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Greedy Interference Avoidance in Non-Collaborative Multi-Base Wireless Systems

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Abstract — **In this paper we present application of greedy interference avoidance to wireless systems with multiple bases which do not collaborate or exchange information. We start with a simple system consisting of two users and two base stations for which we formally state a greedy interference avoidance algorithm and show that its application converges to a simultaneous water filling solution which is also a Nash equilibrium point for the system. We discuss generalization of the algorithm to systems with more than two user-base pairs which do not collaborate, and present also numerical results obtained from simulations.**

I. Introduction

Interference avoidance has emerged in the literature as a method for distributed codeword adaptation in CDMA systems. Introduced originally in the context of a single cell CDMA system with MMSE receivers [2, 27] MMSE interference avoidance was followed by greedy interference avoidance which uses matched filter receivers and a minimum eigenvector approach [21, 22]. For a single cell system interference avoidance was subsequently extended to incorporate non-ideal channels between users and the base station [6, 12, 13].

In a single cell system all users communicate with a single base station which knows codewords for all users in the system and uses them to decode the transmitted information symbols. In general, wireless systems consist of a collection of users and base stations dispersed over a given geographical area, in which individual users are interested in sending information only to a particular base station and each base cares only about decoding the users assigned to it. For wireless systems with multiple base stations interference avoidance was also applied in a collaborative scenario which assumes that information received at all bases is collectively available for decoding [15–17]. Under this scenario the system under consideration can be regarded as a system with multiple inputs and multiple outputs (MIMO), and application of greedy interference avoidance in this scenario leads to a social optimum corresponding to maximum sum capacity.

When no collaboration among base stations operation is assumed, a given user is decoded at its associated base under interference from transmissions by other users intended for other bases. In the most general case, with Gaussian channels and multiple transmitters and receivers such systems are instances of the general interference channel, which is still an open research problem [8, p. 382]. We note that the interference channel problem was formulated originally by Shannon [26] and main results on capacity for the interference channel were established by Ahlswede [1], Carleial [3–5], Sato [23–25], Han and Kobayashi [10], and Costa [7]. We also note that an information theoretic approach to the interference channel problem is beyond the scope of this paper, and we make no attempt to address capacity issues for the interference channel here.

Rather, our work is along the line of more recent work by Yu [28] which proposes a distributed iterative algorithm for performance optimization in a Gaussian interference channel scenario, and in this paper we present application of greedy interference avoidance methods to non-collaborative multi-base wireless systems. We start with a simple system consisting of two bases, each base having only one user associated with it, which is depicted in Figure 1, and which is similar to the one considered by Yu [28]. However, unlike Yu [28] we explicitly assume that users transmit information using a multicode CDMA approach for which we plan to apply greedy interference avoidance to adaptation of user codewords. In this paper we present an iterative algorithm for codeword optimization for the two user-base system which is based on greedy interference avoidance. We look at fixed-point properties of the proposed algorithm and discuss extensions to systems with more than two user-base pairs.

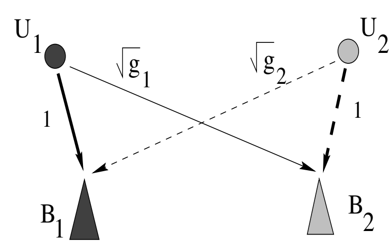

Figure 1: A system with two transmitters and two receivers.

II. System Description and the Greedy Interference Avoidance Algorithm

For the non-collaborative system with two users and two base stations in Figure 1 we assume an arbitrary signal space representation[1] of dimension N, in which during each signaling interval users transmit frames of data using a multicode CDMA approach described schematically in Figure 2. Thus, each user

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[1]Implied by finite bandwidth and finite signaling interval constraints [11].

