A Blind Adaptive Decorrelating Detector for CDMA Systems*

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Abstract: The decorrelating detector is known to eliminate the multi access interference given that the signature sequences of the users are linearly independent, at the cost of enhancing the Gaussian receiver noise. In this paper, we present and study the convergence of a blind adaptive decorrelating detector which is based on the observation of the available statistics. The algorithm is iterative and distributed. Only two parameters are needed to be known for the construction of the receiver filter of a user: the user's signature sequence and the variance of the white Gaussian receiver noise. The computational complexity of the proposed algorithm per iteration is linear in the number of users.

1 Introduction

Code Division Multiple Access (CDMA) systems suffer from the *near-far effect*, because of non-orthogonality of the users' signature sequences. Multiuser detection [1] can be used to overcome the near-far problem. Multiuser detectors exploit the special structure of the multiple access interference to effectively demodulate the non-orthogonal signals of the users. It was shown in [2] that the optimal multiuser detector has a computational complexity which increases exponentially with the number of active users. Several suboptimum detectors, including the decorrelating detector [3], have been proposed to achieve a performance comparable to that of optimum detector while keeping the complexity low.

The decorrelating detector [3] which has linear (in number of users) computational complexity was shown to eliminate the multi access interference totally if the signature sequences of the users are linearly independent at the cost of enhancing the additive white Gaussian noise (AWGN). The decorrelating detector of [3] is centralized and non-iterative. The construction of the decorrelating detector filter for a certain user requires the knowledge of the signature sequences of all interfering users as well as the signature sequence of the user of interest. In addition to that, the construction requires inversion of the $N \times N$ cross correlation matrix, where N is the number of active users. Blind adaptive algorithms are desirable to overcome the need for the knowledge about the parameters of the interfering users and iterative algorithms are needed to avoid the matrix inversion which may have a large computational complexity. A blind adaptive algorithm based on the minimization of the output energy was given in [4]. This algorithm was shown to converge to the MMSE multiuser detector [5].

In [6] an adaptive multiuser detector which converges to the decorrelating detector is proposed. This detector still needs the signature sequences of all the users and transmission of training sequences. In [7] blind algorithms based on signal subspace tracking are investigated and two algorithms which converge to the decorrelating and MMSE multiuser detectors are proposed. The blind adaptive decorrelating detector proposed in [7] needs the information about the variance of the AWGN and the number of users both of which can be estimated again using subspace tracking techniques. The computational complexity of the algorithm of [7] is $O(GN)$ per iteration, where G is the processing gain.

In this paper we present a blind adaptive multiuser detector which uses observables that are readily available at the receiver and which converges to a decorrelating detector. The detector is constructed via a distributed iterative algorithm which updates the receiver filter coefficients of a desired user by using the previous output of the filter under construction. Since the filter output is random due to the randomness of the multi access interference and existence of ambient Gaussian channel noise, the algorithm evolves stochastically. We prove the convergence of the filter coefficients to a decorrelating detector in the mean squared error (MSE) sense. We develop lower and upper bounds for the MSE of the filter coefficients at iteration $n+1$ in terms of the MSE of the filter coefficients at iteration n, and investigate the conditions under which the MSE sequence converges to zero as number of iterations grows to infinity.

The computational complexity of the proposed algorithm is $O(G)$ per iteration. The proposed algorithm requires the knowledge of only two parameters for the construction of the filter coefficients for a desired user: the desired user's signature sequence and the variance of the AWGN. The variance of the AWGN is a fixed quantity (not time varying) and can be estimated easily, perhaps before the information transfer starts when only the samples of AWGN can be observed without any interference at the output of an arbitrary (non-zero) receiver filter. In such a case, a reliable estimate of the variance of AWGN can be obtained by time-averaging the squares of the output of the receiver filter.

