Interference Avoidance for Capacity Optimization in Mutually Interfering Wireless Systems

Otilia Popescu
WINLAB
Rutgers University
Email: otilia@winlab.rutgers.edu

Dimitrie C. Popescu
Department of Electrical Engineering
University of Texas at San Antonio
Email: dimitrie.popescu@utsa.edu

Christopher Rose
WINLAB
Rutgers University
Email: crose@winlab.rutgers.edu

***Abstract*— In this paper we present application of greedy interference avoidance to capacity optimization of wireless systems with multiple transmitters and receivers which interfere with each other. We start with a simple system consisting of two transmitters and two receivers for which we formally state a greedy interference avoidance algorithm, and show that its application converges to a simultaneous water filling solution. We discuss generalization of the algorithm to systems with more than two transmitters and receivers which interfere with each other, and present also numerical results obtained from simulations.**

I. Introduction

Wireless communication systems consist of a collection of nodes and links. Nodes are geographically distributed over some region, and can be either transmitters or receivers. In this paper we refer to transmitters as "users", and to receivers as "bases". Links connect active users with particular bases, and convey information. Under the assumption that the spectrum available for communication is shared by all nodes in the system (as it is the case in unlicensed bands) then the signal of a given user at the base to which it is intended is corrupted also by interference coming from other users in the system.

When bases are colocated, then the wireless communication system can be modeled as a multiple access channel [4], for which a vast body of literature offers numerous methods for capacity optimization. When bases are not colocated, but are allowed to collaborate and exchange information [6], [8], [12], [13], [19] the system can be regarded as a system with multiple inputs and multiple outputs (MIMO), and application of greedy interference avoidance in this scenario leads to a social optimum corresponding to maximum sum capacity [12], [13]. In general, when no cooperation among bases is assumed, the system is an instance of the general interference channel which is still an open research problem [4].

In our paper we present a distributed algorithm for capacity optimization in a Gaussian interference channel scenario consisting of multiple users and bases that do not collaborate. The algorithm, which is based on application of greedy interference avoidance [10], is along the line of recent work by Yu [20] where a different algorithm based on application of iterative water filling [21] is proposed for capacity optimization in a similar scenario.

II. System Description

We consider a wireless system with two bases, each base having only one user associated with it, which is depicted in Figure 1, and which is similar to the one considered by Yu [20]. However, unlike Yu [20] we explicitly assume

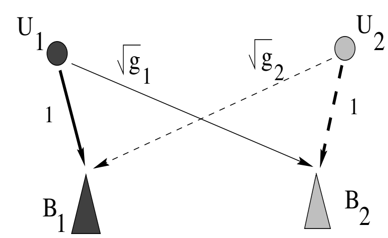

Fig. 1. A system with two transmitters and two receivers.

that users transmit information in frames using a multicode CDMA approach described schematically in Figure 2. Using

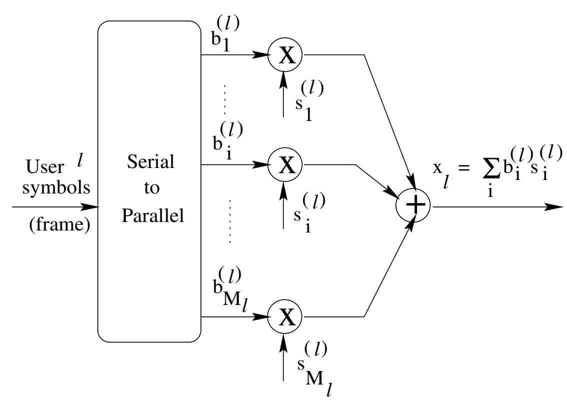

Fig. 2. Multicode CDMA approach for sending frames of information: each symbol in user ℓ's frame is assigned a distinct codeword. The transmitted signal is a superposition of all codewords scaled by their corresponding information symbols.