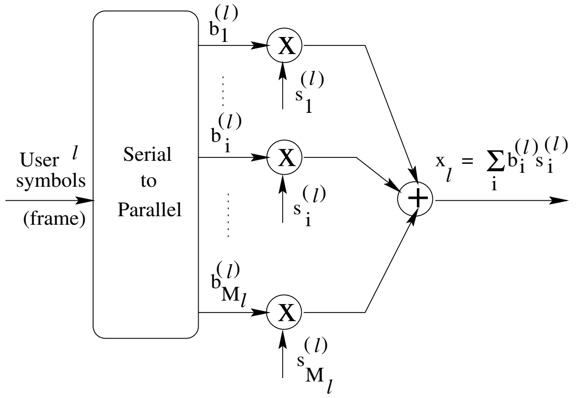

Figure 2: Multicode CDMA approach for sending frames of information: each symbol in user ℓ's frame is assigned a distinct codeword. The transmitted signal is a superposition of all codewords scaled by their corresponding information symbols.

ℓ at a given location transmits the signal

$$\mathbf{x}_\ell = \sum_{m=1}^{M_\ell} \mathbf{s}_m^{(\ell)} b_m^{(\ell)} = \mathbf{S}_\ell \mathbf{b}_\ell, \qquad \forall \ell = 1, \ldots, L \tag{1}$$

where

$$\mathbf{S}_\ell = \begin{bmatrix} | & & | & & | \\ \mathbf{s}_1^{(\ell)} & \cdots & \mathbf{s}_m^{(\ell)} & \cdots & \mathbf{s}_{M_\ell}^{(\ell)} \\ | & & | & & | \end{bmatrix} \tag{2}$$

is the $N \times M_\ell$ codeword matrix corresponding to user ℓ having as columns the unit norm codewords corresponding to user ℓ, and $\mathbf{b}_\ell = [b_1^{(\ell)} \ldots b_{M_\ell}^{(\ell)}]^\top$ is the vector containing the information symbols transmitted by user ℓ. We assume that user ℓ transmits at least as many symbols as signal dimensions, that is $M_\ell \geq N$ so that its corresponding codeword matrix may span the entire signal space. We also assume that each user is received at the base to which it is assigned with unit power, and that all the codewords of a given user are affected by the same channel, which is flat and characterized by gain factor g_ℓ. The received vectors at each base station are

$$\begin{aligned} \mathbf{r}_1 &= \mathbf{S}_1\mathbf{b}_1 + \sqrt{g_2}\mathbf{U}_{21}\mathbf{S}_2\mathbf{b}_2 + \mathbf{n}_1 \\ \mathbf{r}_2 &= \sqrt{g_1}\mathbf{U}_{12}\mathbf{S}_1\mathbf{b}_1 + \mathbf{S}_2\mathbf{b}_2 + \mathbf{n}_2 \end{aligned} \tag{3}$$

where g_1 is the gain corresponding to user 1's signal at base 2, g_2 is the gain corresponding to user 2's signal at base 1, and the unitary matrices $\mathbf{U}_{ij}$ convert the *bases*[2] between the two users' signal spaces. For simplicity and with no loss of generality we will assume $\mathbf{U}_{ij} = \mathbf{I}$. The noise at each base is assumed white, with the same covariance matrix $E[\mathbf{n}_1\mathbf{n}_1^\top] = E[\mathbf{n}_2\mathbf{n}_2^\top] = \eta_0\mathbf{I}_N$. The covariance matrices of the received signals at the two base stations are

$$\begin{aligned} \mathbf{R}_1 &= \mathbf{S}_1\mathbf{S}_1^\top + g_2\mathbf{S}_2\mathbf{S}_2^\top + \eta_0\mathbf{I}_N \\ \mathbf{R}_2 &= g_1\mathbf{S}_1\mathbf{S}_1^\top + \mathbf{S}_2\mathbf{S}_2^\top + \eta_0\mathbf{I}_N \end{aligned} \tag{4}$$

For this simple non-collaborative multibase system we apply interference avoidance to adaptation of user codewords as follows: greedy interference avoidance is sequentially applied for all codewords of one user while the other (interfering) user, whose codewords are kept fixed, is treated as Gaussian noise. This procedure is iterated for both users until a fixed point for the whole system is reached. The algorithm is formally stated below:

[2] Bases as in basis sets.