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2 The Decorrelating Detector

Throughout this paper, a synchronous CDMA system with BPSK modulation is assumed to simplify the analysis. We will use G dimensional vector $\boldsymbol{s}_i$ to denote the pre-assigned unique signature sequence of user i. Let us define an $N \times G$ dimensional matrix $\boldsymbol{S}$ with its (i, j)th element being $(\boldsymbol{s}_i)_j$, the jth component of $\boldsymbol{s}_i$. Therefore, the rows of $\boldsymbol{S}$ (equivalently the columns of $\boldsymbol{S}^\top$) are the signature sequences of the users. For future use, we define a subspace $\mathcal{L}$ in G dimensional vector space to be the subspace spanned by the signature sequences of the users, i.e.,

$$\mathcal{L} = \text{span}\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_N\} = \text{column space of } \boldsymbol{S}^\top \tag{1}$$

We consider the decorrelating detector for the first user without loss of generality and represent its G dimensional receiver filter by $\boldsymbol{c}$. Then $\boldsymbol{c}$ should satisfy the following condition.

$$\boldsymbol{S}\boldsymbol{c} = \alpha \boldsymbol{e}_1 \tag{2}$$

where $\boldsymbol{e}_1$ is the first unit vector in N dimensional space, i.e., $\boldsymbol{e}_1 = [1\ 0\ 0 \ldots\ 0]^\top$, and α is a non-negative real number. It can be easily shown that the bit error rate performance of the decorrelating detector is insensitive to scaling of the filter coefficients as long as the filter eliminates the multi access interference totally. The reason for this is that the scalar factor multiplies both the received power level of the desired user and the AWGN.

We first note that (2) has more than one solution, because it has N equalities and G unknowns, and typically $G \gg N$. The unique decorrelating detector for the first user, $\tilde{\boldsymbol{c}}$, is given [3] as

$$\tilde{\boldsymbol{c}} = \boldsymbol{S}^\top(\boldsymbol{S}\boldsymbol{S}^\top)^{-1}\boldsymbol{e}_1 \tag{3}$$

Let us denote any solution of (2) as $\bar{\boldsymbol{c}}$. Then inserting (2) into (3) we obtain (assuming $\alpha = 1$)

$$\tilde{\boldsymbol{c}} = \boldsymbol{S}^\top(\boldsymbol{S}\boldsymbol{S}^\top)^{-1}\boldsymbol{S}\bar{\boldsymbol{c}} = \boldsymbol{P}\bar{\boldsymbol{c}} \tag{4}$$

Note that $\boldsymbol{P} = \boldsymbol{S}^\top(\boldsymbol{S}\boldsymbol{S}^\top)^{-1}\boldsymbol{S}$ is the projection matrix which projects any vector onto the column space of $\boldsymbol{S}^\top$. Also note that the column space of $\boldsymbol{S}^\top$ is the subspace spanned by $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_N$, the subspace which was denoted as $\mathcal{L}$. Therefore, although (2) has more than one solution, all of the solutions have the same projection onto $\mathcal{L}$; and this projection is equal to the unique decorrelating detector of Lupas and Verdú [3].

Let $\boldsymbol{\Lambda}$ be an $N \times N$ dimensional diagonal matrix with received power of user i, p_i, as its ith diagonal element. Multiplying both sides of (2) first with $\boldsymbol{\Lambda}$, then with $\boldsymbol{S}^\top$ yields

$$\boldsymbol{S}^\top\boldsymbol{\Lambda}\boldsymbol{S}\boldsymbol{c} = \alpha p_1 \boldsymbol{s}_1 \tag{5}$$

We observe that although (5) has G equations in G unknowns it does not have a unique solution for $\boldsymbol{c}$, since $(G - N)$ eigenvalues of $\boldsymbol{S}^\top\boldsymbol{\Lambda}\boldsymbol{S}$ are equal to zero . The solution spaces of (2) and (5) are related as stated in the following Remark.

Remark 1 *If $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_N$ are linearly independent, then all solutions of (2) and (5) coincide.*

Let us define $\boldsymbol{A} = \boldsymbol{S}^\top\boldsymbol{\Lambda}\boldsymbol{S}$. At this point we choose $\alpha = 1/p_1$ and devise the following deterministic gradient descent algorithm:

$$\boldsymbol{c}(n+1) = \boldsymbol{c}(n) - \mu\left(\boldsymbol{A}\boldsymbol{c}(n) - \boldsymbol{s}_1\right) \tag{6}$$

Note that the iterative algorithm given in (6) converges to the solution of (5) and equivalently, by Remark 1, to the solution of (2). The fact that the solution of (2) is not unique is noted earlier. Note that, for any $\boldsymbol{c}$,

$$\boldsymbol{P}\boldsymbol{A}\boldsymbol{c} = \boldsymbol{S}^\top(\boldsymbol{S}\boldsymbol{S}^\top)^{-1}\boldsymbol{S}\boldsymbol{S}^\top\boldsymbol{\Lambda}\boldsymbol{S}\boldsymbol{c} = \boldsymbol{S}^\top\boldsymbol{\Lambda}\boldsymbol{S}\boldsymbol{c} = \boldsymbol{A}\boldsymbol{c} \tag{7}$$

