Minimizing Total Square Correlation with Multiple Receivers

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Abstract

Total square correlation (TSC) is a measure of mutual interference between users in a multiple access system. Here we find the minimum total squared correlation for an uplink cellular system with L transmit locations and B receiving base stations in white and colored noise. A collaborative scenario in which information from each base is centrally pooled and processed is considered. An iterative algorithm which empirically always achieves the minimum TSC is also presented.

1 Introduction

Advances in software radio technology [1, 7] suggest that future wireless systems will be composed of transceivers which can dynamically vary transmitted waveforms and demodulation methods. We envision using such radios with interference avoidance algorithms [11, 13] to adjust modulation/demodulation methods in response to interference variations and provide better performance.

Most previous work on interference avoidance [8–10, 13] has considered only single base stations or grossly simplified and approximated cellular systems [11]. However, a cellular system is a collection of base stations and all users interfere with one another to some extent. This motivates our interest in studying interference avoidance with multiple bases. Unfortunately, we have already seen empirically that simple greedy interference avoidance does not converge when used in a multiple base system where users are decoded at a single base. Furthermore, this general problem of decoding at a single base while interfering with reception at other bases is an instance of a still mostly open information theory problem – the interference channel [2–5, 14].

We therefore consider a simplified problem where information from all bases is used to decode each user and derive the minimum possible mutual interference among all users – the total squared correlation (TSC). Properties of codeword ensembles with minimum TSC are identified, and an iterative algorithm for codeword adaptation and reduction of TSC is presented. As is often the case with interference avoidance [11, 13], the algorithm always seems to converge to the minimum TSC. Though not considered in this paper, our hope is that understanding of minimum TSC solutions over the ensemble of users and base stations will shed some light on base station assignment, power control and signal space partitioning (a soft form of channel assignment) for multiple user multiple base systems.

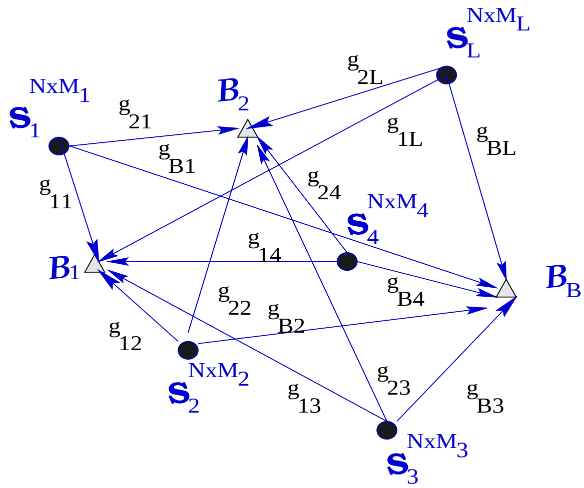


Figure 1: A multibase system with B receiving bases and L transmitting locations, each location k using M_k codewords

2 Problem Statement

We consider an uplink cellular system where users (mobile terminals) are distributed at various locations and communicate with a set of basestations, as is illustrated in figure 1. Usually, each user is interested in sending information only to the basestation to which it is assigned, and does not care what the other basestations receive or understand. Unfortunately regardless of intent, all base stations hear all users to some extent and therefore all users interfere with each other.

So, rather than ignore the information collected about users at bases other than their own, we consider a simpler problem where all information received by the bases is pooled and centrally processed. In addition, we assume that an individual user can simultaneously use as many codewords as desired. Therefore, we associate with each physical location k a matrix of codewords $\mathbf{S}_k$. When couched this way, the problem formulation can be easily adapted to a number of other situations such as MIMO channels and channels with timing jitter, but here we consider only the multiple base problem.

Let B be the number of basestations and L the number of transmitting locations. A codeword matrix $\mathbf{S}_k$ of dimension $N \times M_k$ is associated with each location $k = 1, \ldots, L$, where M_k represents the number of codewords associated with transmit location k. The codeword matrix of the whole system, obtained by forming

$$\mathbf{S} = [\mathbf{S}_1 \; \mathbf{S}_2 \ldots \mathbf{S}_L] \tag{1}$$

is an $N \times M$ matrix, with $M = \sum_{k=1}^{L} M_k$. Each codeword is constrained to have unit energy, $\mathbf{s}_i^\top \mathbf{s}_i = 1$, $\forall i = 1, 2, \ldots, M$ although that need not be true in general. An M_k-dimensional vector of information symbols $\mathbf{b}_k$ is transmitted from each location k, and we denote the vector of all transmitted symbols from all locations $\mathbf{b} = [\mathbf{b}_1^\top \; \mathbf{b}_2^\top \ldots \mathbf{b}_L^\top]^\top$.

