

Semester Report for Fall 2003

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Abstract

This report briefly describes the research activities that I, under the guidance of my advisor Professor Roy D Yates, was involved in at the Wireless Information Network Laboratory (WINLAB) during the period September 2003 to December 2003 (fall semester). My primary research continued on the Network Interference Server project, which began in Summer 2003 as part of the Spectrum Project sponsored by the National Science Foundation (NSF). Taking cue from the role of a Domain Name Server (DNS) in the context of the wired Internet, this project proposes a Network Interference Service (NIS) model capable of providing information about the neighborhood of a wireless node. All nodes within the network that partake of this directory service provide a log of their transmission pattern to the NIS through reliable control channels. A wireless node can request neighborhood information by sending a record of its received signals, and the NIS correlates this sequence of attenuated signal strengths with the uncorrupted data received by itself. The set of linear equations thus set up is solved to estimate the channel coefficients, and hence the “radio distance” between nodes. A simple simulation was performed for an 802.11b network, and the channel coefficients were estimated to a high degree of accuracy. Preliminary results were presented as a poster at the Industrial Advisory Board (IAB) meeting in November 2003.

Contents

1	Introduction	4
2	Analysis	5
3	Algorithm	11
4	Simulation	13
5	Conclusion	14

1 Introduction

The role played by a Domain Name Server (DNS) in the context of the Internet is well-known. It translates the computer name specified by a Uniform Resource Locator (URL) into an IP address, thus making route determination possible. A similar application to determine local network topology in the wireless domain is being proposed in this article. The system utilises a centralized Network Interference Service (NIS) provider that identifies wireless nodes from a study of the interference pattern and the correlation between transmitted and received sequences of data signals. The Network Interference Server is modeled as an intelligent information-gathering source that maintains a database of transmissions in its neighborhood through reliable control channel connections (thus eliminating the possibility of any error) with transmitting nodes. Whenever a wireless node within the network requires neighborhood information, it sends a request to the NIS along with a trace of transmissions it has received in the past. The NIS identifies the requesting node and determines the nodes that lie within its “radio-distance” vicinity by attempting a correlation between the original information sequence and the attenuated sequence received by the requesting node from its neighbors.

The problem at hand encapsulates ideas from channel estimation and equalization, modulation techniques and linear algebra methods. The following section introduces a discrete-time representation of signals with flat fading channel characteristics that is used in the analysis, while Section 3 outlines the algorithm proposed, as well as the simulation results obtained with MATLAB. The paper concludes with a discussion of the results and the scope for future work.

2 Analysis

The Network Interference Service (NIS) model comprises m nodes, all within transmission range of each other, which communicate with the NIS node through reliable control channels. Transmissions occur in discrete time-slots and the transmitted power over a number of time-slots, later defined as a *frame*, is reported to the NIS. The nodes also maintain a record of received power in each time-slot.

Some simplifying assumptions used in the analysis are as follows:

1. Nodes cannot transmit and receive simultaneously.
2. Receiver noise of known power exists.
3. Channel coefficients remain constant over the duration of the analysis.
4. Transmission from node to NIS is error-free.

A discrete-time signal representation is used to describe the channel model that satisfies the above assumptions. For such a system, the i -th user is assigned a finite energy signature waveform, $\{s_i(t), t \in [0, T]\}$, and it transmits a string of bits by modulating that waveform antipodally. To assure synchronization and reduce the information overhead at the NIS, strings of K bits are grouped together as frames at the transmitter end, such that the state of the transmitter remains unchanged within the space of a frame. An activity factor $\alpha_j[l]$ is also introduced that keeps track of the state of the transmitter j in the l -th frame according to the following rule:

$$\alpha_j[l] \triangleq \begin{cases} 1, & \text{if transmitter } j \text{ is on during the } l\text{th frame} \\ 0, & \text{if transmitter } j \text{ is off during the } l\text{th frame} \end{cases} \quad (1)$$

This frame activity factor $\alpha_j[l]$ is reported to the NIS by each transmitter j at each frame interval l .