This means that for any $\boldsymbol{c}(n)$, $\boldsymbol{A}\boldsymbol{c}(n)$ lies entirely in $\mathcal{L}$. Using this result and (6) we obtain

$$\boldsymbol{c}(n+1) = \boldsymbol{P}\boldsymbol{c}(n+1) + \boldsymbol{c}(n) - \boldsymbol{P}\boldsymbol{c}(n) \tag{8}$$

By induction (8) yields

$$\boldsymbol{c}(n) = \boldsymbol{P}\boldsymbol{c}(n) + \boldsymbol{c}(0) - \boldsymbol{P}\boldsymbol{c}(0) \tag{9}$$

Therefore, if the iterative algorithm (6) is started in the subspace $\mathcal{L}$, implying $\boldsymbol{c}(0) = \boldsymbol{P}\boldsymbol{c}(0)$, then from (9) we will have: $\boldsymbol{c}(n) = \boldsymbol{P}\boldsymbol{c}(n)$ for all n. In this case $\boldsymbol{c}(n)$ will always stay in $\mathcal{L}$ and will converge to the scaled version of the unique decorrelating detector solution of [3] $\boldsymbol{c}^* = \frac{1}{p_1}\tilde{\boldsymbol{c}}$ as n goes to infinity. Note that the algorithm converges to the scaled version of $\tilde{\boldsymbol{c}}$, instead of converging to $\tilde{\boldsymbol{c}}$ because α is not chosen to be 1.

The restriction that the iterations should be started in $\mathcal{L}$ for algorithm (6) to converge to a decorrelating detector is fairly mild. Selections $\boldsymbol{c}(0) = \boldsymbol{0}$, $\boldsymbol{c}(0) = \boldsymbol{s}_1$ or any linear combination of the signature sequences imply $\boldsymbol{c}(0) \in \mathcal{L}$, satisfying the convergence condition of (6). The signature sequences of all of the users must be known for the algorithm given in (6). Also, the algorithm of (6) is an off-line algorithm which must be run before the real information transmissions of the users start. After running the algorithm for some time, the filter coefficients would converge to a decorrelating detector filter and then the communication can be started. In this paper, our aim is to develop a blind adaptive algorithm which would converge to a decorrelating detector solution in a stochastic sense by using real random measurements while the users are active and transmitting bits. To this end, we will propose an algorithm which can be viewed as the stochastic version of the deterministic algorithm given in (6).

3 A Blind Adaptive Decorrelating Detector (BADD)

The received base band signal at the input of the receiver filters after chip-matched filtering followed by chip rate sampling is

$$\boldsymbol{r} = \sum_{i=1}^{N} \sqrt{p_i}\, b_i \boldsymbol{s}_i + \boldsymbol{n} \tag{10}$$

where b_i is the information bit (± 1 equiprobably) transmitted by user i and $\boldsymbol{n}$ is a Gaussian random vector with zero mean and $E[\boldsymbol{n}\boldsymbol{n}^\top] = \sigma^2 \boldsymbol{I}$. Note that,

$$E\left[\boldsymbol{r}\boldsymbol{r}^\top\right] = \sum_{i=1}^{N} p_i \boldsymbol{s}_i \boldsymbol{s}_i^\top + \sigma^2 \boldsymbol{I} = \boldsymbol{A} + \sigma^2 \boldsymbol{I} \qquad (11)$$

Therefore, using (11), the deterministic iteration of (6) can be written in an exact form as

$$\boldsymbol{c}(n+1) = \boldsymbol{c}(n) - \mu\left[\left(E\left[\boldsymbol{r}\boldsymbol{r}^\top\right] - \sigma^2 \boldsymbol{I}\right)\boldsymbol{c}(n) - \boldsymbol{s}_1\right] \qquad (12)$$

