Interference Avoidance and Sum Capacity for Multibase Systems

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Abstract

We derive sum capacity bounds for systems with L transmitting locations, B receiving stations and a signal space of dimension N under the assumption that all received signals are pooled at a central location for processing. In addition, we provide an algorithm based on interference avoidance methods which monotonically increases sum capacity to its maximum value.

1 Introduction

We consider a cellular wireless system where users (mobile terminals) are dispersed geographically and communicate with geographically dispersed base stations. We assume that each user's signal can be represented in a shared N-dimensional signals space and that the gains between any two points in space are the same over all dimensions (flat fading). Although each user is usually only interested in sending information to the base to which it has been assigned and each base cares only about decoding users assigned to it, all users interfere with one another at every base according to the given propagation characteristics. This scenario is an instance of the *interference channel* problem [1–3, 13] for which few results have been derived in the half century since it was proposed. However, if we assume that information from all bases is pooled and centrally processed, the problem becomes tractable and represents an upper bound to sum capacity in the interference channel problem as well.

Therefore, we derive sum capacity for systems with multiple basestations in a nonfading (simple path loss) environment under the assumption that the bases can collaborate and exchange information such that received signals from all bases are available for decoding. Along the way, we prove what appears to be a new mathematical result on structural conditions for maximizing the determinant of a block structured matrix with trace constraints on each block, and we apply this result to derive an upper bound on sum capacity for such systems. In addition, we present an interference avoidance based algorithm [10, 11] that increases sum capacity iteratively and is guaranteed to converge to the maximum sum capacity [6] – a result corroborated by simulation.

2 Problem Statement

Let B be the number of basestations and L the number of transmitting locations in our system. A codeword matrix $\mathbf{S}_k$ of dimension $N \times M_k$ is associated with each location $k = 1, \ldots, L$, where M_k represents the number of codewords associated with transmit location k. The codeword matrix of the whole system is obtained by arranging all codeword matrices $\mathbf{S}_k$ in a global codeword matrix

$$\mathbf{S} = [\mathbf{S}_1 \; \mathbf{S}_2 \ldots \mathbf{S}_L] \tag{1}$$

$\mathbf{S}$ is an $N \times M$ matrix, with $M = \sum_{k=1}^{L} M_k$. With no loss of generality, each codeword is constrained to have unit norm, that is $\mathbf{s}_i^\top \mathbf{s}_i = 1$ for all $i = 1, 2, \ldots, M$. An M_k-dimensional vector of information symbols $\mathbf{b}_k$ is transmitted from each location k, and we denote the vector of all transmitted symbols from all locations $\mathbf{b} = [\mathbf{b}_1^\top \; \mathbf{b}_2^\top \ldots \mathbf{b}_L^\top]^\top$.

The received vector at basestation j consists of a superposition of transmitted vectors from all locations $k = 1, \ldots, L$ scaled by some gain factors g_{jk}. Note that these gains affect all signal space dimensions equally – the fading model is assumed flat over the whole signal space. Including additive noise $\mathbf{w}_j$ yields

$$\mathbf{r}_j = \sum_{k=1}^{L} g_{jk} \mathbf{S}_k \mathbf{b}_k + \mathbf{w}_j \tag{2}$$

We also note that the gain factor g_{jk} of the link between transmit location k and basestation j incorporates both the transmitted power of location k and the propagation (path loss) model of the link jk. By defining a gain matrix associated with basestation j as

$$\mathbf{G}_j = \begin{bmatrix} \ddots & & \\ & g_{jk}\mathbf{I}_{M_K} & \\ & & \ddots \end{bmatrix} \quad k = 1, 2, \ldots, L, \;\; j = 1, 2, \ldots, B \tag{3}$$

we can rewrite the received vector at basestation j as

$$\mathbf{r}_j = \mathbf{S}\mathbf{G}_j\mathbf{b} + \mathbf{w}_j \tag{4}$$

and then a composite received signal vector for the system as

$$\mathbf{r}^\top = \begin{bmatrix} \mathbf{r}_1^\top & \mathbf{r}_2^\top & \ldots & \mathbf{r}_B^\top \end{bmatrix} \tag{5}$$

a BN-dimensional vector. The correlation matrix, $E[\mathbf{r}\mathbf{r}^\top]$ is then