an arbitrary signal space representation of dimension N [9], the transmitted signal by user ℓ is expressed as

$$\mathbf{x}_\ell = \sum_{m=1}^{M_\ell} \mathbf{s}_m^{(\ell)} b_m^{(\ell)} = \mathbf{S}_\ell \mathbf{b}_\ell, \qquad \ell = 1, 2 \tag{1}$$

where $\mathbf{S}_\ell = \left[\mathbf{s}_1^{(\ell)} \ldots \mathbf{s}_m^{(\ell)} \ldots \mathbf{s}_{M_\ell}^{(\ell)}\right]$ is the $N \times M_\ell$ codeword matrix corresponding to user ℓ having as columns the unit norm codewords of user ℓ, and $\mathbf{b}_\ell = [b_1^{(\ell)} \dots b_{M_\ell}^{(\ell)}]^\top$ is the vector containing the information symbols transmitted by user ℓ. We assume that user ℓ transmits at least as many symbols as signal dimensions, that is $M_\ell \geq N$, so that its corresponding codeword matrix may span the entire signal space. We also assume that all the codewords of a given user are affected by the same channel, which is flat and characterized by a scalar gain factor. Without loss of generality we take the gain of each user to the base to which it is assigned with to be normalized to unity, and denote by g_1 the gain corresponding to user 1's signal at base 2 and g_2 is the gain corresponding to user 2's signal at base 1. The received vectors at each base station are

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{S}_1\mathbf{b}_1 + \sqrt{g_2}\mathbf{U}\mathbf{S}_2\mathbf{b}_2 + \mathbf{n}_1 \\ \mathbf{r}_2 &= \sqrt{g_1}\mathbf{U}^\top\mathbf{S}_1\mathbf{b}_1 + \mathbf{S}_2\mathbf{b}_2 + \mathbf{n}_2 \end{aligned} \quad (2)$$

where $\mathbf{U}$ is a unitary matrix converting the basis function sets between the two users' signal spaces. For simplicity and with no loss of generality we assume that this is equal to the identity matrix,$\mathbf{U} = \mathbf{I}_N$. The noise at each base is assumed white, with the same scaled identity covariance matrix $E[\mathbf{n}_1\mathbf{n}_1^\top] = E[\mathbf{n}_2\mathbf{n}_2^\top] = \eta_0\mathbf{I}_N$. Individual user capacities are expressed as

$$\begin{aligned} C_1 &= 0.5\log|\mathbf{R}_1| - 0.5\log|g_2\mathbf{S}_2\mathbf{S}_2^\top + \eta_0\mathbf{I}_N| \\ C_2 &= 0.5\log|\mathbf{R}_2| - 0.5\log|g_1\mathbf{S}_1\mathbf{S}_1^\top + \eta_0\mathbf{I}_N| \end{aligned} \quad (3)$$

where

$$\begin{aligned} \mathbf{R}_1 &= \mathbf{S}_1\mathbf{S}_1^\top + g_2\mathbf{S}_2\mathbf{S}_2^\top + \eta_0\mathbf{I}_N \\ \mathbf{R}_2 &= g_1\mathbf{S}_1\mathbf{S}_1^\top + \mathbf{S}_2\mathbf{S}_2^\top + \eta_0\mathbf{I}_N \end{aligned} \quad (4)$$

are the covariance matrices of the received signals at the two base stations.

Our goal is to apply greedy interference avoidance methods to adaptation of user codewords in this system, such that the corresponding user capacity will be optimized.