Note that $\boldsymbol{r}\boldsymbol{r}^\top - \sigma^2\boldsymbol{I}$ is an *unbiased* estimate for $\boldsymbol{A}$ matrix. In order to obtain a practical algorithm we replace the exact expression for $\boldsymbol{A}$ in (12) with its unbiased estimator by simply removing the expectation in (12). Thus we obtain,

$$\boldsymbol{c}(n+1) = \boldsymbol{c}(n) - \mu\left[\left(\boldsymbol{r}\boldsymbol{r}^\top - \sigma^2 \boldsymbol{I}\right)\boldsymbol{c}(n) - \boldsymbol{s}_1\right] \qquad (13)$$

Before analyzing the convergence of (13), we state it in terms of available observations, and list the parameters needed at each iteration. Let $y(n)$ be the output of the receiver filter of the desired user at time n. Then, $y(n) = \boldsymbol{r}^\top(n)\boldsymbol{c}(n)$, where $\boldsymbol{r}(n)$ is the sampled received signal before the receiver filters and $\boldsymbol{c}(n)$ is the current filter of the desired user. Thus, the implementation oriented version of the algorithm (13) is

$$\boldsymbol{c}(n+1) = \left(1 - \sigma^2\mu\right)\boldsymbol{c}(n) - \mu\left(y(n)\boldsymbol{r}(n) - \boldsymbol{s}_1\right) \qquad (14)$$

Since $\boldsymbol{r}(n)$ and $y(n)$ are readily available at the input and output of the receiver filter that is under construction, only two system parameters are needed to be known in order to run the algorithm: the signature sequence of the desired user $\boldsymbol{s}_1$, and the variance of the AWGN, σ^2. The variance of AWGN is a fixed quantity which can be easily estimated before the communication starts as discussed in Section 1.

4 Convergence of the BADD

In this Section we will investigate the convergence of the blind adaptive decorrelating detector proposed in the previous section. Let us define the zero mean random vector $\boldsymbol{\eta}(n)$ as

$$\boldsymbol{\eta}(n) = \left(\boldsymbol{r}\boldsymbol{r}^\top - \boldsymbol{A} - \sigma^2\boldsymbol{I}\right)\boldsymbol{c}(n) \qquad (15)$$

Noting that $\boldsymbol{A}\boldsymbol{c}^* = \boldsymbol{s}_1$, we can write the stochastic iterations (13) and (14) as

$$\boldsymbol{c}(n+1) = \boldsymbol{c}(n) - \mu\left[\boldsymbol{A}\left(\boldsymbol{c}(n) - \boldsymbol{c}^*\right) + \boldsymbol{\eta}(n)\right] \qquad (16)$$

Subtracting $\boldsymbol{c}^*$ from both sides of (16) and defining $\boldsymbol{\epsilon}(n) = \boldsymbol{c}(n) - \boldsymbol{c}^*$ yields

$$\boldsymbol{\epsilon}(n+1) = \boldsymbol{\epsilon}(n) - \mu\left(\boldsymbol{A}\boldsymbol{\epsilon}(n) + \boldsymbol{\eta}(n)\right) \qquad (17)$$

Note that norm of $\boldsymbol{\epsilon}(n)$ is a measure of the distance of the current receiver filter from $\boldsymbol{c}^*$, the convergence point. Squaring of both sides of (17) yields

$$\begin{aligned}\|\boldsymbol{\epsilon}(n+1)\|^2 &= \|\boldsymbol{\epsilon}(n)\|^2 - 2\mu\boldsymbol{\epsilon}(n)^\top\boldsymbol{A}\boldsymbol{\epsilon}(n) + 2\mu^2\boldsymbol{\epsilon}(n)^\top\boldsymbol{A}\boldsymbol{\eta}(n)\\ &\quad -2\mu\boldsymbol{\epsilon}(n)^\top\boldsymbol{\eta}(n) + \mu^2\boldsymbol{\epsilon}(n)^\top\boldsymbol{A}^2\boldsymbol{\epsilon}(n) + \mu^2\|\boldsymbol{\eta}(n)\|^2 \qquad (18)\end{aligned}$$

By taking the conditional expectation of both sides, conditioned on $\boldsymbol{\epsilon}(n)=\boldsymbol{\epsilon}$, and observing that $E[\boldsymbol{\eta}(n) \mid \boldsymbol{\epsilon}(n)=\boldsymbol{\epsilon}]=\boldsymbol{0}$, we obtain

$$\begin{aligned}E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}(n)=\boldsymbol{\epsilon}\right] &= \|\boldsymbol{\epsilon}\|^2 - 2\mu\boldsymbol{\epsilon}^\top\boldsymbol{A}\boldsymbol{\epsilon} + \mu^2\boldsymbol{\epsilon}^\top\boldsymbol{A}^2\boldsymbol{\epsilon}\\ &\quad +\mu^2 E\left[\|\boldsymbol{\eta}(n)\|^2 \mid \boldsymbol{\epsilon}(n)=\boldsymbol{\epsilon}\right] \qquad (19)\end{aligned}$$

We will be using the results of the following Lemmas to develop bounds for the right hand side of (19).