The receiver correlates for each bit in the frame and therefore the initial formulation is in terms of individual bits. This result will later be expressed in terms of frames. However, the frame activity factor $\alpha_j[l]$ is still employed, with the understanding that all bits within the same frame correspond to the same activity factor. Thus, assuming signal synchronization is maintained and users share a white Gaussian multiple-access channel,

the signal received by the i th user at time t corresponding to the k -th bit interval is given by [10]:

$$\begin{aligned} r_i(t) &= \sum_{j=1}^m \alpha_j[l] \sqrt{p_j(k)} \sqrt{h_{ij}} b_j(k) s_j(t - kT) + n_i(t), \quad t \in [kT, (k+1)T] \\ &= \sum_{j=1}^m \alpha_j[l] \sqrt{q_{ij}(k)} b_j(k) s_j(t - kT) + n_i(t), \quad t \in [kT, (k+1)T] \end{aligned} \quad (2)$$

where,

h_{ij} : constant channel coefficient between node i and node j

b_j : j th user information sequence where $b \in \{-1, 1\}$

s_j : signature waveform assigned to user j

n_i : additive white gaussian noise (AWGN) with zero-mean and variance σ^2

$q_{ij}(k) = h_{ij} p_j(k)$ is the power of node j received at node i

The output of matched filter f at receiver i can be obtained by integrating over the interval $[kT, (k+1)T]$. However, if all information sequences are equally likely, it suffices to restrict attention to a specific symbol interval [2]. For notational simplicity, $k = 0$ is chosen without any loss of generality. Thus, the matched filter output may be expressed as:

$$\begin{aligned} y_i^{(f)} &= \int_0^T r_i(t) s_f(t) dt \\ &= \sum_{j=1}^m \alpha_j[l] \sqrt{q_{ij}} b_j \int_0^T s_j(t) s_f(t) dt + \int_0^T n_i(t) s_f(t) dt \end{aligned} \quad (3)$$

Defining

$$\begin{aligned} \rho_{jf} &= \int_0^T s_j(t) s_f(t) dt, \quad \rho_{jj} = 1 \\ n_{if} &= \int_0^T n_i(t) s_f(t) dt, \end{aligned} \quad (4)$$

equation (3) is rewritten as:

$$y_i^{(f)} = \sum_{j=1}^m \alpha_j[l] \sqrt{q_{ij}} b_j \rho_{jf} + n_{if} \quad (5)$$

Conventional receivers consist of matched filters that are matched to the signature sequences of the users, and squares of the matched filter outputs are unbiased estimates for the received energies in the sense that the expected value of the square of a matched filter output is equal to the received energy through the matched filter [9]. The randomness over which the expectation is taken is due to the randomness of the transmitted bit as

well as the channel noise. The square of the matched filter output as well as its theoretical mean and variance are as follows:

$$\left\{y_i^{(f)}\right\}^2 = \sum_{j=1}^m \left[\alpha_j[l]q_{ij}\rho_{jf}^2 + \sum_{n=1, n \neq j}^m \alpha_j[l]\alpha_n[l]\sqrt{q_{ij}}\sqrt{q_{in}}b_j b_n \rho_{jf}\rho_{nf} \right] + n_{if}^2 + 2n_{if} \sum_{j=1}^m \alpha_j[l]\sqrt{q_{ij}}b_j \rho_{jf} \quad (6)$$

$$\mathbb{E} \left[\left\{y_i^{(f)}\right\}^2 \right] = \sum_{j=1}^m \alpha_j[l]q_{ij}\rho_{jf}^2 + \sigma^2 \quad (7)$$

$$\begin{aligned} \text{var} \left[\left\{y_i^{(f)}\right\}^2 \right] &= \mathbb{E} \left[\left\{y_i^{(f)}\right\}^4 \right] - \mathbb{E}^2 \left[\left\{y_i^{(f)}\right\}^2 \right] \\ &= \sum_{j=1}^m \left[\alpha_j[l]q_{ij}^2\rho_{jf}^4 + 2 \sum_{n=1, n \neq j}^m \alpha_j[l]\alpha_n[l]q_{ij}q_{in}\rho_{jf}^2\rho_{nf}^2 \right] + 3\sigma^4 + 4\sigma^2 \sum_{j=1}^m \alpha_j[l]q_{ij}\rho_{jf}^2 \\ &\quad - \sum_{j=1}^m \left[\alpha_j[l]q_{ij}^2\rho_{jf}^4 + \sum_{n=1, n \neq j}^m \alpha_j[l]\alpha_n[l]q_{ij}q_{in}\rho_{jf}^2\rho_{nf}^2 \right] - \sigma^4 - 2\sigma^2 \sum_{j=1}^m \alpha_j[l]q_{ij}\rho_{jf}^2 \\ &= \sum_{j=1}^m \sum_{n=1, n \neq j}^m \alpha_j[l]\alpha_n[l]q_{ij}q_{in}\rho_{jf}^2\rho_{nf}^2 + 2\sigma^4 + 4\sigma^2 \sum_{j=1}^m \alpha_j[l]q_{ij}\rho_{jf}^2 \end{aligned} \quad (8)$$

