A Decision Feedback Decorrelator for a Dual Rate Synchronous DS/CDMA System

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***Abstract*— A dual rate synchronous DS/CDMA system provides service to low bit rate and high bit rate users. In a fixed duration interval, a low rate user transmits one bit while a high rate user transmits M bits. The differences in the bit transmission rates result in different processing gains for each class of user. In this paper, we propose a decision feedback decorrelator for the dual rate synchronous DS/CDMA system which uses modified correlators and initiates the bit decision process at the end of the high rate bit interval. Every step of the decision process is executed by utilizing the decisions of all previously decoded users. This dual rate decision feedback receiver (evaluated by simulation) is found to outperform two types of decorrelators for dual rate CDMA systems. It is also observed that as the interferers grow stronger relative to the desired user, the performance of the decision feedback receiver for decoding the desired user approaches the single user bound.**

I. Introduction

Beginning with the *optimum multiuser receiver* of Verdú [1], multiuser detection for CDMA systems has received a great deal of attention in the past decade. In particular, the computational complexity of the optimum multiuser receiver prompted the development of high performance suboptimal receivers [2]–[6]. All of these proposed multiuser detectors were designed for CDMA systems in which all users transmit at the same bit rate. Among such receivers, the decorrelating detector [3] is perhaps the simplest in structure and is reasonably easy to implement. The decorrelator eliminates the multiuser interference at the cost of increased noise variance. Furthermore, the decorrelator does not require the knowledge of the received signal strengths. A somewhat more complex receiver is the decorrelating decision-feedback detector [2]. The decision feedback receiver achieves better performance by utilizing bit decisions of the stronger users to reduce multiuser interference for the weaker users.

Significant efforts are being made to integrate the cellular network with fixed networks for communicating both voice and data messages. When all users have the same modulation scheme, two access strategies have been proposed for multi-rate DS/CDMA [7]. These are, 1) fixed chip rate, variable processing gain and 2) fixed processing gain, variable chip rate. A comparative study of the above access strategies was performed in [8]. However, in [7], it is found that the access strategy 2 is complicated because the receiver must be synchronized to its particular code rate and the system needs additional frequency planning due to the unequal bandwidth spreading of different users.

Several studies regarding receiver designs for multi-rate CDMA systems have been performed. Multistage multiuser receivers for multi-rate CDMA communications were introduced in [9]. For minimum mean squared error (MMSE) performance criteria, [10] considered multi-user communication with multiple symbol rates. In [11], a successive interference cancellation scheme for multi-rate CDMA was studied by employing complex modulation techniques, such as M-ary PAM. In [12], [13], it is shown that the dual rate system is equivalent to a single rate system in which each high rate user is modeled as M independent low rate users. This equivalent representation permits the specification of a receiver called the *low rate decorrelator* (*LRD*) for the dual rate CDMA system that is simply the decorrelator of [3] applied to the equivalent single rate system. The complexity of the low rate decorrelator prompts consideration of a *high rate decorrelator* (*HRD*), in which during the bit interval of a high rate user, each low rate user is modeled as a high rate user. It is found that the high rate decorrelator achieves a sizable reduction in complexity while suffering a modest penalty in terms of bit error probability. Both low rate and high rate decorrelators preserve the standard decorrelator's near/far resistance properties.

Motivated by the decision feedback detector of [2], this paper develops a decision feedback decorrelating detector for the dual rate synchronous CDMA system. This proposed receiver is referred to as the *high rate decision feedback decorrelator* or *HRDF.* This paper will study the performance of the HRDF and compare that with the performance of the dual rate decorrelators proposed in [12], [13].