Lemma 1 *If* $\boldsymbol{P}\boldsymbol{\epsilon} = \boldsymbol{0}$ *then* $\boldsymbol{\epsilon}^\top\boldsymbol{A}\boldsymbol{\epsilon} = 0$ *and* $\boldsymbol{\epsilon}^\top\boldsymbol{A}^2\boldsymbol{\epsilon} = 0$. *If* $\boldsymbol{P}\boldsymbol{\epsilon} \neq \boldsymbol{0}$ *then there exist* $0 < k_0 \leq k_1 < \infty$, *such that*

$$k_0\|\boldsymbol{\epsilon}\|^2 \leq \quad \boldsymbol{\epsilon}^\top\boldsymbol{A}\boldsymbol{\epsilon} \quad \leq k_1\|\boldsymbol{\epsilon}\|^2 \qquad (20)$$

$$k_0^2\|\boldsymbol{\epsilon}\|^2 \leq \quad \boldsymbol{\epsilon}^\top\boldsymbol{A}^2\boldsymbol{\epsilon} \quad \leq k_1^2\|\boldsymbol{\epsilon}\|^2 \qquad (21)$$

Lemma 2 *There exist* $0 < c_0 \leq c_1 < \infty$, *such that*

$$0 \leq E\left[\|\boldsymbol{\eta}(n)\|^2 \mid \boldsymbol{\epsilon}(n)=\boldsymbol{\epsilon}\right] \leq c_0 + c_1\|\boldsymbol{\epsilon}\|^2 \qquad (22)$$

The outline of the proof of Lemma 1 is as follows. Since $\boldsymbol{A}$ is a positive semidefinite matrix with rank N, $(G-N)$ eigenvalues are equal to zero and remaining N eigenvalues are positive. Since for any $\boldsymbol{x}$, $\boldsymbol{A}\boldsymbol{x} \in \mathcal{L}$ by (7), eigenvectors of $\boldsymbol{A}$ are either completely in $\mathcal{L}$ with positive eigenvalues, or completely out of $\mathcal{L}$ (meaning that their projections onto $\mathcal{L}$ are zero) with zero eigenvalues. In this case Lemma 1 is a simple result of Rayleigh quotient [8]. The proof of Lemma 2 which can be found in [9] is more lengthy and is omitted here.

In what follows, we will denote the conditioning on $\boldsymbol{\epsilon}(n) = \boldsymbol{\epsilon}$, $\boldsymbol{P}\boldsymbol{\epsilon} = \boldsymbol{0}$ and $\boldsymbol{P}\boldsymbol{\epsilon} \neq \boldsymbol{0}$ by $\boldsymbol{\epsilon}$, $\mathcal{P}$ and $\bar{\mathcal{P}}$, respectively. For example $E[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}(n) = \boldsymbol{\epsilon}, \boldsymbol{P}\boldsymbol{\epsilon} = \boldsymbol{0}]$ will be denoted by $E[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \mathcal{P}]$. If $\boldsymbol{P}\boldsymbol{\epsilon}=\boldsymbol{0}$, following lower and upper bounds can be developed for the right hand side of (19) using Lemmas 1 and 2,

$$\begin{aligned}E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \mathcal{P}\right] &\leq \left(1+\mu^2 c_1\right)\|\boldsymbol{\epsilon}\|^2 + c_0\mu^2\\ E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \mathcal{P}\right] &\geq \|\boldsymbol{\epsilon}\|^2 \qquad (23)\end{aligned}$$

And similarly if $\boldsymbol{P}\boldsymbol{\epsilon} \neq \boldsymbol{0}$,

$$\begin{aligned}E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \bar{\mathcal{P}}\right] &\leq \left(1 - 2\mu k_0 + (k_1^2 + c_1)\mu^2\right)\|\boldsymbol{\epsilon}\|^2 + c_0\mu^2\\ E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \bar{\mathcal{P}}\right] &\geq \left(1 - 2\mu k_1 + k_0^2\mu^2\right)\|\boldsymbol{\epsilon}\|^2 \qquad (24)\end{aligned}$$