The received vector $\mathbf{r}_j$ at basestation j is the superposition of transmitted vectors from all locations $k = 1, \ldots, L$ scaled by some gain factors g_{jk} plus additive noise $\mathbf{w}_j$ with covariance matrix $\mathbf{W}_j$

$$\mathbf{r}_j = \sum_{k=1}^{L} g_{jk} \mathbf{S}_k \mathbf{b}_k + \mathbf{w}_j \tag{2}$$

We note here that the gain factor g_{jk} of the link between transmit location k and basestation j incorporates both the transmitted power of location k and the propagation (path loss) model of the link jk. By defining a gain matrix associated with basestation j

$$\mathbf{G}_j = \begin{bmatrix} \ddots & & \\ & g_{jk}\mathbf{I}_{M_K} & \\ & & \ddots \end{bmatrix} \quad k = 1, 2, \ldots, L, \; j = 1, 2, \ldots, B \tag{3}$$

we can rewrite the received vector at basestation j as

$$\mathbf{r}_j = \mathbf{S}\mathbf{G}_j\mathbf{b} + \mathbf{w}_j \tag{4}$$

and by forming

$$\mathbf{r}^\top = \begin{bmatrix} \mathbf{r}_1^\top & \mathbf{r}_2^\top & \ldots & \mathbf{r}_B^\top \end{bmatrix} \tag{5}$$

we obtain an BN-dimensional vector with correlation matrix, $E[\mathbf{r}\mathbf{r}^\top]$,

$$\mathbf{R} = E[\mathbf{r}\mathbf{r}^\top] = \mathbf{Q} + \mathbf{W} \tag{6}$$

where $\mathbf{W}$ is the overall noise covariance

$$\mathbf{W} = E\left\{ \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \vdots \\ \mathbf{w}_B \end{bmatrix} \begin{bmatrix} \mathbf{w}_1^\top & \mathbf{w}_2^\top & \cdots & \mathbf{w}_B^\top \end{bmatrix} \right\} \tag{7}$$

and

$$\mathbf{Q} = \begin{bmatrix} \mathbf{S}\mathbf{G}_1^2\mathbf{S}^\top & \mathbf{S}\mathbf{G}_1\mathbf{G}_2\mathbf{S}^\top & \ldots & \mathbf{S}\mathbf{G}_1\mathbf{G}_B\mathbf{S}^\top \\ \mathbf{S}\mathbf{G}_2\mathbf{G}_1\mathbf{S}^\top & \mathbf{S}\mathbf{G}_2^2\mathbf{S}^\top & \ldots & \mathbf{S}\mathbf{G}_2\mathbf{G}_B\mathbf{S}^\top \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{S}\mathbf{G}_B\mathbf{G}_1\mathbf{S}^\top & \mathbf{S}\mathbf{G}_B\mathbf{G}_2\mathbf{S}^\top & \cdots & \mathbf{S}\mathbf{G}_B^2\mathbf{S}^\top \end{bmatrix} \tag{8}$$

We can also rewrite $\mathbf{R}$ in terms of the contribution from each location as

$$\mathbf{R} = \sum_{k=1}^{L} \mathbf{R}_k + \mathbf{W} \tag{9}$$

where $\mathbf{R}_k$ is the contribution of transmit location k

$$\mathbf{R}_k = \begin{bmatrix} g_{1k}^2\mathbf{S}_k\mathbf{S}_k^\top & g_{1k}g_{2k}\mathbf{S}_k\mathbf{S}_k^\top & \ldots & g_{1k}g_{Bk}\mathbf{S}_k\mathbf{S}_k^\top \\ g_{2k}g_{1k}\mathbf{S}_k\mathbf{S}_k^\top & g_{2k}^2\mathbf{S}_k\mathbf{S}_k^\top & \ldots & g_{2k}g_{Bk}\mathbf{S}_k\mathbf{S}_k^\top \\ & & \ddots & \\ g_{Bk}g_{1k}\mathbf{S}_k\mathbf{S}_k^\top & g_{Bk}g_{2k}\mathbf{S}_k\mathbf{S}_k^\top & \ldots & g_{Bk}^2\mathbf{S}_k\mathbf{S}_k^\top \end{bmatrix} = \mathbf{g}_k\mathbf{g}_k^\top \otimes \mathbf{S}_k\mathbf{S}_k^\top \tag{10}$$