III. The Greedy Interference Avoidance Algorithm

Greedy interference avoidance is sequentially applied for all codewords of one user while the other (interfering) user, whose codewords are kept fixed, is treated as Gaussian noise. This procedure is iterated for both users until a fixed point for the whole system is reached. The algorithm is formally stated below:

Greedy Interference Avoidance for Two Non-Collaborative Bases

1) Initialize user codeword matrices $\mathbf{S}_1$, $\mathbf{S}_2$ and gains g_1, g_2
2) For each codeword of user 1 $\mathbf{s}_m^{(1)}$, $m = 1, \dots, M_1$
   - replace $\mathbf{s}_m^{(1)}$ with minimum eigenvector of $\mathbf{R}_1 - \mathbf{s}_m^{(1)}\mathbf{s}_m^{(1)\top}$
3) Repeat Step 2 until a fixed point is reached
4) For each codeword of user 2 $\mathbf{s}_n^{(1)}$, $n = 1, \dots, M_2$
   - replace $\mathbf{s}_n^{(2)}$ with minimum eigenvector of $\mathbf{R}_2 - \mathbf{s}_n^{(2)}\mathbf{s}_n^{(2)\top}$
5) Repeat Step 4 until a fixed point is reached
6) Repeat Steps 2 – 5 until a fixed point is reached

Numerically, a fixed point of the algorithm is defined with respect to a stopping criterion and is reached when the difference between two consecutive values is within a specified tolerance ϵ. In our simulations we used the individual user capacities given in equation (3) as stopping criteria.

We note that repeated application of greedy interference avoidance at one base ℓ while interference coming from the other base is fixed, is guaranteed to converge to the optimal fixed point which maximizes C_ℓ, and for which user ℓ water fills over its interference-plus-noise covariance matrix while treating the other user as Gaussian interference [10], [16]. We also note that performed iteratively for the two user-base pairs, the greedy interference avoidance algorithm can be regarded as an instance of the general iterative water filling procedure established by Yu et. al [20], [21] and is therefore guaranteed to converge to a fixed point corresponding to a simultaneous water filling distribution for both users in their respective signal spaces [20], [21].

Simulations of the greedy interference avoidance algorithm show the simultaneous water filling distribution achieved by both users in their respective signal spaces, and for illustration we present examples of typical distribution of eigenvalues of the resulting covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ in Figures 3 and 4. We note that different colors are used to distinguish between the two users, and the way their energy contributes to the eigenvalue distribution in the covariance matrices.

For the example in Figure 3 at the end of the algorithm the system reached a simultaneous water filling distribution with complete overlap between users in signal space. As the gains g_1 and g_2 are increased, the simultaneous water filling distribution will correspond to complete user separation in signal space, and a typical example in this sense is presented in Figure 4. We note that simultaneous water filling distributions for which users overlap only partially in signal space are also possible, and that a detailed analysis of the potential signal space configurations corresponding to the simultaneous water filling solution can be found in [11].

We have also looked at the variation of different capacity measures during the algorithm. In addition to user capacities C_1 and C_2 in equation (3), we have also computed the sum capacity C_{sum} of the whole system assuming collaboration as in [12], [13] and compared it with the sum of individual user capacities $C_1 + C_2$. We call this latter measure

$$\mathcal{C} = C_1 + C_2 \quad (5)$$

collective capacity [11] in order to distinguish it from the information-theoretic *sum capacity* computed for the collaborative scenario in [12], [13], and use it as a global performance measure for the system when no collaboration is assumed.

Simulations have shown that the capacity measures mentioned before are usually increased by the algorithm, although the increase is not always monotonic. For cases where the final distribution corresponds to a complete user overlap in signal space, as the one presented in Figure 3, C_{sum} increases monotonically and reaches the upper bound derived in [12],