Greedy Interference Avoidance for Two Non-Collaborative Bases

1. Initialize user codeword matrices $\mathbf{S}_1$, $\mathbf{S}_2$ and gains g_1, g_2
2. For each codeword of user 1 $\mathbf{s}_m^{(1)}$, $m = 1, \ldots, M_1$
   - replace $\mathbf{s}_m^{(1)}$ with minimum eigenvector of $\mathbf{R}_1 - \mathbf{s}_m^{(1)}\mathbf{s}_m^{(1)\top}$
3. Repeat Step 2 until a fixed point is reached
4. For each codeword of user 2 $\mathbf{s}_n^{(1)}$, $n = 1, \ldots, M_2$
   - replace $\mathbf{s}_n^{(2)}$ with minimum eigenvector of $\mathbf{R}_2 - \mathbf{s}_n^{(2)}\mathbf{s}_n^{(2)\top}$
5. Repeat Step 4 until a fixed point is reached
6. Repeat Steps 2 – 5 until a fixed point is reached

A fixed point of the algorithm is defined with respect to a stopping criterion. That is, we say that a fixed point is reached when the difference between two consecutive values of the stopping criterion is within a specified tolerance ϵ. The stopping criterion used is the individual performance of a given user in terms of its capacity considering the other (interfering) user as Gaussian interference. Individual user capacities are expressed as

$$\begin{aligned} C_1 &= \frac{1}{2}\log|\mathbf{R}_1| - \frac{1}{2}\log|g_2\mathbf{S}_2\mathbf{S}_2^\top + \eta_0\mathbf{I}_N| \\ C_2 &= \frac{1}{2}\log|\mathbf{R}_2| - \frac{1}{2}\log|g_1\mathbf{S}_1\mathbf{S}_1^\top + \eta_0\mathbf{I}_N| \end{aligned} \tag{5}$$

We note that repeated application of greedy interference avoidance at one base ℓ while interference coming from the other base is fixed, is guaranteed to converge to the optimal fixed point which maximizes C_ℓ, and for which user ℓ water fills over its interference-plus-noise covariance matrix while treating the other user as Gaussian interference [14, 21]. We also note that performed iteratively for the two user-base pairs, the greedy interference avoidance algorithm can be regarded as an instance of the general iterative water filling procedure established by Yu et. al [28,29] and is therefore guaranteed to converge to a fixed point corresponding to a simultaneous water filling distribution for both users in their respective signal spaces [28,29].

Simulations of the greedy interference avoidance algorithm show the simultaneous water filling distribution achieved by both users in their respective signal spaces, and for illustration we present examples of typical distribution of eigenvalues of the resulting covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ in Figures 3 and 4. We note that different colors are used to distinguish between the two users, and the way their energy contributes to the eigenvalue distribution in the covariance matrices.

For the example in Figure 3 at the end of the algorithm the system reached a simultaneous water filling distribution with complete overlap between users in signal space. As the gains g_1 and g_2 are increased, the simultaneous water filling

