \end{aligned}$$

□

Since $\boldsymbol{T}(\boldsymbol{p})$ is a standard interference function, the power control algorithm (12) converges to $\bar{\boldsymbol{p}} = \boldsymbol{T}(\bar{\boldsymbol{p}})$. The filter coefficients converge to $\bar{\boldsymbol{c}}_i = \arg\min_{\boldsymbol{c}_i} I_i(\bar{\boldsymbol{p}}, \boldsymbol{c}_i)$. Equivalently, the power control algorithm converges to a minimum power solution for the SIR target based power control problem with linear receiver filters; and the linear receiver filter converges to the MMSE multiuser detector.

4 Implementation of the Power Control Algorithm

The power control algorithm (12) is implicitly a two stage algorithm. We will denote the matrix $\boldsymbol{A}_i$ as $\boldsymbol{A}_i(\boldsymbol{p}(n))$ below in order to emphasize its dependency on the power vector. This matrix is calculated by using (9) when $\boldsymbol{p}(n)$ is given. At iteration $n+1$, the MMSE filter $\hat{\boldsymbol{c}}_i$ is constructed by using the current power vector $\boldsymbol{p}(n)$ and then the power vector is updated using the new filter coefficients $\hat{\boldsymbol{c}}_i$. The resulting iterative algorithm for user i is

$$\begin{aligned} \hat{\boldsymbol{c}}_i &= \frac{\sqrt{p_i}}{\left(1 + p_i \boldsymbol{s}_i^\top \boldsymbol{A}_i^{-1}(\boldsymbol{p}(n)) \boldsymbol{s}_i\right)} \boldsymbol{A}_i^{-1}(\boldsymbol{p}(n)) \boldsymbol{s}_i && (23) \\ p_i(n+1) &= \frac{\gamma_i^*}{h_{ii}} \left(\frac{\sum_{j \neq i} p_j(n) h_{ij} (\hat{\boldsymbol{c}}_i^\top \boldsymbol{s}_j)^2 + \sigma^2 \hat{\boldsymbol{c}}_i^\top \hat{\boldsymbol{c}}_i}{(\hat{\boldsymbol{c}}_i^\top \boldsymbol{s}_i)^2} \right) && (24) \end{aligned}$$

It is important to note that the algorithm presented in (23) and (24) is distributed and the quantities needed can be easily estimated from available observables. Although from (23) and (9), it appears that all transmitter powers p_j and channel gains h_{ij} are needed to obtain $\boldsymbol{A}_i$ and hence $\hat{\boldsymbol{c}}_i$, fortunately, this is not the case. In particular, we can estimate $\boldsymbol{A}_i$ by sampling the received signal before the receiver filters and taking empirical averages. From (1), the mutual independence of the zero mean transmitted bits $\{b_n\}$ and the Gaussian noise $\boldsymbol{N}$ implies

$$E\left[\boldsymbol{r}_i \boldsymbol{r}_i^\top\right] = \boldsymbol{A}_i + p_i h_{ii} \boldsymbol{s}_i \boldsymbol{s}_i^\top \qquad (25)$$

Therefore, $\boldsymbol{r}_i \boldsymbol{r}_i^\top - p_i h_{ii} \boldsymbol{s}_i \boldsymbol{s}_i^\top$ is an unbiased estimate for $\boldsymbol{A}_i$. If at the assigned base station of user i, the uplink gain h_{ii} and transmitter power p_i are known, $\boldsymbol{A}_i$ can be estimated by a sample average of $\boldsymbol{r}_i \boldsymbol{r}_i^\top$ over multiple bit intervals. For the adjusted filter coefficients $\hat{\boldsymbol{c}}_i$, equation (2) implies that the average squared filter output for user i under power vector $\boldsymbol{p}(n)$ is

$$E\big[y_i^2(n)\big] = \sum_{j=1}^{N} p_j(n) h_{ij} (\hat{\boldsymbol{c}}_i^\top \boldsymbol{s}_j)^2 + \sigma^2 \hat{\boldsymbol{c}}_i^\top \hat{\boldsymbol{c}}_i \qquad (26)$$

Thus, $y_i^2(n) - p_i(n) h_{ii} (\hat{\boldsymbol{c}}_i^\top \boldsymbol{s}_i)^2$ is an unbiased estimate for the interference, i.e., for the numerator of the term in the parenthesis on the right hand side of (24). A simple measurement based power control algorithm can use a sample average of $y_i^2(n)$ over multiple bit intervals to estimate the interference. We note that the simple estimation methods still require a user to estimate its own uplink gain h_{ii}. This can be done, perhaps roughly, using the downlink transmission of a base station pilot tone.