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \mathbf{Q} + \mathbf{W} \tag{6}$$

where $\mathbf{W}$ is the overall noise covariance

$$\mathbf{W} = E\left\{ \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_B \end{bmatrix} \begin{bmatrix} \mathbf{w}_1^\top & \mathbf{w}_2^\top & \cdots & \mathbf{w}_B^\top \end{bmatrix} \right\} \tag{7}$$

and

$$\mathbf{Q} = \begin{bmatrix} \mathbf{S}\mathbf{G}_1^2\mathbf{S}^\top & \mathbf{S}\mathbf{G}_1\mathbf{G}_2\mathbf{S}^\top & \ldots & \mathbf{S}\mathbf{G}_1\mathbf{G}_B\mathbf{S}^\top \\ \mathbf{S}\mathbf{G}_2\mathbf{G}_1\mathbf{S}^\top & \mathbf{S}\mathbf{G}_2^2\mathbf{S}^\top & \ldots & \mathbf{S}\mathbf{G}_2\mathbf{G}_B\mathbf{S}^\top \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}\mathbf{G}_B\mathbf{G}_1\mathbf{S}^\top & \mathbf{S}\mathbf{G}_B\mathbf{G}_2\mathbf{S}^\top & \cdots & \mathbf{S}\mathbf{G}_B^2\mathbf{S}^\top \end{bmatrix} \tag{8}$$

We can also rewrite $\mathbf{R}$ in terms of the contribution from each location as

$$\mathbf{R} = \sum_{k=1}^{L} \mathbf{R}_k + \mathbf{W} \tag{9}$$

where $\mathbf{R}_k$ is the contribution of transmit location k

$$\mathbf{R}_k = \begin{bmatrix} g_{1k}^2 \mathbf{S}_k \mathbf{S}_k^\top & g_{1k} g_{2k} \mathbf{S}_k \mathbf{S}_k^\top & \cdots & g_{1k} g_{Bk} \mathbf{S}_k \mathbf{S}_k^\top \\ g_{2k} g_{1k} \mathbf{S}_k \mathbf{S}_k^\top & g_{2k}^2 \mathbf{S}_k \mathbf{S}_k^\top & \cdots & g_{2k} g_{Bk} \mathbf{S}_k \mathbf{S}_k^\top \\ \vdots & \vdots & \ddots & \vdots \\ g_{Bk} g_{1k} \mathbf{S}_k \mathbf{S}_k^\top & g_{Bk} g_{2k} \mathbf{S}_k \mathbf{S}_k^\top & \cdots & g_{Bk}^2 \mathbf{S}_k \mathbf{S}_k^\top \end{bmatrix} = \mathbf{g}_k \mathbf{g}_k^\top \otimes \mathbf{S}_k \mathbf{S}_k^\top \tag{10}$$

where we define the gain vector for transmit location k as

$$\mathbf{g}_k = \begin{bmatrix} g_{1k} \\ g_{2k} \\ \vdots \\ g_{Bk} \end{bmatrix} \tag{11}$$

and $\mathbf{A} \otimes \mathbf{B} = \{a_{ij}\mathbf{B}\}$ is the Kronecker product of $\mathbf{A}$ and $\mathbf{B}$ [5]. Thus, a compact way to write correlation matrix is

$$\mathbf{R} = \sum_{k=1}^{L} \mathbf{g}_k \mathbf{g}_k^\top \otimes \mathbf{S}_k \mathbf{S}_k^\top + \mathbf{W} \tag{12}$$

Interference avoidance algorithms [7–11, 14] operate at the single codeword level – each codeword is modified to increase sum capacity or to reduce a quantity called total square correlation – a measure of mutual interference among codewords. The replacement codeword is assumed to have the same (unit) norm as the original codeword. Thus, the set of codewords associated with each location satisfies trace$(\mathbf{S}_i \mathbf{S}_i^\top) = M_i$, the number of codewords associated with location i. This leads to trace constraints on each $N \times N$ sub-block of $\mathbf{R}$

$$\text{Trace}\left[\sum_{k=1}^{L} g_{ik} g_{jk} \mathbf{S}_k \mathbf{S}_k^\top + \mathbf{W}_{ij}\right] = \sum_{k=1}^{L} M_k g_{ik} g_{jk} + \text{Trace}\left[\mathbf{W}_{ij}\right] \tag{13}$$