II. Dual Rate Synchronous System

In our dual rate CDMA system model, each bit results in the baseband transmission of a sequence of pulses, or chips, $p[t]$, each pulse having a duration of one chip period T_c. These pulses are sent over an additive white Gaussian noise channel in which the noise $n(t)$ has power spectral density $N_0/2$. Each user group will be classified by its bit rate. The transmission rate of users of group g is denoted by R_g, where $g = 0, 1$ and $R_1 = MR_0$ for an integer $M > 1$. For the K_g group g users, the transmission time of a bit is $T_g = 1/R_g$ and the processing gain is $L_g = T_g/T_c$. The signature waveform of the nth group g customer is

$$S_{n,g}(t) = \sum_{m=1}^{L_g} \{a_{n,g}(m) \frac{1}{\sqrt{L_g}} p[t-(m-1)T_c]\}$$

where $a_{n,g}(m) \in \{-1, 1\}$ denotes the pseudo noise (PN) sequence of user n of group g. The energy of the pulse $p[t]$ is normalized so that $\int_0^{T_g} [S_{n,g}(t)]^2 dt = 1$, $g = 0, 1; n = 1, \ldots, K_g$. In the interval $[0, T_0]$, each low rate user transmits one bit while each high rate user transmits M bits. The jth user from group 0 transmits bit $b_{j,0} \in \{-1, 1\}$ with received energy $E_{j,0}$ in the interval $[0, T_0]$. Similarly, the kth user from group 1 transmits its ith bit $b^i_{k,1} \in \{-1, 1\}$ with received $E^i_{k,1}$ in the ith subinterval $[(i-1)T_1, iT_1]$ using the signature waveform $S^i_{k,1}(t) = S_{k,1}(t-(i-1)T_1)$. Over the interval $[0, T_0]$, the received baseband signal $r(t)$ can be written as

$$\sum_{j=1}^{K_0} \sqrt{E_{j,0}} b_{j,0} S_{j,0}(t) + \sum_{k=1}^{K_1} \left\{ \sum_{i=1}^{M} \sqrt{E^i_{k,1}} b^i_{k,1} S^i_{k,1}(t) \right\} + n(t)$$

The $K = K_0 + MK_1$ bits transmitted in the interval $[0, T_0]$ can be written as the K bit vector $b = [b_1, b_2, \ldots, b_K]$ or

$$[\overbrace{\underbrace{b_{1,0}, \cdots, b_{K_0,0}}_{\text{in } [0,T_0]}}^{\text{group 0 users}}, \overbrace{\underbrace{b^{(1)}_{1,1}, \ldots, b^{(1)}_{K_1,1}}_{\text{in } [0,T_1]}, \quad \cdots \quad \underbrace{, b^{(M)}_{1,1}, \ldots, b^{(M)}_{K_1,1}}_{\text{in } [(M-1)T_1, MT_1]}}^{\text{group 1 users}}]^\top$$

It has been observed [3] that finding the maximum-likelihood estimate of b is an NP-hard problem. Thus, the use of sub-optimum receivers for the dual rate CDMA system is explored. In the following section III., we develop the high rate decision feedback decorrelator (HRDF) for the dual rate synchronous DS/CDMA system.

III. Decision Feedback Decorrelator

The main goal of this paper is to develop the structure of the high rate decision feedback decorrelator, or HRDF, which retains the desirable properties of the HRD. The HRDF exploits the fact that a group 0 user transmits the same bit during each subinterval i in which a group 1 user transmits one bit. The working principle of our proposed receiver of Figure 1 is as follows:

1. At the end of subinterval $i < M$:
   (a) Examine the received signal over $[0, iT_1]$.
   (b) Decorrelate the partial signature sequences of the group 0 users.
   (c) Subtract away group 1 user for the interval $[0, (i-1)T_1]$ based on bit decisions in each of the $(i-1)$ subintervals.
   (d) Using decision feedback, make bit decisions for the group 1 users in the subinterval i.
2. At the end of subinterval $i = M$:
   (a) Repeat step (1a) and (1c) and use decision feedback to decode the group 0 user bits in $[0, MT_1]$ together with the group 1 user bits in $[(M-1)T_1, MT_1]$.

