Power Control for an Asynchronous Multi-rate Decorrelator

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Abstract

The varied Quality of Service (QoS) or bit error rate (BER) requirements of multimedia traffic require the use of power control for CDMA wireless systems employing multiuser detection. Using a decorrelator in an asynchronous multi-rate DS/CDMA system, it may be necessary for different users to combat the noise enhancement and the propagation losses to varying degrees depending on individual requirements. In this context, we propose a power control algorithm for a multi-rate decorrelator that is suitable for a class of BER based link quality objectives. If the uplink channel gain of the desired user is known, then it is simple for each user to choose the transmitter power needed to meet its target BER objective. In practice, however, the uplink channel gain is often difficult to measure. To avoid this measurement, we employ stochastic approximation methods to develop a simple, distributed, iterative power control algorithm. In this algorithm, each mobile will use the output of its own decorrelator to update its transmitted power in order to achieve its QoS objective. We will show that when a user's bits have nonzero asymptotic efficiencies, then the power control algorithm converges quickly in the mean square sense to an optimal power at which the desired user achieves its QoS objective.

1 Introduction

Extensive efforts are being made to integrate code division multiple access (CDMA) cellular networks with fixed networks for communicating both voice and data messages. Receiver design for an asynchronous multi-rate CDMA system has become a new area of investigation [1–4]. Reference [2] proposed a decorrelator called the *asynchronous multi-rate decorrelator* or *AMD* for an asynchronous multi-rate CDMA system. In the asynchronous multi-rate system, variable processing gains were used to implement distinct bit rates. The AMD is decentralized and decodes users by employing sliding finite length observation windows. It was proven in [2] that the performance of any finite window decorrelator improves monotonically with the observation window length, although a longer observation window results in greater computational complexity. The AMD operates by employing a set of modified correlators whose signals are simply time shifted versions of one another and can decode a user by suppressing the multiuser interference as long as the desired user has non-zero asymptotic efficiency [5]. As with all decorrelators, the multiuser interference is suppressed at the cost of reduced received signal energy. As a consequence of supporting multiple rates in an asynchronous system, different bits of an individual user are subject to varying degrees of energy degradation and, as a consequence, may experience different probabilities of error.

Recently, several studies [6, 7] have been performed in order to integrate power control with multiuser detectors. The motivation of these works was to achieve a performance gain over multiuser detection by providing power control for multiuser detection. Reference [6] addressed power control for the linear MMSE multiuser receiver in a single-rate system. The proposed schemes are synchronous and require SIR (signal-to-interference ratio) measurements for the power updates.

In this paper, a class of QoS objectives for the AMD are proposed. The choice of a QoS objective will dictate a corresponding power control algorithm. If the uplink channel gain of the mobile is known then it is simple to provide each user its QoS objective. In practice, however, the measurement of the channel gain is non-trivial and neglects the stochastic behavior of the channel. This practical situation leads us to use stochastic approximation methods [8] to develop an asynchronous, distributed, iterative power control algorithm for the AMD that finds the correct powers without knowledge of the uplink gain. In this stochastic power control algorithm, a mobile needs to know only the output of the AMD to update its transmitted power in order to achieve its QoS objective. The proposed power control algorithm is feasible as long as asymptotic efficiencies of the bits of the desired user are nonzero. In this case, the algorithm converges in the mean square sense to an optimal power at which the user achieves its QoS requirement.

2 Asynchronous multi-rate system

In the asynchronous multi-rate DS/CDMA system model of [2], each bit results in the baseband transmission of a sequence of pulses, or chips, $p[t]$, each pulse having a duration of one chip period T_c. These pulses are sent over an additive white Gaussian noise channel in which the noise $N(t)$ has power spectral density σ^2.