We have denoted the gain vector associated with each transmit location k as $\mathbf{g}_k = [g_{1k}\; g_{2k}, \ldots, g_{Bk}]^\top$, and $\mathbf{A} \otimes \mathbf{B} = [a_{ij}\mathbf{B}]$ is the Kronecker product of the two matrices [6]. Thus, a compact way to write correlation matrix is

$$\mathbf{R} = \sum_{k=1}^{L} \mathbf{g}_k\mathbf{g}_k^\top \otimes \mathbf{S}_k\mathbf{S}_k^\top + \mathbf{W} \tag{11}$$

TSC is defined in terms of correlation matrix

$$\text{TSC} = \text{Trace}\left[\mathbf{R}^2\right] = \sum_i \lambda_i^2 \tag{12}$$

with $\{\lambda_i\}_{i=1}^{BN}$ the eigenvalues of $\mathbf{R}$. Our goal is to find the absolute minimum value that the TSC can achieve. This goal can be formulated as a constrained optimization problem

$$\min_{\{\mathbf{S}_i\}, i=1,\ldots L} (\text{TSC}) \tag{13}$$

subject to trace constraints imposed on matrix $\mathbf{R}$

$$\text{Trace}\,[\mathbf{R}] = \sum_{k=1}^{L} M_k \|\mathbf{g}_k\|^2 + \sum_{i=1}^{B} \text{Trace}\,[\mathbf{W}_{ii}] \tag{14}$$

as well as on all $N \times N$ sub-blocks of $\mathbf{R}$

$$\text{Trace}\left[\sum_{k=1}^{L} g_{ik} g_{jk} \mathbf{S}_k \mathbf{S}_k^\top + \mathbf{W}_{ij}\right] = \sum_{k=1}^{L} M_k g_{ik} g_{jk} + \text{Trace}\,[\mathbf{W}_{ij}] \tag{15}$$

3 The White Noise Case

In this section we assume independent white noise at each base $\mathbf{W}_{ii} = \mathbf{I}_N$ for $i = 1, 2, \ldots, B$. We will start with the case of only one transmit location with B basestations followed by the general case of L transmit locations and B basestations.

Let $\mathbf{S}$ denote the matrix of codewords associated with the given location and let $\mathbf{g} = [g_1\ g_2 \ldots, g_B]^\top$ be the gain vector, with g_i the gain from the transmitting location to basestation i. The correlation matrix of the received vector is

$$\mathbf{R} = \begin{bmatrix} g_1^2 \mathbf{S}\mathbf{S}^\top & g_1 g_2 \mathbf{S}\mathbf{S}^\top & \ldots & g_1 g_B \mathbf{S}\mathbf{S}^\top \\ g_2 g_1 \mathbf{S}\mathbf{S}^\top & g_2^2 \mathbf{S}\mathbf{S}^\top & \ldots & g_2 g_B \mathbf{S}\mathbf{S}^\top \\ & \ddots & & \\ g_B g_1 \mathbf{S}\mathbf{S}^\top & g_B g_2 \mathbf{S}\mathbf{S}^\top & \ldots & g_B^2 \mathbf{S}\mathbf{S}^\top \end{bmatrix} + \mathbf{I}_{BN} = \mathbf{g}\mathbf{g}^\top \otimes \mathbf{S}\mathbf{S}^\top + \mathbf{I}_{BN} = \mathbf{Q} + \mathbf{I}_{BN} \tag{16}$$

where $\mathbf{Q} = \mathbf{g}\mathbf{g}^\top \otimes \mathbf{S}\mathbf{S}^\top$. The traces of the $N \times N$ sub-blocks of $\mathbf{R}$ are $g_i g_j M + \delta_{ij} N$ and

$$\text{Trace}\,[\mathbf{R}] = M\|\mathbf{g}\|^2 + BN \tag{17}$$

where $\|\mathbf{g}\|^2 = \mathbf{g}^\top \mathbf{g} = \sum_{i=1}^{B} g_i^2$ is the norm of the gain vector.