In particular, during the ith subinterval, each group 0 user j transmits the ith segment of the signature waveform $S_{j,0}(t)$ denoted as

$$S^{(i)}_{j,0}(t) = \sum_{m=(i-1)L_1+1}^{iL_1} \{a_{j,0}(m) \frac{1}{\sqrt{L_0}} p[t-(m-1)T_c]\}$$

Over the interval $[0, iT_1]$, the jth group 0 user has transmitted using the *partial signature*

$$\tilde{S}^{(i)}_{j,0}(t) = \sum_{m=1}^{i} S^{(m)}_{j,0}(t), 1 \le j \le K_0$$

The baseband signal $r^{(i)}(t)$ which is received during the interval $[0, iT_1]$ can be written as

$$\sum_{j=1}^{K_0} \sqrt{E_{j,0}} b_{j,0} \tilde{S}^{(i)}_{j,0}(t) + \sum_{k=1}^{K_1} \left\{ \sum_{m=1}^{i} \sqrt{E^{(m)}_{k,1}} b^{(m)}_{k,1} S^{(m)}_{k,1}(t) \right\} + n(t)$$

The group 0 users' partial signatures segments transmitted during the interval $[0, iT_1]$, where $i < M$, are not normalized and may not be linearly independent. Therefore, we generate the effective group 0 users from the actual group 0 users by applying the Gram-Schmidt procedure to the group 0 partial signatures. Let us assume that the Gram-Schmidt procedure on the partial signatures of the group 0 users in the interval $[0, iT_1]$, where $i < M$, yields $K^{(i)}_0$ basis functions, $\{\Psi^{(i)}_{j,0}(t) : 1 \le j \le K^{(i)}_0\}$, where $\Psi^{(i)}_{j,0}(t)$ is normalized to have unit energy on the interval $[0, iT_1]$. Using these basis functions, we can write each group 0 partial signature as a linear combination $\tilde{S}^{(i)}_{j,0}(t) = \sum_{j'=1}^{K^{(i)}_0} a^{(i)}_{jj'} \Psi^{(i)}_{j',0}(t)$. Thus, for $i < M$, the received signal $r^{(i)}(t)$ over $[0, iT_1]$, can be written

$$\sum_{j'=1}^{K^{(i)}_0} \sqrt{\epsilon^{(i)}_{j',0}} \beta^{(i)}_{j',0} \Psi^{(i)}_{j',0}(t) + \sum_{k=1}^{K_1} \sum_{m=1}^{i} \sqrt{E^{(m)}_{k,1}} b^{(m)}_{k,1} S^{(m)}_{k,1}(t) + n(t) \tag{1}$$

where

$$\epsilon^{(i)}_{j',0} = (\sum_{j=1}^{K_0} a^{(i)}_{jj'} E^{\frac{1}{2}}_{j,0} b_{j,0})^2 \quad \beta^{(i)}_{j',0} = \operatorname{sgn}(\sum_{j=1}^{K_0} a^{(i)}_{jj'} E^{\frac{1}{2}}_{j,0} b_{j,0})$$

Effectively, we have reduced K_0 group 0 users with possibly dependent partial signatures to $K^{(i)}_0$ effective group 0 users with orthogonal signatures. Each effective group 0 user transmits one bit $\beta^{(i)}_{j,0}$ with energy $\epsilon^{(i)}_{j,0}$ in the interval $[0, iT_1]$, where $i < M$. We observe that the energy $\epsilon^{(i)}_{j,0}$ of a group 0 effective user depends on the bits transmitted by the group 0 users and also on the projections of the partial signatures on the basis function $\Psi_{j,0}^{(i)}$. Thus, the energies of the group 0 effective users are not known. The proposed receiver will decode each group 1 bit after the end of its transmission and therefore, it models the received signal over the interval $[0, iT_1]$ as the sum of the following three components: i) the baseband signals due to the $K^{(i)} = K_0^{(i)} + K_1$ effective users, ii) cumulative interference from group 1 bits of the previous subintervals and iii) the additive Gaussian noise. For the group 1 user, let us denote $k_1^{(i)}[n] = n - K_0^{(i)}$ if $n > K_0^{(i)}$, $i < M$ and redefine the $K^{(i)}$ effective users' signature waveforms, energies and transmitted bits of equation (1) as follows:

$$s_n^{(i)}(t) = \begin{cases} \Psi_{n,0}^{(i)}(t) & 1 \leq n \leq K_0^{(i)} \\ S_{k_1^{(i)}[n],1}^{(i)}(t) & K_0^{(i)} < n \leq K^{(i)} \end{cases} \quad (2)$$

$$E_n^{(i)} = \begin{cases} \sqrt{\epsilon_{n,0}^{(i)}} & 1 \leq n \leq K_0^{(i)} \\ \sqrt{E_{k_1^{(i)}[n],1}^{(i)}} & K_0^{(i)} < n \leq K^{(i)} \end{cases} \quad (3)$$

$$b_n^{(i)} = \begin{cases} \beta_{n,0}^{(i)} & 1 \leq n \leq K_0^{(i)} \\ b_{k_1^{(i)}[n],1}^{(i)} & K_0^{(i)} < n \leq K^{(i)} \end{cases} \quad (4)$$

Using equations (2)-(4), $r^{(i)}(t)$, the received signal over $[0, iT_1]$ from equation (1), becomes

$$\sum_{n=1}^{K^{(i)}} \sqrt{E_n^{(i)}} b_n^{(i)} s_n^{(i)}(t) + \sum_{m=1}^{i-1} \left\{ \sum_{k=1}^{K_1} \sqrt{E_{k,1}^{(m)}} b_{k,1}^{(m)} S_{k,1}^{(m)}(t) \right\} + n(t) \quad (5)$$

At the end of the subinterval M, the group 0 bits of the interval $[0, MT_1]$ and group 1 bits of the subinterval $[(M-1)T_1, MT_1]$ will be jointly decoded. At this time, this joint decoding will use the actual signature waveform, energy, transmitted bit of each group 0 user. In this case, the effective number of users $K^{(M)}$ equals the actual number of users $K_0 + K_1$. In this regard, it is assumed that during the interval $[0, MT_1]$, the $K^{(M)}$ signature waveforms of the effective users will be linearly independent.

Let us assume that the Gram-Schmidt procedure on the signature waveforms of the effective users of equation (5) yields the set of basis functions $\{\Psi_j^{(i)}(t) : 1 \leq j \leq K^{(i)}\}$, where $\Psi_j^{(i)}(t) = \Psi_{j,0}^{(i)}(t)$, $1 \leq j \leq K_0^{(i)}$. Now, let us modify the correlators so that the received signal will be correlated with $\{\Psi_j^{(i)}(t)\}$. In a subinterval $i < M$, we want to decode the group 1 bits only; therefore, the received signal will be correlated with $\{\Psi_j^{(i)}(t) : K_0^{(i)} + 1 \leq j \leq K^{(i)}\}$. For $i = M$, the group 0 users will be decoded together with the group 1 users and the complete set of basis functions $\{\Psi_j^{(i)}(t)\}$ will be used. The sampled output of the bank of modified correlators at the ith subinterval can be written as the vector

$$r^{(i)} = \Phi^{(i)} \Lambda^{(i)} b^{(i)} + \Upsilon^{(i)} + n^{(i)} \quad (6)$$

By defining a function $F(i)$ such that $F(i) = K_1$, for $i < M$ and $F(M) = K^{(M)}$, we note that $\Phi^{(i)}$ is an $F(i) \times F(i)$

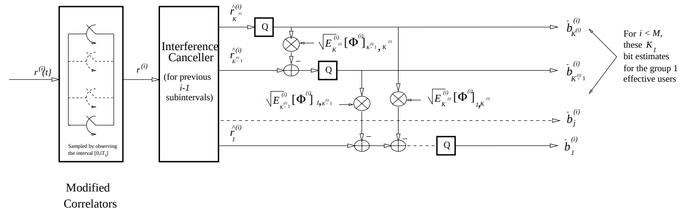

Fig. 1. This diagram shows the operational principle of the high rate decision feedback decorrelating detector, HRDF.