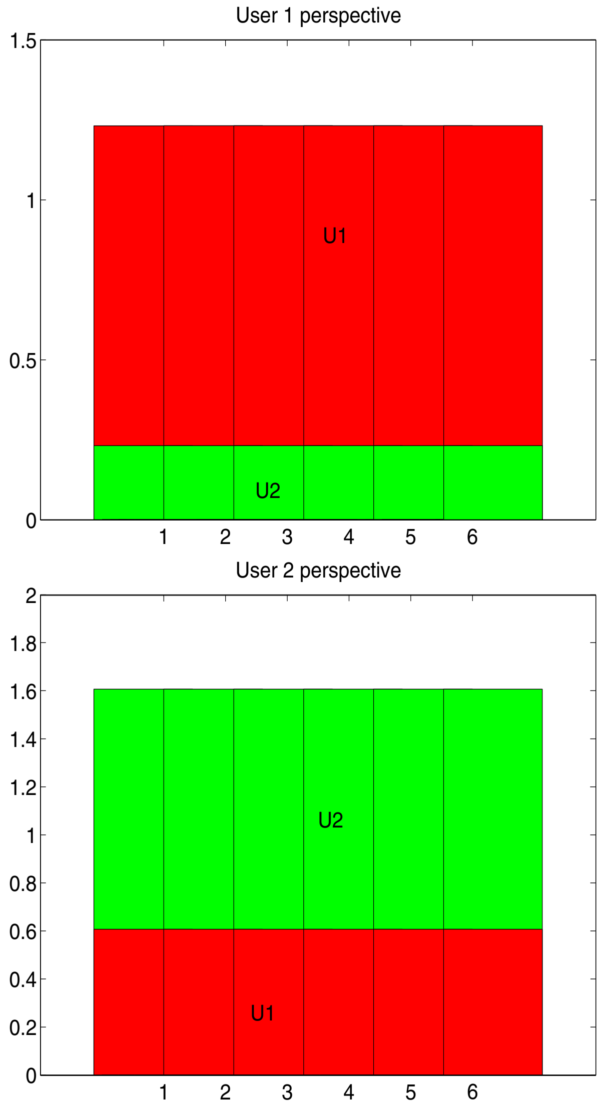

Fig. 3. Distribution of eigenvalues of covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ corresponding to simultaneous water filling for a two user-base system with $N = 6$, $g_1 = 0.2311$, $g_2 = 0.6068$, and $\eta_0 = 1$. Users overlap in all signal dimensions in this case.

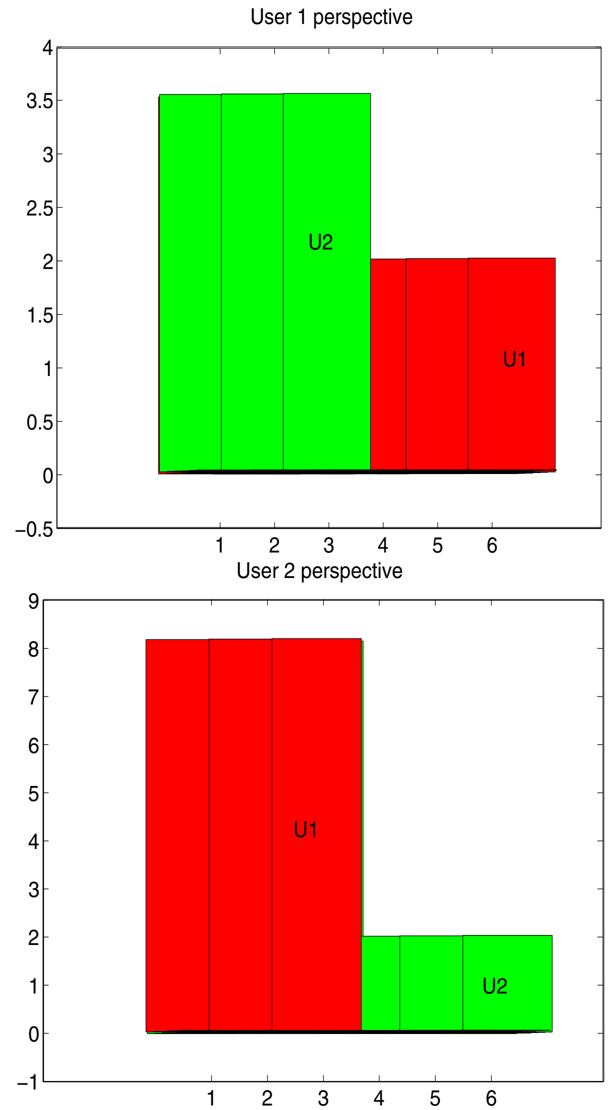

Fig. 4. Distribution of eigenvalues of covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ corresponding to simultaneous water filling for a two user-base system with $N = 6$, $g_1 = 1.7643$, $g_2 = 4.0658$, and $\eta_0 = 1$. Users reside in orthogonal, non-overlapping signal subspaces in this case.