We seek to maximize the sum capacity for this system – the maximum of the mutual information between the input and output over all distributions on the input that satisfy the constraints. For the covariance matrix defined in equation (6), assuming Gaussian noise, the sum capacity is given by

$$C = \frac{1}{2}\log(\det(\mathbf{R})) - \frac{1}{2}\log(\det(\mathbf{W})) \tag{14}$$

as shown in [4]. Our goal is to maximize equation (14) under the block trace constraints of equation (13) imposed on $\mathbf{R}$. That is, we must find codeword sets $\mathbf{S}_k$ that maximize sum capacity.

3 Sum Capacity Maximization

For an $N \times N$ positive definite matrix $\mathbf{X}$ with constant trace it is easily shown that $|\mathbf{X}|$ is maximized when $\mathbf{X}$ is a scaled identity matrix [12]. However, it is obvious that unless the off-diagonal blocks of $\mathbf{R}$ have zero trace, the covariance can NEVER be a scaled identity matrix. So, we seek the structure of $\mathbf{R}$ which maximizes the determinant subject to the imposed block trace constraints.

We first define a class of positive definite ($\in \mathcal{M}^+$) matrices $\mathcal{Q}_{N,J}$ where the subscript N denotes the size of the square sub-blocks and J denotes the number of vertical and horizontal sub-blocks. The matrices in this class are of the form $\mathbf{Q} = \{\mathbf{Q}_{i,j}\}$ for $i, j = 1, \ldots, J$ where the square $N \times N$ submatrices are symmetric ($\mathbf{Q}_{ij} = \mathbf{Q}_{ji}^\top$) and have trace $\text{Trace}\,[\mathbf{Q}_{ij}] = E_{ij}$. For this class of matrices we have the following theorem:

Theorem 1 *Let*

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & \cdots & \mathbf{Q}_{1J} \\ \mathbf{Q}_{21} & \mathbf{Q}_{22} & & \vdots \\ \vdots & & \ddots & \vdots \\ \mathbf{Q}_{J1} & \cdots & \cdots & \mathbf{Q}_{JJ} \end{bmatrix} \tag{15}$$

where $\mathbf{Q} \in \mathcal{Q}_{N,J}$. *Let the trace constraints of the sub-blocks be* $\text{Trace}\,[\mathbf{Q}_{ij}] = E_{ij}$. *Then the determinant of* $\mathbf{Q}$ *is maximized when each block is of the form*

$$\mathbf{Q}_{ij} = \frac{E_{ij}}{N}\mathbf{I}_N \tag{16}$$

and the value of the maximum determinant is

$$\max_{\mathbf{Q} \in \mathcal{Q}_{N,J}} |\mathbf{Q}| = \frac{1}{N^{NJ}} |\mathbf{E}|^N \tag{17}$$

where the elements of the square matrix $\mathbf{E}$ *are* $\{E_{ij}\}$.

A proof is available upon request.

Thus, the covariance matrix $\mathbf{R}$ for a system with B bases belongs to the class $\mathcal{Q}_{N,B}$, and to maximize $|\mathbf{R}|$ each of the sub-blocks of $\mathbf{R}$ should be a scaled identity matrix if possible. The question is whether a set of codewords exists which renders $\mathbf{R}$ in the proper form, an issue we explore next.