For a system with K users, the transmission rate of user j is denoted by $R_j = M_j R$ for an integer $M_j \geq 1$ where R denotes a system base rate. The bit transmission time of user j is $T_j = 1/R_j$ and the processing gain is $L_j = T_j/T_c$. The jth user has signature waveform $S_j(t) = \sum_{m=1}^{L_j} \{a_j(m) \frac{1}{\sqrt{L_j}} p[t-(m-1)T_c]\}$, where $t \in [0, T_j]$ and $a_j(m) \in \{-1, 1\}$ denotes the signature sequence of user j. The energy of the pulse $p[t]$ is normalized so that each $S_j(t)$ has unit energy over $[0, T_j]$. Let us denote Δ_j as the delay of the jth user. In the asynchronous channel, the received signal due to the jth user is given by

$$r_j(t) = \sum_{i=-\infty}^{+\infty} b_j^{(i)} \sqrt{E_j^{(i)}} S_j(t - iT_j - \Delta_j) \qquad 1 \leq j \leq K \tag{1}$$

where $b_j^{(i)} \in \{-1, +1\}$ is the ith bit and $E_j^{(i)}$ is the received energy of the jth user. Since the transmitted power is proportional to the received energy, we will refer to $E_j^{(i)}$ as power. It is assumed that the receiver knows the time delay Δ_j for each user j and the received signal is

$$r(t) = \sum_{k=1}^{K} r_k(t) + N(t) \tag{2}$$

We now describe how the ith bit $b_j^{(i)}$, $0 \leq i \leq M_j - 1$, of user j is decoded. In order to decode $b_j^{(i)}$, we assume $\Delta_j = 0$ and that the other Δ_k are computed relative to user j. It may be desirable for different bits to be decorrelated using different window sizes. The bit $b_j^{(i)}$ will be decoded by processing the windowed received signal

$$R_j^{(i)}(t) = r(t)\Big[u\left(t - [i - n_j^{(i)}]T_j\right) - u\left(t - [i + 1 + d_j^{(i)}]T_j\right)\Big] \tag{3}$$

where $u(t)$ is the unit step function. With respect to bit $b_j^{(i)}$, the window of $R_j^{(i)}(t)$ extends $n_j^{(i)}$ bits into the past and $d_j^{(i)}$ bits into the future and thus covers $n_j^{(i)} + d_j^{(i)} + 1$ bits of user j.

3 Asynchronous multi-rate decorrelator, AMD

The decorrelator [5] can be implemented in a decentralized fashion using modified correlator signals. In [2], it was mentioned that the modified correlators of a decorrelator can be generated by applying the Gram-Schmidt orthogonalization to the interfering users' signature sequences. Reference [2] focused its attention on the Gram-Schmidt procedure to develop the AMD which we now briefly describe.

Among the M_j bits of user j, we present the AMD decoding technique for bit $b_j^{(i)}$. Let $I_{n_j^{(i)},d_j^{(i)}}$ denote the set of signature waveforms that are generated by the interfering users within the observation window $\left[(i-n_j^{(i)})T_j,(i+1+d_j^{(i)})T_j\right]$. The AMD decodes the bit $b_j^{(i)}$ using a modified correlator $\Phi_0^{(i)}(t)$ such that $\Phi_0^{(i)}(t)$ is orthogonal to each interfering signal in the set $I_{n_j^{(i)},d_j^{(i)}}$. The output of the modified correlator $\Phi_0^{(i)}(t)$ is

$$r_j^{(i)} = \int_{(i-n_j^{(i)})T_j}^{(i+1+d_j^{(i)})T_j} R_j^{(i)}(t)\Phi_0^{(i)}(t)\,dt = \sqrt{\zeta_j^{(i)}E_j^{(i)}}b_j^{(i)} + N_j^{(i)} \tag{4}$$

where $N_j^{(i)}$ is a Gaussian random variable with mean zero and variance σ^2 and

$$\zeta_j^{(i)} = \left\{\int_{(i-n_j^{(i)})T_j}^{(i+1+d_j^{(i)})T_j} \Phi_0^{(i)}(t)S_j(t-iT_j)\,dt\right\}^2 \tag{5}$$

is the asymptotic efficiency of the AMD for decoding the bit $b_j^{(i)}$ in the observation window $\left[(i-n_j^{(i)})T_j,(i+1+d_j^{(i)})T_j\right]$. The corresponding decoding rule will be $\bar{b}_j^{(i)} = \mathrm{sgn}(r_j^{(i)})$ and the probability that the bit $b_j^{(i)}$ is decoded incorrectly is

$$q_j^{(i)} = Q\left(\sqrt{\zeta_j^{(i)}E_j^{(i)}/\sigma^2}\right) \tag{6}$$

where $Q(\cdot)$ is the standard normal complementary CDF.