[13]. The collective capacity achieved at the end of the algorithm in this case, $\mathcal{C} = 3.2353$ is smaller than $C_{\text{sum}} = 4.3260$ which is equal to the maximum bound. For cases when the final distribution corresponds to separation of users in signal space, as the one presented in Figure 4, the increase in sum capacity is no longer monotonic. For the particular example in Figure 4 the final sum capacity value is 6.4290 and is smaller than the upper bound derived in [12], [13] which in this case is 7.3651, but is again larger than the collective capacity which in this case is 3.2958.

Application of the greedy interference avoidance algorithm can be generalized in a straightforward way to a system with more than two user-base pairs which do not collaborate, and we formally state the generalized algorithm here:

Greedy Interference Avoidance for Multiple Non-Collaborative Bases

1) Initialize user codeword matrices $\{\mathbf{S}_\ell\}$ and gains $\{g_{ij}\}$ between users and bases[1]
2) For each user-base pair $i = 1, \ldots, B$
   a) For each codeword $\mathbf{s}_j^{(i)}$ of $\mathbf{S}_i$, $j = 1, \ldots, M_i$
      - replace $\mathbf{s}_j^{(i)}$ with minimum eigenvector of $\mathbf{R}_i - \mathbf{s}_j^{(i)} \mathbf{s}_j^{(i)\top}$
   b) Repeat Step 2a until a fixed point is reached
3) Repeat Step 2 until a fixed point is reached.

[1]We still consider the system normalized with gains from a given user i to its associated base i $g_{ii} = 1$.

The same iterative water filling argument applied in the case of two bases can be applied in the case of multiple bases, and ensures that the greedy interference avoidance algorithm achieves a fixed point which corresponds to a simultaneous water filling distribution for all users in their respective signal spaces while regarding the other users as Gaussian interference.

Simulations of the greedy interference avoidance algorithm with multiple bases show the simultaneous water filling distribution achieved at a fixed point, and for illustration we present in Figure 5 the distribution of eigenvalues of the resulting covariance matrices for a system with three user-base pairs. In this example $N = 9$, $\eta_0 = 1$ and the gains, generated randomly, are given by

$$\mathbf{G} = \begin{bmatrix} 1.0000 & 0.0150 & 0.9901 \\ 0.3050 & 1.0000 & 0.7889 \\ 0.8744 & 0.9708 & 1.0000 \end{bmatrix}$$

with $G(i, j)$ the gain from user i to base j.

By varying the gain values between users and base stations one can observe the three possible signal space configurations corresponding to simultaneous water filling analyzed in [11]. In the case of weak and moderate interference [3] users overlap completely or partially in signal space: when the gains are small (all of them smaller than 1) users completely overlap in all dimensions of the signal space, and as the gains increase (some of the gains getting close or over 1) the simultaneous water filling structure becomes more complex in the signal space, with users overlapping only partially. In the case of strong interference [1], [2], [7], [17], [18] (all gains larger than 1) users become completely separated.

We close this section by noting that users in the non-collaborative multi-base wireless system considered in this paper can also be regarded as players in a non-cooperative game in which they compete for data rates with the sole objective of maximizing individual performance regardless of the other users [20]. Different strategies can be employed by users for performance maximization in this non-cooperative game. In the case of our proposed algorithm data rate maximization is achieved by adjusting user codewords using greedy interference avoidance, whereas in [20] this is achieved by applying iterative water filling and adjusting the power allocation over frequencies through adaptation of user transmit covariance matrices.