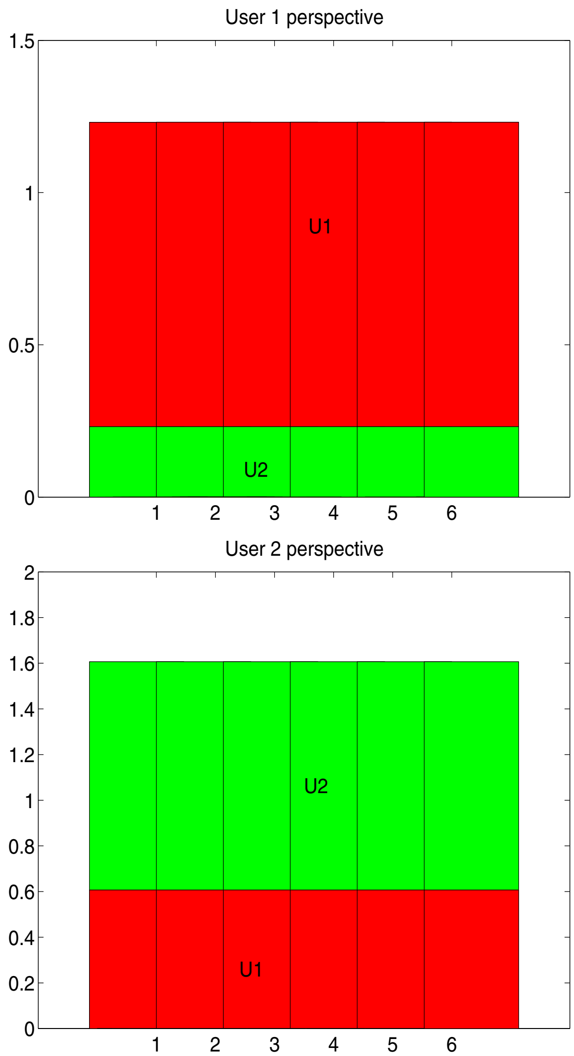

Figure 3: Distribution of eigenvalues of covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ corresponding to simultaneous water filling for a two user-base system with $N = 6$, $g_1 = 0.2311$, $g_2 = 0.6068$, and $\eta_0 = 1$. Users overlap in all signal dimensions in this case.

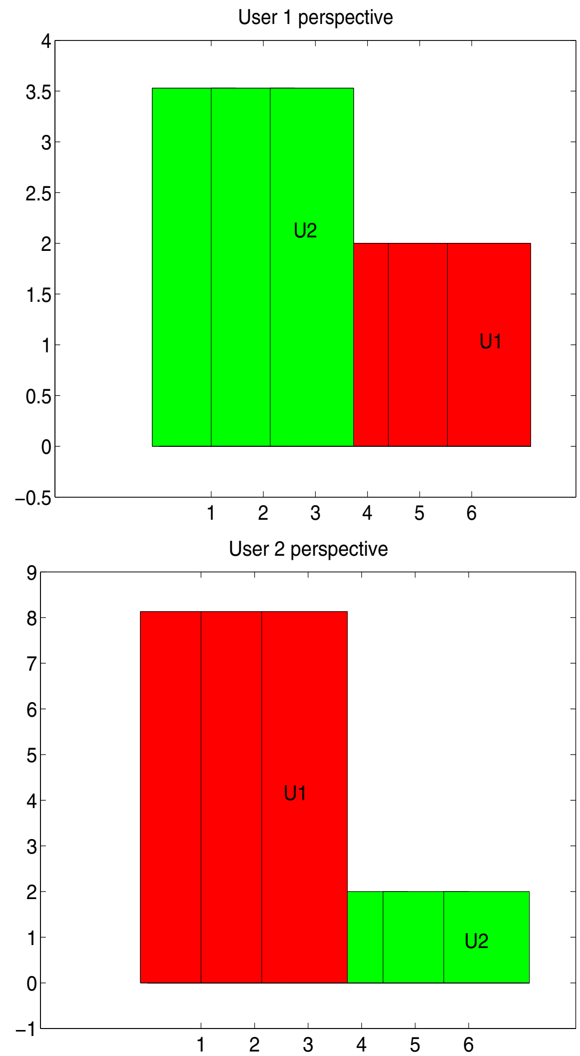

Figure 4: Distribution of eigenvalues of covariance matrices $\mathbf{R}_1$ and $\mathbf{R}_2$ corresponding to simultaneous water filling for a two user-base system with $N = 6$, $g_1 = 1.7643$, $g_2 = 4.0658$, and $\eta_0 = 1$. Users reside in orthogonal, non-overlapping signal subspaces in this case.

distribution will correspond to complete user separation in signal space, and a typical example in this sense is presented in Figure 4. We note that simultaneous water filling distributions for which users overlap only partially in signal space are also possible, and that a detailed analysis of the potential signal space configurations corresponding to the simultaneous water filling solution can be found in [18].

We have also looked at the variation of different capacity measures during the algorithm. In addition to user capacities C_1 and C_2 in equation (5), we have also computed the sum capacity C_{sum} of the whole system assuming collaboration as in [15, 16] and compared it with the sum of individual user capacities $C_1 + C_2$. We call this latter measure

$$\mathcal{C} = C_1 + C_2 \tag{6}$$

collective capacity [15, 18] in order to distinguish it from the information-theoretic *sum capacity* computed for the collaborative scenario in [15, 16], and use it as a global performance measure for the system when no collaboration is assumed.