Equation (7) follows from the assumption that $\mathbb{E}[b_f(k)] = 0$, $\mathbb{E}[n_{il}(k)] = 0$ and bit-sequences for different nodes are independent.

Assuming perfect frame synchronicity, the received signal is now also grouped into frames of K bits. Since the activity state of each transmitter remains unchanged over a frame, the signal power received by the i th node over a frame may simply be obtained by averaging the received power associated with each bit in the frame. Thus, the power in the f th matched filter of the i th receiver over frame l , denoted by $x_i^{(f)}[l]$, is defined as:

$$x_i^{(f)}[l] = \frac{1}{K} \sum_{k=(l-1)K+1}^{lK} \left\{y_i^{(f)}(k)\right\}^2 \quad (9)$$

Also, the expected value of the received power over the frame l is:

$$\begin{aligned} \mathbb{E} \left[x_i^{(f)}[l] \right] &= \mathbb{E} \left[\left\{y_i^{(f)}(k)\right\}^2 \right] \\ &= \sum_{j=1}^m \alpha_j[l]q_{ij}\rho_{jf}^2 + \sigma^2 \end{aligned} \quad (10)$$

It may be pointed out as an aside that in the case of IEEE 802.11, each transmitter uses the same signature

waveform (*i.e.*, $\rho_{jf}(k) = 1, \forall j, f$), then the equation simplifies further to:

$$\mathbb{E} \left[x_i^{(f)}[l] \right] - \sigma^2 = \sum_{j=1}^m \alpha_j[l] q_{ij} \quad (11)$$

As has been discussed in [3], measurements at the receiver include the total received signal power $x_i^{(f)}[l]$ and the background receiver noise σ^2 . Thus, a procedure will now be outlined to evaluate the channel coefficients h_{ij} , provided there is a large number of received power measurements.

Assuming a sufficiently large number of bits in each frame, equations (9) and (10) may be approximated as:

$$x_i^{(f)}[l] - \sigma^2 = \sum_{j=1}^m \alpha_j[l] q_{ij} \rho_{jf}^2[l] \quad (12)$$

In matrix notation,

$$\begin{bmatrix} x_1^{(f)}[l] - \sigma^2 \\ x_2^{(f)}[l] - \sigma^2 \\ \vdots \\ x_m^{(f)}[l] - \sigma^2 \end{bmatrix} = \begin{bmatrix} q_{11} & q_{12} & \cdots & q_{1m} \\ q_{21} & q_{22} & \cdots & q_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ q_{m1} & q_{m2} & \cdots & q_{mm} \end{bmatrix} \begin{bmatrix} \alpha_1[l] \rho_{1f}^2[l] \\ \alpha_2[l] \rho_{2f}^2[l] \\ \vdots \\ \alpha_m[l] \rho_{mf}^2[l] \end{bmatrix} \quad (13)$$