We have described the decoding technique for the ith bit, $i = 1,\ldots,M_j-1$, of user j. After an interval of M_j bits, user j will observe the same pattern of interfering signature waveforms. Thus, the receiver dedicated to the user j needs to know the specifications of M_j modified correlators. Furthermore, user j will experience M_j different asymptotic efficiencies or BERs.

4 Quality of Service for the AMD

In this section, we will propose a class of QoS objectives for the AMD. Let us denote the uplink channel gain of user j by h_j and its transmitted power in the ith slot by $P_j^{(i)}$. Then the received power at the ith slot becomes $E_j^{(i)} = h_jP_j^{(i)}$. The criterion that is used for power control is to guarantee each user's link quality objective or QoS requirement. We will consider the following link quality objectives.

- *The fixed BER (FB) objective:* Provide each bit of user j with a target probability of error q_j^*. Mathematically, we must choose a set of transmitter powers $P_j^{(i)}$, $i = i, \ldots, M_j - 1$, such that

$$Q\left(\sqrt{\zeta_j^{(i)} h_j P_j^{(i)} / \sigma^2}\right) = q_j^* \qquad i = 0, \ldots, M_j - 1 \tag{7}$$

Note that the FB objective requires that the transmitter power of user j be adjusted from bit to bit. When the target BER q_j^* is low, then small variations in $\zeta_j^{(i)}$ will significantly change the transmitted power. Thus it may not be possible for user j to adjust the transmitter power quickly enough. This situation leads us to propose the following link quality objective.

- *Average BER (AB) objective:* Using a fixed transmitted power, guarantee user j an average bit error target q_j^* over M_j transmitted bits. Here $P_j^{(i)} = P_j$ for each bit $b_j^{(i)}$ and the link quality objective of user j is

$$\frac{1}{M_j} \sum_{i=0}^{M_j - 1} Q\left(\sqrt{\zeta_j^{(i)} h_j P_j / \sigma^2}\right) = q_j^* \tag{8}$$

We observe that the AB objective achieves the same average BER as the FB objective by the use of a fixed transmitter power. The AB objective yields different BERs for different bits. The bit with the lowest asymptotic efficiency will experience the highest BER. In an asynchronous multi-rate channel, several bits of user j may experience significantly lower asymptotic efficiencies than others. In this case, user j may prefer to protect those bits which are suffering from low asymptotic efficiencies. To handle this situation, we propose the following link quality objective.

- *Worst case BER (WB) objective:* Using a fixed transmitted power, provide target BER q_j^* to the bit with the lowest asymptotic efficiency. That is, we choose P_j such that

$$\max_i Q\left(\sqrt{\zeta_j^{(i)} h_j P_j / \sigma^2}\right) = q_j^* \tag{9}$$

Compared to the AB objective, the WB objective will demand higher transmitted power from the mobile. We will see that the same power control algorithm can be used to achieve either the AB or WB objective. For either objective, user j will have a target received power E_j^* that guarantees the corresponding link QoS. Of course, E_j^* will depend on whether user j desires the AB or WB objective. Since $Q(\cdot)$ is a monotone decreasing function, a straightforward offline calculation can determine E_j^* for either objective. If the uplink channel gain h_j is known, then the target transmitted power P_j^* is simply E_j^*/h_j.