upper triangular matrix with (n, j)th component

$$\Phi_{j,n}^{(i)} = \begin{cases} 0 & j > n \\ \int_0^{iT_1} s_j^{(i)}(t) \Psi_n^{(i)}(t)\, dt > 0 & j \leq n \end{cases}$$

The term $\Lambda^{(i)}$ is an $F(i) \times F(i)$ diagonal matrix with $\Lambda_{n,n}^{(i)} = [E_{n+K^{(i)}}^{(i)}]^{\frac{1}{2}}$ for $i < M$ and $\Lambda_{n,n}^{(M)} = [E_n^{(M)}]^{\frac{1}{2}}$, and $n^{(i)}$ is a Gaussian noise vector of size $F(i)$ with cross-correlation matrix $\frac{N_0}{2} I$. The term $\Upsilon^{(i)}$, is an $F(i) \times 1$ column vector which is the residual multiuser interference resulting from the group 1 users of the previous $i-1$ subintervals. The nth component of $\Upsilon^{(i)}$ is

$$\Upsilon_n^{(i)} = \begin{cases} 0 & i = 1 \\ \sum_{m=1}^{i-1} \left[\sum_{j=1}^{K_1} \phi_{j,n}^{(mi)} \sqrt{E_{j,1}^{(m)}} b_{j,1}^{(m)} \right] & i > 1 \end{cases}$$

where

$$\phi_{j,n}^{(mi)} = \begin{cases} \int_0^{iT_1} S_{j,1}^{(m)}(t) \Psi_{n+K_0^{(i)}}^{(i)}(t)\, dt & 1 < i < M \\ \int_0^{MT_1} S_{j,1}^{(m)}(t) \Psi_n^{(M)}(t)\, dt & i = M \end{cases}$$

We have assumed that we know the actual bit energies $E_{j,0}$ and $E_{k,1}$ of each group 0 user j and group 1 user k. At the receiver end, the users will be ordered and decoded sequentially in a decreasing order. The bit decision process will exploit previous users' decisions to decode the current user. Therefore, in subinterval i, the group 1 bit decisions from the previous $(i-1)$ subintervals are known and during the ith subinterval, if we have already made bit decisions $\bar{b}_{k+1}^{(i)}, \ldots, \bar{b}_{K^{(i)}}^{(i)}$, then, we can write the decision statistic for effective user k as $\bar{r}_k^{(i)}$, where $1 \leq k \leq F(i)$ and $\bar{r}_k^{(i)}$ equals to

$$r_k^{(i)} - \sum_{m=1}^{i-1} \left[\sum_{j=1}^{K_1} \phi_{j,k}^{(mi)} \sqrt{E_{j,1}^{(m)}} \bar{b}_{j,1}^{(m)} \right] - \sum_{j=k+1}^{K^{(i)}} [\Phi^{(i)}]_{k,j} \sqrt{E_j^{(i)}} \bar{b}_j^{(i)}$$

The corresponding decoding rule is $\bar{b}_k^{(i)} = \mathrm{sgn}[\bar{r}_k^{(i)}]$.

In [2], [4], [5], it is observed that the feedback in the receiver of the single rate CDMA system is primarily beneficial if the interfering users are stronger. However, in our receiver design, we rank the $K^{(i)}$ users as follows: We define the *orthogonal energy*, or *OE*, of a user as the fraction of it's received energy associated with subspace orthogonal to interfering users. We write the orthogonal energy as $\mathrm{OE}_n^{(i)} = E_n^{(i)} / \left(\bar{\Gamma}^{(i)}\right)_{n,n}^{-1}$, where $\Gamma^{(i)}$ denotes $K^{(i)} \times K^{(i)}$