3.1 Independent White Noise At Each Base

If we assume independent white noise of different amplitudes at each base, the overall noise covariance is block diagonal and each block is of the form $\mathbf{W}_{ii} = \frac{\omega_{ii}}{N}\mathbf{I}_N$, $i = 1, \ldots, B$ where $\omega_{ii} = \text{Trace}\,[\mathbf{W}_{ii}]$, the overall noise energy at base i. From equation (13) we have the sub-block traces of $\mathbf{R}$ as

$$E_{ij} = \sum_{k=1}^{L} M_k g_{ik} g_{jk} + \delta_{ij}\omega_{ii} \tag{18}$$

Direct application of Theorem 1 to $\mathbf{R}$ results in the condition:

$$\sum_{k=1}^{L} g_{ik} g_{jk} \mathbf{S}_k \mathbf{S}_k^\top + \delta_{ij}\frac{\omega_{ii}}{N}\mathbf{I}_N = \frac{E_{ij}}{N}\mathbf{I}_N$$

which can always satisfied if each of the $\mathbf{S}_k\mathbf{S}_k^\top$ is a scaled identity matrix (although there may be other solutions as well). We state this result as a theorem:

Theorem 2 *For independent white noise at each base i with total energy ω_{ii}, sum capacity is maximized when all the codeword covariances are*

$$\mathbf{S}_k\mathbf{S}_k^\top = \mathbf{X}_k = \frac{M_k}{N}\mathbf{I}_N$$

and the corresponding sum capacity value is

$$C_{\max} = \frac{N}{2}\left[\log|\mathbf{E}| - \sum_{i=1}^{B}\log\omega_{ii}\right] \tag{19}$$

3.2 Colored Base and Other-System Noise

We will consider two potential noise sources in a multiple base system. First we have independent colored noise sources at the base receivers and we will let the noise covariance at each base i be $\boldsymbol{\mathcal{V}}_i$. The second noise source is assumed to come from random emitters (possibly from other systems) at discrete geographic locations. These noise sources have the same form of gains to the bases as the transmitting locations with gains $\mathbf{g}_i$ and will therefore produce the same type of covariance structure. So, suppose each noise source has covariance $\boldsymbol{\mathcal{W}}_n$, $n = 1, 2, \cdots, \mu$, then adding these two independent noise sources yields the total noise covariance

$$\mathbf{W} = \begin{bmatrix} \boldsymbol{\mathcal{V}}_1 & & & \\ & \boldsymbol{\mathcal{V}}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{\mathcal{V}}_B \end{bmatrix} + \sum_{n=1}^{\mu}\mathbf{h}_n\mathbf{h}_n^\top \otimes \boldsymbol{\mathcal{W}}_n \tag{20}$$

where the $\{\mathbf{h}_n\}$ are the gain vectors (defined similarly to the transmit location gains $\{\mathbf{g}_k\}$) associated with the geographically distributed noise sources. Under these assumptions, we notice that $\mathbf{W}$ is composed of $N \times N$ blocks $\mathbf{W}_{ij}$

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} & \cdots & \mathbf{W}_{1B} \\ \mathbf{W}_{21} & \mathbf{W}_{22} & \cdots & \mathbf{W}_{2B} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{W}_{B1} & \mathbf{W}_{B2} & \cdots & \mathbf{W}_{BB} \end{bmatrix} \tag{21}$$

where $\mathbf{W}_{ij} = \mathbf{W}_{ij}^\top$ and $\mathbf{W}_{ij} = \mathbf{W}_{ji}$.

Therefore, if we define trace constraints as

$$E_{ij} = \sum_{k=1}^{L} M_k g_{ik} g_{jk} + \omega_{ij} \tag{22}$$

where $\omega_{ij} = \text{Trace}\left[\mathbf{W}_{ij}\right]$, then Theorem 1 requires that

$$\sum_{k=1}^{L} g_{ik} g_{jk}\mathbf{X}_k + \mathbf{W}_{ij} = \frac{E_{ij}}{N}\mathbf{I}_N \tag{23}$$

where $\mathbf{X}_k = \mathbf{S}_k\mathbf{S}_k^\top$. Owing to the structure of the covariance matrix, equation (23) can be interpreted as a particularly simple set of $\frac{1}{2}B(B+1)$ equations in L unknown covariances $\{\mathbf{X}_k\}$.