In practice, the uplink channel gain is often difficult to measure. The base station may require training sequences from the mobile in order to measure h_j. We develop a stochastic approximation method [8] that uses the AMD outputs to iteratively converge to a target received power E_j^* without the knowledge of the uplink channel gain h_j. Let us refer this stochastic approximation method as the *target power* or *TP* algorithm. Based on the value of E_j^*, the TP algorithm will achieve the AB or WB objective. In the TP algorithm, the base station will use the modified correlator outputs to estimate the average received power and the mobile uses this estimated average received power to

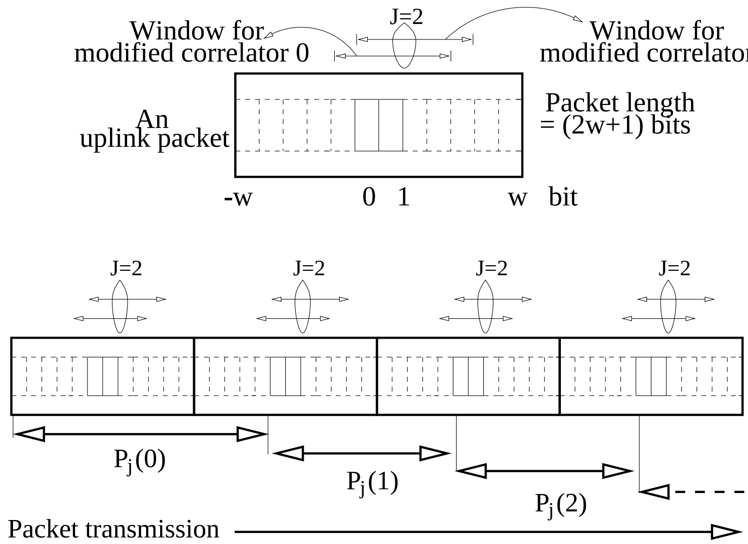

Figure 1: The modified correlators whose outputs are used in different power control iterations operate over different time intervals. Therefore, random noise vectors $\mathbf{N_j}(m)$, $m = 1, 2, \ldots,$ are identically distributed and statistically independent.

update its transmitted power in order to meet the link quality objective. It is easy to verify that the TP algorithm is feasible as long as the asymptotic efficiencies of user j are nonzero.

In the TP algorithm, we assume that both in the uplink (from the mobile to the base) and the downlink (from the base to the mobile), bits are transmitted in packets. It is also assumed that the base station measures J modified correlator outputs and estimates the average received power over J bits. Let us concentrate on the J modified correlators which are used to decode the bits numbered from 0 through $J-1$. From equation (4), the output for the ith bit of user j is $r_j^{(i)}$ then

$$r_j^{(i)} = \sqrt{\zeta_j^{(i)} h_j P_j} b_j^{(i)} + N_j^{(i)} \tag{10}$$

Under the transmitted power P_j, the power of user j, averaged over the outputs of J modified correlators of user j, is

$$\overline{Y}_j(P_j) = \frac{h_j P_j}{J} \sum_{i=0}^{J-1} \zeta_j^{(i)} + \sigma^2 \tag{11}$$

In equation (11), we see that $P_j = P_j^*$ iff $\overline{Y}_j(P_j) = \overline{Y}_j(P_j^*) = Y_j^*$. Therefore, the target power P_j^* will be achieved if the average power at the output of modified correlators equals Y_j^*. In the TP algorithm, the base station will transmit the average of the squared values of the J modified correlator outputs to user j who will use this value to update its transmitted power in order to achieve the target average power, Y_j^*.

Squaring equation (10), we get

$$\left(r_j^{(i)}\right)^2 = \zeta_j^{(i)} h_j P_j + 2\sqrt{\zeta_j^{(i)} h_j P_j} b_j^{(i)} N_j^{(i)} + \left(N_j^{(i)}\right)^2 \tag{12}$$

The noise components at the J modified correlator outputs can be represented by the noise vector $\mathbf{N_j} = \left[N_j^{(0)}, \ldots, N_j^{(J-1)}\right]$. Note that the autocorrelation matrix of the noise vector $\mathbf{N_j}$ will be a function of the cross-correlations of the J modified correlators. We will need to assume that the AMD and the mobile terminal are synchronized such a way that random noise vectors $\mathbf{N_j}(m)$, after $m = 1, 2, \ldots,$ power control iterations, are independent and identically distributed (iid). A set of iid noise vectors $\mathbf{N_j}(m)$, $m = 1, 2, \ldots,$ can be obtained by carefully choosing the J bits used for power control. One such choice of J bits is depicted in Figure 1. The number of outputs, J will depend on the lengths of the uplink and downlink packets, the length of modified correlators and the quantization levels of the modified correlator outputs.