According to the model, the NIS has access to the transmission sequences $\alpha_j[l]$ for all nodes j over all frame-intervals l . However, it receives the temporal received power values $x_i^{(f)}[l] - \sigma^2$ from only one node i . This dynamic behavior for a single node i for n frame-intervals may be represented by:

$$\begin{bmatrix} x_i^{(f)}[1] - \sigma^2 \\ x_i^{(f)}[2] - \sigma^2 \\ \vdots \\ x_i^{(f)}[n] - \sigma^2 \end{bmatrix} = \begin{bmatrix} \alpha_1[1] \rho_{1f}^2[1] & \alpha_2[1] \rho_{2f}^2[1] & \cdots & \alpha_m[1] \rho_{mf}^2[1] \\ \alpha_1[2] \rho_{1f}^2[2] & \alpha_2[2] \rho_{2f}^2[2] & \cdots & \alpha_m[2] \rho_{mf}^2[2] \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1[n] \rho_{1f}^2[n] & \alpha_2[n] \rho_{2f}^2[n] & \cdots & \alpha_m[n] \rho_{mf}^2[n] \end{bmatrix} \begin{bmatrix} q_{i1} \\ q_{i2} \\ \vdots \\ q_{im} \end{bmatrix} \quad (14)$$

The channel coefficients h_{ij} are subsequently obtained by solving this set of equations using the algorithm outlined in the following section. Finally, if the path-loss coefficient is known, the inter-node distances with respect to node i may be estimated.

A similar analysis can be performed in terms of a correlation receiver, as in [8], instead of the matched filter implementation. Assuming that each transmitter j has a pre-assigned unique signature sequence given by $s_j(t)$,

the waveform transmitted by it in a single frame-interval is given by:

$$x_j(t) = \alpha_j[l]\sqrt{p_j}s_j(t), \quad t \in [lKT, (l+1)KT]$$

where, l = frame number,

K = number of bits in each frame,

T = bit interval

(15)

The baseband received signal, $r_i(t)$, in one bit-interval at the front end of the receiver filters at the assigned base of user i is thus given by:

$$r_i(t) = \sum_{j=1; j \neq i}^m \alpha_j[l]\sqrt{p_j h_{ij}}s_j(t) + n(t)$$

where, h_{ij} = channel coefficient between nodes i and j

$n(t)$ = additive white Gaussian noise process

(16)

If \mathbf{c}_i is now defined to be the receiver filter for user i at its assigned base station, then the receiver filter output of user i will be:

$$\begin{aligned} y_i &= \sum_{j=1; j \neq i}^m \alpha_j[l]\sqrt{p_j h_{ij}}(\mathbf{c}_i^\top \mathbf{s}_j) + \mathbf{c}_i^\top \mathbf{n} \\ &= \sum_{j=1; j \neq i}^m \alpha_j[l]\sqrt{q_{ij}}\rho_{ij} + n' \end{aligned} \quad (17)$$

where, n' = Gaussian random variable with zero mean and variance $\sigma^2 \mathbf{c}_i^\top \mathbf{c}_i$

The received energy may be obtained by squaring the output of the filter, and results similar to equations (6), (7) and (8) are obtained.

$$y_i^2 = \sum_{j=1}^m \left[\alpha_j^2[l]q_{ij}\rho_{ij}^2 + \sum_{k=1; k \neq j}^m \alpha_j[l]\alpha_k[l]\sqrt{q_{ij}q_{ik}}\rho_{ij}\rho_{ik} \right] + n'^2 + 2n' \sum_{j=1}^m \alpha_j[l]\sqrt{q_{ij}}\rho_{ij} \quad (18)$$

$$\mathbb{E}[y_i^2] = \sum_{j=1}^m \alpha_j^2[l]q_{ij}\mathbb{E}[\rho_{ij}^2] + \sigma^2 \quad (19)$$

$$\text{var}[y_i^2] = \sum_{j=1}^m \alpha_j^2[l]q_{ij}^2 \text{var}[\rho_{ij}^2] + 2\sigma^4 + 4\sigma^2 \sum_{j=1}^m \alpha_j^2[l]q_{ij}\mathbb{E}[\rho_{ij}^2] \quad (20)$$

The purpose of this exercise was to establish that although matched filters had been employed in the analysis of the NIS problem, this does not result in any loss of generality, and any other receiver scheme could equally

well be used. In all cases, a set of linear equations similar to (14) would eventually have to be solved.

In the study of solutions to linear equations [7, 4], three different cases are identified based on the dimensions of matrices.

1. *Overdetermined case:* When the number of measurements exceeds the number of free parameters, an exact fit of the model to the data is not possible. In this case, the number of columns in the transmission matrix \mathbf{S} is less than the number of rows.
2. *Determined case:* When the number of free parameters equals the number of measurements, the measurements can be fitted exactly as long as there exists no linear dependence among the rows of the square matrix \mathbf{S} .
3. *Underdetermined case:* When the number of free parameters exceeds the number of measurements, there is not enough data for a unique solution. In this case, the number of columns in \mathbf{S} is greater than the number of rows.