Simulations have shown that the capacity measures mentioned before are usually increased by the algorithm, although the increase is not always monotonic. For cases where the final distribution corresponds to a complete user overlap in signal space, as the one presented in Figure 3, C_{sum} increases monotonically and reaches the upper bound derived in [15, 16]. The collective capacity achieved at the end of the algorithm in this case, $\mathcal{C} = 3.2353$ is smaller than $C_{\text{sum}} = 4.3260$ which is equal to the maximum bound. For cases when the final distribution corresponds to separation of users in signal space, as the one presented in Figure 4, the increase in sum capacity is no longer monotonic. For the particular example in Figure 4 the final sum capacity value is 6.4290 and is smaller than the upper bound derived in [15, 16] which in this case is 7.3651, but is again larger than the collective capacity which in this case is 3.2958.

Application of the greedy interference avoidance algorithm can be generalized in a straightforward way to a system with more than two user-base pairs which do not collaborate, and we formally state the algorithm here:

Greedy Interference Avoidance for Multiple Non-Collaborative Bases

1. Initialize user codeword matrices $\{\mathbf{S}_\ell\}$ and gains $\{g_{ij}\}$ between users and bases[3]
2. For each user-base pair $i = 1, \ldots, B$
   (a) For each codeword $\mathbf{s}_j^{(i)}$ of $\mathbf{S}_i$, $j = 1, \ldots, M_i$
      - replace $\mathbf{s}_j^{(i)}$ with minimum eigenvector of $\mathbf{R}_i - \mathbf{s}_j^{(i)}\mathbf{s}_j^{(i)\top}$

   (b) Repeat Step 2a until a fixed point is reached
3. Repeat Step 2 until a fixed point is reached.

The same iterative water filling argument applied in the case of two bases can be applied in the case of multiple bases and ensures that the greedy interference avoidance algorithm achieves a fixed point which corresponds to a simultaneous water filling distribution for all users in their respective signal spaces while regarding the other users as Gaussian interference.

Simulations of the greedy interference avoidance algorithm with multiple bases show the simultaneous water filling distribution achieved at a fixed point, and for illustration we present in Figure 5 the distribution of eigenvalues of the resulting covariance matrices for a system with three user-base pairs. In this example $N = 9$, $\eta_0 = 1$ and the gains, generated randomly, are given by

$$\mathbf{G} = \begin{bmatrix} 1.0000 & 0.0150 & 0.9901 \\ 0.3050 & 1.0000 & 0.7889 \\ 0.8744 & 0.9708 & 1.0000 \end{bmatrix}$$

with $G(i, j)$ the gain from user i to base j.

By varying the gain values between users and base stations one can observe the three possible signal space configurations corresponding to simultaneous water filling analyzed in [18]. In the case of weak and moderate interference [7] users overlap completely or partially in signal space: when the gains are small (all of them smaller than 1) users completely overlap in all dimensions of the signal space, and as the gains increase (some of the gains getting close or over 1) the simultaneous water filling structure becomes more complex in the signal space, with users overlapping only partially. In the case of strong interference [3, 4, 10, 24, 25] (all gains larger than 1) users become completely separated.

III. Conclusions

In this paper we presented application of greedy interference avoidance for wireless systems with multiple base stations which do not collaborate. Greedy interference avoidance was introduced for a simple system with two users and two base stations similar to the one considered in [28], and was subsequently generalized for any number of user and bases. We showed that the algorithm converges to a simultaneous water filling solution [29] for which each user water fills its corresponding signal space by regarding the other (interfering)