Using this game theoretic approach it has been shown [20] that the simultaneous water filling solution, which is a fixed point for both greedy interference avoidance and iterative water filling algorithms, is a Nash equilibrium for the non-collaborative multi-base system. Thus, at a simultaneous water filling point each user's strategy is an optimal response to the other user's strategy, and no unilateral strategy change by any user will be able to improve individual user performance [5, Ch. 1]. In general this Nash equilibrium point is not unique, and the algorithm may converge to different fixed points [11] for which a complete analysis is presented in [14], [15].

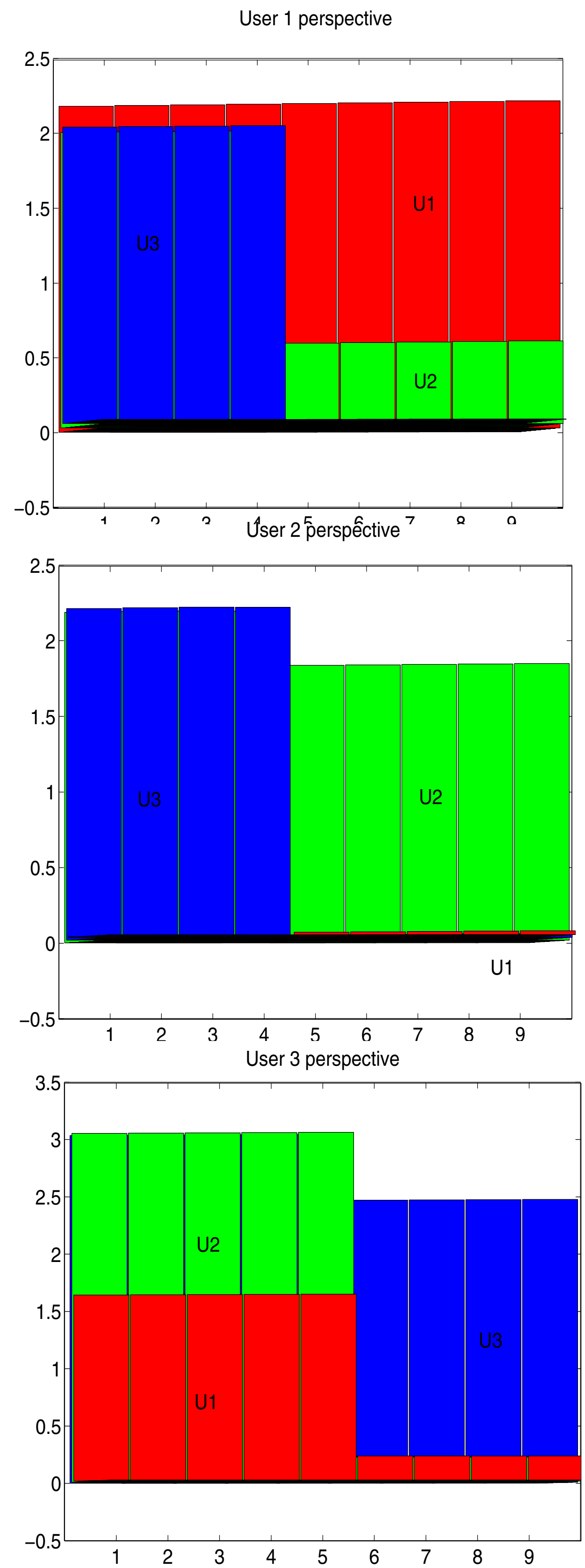

Fig. 5. Distribution of Eigenvalues of Covariance Matrices $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ Corresponding to Simultaneous Water Filling for a Three User-Base System