Thus, the channel coefficients h_{ij} for node i may be uniquely determined when a full-rank matrix \mathbf{S} of dimension $m \times m$ is obtained.

However, for rank-deficient matrices it is not possible to compute the channel coefficient vector deterministically. Instead, techniques need to be borrowed from estimation theory to obtain expected values of the channel coefficients and hence expected inter-node distances. Several estimation techniques are outlined in the literature that allow for blind equalization of a communication channel. The most common ones include maximum likelihood (ML) and minimum mean squared error estimates (MMSE) [5], both of which are based on the expectation maximization (EM) algorithm [1].

3 Algorithm

The proposed model for the Network Interference Server (NIS) is theoretically capable of obtaining inter-node distances for any network where a model for the transmission activity is known. However, with increase in the number of nodes within the network, the behavior of the algorithm quickly deteriorates. Since a typical network may potentially contain hundreds of nodes, there should be a way to eliminate distant nodes that do not contribute much interference to the receiver at hand. Such a technique is now described.

Since the frame activity sequence of each of the nodes, *i.e.*, $\alpha_j[l]$, is known to the NIS, it can evaluate the correlation between the received signal over a certain number of frame-intervals and the activity of each of the other $m - 1$ nodes. The resulting column matrix of $m - 1$ elements gives an indication of the effect of the transmitter nodes at the receiver since the nearest nodes will have the maximum correlation with the received power.

Using the approximation for the received power introduced in (13), the covariance between the sample mean of received power at requesting node i over L frames and the activity of the j th node over the same number of frames is given by:

$$\begin{aligned} \text{cov} [x_i^{(f)}, \alpha_j] &= \text{E} \left[\left\{ x_i^{(f)} - \text{E} \left(x_i^{(f)} \right) \right\} \left\{ \alpha_j - \text{E} \left(\alpha_j \right) \right\} \right] \\ &\approx \frac{1}{L} \sum_{u=l-L}^l \left[\left(x_i^{(f)}[u] - \overline{x_i^{(f)}[l]} \right) \left(\alpha_j[u] - \overline{\alpha_j[l]} \right) \right] \end{aligned} \quad (21)$$

where,

$$\begin{aligned} \overline{x_i^{(f)}[l]} &\triangleq \frac{1}{L} \sum_{u=l-L}^l x_i^{(f)}[u] \\ \overline{\alpha_j[l]} &\triangleq \frac{1}{L} \sum_{u=l-L}^l \alpha_j[u] \end{aligned} \quad (22)$$

Then, the corresponding correlation coefficient is defined as:

$$\text{cor} [x_i^{(f)}[l], \alpha_j[l]] = \frac{\text{cov} [x_i^{(f)}[l], \alpha_j[l]]}{\sqrt{\text{var}(x_i^{(f)}[l])} \sqrt{\text{var}(\alpha_j[l])}} \quad (23)$$

where,

$$\begin{aligned}
\text{var}(x_i^{(f)}[l]) &\triangleq \text{E} \left[\left\{ x_i^{(f)} - \text{E} \left(x_i^{(f)} \right) \right\}^2 \right] \\
&\approx \frac{1}{L} \sum_{u=l-L}^l \left[\left\{ x_i^{(f)}[u] - \overline{x_i^{(f)}[l]} \right\}^2 \right] \\
\text{var}(\alpha_j[l]) &\triangleq \text{E} \left[\left\{ \alpha_j - \text{E}(\alpha_j) \right\}^2 \right] \\
&\approx \frac{1}{L} \sum_{u=l-L}^l \left[\left\{ \alpha_j[u] - \overline{\alpha_j[l]} \right\}^2 \right]
\end{aligned} \tag{24}$$

and the sample means, $\overline{x_i^{(f)}[l]}$ and $\overline{\alpha_j[l]}$, are defined in (16).