The question then is whether there exists a realizable/feasible set $\{\mathbf{X}_k\}$ which satisfies equation (23), and if no such set $\{\mathbf{X}_k\}$ exists, what the actual optimizing set should be. We do not consider the latter question here and simply assume that either a feasible solution to equation (23) exists, or if it does not, our solution serves as an upper bound.

Regardless, the maximum value for $|\mathbf{R}|$ is then

$$\max|\mathbf{R}| \leq \frac{1}{N^{NB}}|\mathbf{E}|^N \tag{24}$$

where $\mathbf{E}$ is the $B \times B$ matrix of traces with elements given in equation (22). As before we can formulate the result as a theorem:

Theorem 3 *For a multiple base system with B basestations, L transmitting locations and $NB \times NB$ noise covariance matrix $\mathbf{W}$ whose $N \times N$ sub-blocks $\mathbf{W}_{ij}$ are symmetric, the maximum sum capacity is achieved when each set of codeword covariances $\mathbf{X}_k = \mathbf{S}_k\mathbf{S}_k^\top$ satisfies*

$$\sum_{k=1}^{L} g_{ik}g_{jk}\mathbf{X}_k + \mathbf{W}_{ij} = \frac{E_{ij}}{N}\mathbf{I}_N$$

and the associated sum capacity value is

$$C_{\max} = \frac{1}{2}\left[N\log|\mathbf{E}| - NB\log N - \log|\mathbf{W}|\right] \tag{25}$$

If no feasible solution for such a set $\{\mathbf{X}_k\}$ exists, then $C_{\max}$ serves as an upper bound.

4 An Algorithm for Increasing Sum Capacity

We maximize sum capacity in equation (14) by maximizing $|\mathbf{R}|$. As we have seen, the covariance matrix $\mathbf{R}$ has certain structural properties which we will formally define as $\mathbf{R} \in \mathcal{Q}_{N,B}$ at the end of the following development.

From the perspective of a single location k using (9)we can write the covariance matrix as

$$\mathbf{R} = \mathbf{R}_k + \sum_{i=1,i\neq k}^{L} \mathbf{R}_i + \mathbf{W}$$

Using an eigenvalue-eigenvector decomposition for $\mathbf{S}_k\mathbf{S}_k^\top$, contribution of location k to covariance matrix, equation (10), can be written as

$$\mathbf{R}_k = \begin{bmatrix} g_{1k}^2\mathbf{\Phi}_k\mathbf{D}_k\mathbf{\Phi}_k^\top & g_{1k}g_{2k}\mathbf{\Phi}_k\mathbf{D}_k\mathbf{\Phi}_k^\top & \cdots & g_{1k}g_{Bk}\mathbf{\Phi}_k\mathbf{D}_k\mathbf{\Phi}_k^\top \\ g_{2k}g_{1k}\mathbf{\Phi}_k\mathbf{D}_k\mathbf{\Phi}_k^\top & g_{2k}^2\mathbf{\Phi}_k\mathbf{D}_k\mathbf{\Phi}_k^\top & \cdots & \vdots \\ \vdots & \cdots & \ddots & \vdots \\ g_{Bk}g_{1k}\mathbf{\Phi}_k\mathbf{D}_k\mathbf{\Phi}_k^\top & \cdots & \cdots & g_{Bk}^2\mathbf{\Phi}_k\mathbf{D}_k\mathbf{\Phi}_k^\top \end{bmatrix} \tag{26}$$

or

$$\mathbf{R}_k = \begin{bmatrix} \frac{g_{1k}}{e_k}\mathbf{\Phi}_k \\ \vdots \\ \vdots \\ \frac{g_{Bk}}{e_k}\mathbf{\Phi}_k \end{bmatrix} \left[\ e_k^2\mathbf{D}_k\ \right] \left[\ \frac{g_{1k}}{e_k}\mathbf{\Phi}_k^\top \quad \cdots \quad \cdots \quad \frac{g_{Bk}}{e_k}\mathbf{\Phi}_k^\top\ \right] \tag{27}$$

where

$$e_k^2 = \sum_{j=1}^{B} g_{jk}^2 = \|\mathbf{g}_k\|^2 \tag{28}$$

is the total energy received from the unit energy user codewords at location k.