Using equations (12) and $\mathbf{N_j}$, let us denote the sample mean of the average power at the outputs of modified correlators as $Y_j[P_j, \mathbf{N_j}]$ where

$$Y_j[P_j, \mathbf{N_j}] = \frac{1}{J} \sum_{i=0}^{J-1} \left\{ \zeta_j^{(i)} h_j P_j + 2\sqrt{\zeta_j^{(i)} h_j P_j} b_j^{(i)} N_j^{(i)} + \left(N_j^{(i)} \right)^2 \right\} \tag{13}$$

Taking the conditional expectation on both sides of equation (13), we can write the average power at the outputs of modified correlators as

$$E[Y_j[P_j, \mathbf{N_j}] | P_j = \xi] = \frac{h_j \xi}{J} \sum_{i=0}^{J-1} \zeta_j^{(i)} + \sigma^2 = \overline{Y}_j(\xi) \tag{14}$$

When $\xi = P_j^*$, we obtain the target average power, Y_j^* at the outputs of modified correlators. We have mentioned earlier that in the TP algorithm, our goal is to obtain $\overline{Y}_j(\xi) = Y_j^*$ to guarantee user j the target power P_j^*. Note that exact calculation of Y_j^* requires perfect estimation of the noise power or variance, σ^2. Using $Y_j[P_j, \mathbf{N_j}]$ and Y_j^*, the TP algorithm for user j implements the stochastic approximation method

$$P_j(m+1) = P_j(m) - a_m \left(Y_j[P_j(m), \mathbf{N_j}(m)] - Y_j^* \right) \tag{15}$$

Note that a_m may be a constant or a function of the iteration index m. The properties of a_m that are necessary for the convergence of the proposed equation (15) will be discussed shortly. Note that either AB or WB objectives can be achieved by appropriately setting the target average received power Y_j^*. The mobile executes the TP algorithm without knowledge of the asymptotic efficiencies $\zeta_j^{(i)}$.

5 Convergence of the TP algorithm

Let us denote the *mean square error* of the transmitter power at the mth iteration by

$$\Theta_m = E\left[\left[P_j(m) - P_j^* \right]^2 \right] \tag{16}$$

Our goal will be to verify that the TP algorithm converges by showing that

$$\lim_{m \to \infty} \Theta_m = 0 \tag{17}$$

The evolution of the stochastic power control algorithm is dictated by the received power error $X_j(m) = Y_j[P_j(m), \mathbf{N_j}(m)] - Y_j^*$. We will use upper bounds on the first and second conditional moments of $X_j(m)$ given $P_j(m)$ in order to prove convergence. Using $\mu_k(\xi) = E\left[X_j^k(m) | P_j(m) = \xi \right]$ to denote the kth conditional moment, we will need the following lemmas. Due to space constraints, proofs will be found in [9].

Lemma 1 *There exists a positive constant c_0 such that*

$$\mu_1(\xi) = E[X_j(m) | P_j(m) = \xi] = c_0 \left(\xi - P_j^* \right) \tag{18}$$

Lemma 2 *There exists positive constants c_1 and c_2 such that*

$$\mu_2(\xi) = E\left[X_j^2(m)|P_j(m) = \xi\right] \leq c_1 \left(\xi - P_j^*\right)^2 + c_2 \tag{19}$$

To prove the convergence of $\Theta(m)$, the following intermediate result will be used.