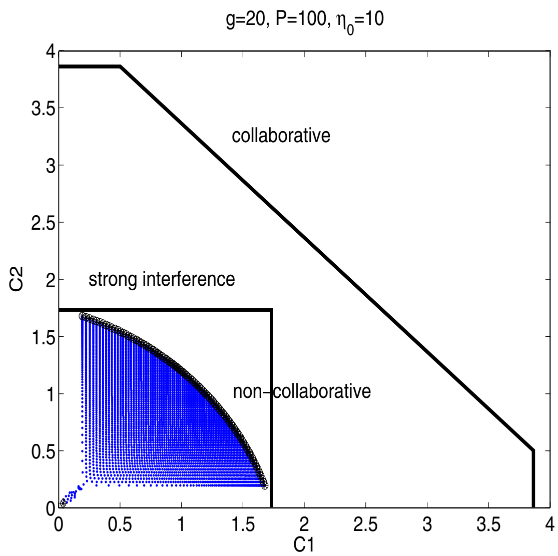

Fig. 6. Comparing the capacity performances between collaborative and non-collaborative multi-base systems.

IV. Discussion and Conclusions

In this paper we presented application of interference avoidance methods for capacity optimization in mutually interfering wireless systems. We introduced greedy interference avoidance for a simple system with two users and two bases similar to the one considered in [20], and subsequently generalized it for any number of users and bases. We also showed that the algorithm converges to a simultaneous water filling solution [21] for which each user water fills its corresponding signal space by regarding the other (interfering) users as Gaussian noise, and provided numerical examples illustrating this solution.

We conclude our paper with a brief comparison of the considered non-collaborative scenario in which users interfere with each other, with a collaborative scenario as in [12], [13] in which bases exchange information and perform joint decoding. For illustration we consider a two user symmetric system, with user powers $P_1 = P_2 = P = 100$, noise level $\eta_0 = 10$, and user gain to the other base $g_1 = g_2 = 20$. This value of the gain corresponds to a strong interference system as defined in [1]. Application of greedy interference avoidance in the non-collaborative scenario as presented in this paper leads to a fixed point situated inside the "non-collaborative" simultaneous water filling region [11] in Figure 6. Subtracting interference as suggested in [1] users can improve performance and achieve capacities larger than those corresponding to the simultaneous water filling fixed points which are situated inside the capacity region of the strong interference case (the rectangle denoted "strong interference" in Figure 6 with corners $C_i = \frac{1}{2}\log\left(1+\frac{P_i}{\eta_0}\right)$, $i = 1, 2$). Further improvement in user capacities is possible if collaboration is allowed: the capacity region in this case is the pentagon denoted "collaborative" in Figure 6 (computed according to [4, p. 407]) with corners having coordinates $\left(\frac{1}{2}\log\left(1+\frac{P_1\prime}{P_2\prime+\eta_0}\right), \frac{1}{2}\log\left(1+\frac{P_2\prime}{\eta_0}\right)\right)$ respectively $\left(\frac{1}{2}\log\left(1+\frac{P_1\prime}{\eta_0}\right), \frac{1}{2}\log\left(1+\frac{P_2\prime}{P_1\prime+\eta_0}\right)\right)$, where in our example $P_1\prime = P_2\prime = gP + P$).

We note that as the gains increase, the pentagon corresponding to the collaborative scenario becomes wider. This implies a larger difference in performance between the collaborative and non-collaborative cases. Therefore, when the interference level is high, collaboration is far superior to any non-collaborative method.

Acknowledgements

This work was supported in part by the National Science Foundation under grant CCR-0312323, and by the Texas Higher Education Coordinating Board under Advanced Technology Program grant 000512-0261-2003.