An important observation toward the convergence proof is that the projection of $\boldsymbol{c}(n)$ (equivalently the projection of $\boldsymbol{\epsilon}(n)$) on $\mathcal{L}$ would be non-zero *almost surely (a.s.)* [10]. This means that for any n the probability of the event $[\boldsymbol{P}\boldsymbol{\epsilon}(n) = \boldsymbol{0}]$ is zero. Similar to the deterministic result in (9), it can be shown using induction on (17) that (see also [11, Eqn. (16)]),

$$\boldsymbol{\epsilon}(n) = \boldsymbol{P}\boldsymbol{\epsilon}(n) + \boldsymbol{\epsilon}(0) - \boldsymbol{P}\boldsymbol{\epsilon}(0) \qquad \text{a.s.} \qquad (25)$$

Thus, if $\boldsymbol{\epsilon}(0) \in \mathcal{L}$ then $\boldsymbol{\epsilon}(0) = \boldsymbol{P}\boldsymbol{\epsilon}(0)$ and (25) implies $\boldsymbol{\epsilon}(n) = \boldsymbol{P}\boldsymbol{\epsilon}(n)$ a.s. An upper bound for $E[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}]$ can be developed as

$$
\begin{aligned}
E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}\right] &= E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \mathcal{P}\right] \text{Prob}\{\boldsymbol{P\epsilon} = \mathbf{0}\} \\
&\quad + E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \bar{\mathcal{P}}\right] \text{Prob}\{\boldsymbol{P\epsilon} \neq \mathbf{0}\} \\
&= E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}, \bar{\mathcal{P}}\right] \\
&\leq \left(1 - 2\mu k_0 + (k_1^2 + c_1)\mu^2\right) \|\boldsymbol{\epsilon}\|^2 + c_0\mu^2 \qquad (26)
\end{aligned}
$$

where we used $\text{Prob}\{\boldsymbol{P\epsilon} = \mathbf{0}\} = 0$ and $\text{Prob}\{\boldsymbol{P\epsilon} \neq \mathbf{0}\} = 1$. By similar arguments a lower bound can be found to be

$$E\left[\|\boldsymbol{\epsilon}(n+1)\|^2 \mid \boldsymbol{\epsilon}\right] \geq \left(1 - 2\mu k_1 + k_0^2\mu^2\right) \|\boldsymbol{\epsilon}\|^2 \qquad (27)$$

Taking the expectation of both sides of the inequalities in (26) and (27) with respect to $\boldsymbol{\epsilon}(n)$ and letting $b_n = E[\|\boldsymbol{\epsilon}(n)\|^2]$, we obtain the following bounds for b_n, the mean squared error (MSE) of the filter coefficients at iteration n from $\boldsymbol{c}^*$,

$$
\begin{aligned}
b_{n+1} &\geq \left(1 - 2\mu k_1 + k_0^2\mu^2\right) b_n \qquad (28) \\
b_{n+1} &\leq \left(1 - 2\mu k_0 + (k_1^2 + c_1)\mu^2\right) b_n + c_0\mu^2 \qquad (29)
\end{aligned}
$$

Defining, $\alpha_0 = 1 - 2\mu k_0 + (k_1^2 + c_1)\mu^2$ and $\alpha_1 = 1 - 2\mu k_1 + k_0^2\mu^2$, we can rewrite Equations (28) and (29) as

$$\alpha_1 b_n \leq b_{n+1} \leq \alpha_0 b_n + c_0\mu^2 \qquad (30)$$

We observe from (30) that the nonnegative sequence b_n is sandwiched between the two sequences generated according to $b'_{n+1} = \alpha_0 b'_n + c_0\mu^2$ and $b''_{n+1} = \alpha_1 b''_n$. These two sequences converge to finite numbers if and only if μ is chosen such that $\mid \alpha_0 \mid < 1$ and $\mid \alpha_1 \mid < 1$. Note that both α_0 and α_1 are equal to 1 at $\mu = 0$. We also note that both α_0 and α_1 are locally decreasing as μ increases, since

$$\left.\frac{d\alpha_0}{d\mu}\right|_{\mu=0} = -2k_0 < 0 \quad \text{and} \quad \left.\frac{d\alpha_1}{d\mu}\right|_{\mu=0} = -2k_1 < 0 \qquad (31)$$