Theorem 1 [1] *For 1 transmit location, B basestations and white noise with the same average power at each base, TSC = Trace$[\mathbf{R}^2]$ is minimized subject to the constraint Trace$[\mathbf{R}] = M\|\mathbf{g}\|^2 + BN$ when $\mathbf{S}\mathbf{S}^\top = (M/N)\mathbf{I}_N$, and with this choice the minimum TSC value is*

$$TSC_{\min} = \frac{M^2}{N}\|\mathbf{g}\|^4 + 2M\|\mathbf{g}\|^2 + BN$$

[1]Proofs for all theorems in the paper are omitted due to space constraints but they are available for review upon request.

In the case of L transmit locations and B basestations, all locations contribute to the full correlation matrix which for white noise case becomes (see equation (11))

$$\mathbf{R} = \sum_{k=1}^{L} \mathbf{g}_k \mathbf{g}_k^\top \otimes \mathbf{S}_k \mathbf{S}_k^\top + \mathbf{I}_{BN} \tag{18}$$

The following theorem can be formulated in this case

Theorem 2 *For the case of L transmit locations and B basestations and white noise with the same average power at each basestation $TSC = \text{Trace}[\mathbf{R}^2]$ is minimized subject to the constraint $\text{Trace}[\mathbf{R}] = \sum_{k=1}^{L} M_k \|\mathbf{g}_k\|^2 + BN$ when all $\mathbf{S}_k \mathbf{S}_k^\top = (M_k/N)\mathbf{I}_N$, $k = 1, \ldots, L$. Minimum TSC that is achieved with these codeword ensembles is*

$$TSC_{\min} = \frac{1}{N} \sum_{i,j=1}^{B} \left(\sum_{k=1}^{L} g_{ik} g_{jk} M_k \right)^2 + 2 \sum_{k=1}^{L} M_k \|\mathbf{g}_k\|^2 + BN \tag{19}$$

Note: the solution for $\mathbf{S}_k \mathbf{S}_k^\top$ is not necessary unique.

4 The Colored Noise Case

We will consider two potential noise sources in a multiple base system. First we have independent colored noise sources at the base receivers and we will let the noise covariance at each base i be $\boldsymbol{\mathcal{V}}_i$. The second noise source is assumed to come from random emitters (possibly from other systems) at discrete geographic locations. These noise sources have the same form of gains to the bases as the transmitting locations with gains $\mathbf{g}_i$ and will therefore produce the same type of covariance structure. So, suppose each noise source has covariance $\boldsymbol{\mathcal{W}}_n$, $n = 1, 2, \cdots, \mu$, then adding these two independent noise sources yields the total noise covariance

$$\mathbf{W} = \begin{bmatrix} \boldsymbol{\mathcal{V}}_1 & & & \\ & \boldsymbol{\mathcal{V}}_2 & & \\ & & \ddots & \\ & & & \boldsymbol{\mathcal{V}}_B \end{bmatrix} + \sum_{n=1}^{\mu} \mathbf{h}_n \mathbf{h}_n^\top \otimes \boldsymbol{\mathcal{W}}_n \tag{20}$$

where the $\{\mathbf{h}_n\}$ are the gain vectors (defined similarly to the transmit location gains $\{\mathbf{g}_k\}$) associated with the geographically distributed noise sources. Under these assumptions, we notice that $\mathbf{W}$ is composed of $N \times N$ blocks $\mathbf{W}_{ij}$

$$\mathbf{W} = \begin{bmatrix} \mathbf{W}_{11} & \mathbf{W}_{12} & \cdots & \mathbf{W}_{1B} \\ \mathbf{W}_{21} & \mathbf{W}_{22} & \cdots & \mathbf{W}_{2B} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{W}_{B1} & \mathbf{W}_{B2} & \cdots & \mathbf{W}_{BB} \end{bmatrix} \tag{21}$$

where $\mathbf{W}_{ij} = \mathbf{W}_{ij}^\top$ and $\mathbf{W}_{ij} = \mathbf{W}_{ji}$.

We again start with a single transmit location to gain insight and then present the more general case of L transmit locations and B basestations.