Theorem 1 *The mean square error Θ_{m+1} after $m+1$ iterations satisfies the upper bound*

$$\Theta_{m+1} \leq \left(1 - 2a_m c_0 + a_m^2 c_1\right) \Theta_m + a_m^2 c_2 \tag{20}$$

Theorem 1 implies that the limit (17) will hold if

$$\lim_{m\to\infty} \left[\left(1 - 2a_m c_0 + a_m^2 c_1\right) \Theta_m + a_m^2 c_2\right] = 0 \tag{21}$$

In this study, we will establish the limit (17) by proving the convergence of (21).

To study the convergence of the TP algorithm for a fixed a_m, we prove the following theorem.

Theorem 2 *If $a_m = a$ and $|1 - 2ac_0 + a^2 c_1| \leq 1$, then*

$$\lim_{m\to\infty} \Theta_m \leq \frac{ac_2}{2c_0 - ac_1} \tag{22}$$

Note that $|1-2ac_0+a^2c_1| \leq 1$ implies $2c_0 \geq ac_1$ and thus, the upper bound of Theorem 2 is always positive. Theorem 2 implies that choosing an arbitrarily small a, we can obtain $\frac{ac_2}{2c_0-ac_1} \approx 0$. However, as a approaches zero, $|1 - 2ac_0 + a^2c_1|$ approaches 1 and the convergence speed slows down. This problem also arises when c_0 and c_1 are very small. In this case, we need to choose a very large a to achieve $|1 - 2ac_0 + a^2c_1| < 1$. Suitable selection of a requires the knowledge of c_0, c_1 and c_2. As c_0 and c_1 are functions of the uplink channel gain h_j, thus, the selection of a is as difficult as the measurement of h_j.

This practical situation leads us to search a coefficient sequence a_m which will always guarantee $\lim_{m\to\infty} \Theta_m = 0$. In the following theorem, we show that if a_m satisfies two simple conditions, then $\lim_{m\to\infty} \Theta_m = 0$.

Theorem 3 *If the sequence a_m satisfies*

$$\sum_{m=1}^{\infty} a_m = \infty \qquad \sum_{m=1}^{\infty} a_m^2 < \infty$$

then $\lim_{m\to\infty} \Theta_m = \lim_{m\to\infty} E\left[\left(P_j(m) - P_j^*\right)^2\right] = 0.$

Note that $a_m = a/m$ meets both conditions of Theorem 3. The proof of Theorem 3 follows Sakrison's procedure [8, pages 60-61].

Although the TP algorithm will converge, a number of issues must be resolved in order to implement the algorithm. In practice, the uplink channel gain h_j and the noise variance σ^2 are very small. Therefore, the received power measurement $Y_j[P_j, \mathbf{N_j}(m)]$ and the target received power Y_j^* in equation (15) will be very small. The TP algorithm uses these small values to update the transmitted power which typically is several orders of magnitude larger. If the initial transmitted power varies significantly from P_j^*, then the convergence speed of the TP algorithm will be extremely slow. For fast convergence, we need to choose a sufficiently large. The heuristic procedure for the selection of a will be discussed in the following section.

A second practical concern is that the channel gain h_j, the time offsets of equation (1), and the asymptotic efficiencies of user j can vary with time. Therefore, to track these variations, we would initialize the TP algorithm using $a_m = a/m$. When a_m becomes small, we would use a small fixed value for a_m. For constructing the blind MMSE receiver, a similar technique is also used in [10].

6 Empirical results

We modeled a circular single cell of a dual-rate rate asynchronous DS/CDMA system. The radius of the cell was chosen to be $r_0 = 1000$ meters. It was assumed that the mobiles were uniformly distributed in the cell. This assumption yielded a probability density function $f(r) = 2r/r_0^2$ for the distance of a user from the base station. We used a path loss exponent $\alpha = 4$. The height of the base station was 30 meters so that the uplink channel gain of user j, h_j was $1/\left(r^2 + 30^2\right)^{\alpha/2}$. It was assumed that there were five low rate and five high rate users in the system. The processing gain of the low rate users was 64 and the processing gain of the high rate users was 8. The signature waveforms were generated using random signature sequences. The time offsets of different users were randomly generated. We assumed that the time offsets were multiples of the chip duration. The Gaussian noise had power spectral density 6×10^{-14} W/Hz.