We can rewrite equation (27) as

$$\mathbf{R}_k = \mathbf{Z}_k \left(e_k^2 \mathbf{S}_k \mathbf{S}_k^\top \right) \mathbf{Z}_k^\top$$

where

$$\mathbf{Z}_k = \begin{bmatrix} \frac{g_{1k}}{e_k}\mathbf{I}_N \\ \vdots \\ \frac{g_{Bk}}{e_k}\mathbf{I}_N \end{bmatrix} \tag{29}$$

a matrix with N orthonormal columns. When $\mathbf{R}_k$ is written this way, it is obvious that the signal from location k resides in a subspace of $\mathbb{R}^{NJ}$ spanned by the N orthonormal columns of $\mathbf{Z}_k$.

Using singular value decomposition [5] of matrix $\mathbf{Z}_k$ we have

$$\mathbf{Z}_k = \mathbf{U}_k \mathbf{T}_k \mathbf{V}_k^\top$$

where

$$\mathbf{U}_k = [\mathbf{Z}_k \bar{\mathbf{Z}}_k]_{NB \times NB} \qquad \mathbf{T}_k = \begin{bmatrix} \mathbf{I}_N \\ 0_{(B-1)N \times N} \end{bmatrix} \qquad \mathbf{V}_k = \mathbf{I}_N$$

so that $\mathbf{U}_k \mathbf{U}_k^\top = \mathbf{U}_k^\top \mathbf{U}_k = \mathbf{I}_{BN}$, and $\bar{\mathbf{Z}}_k$ is the orthonormal complement of $\mathbf{Z}_k$.

We can then perform a similarity transformation on matrix $\mathbf{R}$ using $\mathbf{U}_k$:

$$\mathbf{R}^{(k)} = \mathbf{U}_k^\top \; \mathbf{R} \; \mathbf{U}_k = \begin{bmatrix} \mathbf{Z}_k^\top \mathbf{R} \mathbf{Z}_k & \mathbf{Z}_k^\top \mathbf{R} \bar{\mathbf{Z}}_k \\ \bar{\mathbf{Z}}_k^\top \mathbf{R} \mathbf{Z}_k & \bar{\mathbf{Z}}_k^\top \mathbf{R} \bar{\mathbf{Z}}_k \end{bmatrix} \tag{30}$$

which preserves the determinant $\det(\mathbf{R}) = \det(\mathbf{R}^{(k)})$. Now we seek to maximize $\det(\mathbf{R}^{(k)})$.

The upper left corner of $\mathbf{R}^{(k)}$ is an $N \times N$ matrix and is the only block in this decomposition which depends on $\mathbf{S}_k$. Thus, we can write

$$\mathbf{R}^{(k)} = \begin{bmatrix} e_k^2 \mathbf{S}_k \mathbf{S}_k^\top + \mathbf{A}_k & \mathbf{B}_k \\ \mathbf{B}_k^\top & \mathbf{C}_k \end{bmatrix} \tag{31}$$

where matrices $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{C}_k$ do not depend on $\mathbf{S}_k$.

Using the partitioning of matrix $\mathbf{R}^{(k)}$ from equation (31) we can write the determinant as

$$\det(\mathbf{R}^{(k)}) = \det(\mathbf{C}_k)\det(e_k^2 \mathbf{S}_k \mathbf{S}_k^\top + \mathbf{A}_k - \mathbf{B}_k \mathbf{C}_k^{-1} \mathbf{B}_k^\top)$$

or

$$\det(\mathbf{R}^{(k)}) = e_k^{2N} \det(\mathbf{C}_k)\det(\mathbf{S}_k \mathbf{S}_k^\top + \mathbf{Y}_k) \tag{32}$$

where we define

$$\mathbf{Y}_k = \frac{1}{e_k^2}(\mathbf{A}_k - \mathbf{B}_k \mathbf{C}_k^{-1} \mathbf{B}_k^\top) \tag{33}$$

And since matrices $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{C}_k$ are independent of $\mathbf{S}_k$ we can maximize determinant of $\mathbf{R}^{(k)}$ (and hence sum capacity) via an interference avoidance algorithm [10,11] performed for the second determinant in equation (32).