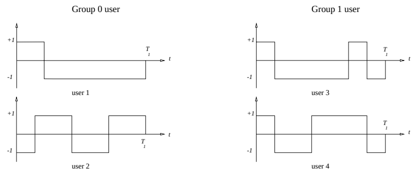

Fig. 2. Gold signature sequences of length seven for the dual rate DS/CDMA system.

cross-correlation matrix. For the interval $[0, iT_1]$, where $i < M$, we order the K_1 group 1 users such that $\mathrm{OE}_n^{(i)}$ is increasing in n. However, for $i = M$, group 0 users will be decoded together with group 1 users. In particular, without classifying the user by it's rate, all $K^{(M)} = K_0 + K_1$ users will be ranked according to non decreasing OEs. This heuristic ordering procedure is motivated by the known performance of the decorrelator. As we have seen that the bit error rate of the decorrelator is a decreasing function of OE and in a bandwidth efficient CDMA system with many users, for decoding the first few users, the performance of the HRDF will be almost equivalent to the decorrelator. Therefore, we rank the users by their decreasing OEs rather than their energies in order to reduce the likelihood of early decision feedback errors which have an impact on the decoding of higher ordered users.

A. Properties of HRDF

The implementation of the HRDF requires $K_0 + MK_1$ modified correlators and two interference cancellers. We can implement the HRDF by using $\sum_{i=1}^{M} K_0^{(i)} + K_1$ correlators matched to the effective signature waveforms rather than using $K_0 + MK_1$ modified correlators. However, this technique requires an additional linear transformation $([\Phi^{(i)}]^\top)^{-1}$ at the outputs of the correlators. Note that this alternative method is used in the decision feedback receiver of the single rate system [2]. In order to process the current bit decision, the first interference canceller cancels out the multiuser interference caused by the high rate bits of the previous subintervals and the second one subtracts the multiuser interference caused by the previously decoded users of the current subinterval. Every step of the bit decision process is executed by measuring the cross-correlation among the current modified correlator signal and the signature waveforms corresponding to the bits which are decoded previously. However, these operations of interference cancellers can be observed by adapting the following method applied in [4]. At the end of each bit decision, the corresponding baseband signal will be regenerated and subtracted from the received signal, and then, this new signal will be correlated with the correlator corresponding to the bit which will be decoded next.

In [12], [13], it has been observed that to build the low rate decorrelator, it is necessary that $K_0+MK_1 \leq L_0$, since otherwise it will not be possible to find $K_0 + MK_1$ independent code words for the users of the equivalent single rate system. Thus, for the low rate decorrelator, removing a group 1 user will permit adding M group 0 users.

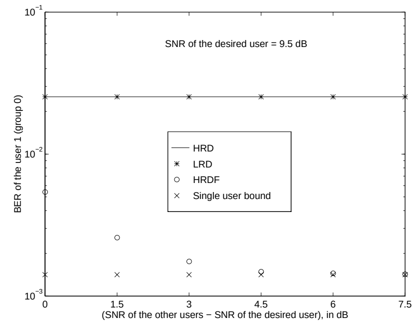

Fig. 3. Four user system: BER of the user 1 (group 0) when $K_0 = 2$, $K_1 = 2$, $L_0 = 56$ and $L_1 = 7$.

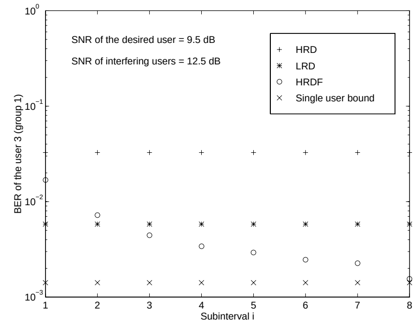

Fig. 4. Four user system: BER of the user 3 (group 1) when $K_0 = 2$, $K_1 = 2$, $L_0 = 56$ and $L_1 = 7$.