In the experiment, user j was a high rate customer. The length of the observation window was $(n_j + d_j + 1)$, where n_j denoted the number of desired users' bits into the past and d_j is the number of its bits into the future and $n_j = d_j = 7$. After an interval of 8 bits, user j observed the same pattern of the signature waveforms from the other interfering users. Thus, the receiver dedicated to user j needed to know the specifications of 8 modified correlators and as a result, user j experienced 8 different asymptotic efficiencies. In our experiments, the following asymptotic efficiencies were used

$$\left\{\zeta_j^{(i)} | 0 \le i \le 7\right\} = \left\{0.3573 \;\; 0.3319 \;\; 0.3392 \;\; 0.3782 \;\; 0.4543 \;\; 0.3298 \;\; 0.3329 \;\; 0.2765\right\}$$

We assumed that the user j applied the AB link quality objective. The target average BER q_j^* was 10^{-2} and the corresponding target received power E_j^* was 9.5356×10^{-13}. The technique shown in Figure 1 was used to satisfy the requirement of iid noise vectors $\mathbf{N_j}(m)$.

In the first experiment, the average BER was determined as a function of the number of power control iterations for fixed a_m as well as $a_m = 10^{13}/m$; see Figure 2. Here in each iteration, the output of modified correlator 0 was used to update the transmitted power or $J = 1$. The mobile initialized the transmitted power from zero or $P_j(0) = 0$. The empirical results showed that for $a_m = 10^{13}/m$, the mobile achieved its QoS objective quickly compared to the fixed a_m of 0.1×10^{13}, or 0.01×10^{13}. These large values of a were needed, because the $|Y_j[P_j, \mathbf{N_j}(m)] - Y_j^*|$ of equation (15) were very small and $P_j^* \gg P_j(0)$. In this experiment, we found that in the case of fixed $a_m = a$, smaller values of a suffered from slow convergence, as suggested in the discussion of section 5. In the following experiments, we studied only the sequence $a_m = a/m$.

In Figure 3, we plot the average BER as a function of the number power control iterations for $J = 1, 8, 16$, where $a = 10^{13}$ and $P_j(0) = 0$. In this experiment, we observed that for $J = 8$, the TP algorithm converged quickly. However, for $J = 16$, the mobile increased the transmitted power uniformly to achieve q_j^*. Similar results were also observed in [11] where stochastic power control method for the matched filter receiver

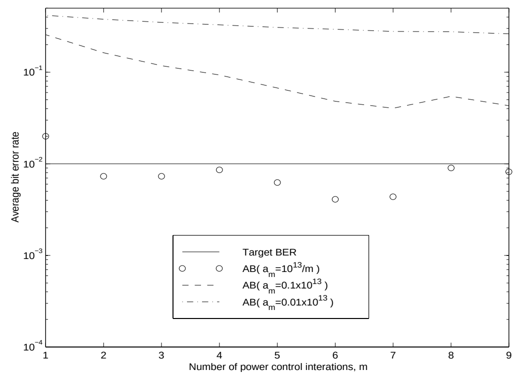

Figure 2: The average BER versus m for fixed a_m as well as $a_m = 10^{13}/m$. Here $J = 1$ and $P_j(0) = 0$.

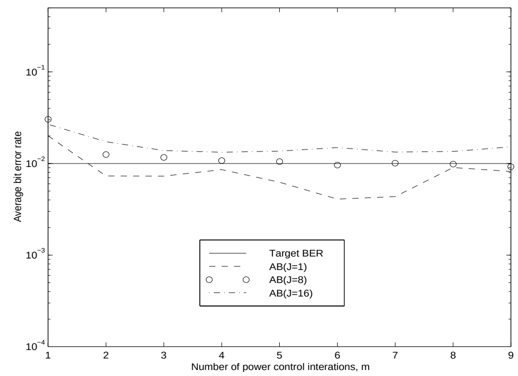

Figure 3: The average BER versus m for variable J. Here $a_m = 10^{13}/m$ and $P_j(0) = 0$.