We may apply this reasoning to each location and derive an iterative algorithm:

1. Initialization: state space dimension N; number of basestations B; number of locations L; number of codewords for each location $M_1 + M_2 + \ldots + M_L = M$; noise covariance matrix: $\mathbf{W}$; gain matrices associated to each base: $\mathbf{G}_i,\ i = 1, \ldots, B$; initial codewords: $\mathbf{S}_0$;

2. For each location $k = 1, \ldots, L$ do
   - determine $\mathbf{Z}_k$ and $\mathbf{U}_k$ and compute $\mathbf{R}^{(k)}$ and its blocks: $\mathbf{A}_k$, $\mathbf{B}_k$ and $\mathbf{C}_k$
   - perform interference avoidance on codeword matrix $\mathbf{S}_k$ with noise $\mathbf{Y}_k$:
     replace each codeword $\mathbf{s}_i^{(k)}$, $i = 1, \ldots, M_k$, of $\mathbf{S}_k$ with minimum
     eigenvalue-eigenvector of $\mathbf{S}_k\mathbf{S}_k^\top + \mathbf{Y}_k - \mathbf{s}_i^{(k)}\mathbf{s}_i^{(k)\top}$.

3. Repeat step 2 until the codewords $\mathbf{S}$ stop changing.

Now, interference avoidance algorithms waterfill the dimensions over which they operate [10]. It has recently been shown [6, 15] that iterative waterfilling always converges to maximum sum capacity. Therefore our algorithm will always achieve maximum sum capacity.

This algorithm was implemented in MATLAB and always converged both in sum capacity and in the sense that we reach the point where codewords stopped changing to within a tolerance of 10^{-6}. The capacity attained with white noise at each base was the same as that predicted analytically, and in addition, each $\mathbf{S}_k\mathbf{S}_k^\top$ were scaled identity matrices as indicated by Theorem 1. For colored noise, the final structure of $\mathbf{R}$ after interference avoidance is sometimes but not always a set of scaled identity blocks as one would suspect from Theorem 3. However, if the user energy is enough in the system the algorithm is able to waterfill the entire signal space and reach the maximum capacity bound.

5 Simulations

Due to the limited space of the paper we present a single example of an white noise case. We consider $B = 2$ basestations and $L = 3$ locations each with 3 associated codewords. The signal space dimension is $N = 3$. The gain matrix is

$$\mathbf{G} = \begin{bmatrix} \mathbf{g}_1^\top \\ \mathbf{g}_2^\top \\ \mathbf{g}_3^\top \end{bmatrix} = \begin{bmatrix} 0.0043 & 0.0224 \\ 0.2355 & 0.1890 \\ 0.3546 & 0.0063 \end{bmatrix}$$

where $\mathbf{g}_k$ is the gain vector for transmit location k defined in (11); thus, G_{ij} is the gain from location i to base j. The noise covariance matrices at each basestation are $\mathbf{W}_1 = 2\mathbf{I}_3 \qquad \mathbf{W}_2 = 1.5\mathbf{I}_3$. We start with unit norm random codewords

$$\mathbf{S}_0 = \begin{bmatrix} -0.1449 & 0.8220 & 0.6212 & 0.6644 & 0.6143 & -0.0802 & 0.2995 & -0.7675 & -0.5776 \\ -0.6697 & 0.0099 & 0.1786 & 0.1436 & -0.3693 & -0.3637 & 0.8956 & -0.2233 & -0.8123 \\ -0.7284 & -0.5695 & -0.7631 & 0.7335 & -0.6974 & -0.9281 & -0.3288 & 0.6009 & -0.0811 \end{bmatrix}$$

The matrix of traces is

$$\mathbf{E} = \begin{bmatrix} 6.5437 & 0.1405 \\ 0.1405 & 4.6088 \end{bmatrix}$$

and via Theorem 2 the maximum capacity bound is $C_{\max} = 0.1650$. All capacity values are expressed in nats/channel use.