The correlation vector between the power received at the i th node and the activity of each of the m transmitters over the interval corresponding to L frames is then denoted by the following $m \times 1$ matrix:

$$\left[\text{cor} \left\{ x_i^{(f)}[l], \alpha_1[l] \right\}, \text{cor} \left\{ x_i^{(f)}[l], \alpha_2[l] \right\}, \dots, \text{cor} \left\{ x_i^{(f)}[l], \alpha_m[l] \right\} \right]^\top \tag{25}$$

The correlation coefficients may now be sorted and only the nodes corresponding to correlation coefficient values that satisfy a pre-defined threshold are considered for neighborhood estimation.

Thus, the steps involved in obtaining the inter-node distances with respect to a requesting node i may be summarised as follows:

1. Each node records the average received power in the frames in which it itself is not transmitting.
2. The sequence of activity periods for each of the nodes is recorded by the NIS through reliable control channels.
3. When a wireless node needs a neighborhood map, it sends a request to the NIS with a history of received powers.
4. After identifying the requesting node, the NIS correlates the requesting node's received power values with the activity log of all nodes in its database to estimate a subset of nodes that affect the power measurements at the receiver and hence lie within a certain radio-distance from the receiver.
5. Finally, the NIS solves the channel coefficients (and hence obtains the distances from the requesting node) as long as the number of independent and non-zero measurements sent by the requesting node is equal to the number of other nodes in the sub-network evaluated in the previous step, *i.e.*, the matrix in (15) is full-ranked and forms a determinable system.

4 Simulation

To verify the operation of the NIS algorithm, a simple simulation experiment was designed. This comprised a single NIS server and five nodes uniformly distributed in a square of side 200m. Nodes were assumed to transmit data packets at a constant bit-rate (*i.e.*, CBR traffic) of 1 Mbps, and contention was decided by the CSMA/CA (carrier sense multiple-access/collision avoidance) mechanism of the IEEE 802.11b protocol [6].

The distribution of the activity factors was obtained by running the network simulator and these transmission time values were then used to execute the NIS algorithm in MATLAB. Since nodes were placed at different distances from the NIS, it was possible to estimate the channel coefficients as well as the inter-node distances with different degrees of accuracy. Fig. 1 shows the variation in channel coefficient estimation error (in percentage) with the increase in frame-length at different inter-node distances. The signal-to-noise ratio (SNR) was fixed at 40 dB at a distance of 1 m from the transmitter.

It was also observed that the estimation improved with longer frame-lengths. This follows from the fact that the channel coefficients are obtained by solving the set of deterministic simultaneous equations (14), which was approximated from (12) by assuming that each frame contained a large number of bits.

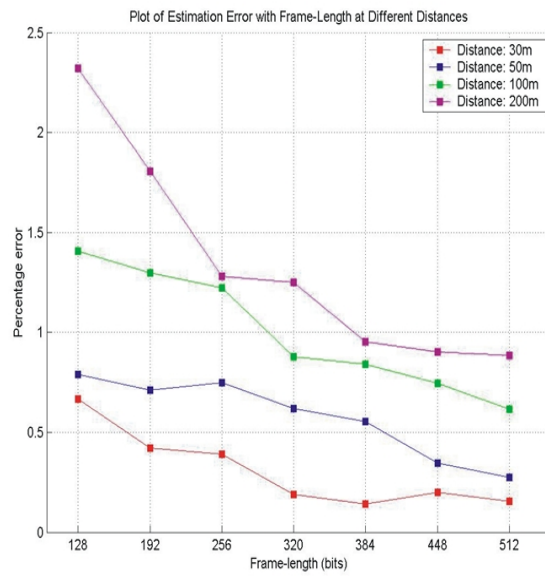


Figure 1: MATLAB plot of percentage error in estimating the channel coefficient against different frame-lengths for different inter-node distances.

5 Conclusion

This paper describes a theoretical framework to obtain inter-node distances within a network where a centralized Network Interference Server is allowed to keep track of the transmission activity of all nodes that use its service. Ideal channel coefficients are assumed in the analysis, including a known background noise variance and time-invariant channel coefficients. An improvement of the current thesis over previous formulations is that it roughly estimates the node neighborhood from the correlation between received power and transmitter activity *before* evaluating the channel coefficients, thus allowing the algorithm to be applied even to a densely populated network.

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