5 Simulation Results

In our simulations, we consider a single cell of a wireless cellular CDMA system and ignore the intercell interference. We assume that the mobiles are uniformly distributed on a disk of radius R_0 (chosen as $R_0 = 1000$ meters) around the base station. The propagation constant is taken to be $\alpha = 4$ in the simulations. At the beginning of the iterations, the power vector is initialized to zero, and the filter coefficients are initialized to the signature sequences of the users (i.e., $p_i(0) = 0$ and $\boldsymbol{c}_i(0) = \boldsymbol{s}_i$, for all i).

We compared the performance of the conventional power control algorithm which assumes a conventional detector structure composed of the filters matched to the signature sequences of the users, and the power control algorithm proposed in this paper which optimizes the filter coefficients in addition to updating the powers. Since the filter coefficients are always chosen to be the MMSE detector, we call the proposed algorithm the MMSE power control. We chose the processing gain to be $G = 150$ and a random signature of length G chips was assigned to each user. Although the convergence theorems permit individual SIR targets γ_i^* for each user i, for the simulations we chose a common SIR target $\gamma_i^* = 4$ ($\approx$ 6 dB) for all users. The AWGN noise power equaled $\sigma^2 = 10^{-13}$, corresponding roughly to a 1 MHz bandwidth. The number of users was varied from 10 to 60 by increments of 10.

For $N = 10, 20$ and 30 users, Figure 1 shows in log scale the total transmitter power $\sum_{i=1}^{N} p_i$, as a function of the iteration index, for the MMSE and conventional power control algorithms. The same graphs are plotted in Figure 2 for $N = 40, 50$ and 60 users. We observe that the MMSE power control outperforms the conventional power control in terms of total received power, and convergence rate. Using MMSE power control, the total transmitter power is much less (two orders of magnitude in our experiments) than that needed for the conventional detector. Also, the MMSE power control algorithm converges to the optimal power vector faster than the conventional power control algorithm.

In Figure 2, the steadily increasing transmitter power curves for conventional power control with $N \geq 40$ occur because the conventional power control problem is infeasible. In these instances updating the receiver filter coefficients converted the infeasible conventional power control problems into feasible problems.

In order to observe the convergence of the SIRs to the common target SIR, we plotted the SIRs of all of the users in Figures 3 and 4 for the conventional power control algorithm and the MMSE power control algorithm, respectively, for $N = 20$ users. Figures 5 and 6 show the same graphs produced for $N = 60$ users.

We observe from Figures 3 and 4 that when the MMSE power control is used, the SIRs converge to the common target SIR faster than with the conventional power control. We again observe the infeasibility of the target SIR from Figure 5, by noting that the SIRs of the users converge to the values which are less than the target value ($\gamma_i^* = 4$). We note that the SIRs converge to the maximum achievable common SIR target with fixed system parameters such as channel gains, cross correlations between the signature sequences; see [2,15].

6 Conclusion

We proposed an iterative and distributed power control algorithm which updates the power levels and linear receiver filters of the individual users. We showed that the proposed algorithm converges to a minimum power solution where all the users satisfy their SIR-based quality of service requirements; and that the linear receiver filter converges to an MMSE multiuser detector.

We observed that the MMSE power control is superior in terms of the total transmitter power and convergence rate when compared with the conventional power control algo-

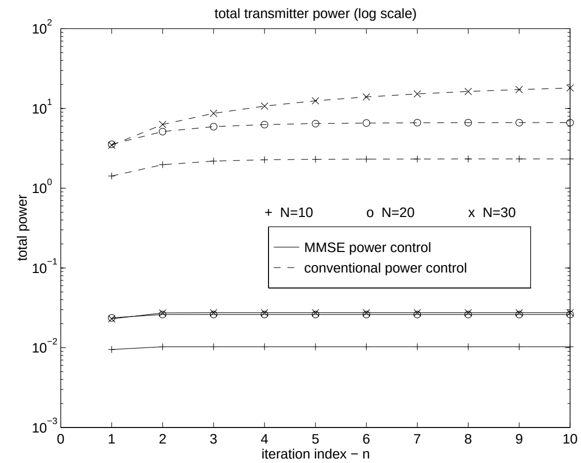

Figure 1: Total transmitter power for the conventional and the MMSE power control algorithms for $N = 10, 20$ and 30.

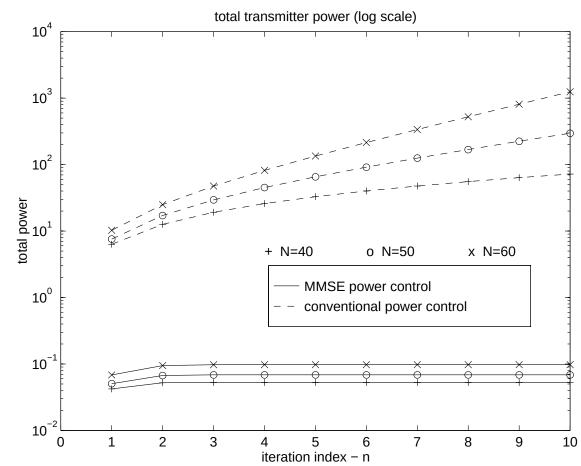

Figure 2: Total transmitter power for the conventional and the MMSE power control algorithms for $N = 40, 50$ and 60. (Infeasible with conventional receivers.)