The initial sum capacity is $C_0 = 0.1624$. The algorithm stops when codewords stop changing with a precision of $1e-06$. The final sum capacity is $C_f = 0.1650$ and the variation after each location changes its set of codewords is presented in figure 1. The final codewords found by the algorithm are

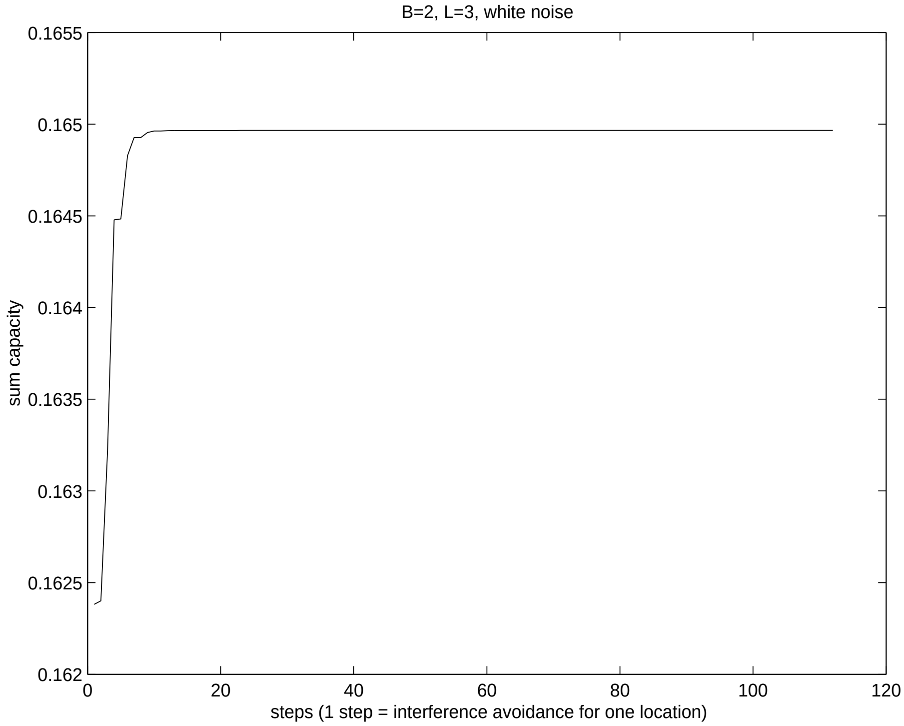

Figure 1: Channel capacity plot obtained using the maximum capacity algorithm for 2 basestations and 2 locations with white noise

$$\mathbf{S}_f = [\mathbf{S}_1 \ \mathbf{S}_2 \ \mathbf{S}_3] =$$

$$\begin{bmatrix} 0.0578 & -0.0092 & -0.9983 & -0.1694 & -0.9302 & -0.3255 & -0.2906 & -0.7011 & -0.6512 \\ -0.8994 & -0.4344 & -0.0480 & 0.6677 & 0.1346 & -0.7322 & 0.8396 & 0.1395 & -0.5249 \\ -0.4332 & 0.9007 & -0.0334 & -0.7249 & 0.3414 & -0.5983 & -0.4589 & 0.6993 & -0.5481 \end{bmatrix}$$

and they obey the conditions $\mathbf{S}_1\mathbf{S}_1^\top = \mathbf{S}_2\mathbf{S}_2^\top = \mathbf{S}_3\mathbf{S}_3^\top = \mathbf{I}_3$ established in the paper. Also the structure of the final correlation matrix has the structure in Theorem 1 for maximum determinant

$$\mathbf{R}_f = \begin{bmatrix} 2.1812\mathbf{I}_3 & 0.0468\mathbf{I}_3 \\ 0.0468\mathbf{I}_3 & 1.5363\mathbf{I}_3 \end{bmatrix} = \frac{1}{N}\mathbf{E} \otimes \mathbf{I}_N$$

6 Conclusions

In this paper we derived sum capacity bounds for a system with multiple transmitting locations and receiving (base) locations assuming frequency non-selective channels and decoding with access to information from all bases. In addition, we provided an interference avoidance algorithm which through iterative waterfilling [6, 15] is guaranteed to maximize sum capacity.

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