References

[1] A. B. Carleial. A Case Where Interference Does Not Reduce Capacity. *IEEE Transactions on Information Theory*, 21(6):569–570, Sept. 1975.
[2] A. B. Carleial. Interference Channels. *IEEE Transactions on Information Theory*, 24(1):60–70, January 1978.
[3] M. Costa. On the Gaussian Interference Channel. *IEEE Transactions on Information Theory*, 31(5):607–615, September 1985.
[4] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley-Interscience, New York, NY, 1991.
[5] D. Fudenberg and J. Tirole. *Game Theory*. MIT Press, 1991.
[6] A. Goldsmith, S. Jafar, and G. J. Foschini. Exploring Optimal Multicellular Multiple Antenna Systems. In *Proceedings* 56^{th} *IEEE Vehicular Technology Conference – VTC'02 Fall*, volume 1, pages 261–265, Vancouver, Canada, September 2002.
[7] T. S. Han and K. Kobayashi. A New Achievable Rate Region for the Interference Channel. *IEEE Transactions on Information Theory*, 27(1):49 – 60, January 1981.
[8] S. V. Hanly and P. A. Whiting. Information-Theoretic Capacity of Multi-Receiver Networks. *Telecommunications Systems*, 1(1):1–42, 1993.
[9] H. J. Landau and H. O. Pollack. Prolate Spheroidal Wave Functions, Fourier Analysis and Uncertainty – III: The Dimension of the Space of Essentially Time- and Band-Limited Signals. *The Bell System Technical Journal*, 41(4):1295–1335, July 1962.
[10] D. C. Popescu and C. Rose. *Interference Avoidance Methods for Wireless Systems*. Kluwer Academic Publishers, New York, NY, 2004.
[11] O. Popescu and C. Rose. Water Filling May Not Good Neighbors Make. In *Proc. 2003 IEEE Global Telecommunications Conference – GLOBECOM '03*, vol. 3, p. 1766–1770, San Francisco, CA, Dec. 2003.
[12] O. Popescu and C. Rose. Greedy SINR Maximization in Collaborative Multi-Base Wireless Systems. *EURASIP Journal on Wireless Communications and Networking*, 2004(2):201–209, December 2004.
[13] O. Popescu and C. Rose. Sum Capacity and TSC Bounds in Collaborative Multi-Base Wireless Systems. *IEEE Transactions on Information Theory*, 50(10):2433–2438, October 2004.
[14] O. Popescu, C. Rose, and D. C. Popescu. Signal Space Partitioning Versus Simultaneous Water Filling for Mutually Interfering Systems. In *Proc. 2004 IEEE Global Telecommunications Conference – GLOBECOM'04*, vol. 5, p. 3128–3132, Dallas, TX, Dec. 2004.
[15] O. Popescu, C. Rose, and D. C. Popescu. Strong Interference and Spectrum Warfare. In *Proceedings* 38^{th} *Conference on Information Sciences and Systems – CISS 2004*, Princeton, NJ, March 2004.
[16] C. Rose. CDMA Codeword Optimization: Interference Avoidance and Convergence Via Class Warfare. *IEEE Transactions on Information Theory*, 47(6):2368–2382, Sept. 2001.
[17] H. Sato. On the Capacity Region of a Discrete Two-User Channel for Strong Interference. *IEEE Transactions on Information Theory*, 24(3):377–779, May 1978.
[18] H. Sato. The Capacity of the Gaussian Interference Channel Under Strong Interference. *IEEE Transactions on Information Theory*, 27(6):786–788, Nov. 1981.
[19] D. Tse and S. Hanly. Multi-access Fading Channels. Part I: Polymatroid Structure, Optimal Resource Allocation and Throughput Capacities. *IEEE Transaction on Information Theory*, 44(7):2796 – 2815, Nov. 1998.
[20] W. Yu. *Competition and Cooperation in Multiuser Communication Environments*. PhD thesis, Stanford University, Department of Electrical Engineering, 2002. Principal Advisor: Prof. J. M. Cioffi.
[21] W. Yu, W. Rhee, S. Boyd, and J. M. Cioffi. Iterative Water-Filling for Gaussian Vector Multiple-Access Channels. *IEEE Transactions on Information Theory*, 50(1):145–152, January 2004.