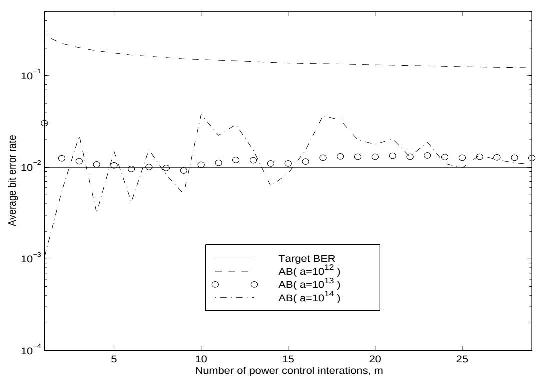

Figure 4: The average BER versus m for variable a when $a_m = a/m$. Here $P_j(0) = 0$.

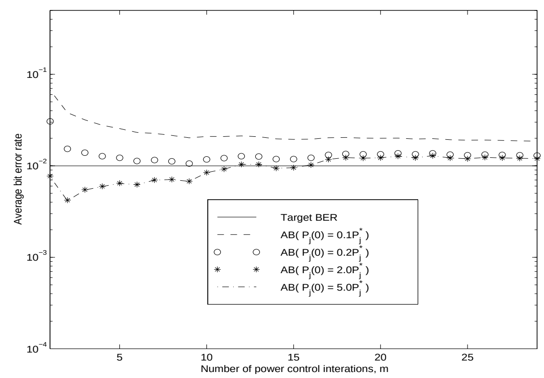

Figure 5: The average BER versus m for various $P_j(0)$, when equation (25) was used to select a. Here $J = 8$.

was studied.

Figure 4 describes the effect of a on the convergence when $J = 8$ and $P_j(0) = 0$. As expected, our empirical results showed that smaller values of a resulted in a slower convergence speed. On the other hand, larger values of a led to a wide variation in the initial transmitted power. These experimental results implied that neither a large a nor a small a was desirable for the mobile. This practical situation leads us to propose the following heuristic procedure to select the parameter a.

Let us assume that $P_j(0) > 0$. Using the target average power, Y_j^* and equation (13), we can estimate or approximate the target transmitted power, P_j^* as follows.

$$\hat{P}_j^* = \frac{Y_j^*}{Y_j[P_j(0), \mathbf{N_j}(0)]} P_j(0) \tag{23}$$

where $\hat{P}_j^*$ is the estimate of P_j^*. In order to be in the vicinity of P_j^* in the very first iteration, we will choose a so that

$$P_j(1) = \hat{P}_j^* \tag{24}$$

Equations (15) and (23) imply that $P_j(1) = \hat{P}_j^*$ will be satisfied if

$$a = \frac{P_j(0)}{Y_j[P_j(0), \mathbf{N_j}(0)]} \tag{25}$$

In equation (25), since $\mathbf{N_j}(0)$ is a random vector, a is a random variable and its magnitude depends on the initial transmitted power $P_j(0)$. For large J and $P_j(0)$, equation (13) implies that $a \approx \left(J/\sum_{i=0}^{J-1} \zeta_j^{(i)}\right) \frac{1}{h_j}$ which is proportional to $\frac{1}{h_j}$. Thus equation (25) can be used to approximate h_j.

In Figure 5, the effect of $P_j(0)$ on the TP algorithm is shown when equation (25) was used to select a. Here $J = 8$ and $P_j(0) = \kappa P_j^*$, where $\kappa = 0.1, 0.2, 2.0, 5.0$. Larger values of $P_j(0)$ yielded better estimates of P_j^* or smaller values of $|P_j(1) - P_j^*|$. As a result, the convergence speed of the power control was improved as $P_j(0)$ increased. For all values of $P_j(0)$, we observed that the mobile updated the transmitted power uniformly to achieve q_j^*.

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