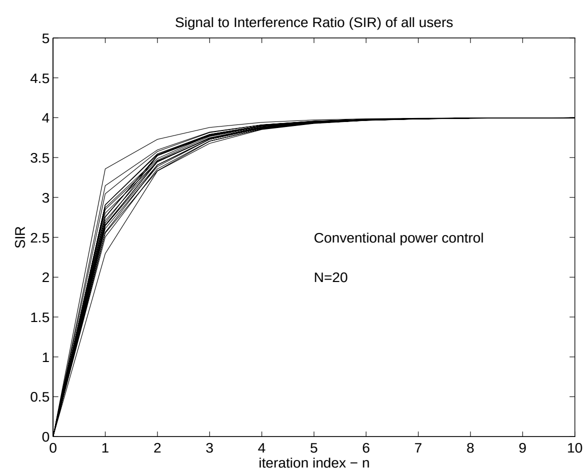

Figure 3: SIRs of all the users versus n for the conventional power control algorithm. Common SIR target value $\gamma^* = 4$ ($\approx$ 6 dB), number of users $N = 20$.

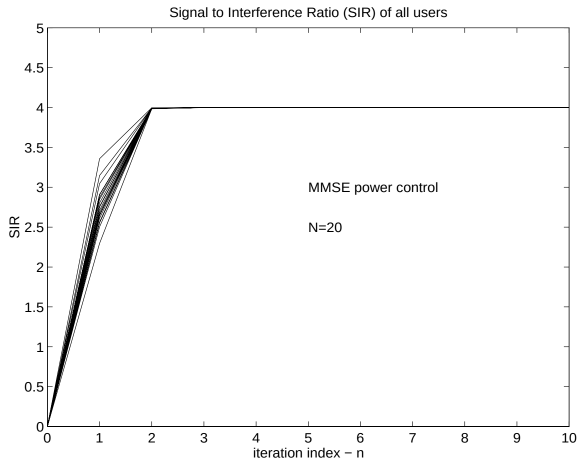

Figure 4: SIRs of all the users versus n for the MMSE power control algorithm. Common SIR target value $\gamma^* = 4$ (≈ 6 dB), number of users $N = 20$.

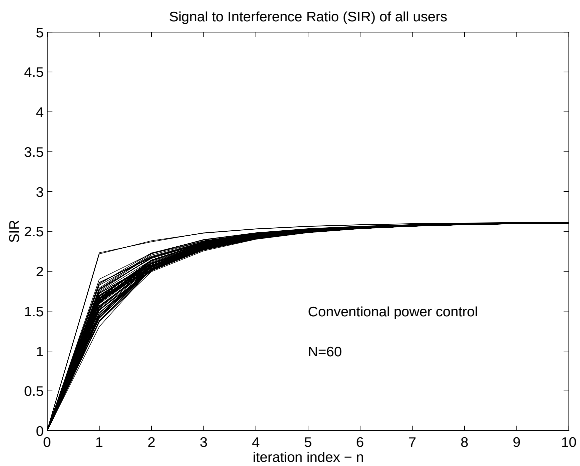

Figure 5: SIRs of all the users versus n for the conventional power control algorithm. Common SIR target value $\gamma^* = 4$ (≈ 6 dB), number of users $N = 60$.

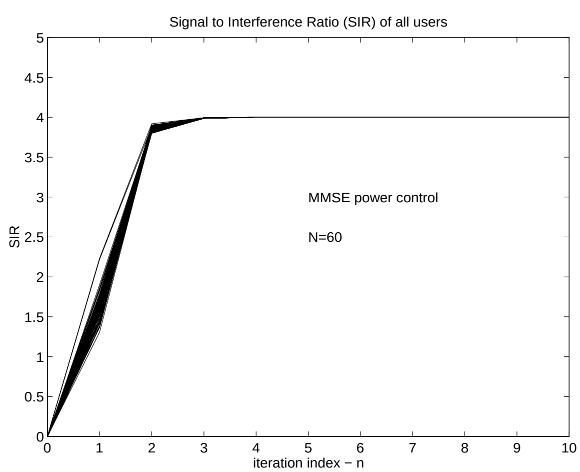

Figure 6: SIRs of all the users versus n for the MMSE power control algorithm. Common SIR target value $\gamma^* = 4$ (≈ 6 dB), number of users $N = 60$.

rithm. With MMSE power control, the same system performance is achieved with less total transmitter power increasing the capacity of the CDMA system when compared with the conventional power control. Since MMSE power control can convert a power control problem that is infeasible with conventional power control into a feasible one, it increases the system capacity by allowing the SIR targets of the users to be higher, or by increasing the number of users supportable at a fixed SIR level.

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