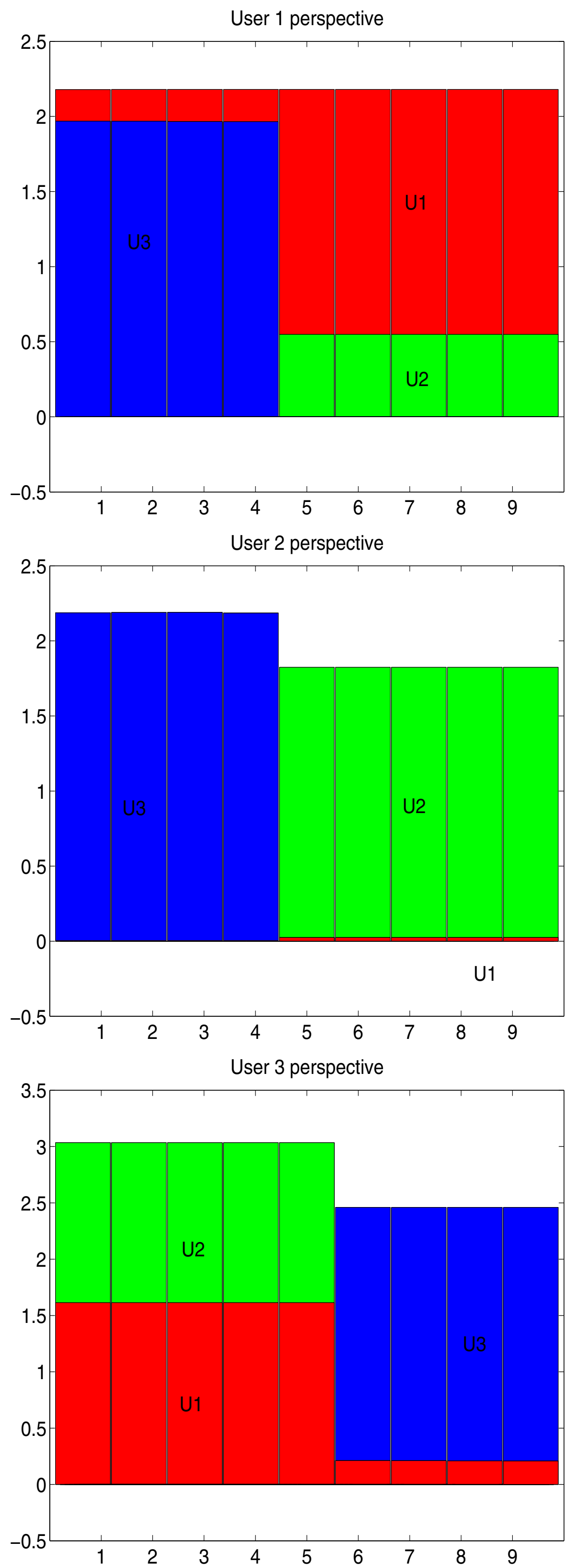

Figure 5: Distribution of Eigenvalues of Covariance Matrices $\mathbf{R}_1$, $\mathbf{R}_2$, and $\mathbf{R}_3$ Corresponding to Simultaneous Water Filling for a Three User-Base System

[3]We still consider the system normalized with gains from a given user i to its associated base i $g_{ii} = 1$.

users as Gaussian noise, and provided numerical examples that illustrated this solution.

We conclude our paper by noting that users in the non-collaborative multi-base wireless system considered in this paper can also be regarded as players in a non-cooperative game in which they compete for data rates with the sole objective of maximizing individual performance regardless of the other users [28]. Different strategies can be employed by users for performance maximization in this non-cooperative game. In the case of our proposed algorithm data rate maximization is achieved by adjusting user codewords using greedy interference avoidance, whereas in [28] this is achieved by applying iterative water filling [29] and adjusting the power allocation over frequencies through adaptation of user transmit covariance matrices.

Using this game theoretic approach it has been shown [28] that the simultaneous water filling solution, which is a fixed point for both greedy interference avoidance and iterative water filling algorithms, is a Nash equilibrium for the considered non-collaborative multi-base system. Thus, at a simultaneous water filling point each user's strategy is an optimal response to the other user's strategy, and no unilateral strategy change by any user will be able to improve individual user performance [9, Ch. 1]. In general this Nash equilibrium point is not unique, and the algorithm may converge to different fixed points [18] for which a complete analysis can be found in [15, 19, 20].

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