This means that we can always choose μ small enough so that $|\alpha_0| < 1$ and $|\alpha_1| < 1$, in which case the sequences b'_n and b''_n converge and the limiting MSE, i.e., $\lim_{n\to\infty} b_n$, has finite lower and upper bounds. From the sandwich theorem,

$$0 \leq \lim_{n\to\infty} b_n \leq \frac{c_0\mu^2}{1 - \alpha_0} \qquad (32)$$

The upper bound can be evaluated as $\mu \to 0$ as,

$$\lim_{\mu\to 0} \frac{c_0\mu^2}{1 - \alpha_0} = \lim_{\mu\to 0} \frac{c_0\mu}{2k_0 - (k_1^2 + c_1)\mu} = 0 \qquad (33)$$

Therefore, if the step size (μ) is chosen extremely small then the MSE of the filter coefficients from the decorrelating filter coefficients goes to zero as the number of iterations grows to infinity. But note that as $\mu \to 0$, the numbers α_0 and α_1 go to 1 in which case the convergence rate goes to zero. Thus, we observe the trade off between the limiting value of the MSE and the convergence rate. If a large value is chosen as the step size then the convergence rate is faster but the limiting MSE is larger; and if a small value is chosen as the step size the limiting MSE is smaller but the convergence rate is slower too. Hence, a time dependent step size sequence which takes large values at the beginning and smaller values at the end may be preferable. An iteration index (n) dependent step size sequence can be used to accomplish this. Replacing the fixed step size μ in (14) with the time varying step size sequence a_n we obtain the new algorithm to be:

$$\boldsymbol{c}(n+1) = \left(1 - \sigma^2 a_n\right) \boldsymbol{c}(n) - a_n \left(y(n)\boldsymbol{r}(n) - \boldsymbol{s}_1\right) \qquad (34)$$

The convergence of (34) is stated in the following Lemma. The arguments of the proof which can be found in [9] follow those made in [12].

Lemma 3 *If a_n satisfies*

$$\sum_n^\infty a_n = \infty \qquad \text{and} \qquad \sum_n^\infty a_n^2 < \infty$$

then the stochastic iteration given in (34) converges to $\boldsymbol{c}^*$ in the MSE sense, i.e., $\lim_{n\to 0} E[\|\boldsymbol{c}(n) - \boldsymbol{c}^*\|^2] = 0$.

Note that $a_n = a/(n + n_0)$, for any $a > 0$ and $n_0 > 0$ satisfy the conditions of Lemma 3.

5 Simulation Results

In the simulations we consider a system with $N = 6$ users using randomly generated signature sequences with a processing gain of $G = 31$. Powers of the users are assigned so that the final signal to interference ratio (SIR) of all users will be 20 dB. Thus, the power of the ith user is found by $p_i = 100\sigma^2[\boldsymbol{\Gamma}^{-1}]_{ii}$ where $\boldsymbol{\Gamma}$ is the cross correlation matrix which is equal to $\boldsymbol{SS}^\top$. The algorithm is run for 100 times and averaged results are plotted in the figures. In Figure 1, we plot the averaged normalized squared error (NSE) of the filter coefficients of the desired (first) user versus iteration index n, where NSE at iteration n is defined as $\text{NSE}(n) = \|\boldsymbol{c}(n) - \boldsymbol{c}^*\|^2 / \|\boldsymbol{c}^*\|^2$. Various curves in Figure 1 correspond to the blind adaptive decorrelating detector algorithms with fixed step size for different step size values and that with a time dependent step size sequence of the form of $a_n = 1/(n + n_0)$ with $n_0 = 5$. We observe that if the step size μ is too large, then the algorithm does not converge; see increasing NSE curve for $\mu = 0.1$ in Figure 1. We also observe that for large step size values the convergence rate is fast but the limiting NSE is large too; compare the curves corresponding to $\mu = 0.01$ and $\mu = 0.001$ in Figure 1.