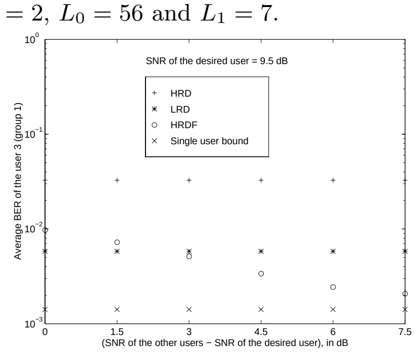

Fig. 5. Four user system: Average BER of the user 3 (group 1) when $K_0 = 2$, $K_1 = 2$, $L_0 = 56$ and $L_1 = 7$.

On the other hand, for the high rate decorrelator, we must have $K_0 + K_1 \leq L_1$. In this case, removing one group 1 user permits us to carry only one additional group 0 user. Hence, the low rate decorrelator is more bandwidth efficient for the group 0 users. However, the HRDF system model allows us to support M group 0 users by using one group 1 signature sequence. In other words, using a group 1 signature sequences of length L_1, we can generate M independent group 0 signature sequences of length L_0. It is always possible to have a $M \times M$ matrix A, where $A_{j,i} \in \{1, -1\}$, with full rank M. Therefore, if the row vector $S_{k,1}^\top$ of size $1 \times L_1$ denotes the signature sequences of the kth group 1 user, then, there always exist M independent row vectors $\{S_{j,0}^\top : j = 1, \ldots, M\}$ of size $1 \times L_0$ such that $S_{j,0}^\top = \left[A_{j,1}S_{k,1}^\top, A_{j,2}S_{k,1}^\top, \ldots, A_{j,M}S_{k,1}^\top\right]$.

IV. Performance of HRDF

Now, the performance of the simulated HRDF system will be compared with the evaluated performance of LRD and HRD. For the purpose of comparisons, we will use a set of signature waveforms derived from Gold sequences of length seven[2], [5]; see Figure 2. As the HRDF system model permit us to use group 1 signature sequences to support group 0 users, we will use the same set of signature sequences to support the group 1 users as well group 0 users and assume four users are active. Among the four users, there are $K_0 = 2$ group 0 users, and $K_1 = 2$ group 1 users. Users 1 and 2 are group 0 users while users 3 and 4 are group 1 users. It is also assumed that the group 0 users have the same signature sequences in every subinterval of duration of T_1. A group 1 user transmits $M = 8$ bits while a group 0 user transmits one bit. The SNR of the kth user of the system will be $10\log(\frac{E_k}{N_0/2})$ and it is assumed that the received energies of group 1 users will not vary from bit to bit.

In Figure 3, the bit error rate (BER) of the group 0 user 1 is plotted as a function of the SNR of the other users, while the SNR of the desired group 0 user is fixed at 9.5 dB. Here, it is observed that the proposed receiver, HRDF significantly outperforms the LRD and HRD. As the other users become stronger relative to the desired user, the feedback data for decoding the desired user becomes more accurate. As a consequence, the BER of the user 1 approaches the single user bound; see Figure 3. Note that the high rate and low rate decorrelators yield the same bit error probability for the group 0 user. This was observed in [12], [13] to be a consequence of using the same cross-correlation matrix in every subinterval i. From Figure 3, we see that the HRDF performs better than either of the dual rate decorrelators of [12], [13] for all operating points we have considered. Until the Mth subinterval, the group 1 users are decoded by decorrelating the group 0 users which enhances the noise variance of the group 1 users. However, in this specific example, the enhancement of noise variance becomes smaller as i increases. In other words, in successive subintervals, the group 1 user experiences less multi-user interference from group 0 users and hence decreasing BER; see Figure 4 where the BER of the user 3 (a group 1 user) in different subintervals, i, is observed when the SNR of the other users is 12.5 dB and the SNR of the desired group 1 user is fixed at 9.5 dB. Here, it is also noted that the user 3 observes sharp improvement in the BER at the Mth subinterval compared to the $(M-1)$th subinterval. This result shows the gain from using decision feedback for the group 0 users over decorrelating the group 0 users at the Mth subinterval.

In order to quantify the relative performance of the HRDF respect to the HRD and the LRD for decoding the group 1 user, let us define a performance measure called *average BER* of the kth group 1 user as $\bar{P}_{k,1} = \frac{1}{M}(\sum_{i=1}^{M} P_{k,1}^{(i)})$, where $P_{k,1}^{(i)}$ denotes the BER of the kth group 1 user at the ith subinterval under the HRDF system. In Figure 5, we have plotted the average BER of user 3 as a function of the SNR of the interfering users. Here, it is noticed that as the interfering users become stronger, outperforming the LRD, the performance of the HRDF approaches to the single user bound.

V. Conclusion

We have found that similar to the high rate decorrelator, HRD, our proposed high rate decision feedback decorrelating detector, HRDF, eliminates the bit processing delay of the low rate decorrelator, LRD and outperforms both the LRD and the HRD. It is observed that as the interfering users become stronger, the bit error rate of the weaker user approaches the single user bound. In addition, it is also observed that HRDF system is as bandwidth efficient as the LRD system.

References

[1] S. Verdú. *Optimum Multi-user Signal Detection.* PhD thesis, University of Illinois at Urbana-Champaign, 1984.

[2] A. Duel-Hallen. Decision-Feedback Multiuser Detector Synchronous for Code-Division Multiple Access Channel. *IEEE Transactions on Communications*, 41:285–290, February 1993.

[3] L. Lupas and S. Verdú. Linear Multiuser Detectors for Synchronous Code-Division Multiple-Access Channels. *IEEE Transactions On Information Theory*, 35(1):123–136, January 1989.

[4] P. Patel and J. Holtzman. Analysis of a Simple Successive Interference Cancellation Scheme in a DS/CDMA. *IEEE Journal on Selected Areas in Communications*, 12:796–807, June 1994.

[5] M. K. Varanasi and B. Aazhang. Near-Optimum Detection in Synchronous Code-Division Multiple-Access Systems. *IEEE Transactions On Communications*, pages 725–736, May 1991.

[6] U. Madhow and M. L. Honig. MMSE Interference Suppression for Direct-Sequence Spread-Spectrum CDMA. *IEEE Transactions on Communications*, 42(12):3178–3188, December 1994.

[7] R. Wyrwas, M. J. Miller, R. Anjaria, and W. Zhang. Multiple Access Options for Multi-media Wireless Systems. In *Proc. of the 3rd Workshop on Third Generation Wireless Information Networks, WINLAB, Rutgers University*, pages 289–294, April 1992.

[8] T.-H. Wu and E. Geraniotis. CDMA with Multiple Chip Rates for Multi-Media Communications. In *Proc. of the 28th Annual Conference on Information Sciences and Systems, Princeton University*, pages 992–997, March 1994.

[9] A. Hottinen, H. Holma, and A. Toskala. Multiuser Detection for Multirate CDMA Communications. In *Proc. of IEEE International Conference on Communications*, 1996.

[10] M. L. Honig and S. Roy. Multi-user Communication with Multiple Symbol Rates. In *Proc. of the IEEE Symposium on Information Theory, Whistler, BC, Canada*, page 381, September 1995.

[11] A. Johansson and A. Svensson. Successive Interference Cancellation Schemes in Multi-rate DS/CDMA Systems. In *Proc. of 45th IEEE Vehicular Technology Conference*, volume 2, pages 704–708. IEEE, July 1995.

[12] M. Saquib, R. Yates, and N. Mandayam. Decorrelating Detectors for a Dual Rate Synchronous DS/CDMA System. In *Proc. of 46th IEEE Vehicular Technology Conference*, volume 1, pages 377–381, April 28 - May 1 1996.

[13] M. Saquib, R. Yates, and N. Mandayam. Decorrelating Detectors for a Dual Rate Synchronous DS/CDMA System. *Wireless Personal Communications.* (to appear).