Averaged (over 100 runs) SIR of the desired (first) user is plotted in Figure 2 for the same system. At iteration n, the SIR of the desired user is calculated as

$$\text{SIR}(n) = \frac{p_1(\boldsymbol{s}_1^\top \boldsymbol{c}(n))^2}{\sum_{j\neq 1} p_j(\boldsymbol{s}_j^\top \boldsymbol{c}(n))^2 + \sigma^2(\boldsymbol{c}^\top(n)\boldsymbol{c}(n))} \qquad (35)$$

We observe that the convergence properties of the SIRs to the *target* SIR (which is 20 dB in this experiment) is similar to the properties of the convergence of the NSE to zero.

The blind adaptive decorrelating detector of [7] is also implemented for the same system and the SIR of the desired user is plotted as a function of iteration index in Figure 3 for different values of the forgetting factor.

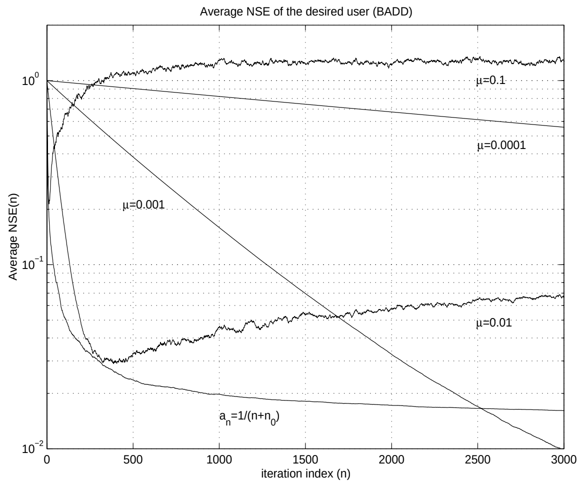

Figure 1: Averaged normalized squared error (NSE) of the desired user.

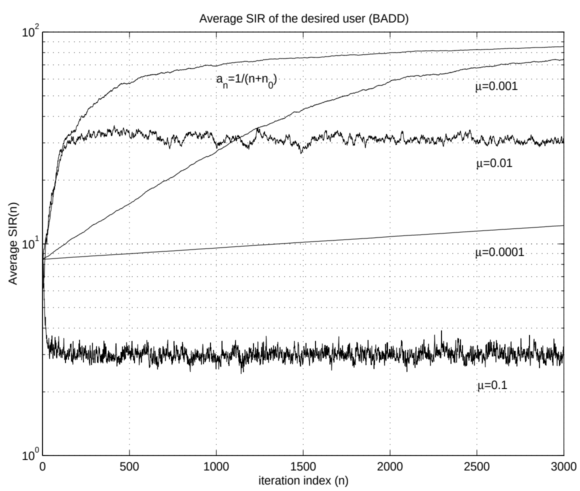

Figure 2: Averaged SIR of the desired user.

6 Conclusion

In this paper we presented an iterative and distributed adaptive decorrelating detector algorithm which is based on the observation of the available statistics, and studied its convergence. The update equation of the algorithm needs the observation of the chip sampled input signal before the receiver filter and the output of the filter with the current filter coefficients. For the implementation of the algorithm to construct the decorrelating receiver filter of a user only two parameters are needed to be known: the user's signature sequence and the variance of the additive white Gaussian receiver noise.

We studied the convergence of the proposed algorithm both for a fixed step size sequence and for a time varying step size

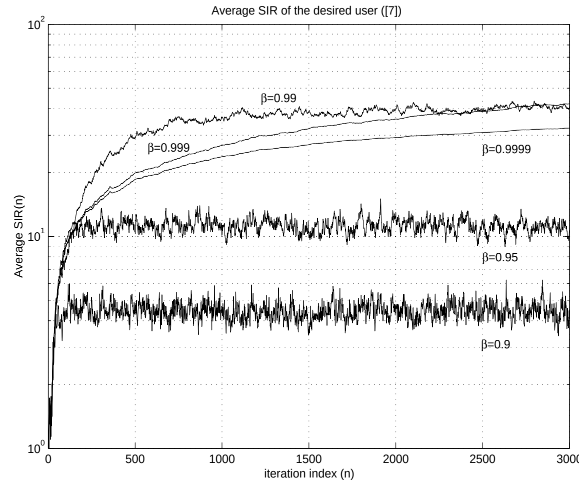

Figure 3: Averaged SIR of the desired user (algorithm of [7]).

sequence. For the first case we developed the conditions of having lower and upper bounds on the MSE and showed that as the step size goes to zero the algorithm converges in the MSE. For the second case we directly proved the convergence in the MSE.

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