For the case of one location and B basestations with colored noise, with the same notations as for white noise, the correlation matrix becomes

$$\mathbf{R} = \begin{bmatrix} g_1^2\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{11} & g_1 g_2\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{12} & \dots & g_1 g_B\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{1B} \\ g_2 g_1\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{21} & g_2^2\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{22} & \dots & g_2 g_B\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{2B} \\ \vdots & & \ddots & \vdots \\ g_B g_1\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{B1} & g_B g_2\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{B2} & \dots & g_B^2\mathbf{S}\mathbf{S}^\top + \mathbf{W}_{BB} \end{bmatrix} \tag{22}$$

with a constant trace given by

$$\mathrm{Trace}\left[\mathbf{R}\right] = M\|\mathbf{g}\|^2 + \sum_{i=1}^{B} \mathrm{Trace}\left[\mathbf{W}_{ii}\right] \tag{23}$$

The TSC for this system is

$$\mathrm{TSC} = \mathrm{Trace}\left[\mathbf{R}^2\right] = \tag{24}$$

$$= \mathrm{Trace}\left[\left(\sum_{i=1}^{B} g_i^2\right)^2 (\mathbf{S}\mathbf{S}^\top)^2 + \left(\sum_{i,j=1}^{B} g_i g_j \mathbf{W}_{ij}\right)\mathbf{S}\mathbf{S}^\top + \mathbf{S}\mathbf{S}^\top\left(\sum_{i,j=1}^{B} g_i g_j \mathbf{W}_{ij}\right) + \sum_{i,j=1}^{B} \mathbf{W}_{ij}^2\right]$$

which can also be written as

$$\mathrm{TSC} = \alpha^2 \mathrm{Trace}\left[(\mathbf{S}\mathbf{S}^\top + \Gamma)^2\right] - \alpha^2 \mathrm{Trace}\left[\Gamma^2\right] + \sum_{i,j=1}^{B} \mathrm{Trace}\left[\mathbf{W}_{ij}^2\right] \tag{25}$$

with

$$\alpha = \sum_{i=1}^{B} g_i^2 = \|\mathbf{g}\|^2 \qquad \Omega = \sum_{i,j=1}^{B} g_i g_j \mathbf{W}_{ij} \tag{26}$$

and

$$\Gamma = \alpha^{-2}\Omega = \frac{\sum_{i,j=1}^{B} g_i g_j \mathbf{W}_{ij}}{\left(\sum_{i=1}^{B} g_i^2\right)^2} \tag{27}$$

Theorem 3 *In a colored noise environment, TSC for a system with 1 transmit location and B basestations is minimized when the codeword ensemble satisfies*

$$\mathbf{S}\mathbf{S}^\top + \Gamma = \frac{M + \mathit{Trace}\left[\Gamma\right]}{N}\mathbf{I}_N \tag{28}$$

and minimum TSC is

$$\mathit{TSC}_{\min} = \frac{\alpha^2\left(M + \mathit{Trace}\left[\Gamma\right]\right)^2}{N} - \alpha^2\,\mathit{Trace}\left[\Gamma^2\right] + \sum_{i,j=1}^{B} \mathit{Trace}\left[\mathbf{W}_{ij}^2\right] \tag{29}$$

For L locations and B basestations, the correlation matrix $\mathbf{R}$ of the system is given in equation (9)and the TSC can rewritten as

$$\mathrm{TSC} = \mathrm{Trace}\left[\mathbf{R}^2\right] = \mathrm{Trace}\left[\sum_{i=1}^{L}\sum_{j=1}^{L}\left(\sum_{k=1}^{B} g_{ki} g_{kj}\right)^2 (\mathbf{S}_i\mathbf{S}_i^\top)(\mathbf{S}_j\mathbf{S}_j^\top)\right] + \tag{30}$$

$$+\text{Trace}\left[\sum_{k=1}^{L}\left(\sum_{i,j=1}^{B} g_{ik}g_{jk}\mathbf{W}_{ij}\right)\mathbf{S}_k\mathbf{S}_k^\top + \sum_{k=1}^{L}\mathbf{S}_k\mathbf{S}_k^\top\left(\sum_{i,j=1}^{B} g_{ik}g_{jk}\mathbf{W}_{ij}\right) + \sum_{i,j=1}^{B}\mathbf{W}_{ij}^2\right]$$

which reduces after some algebra to

$$\text{TSC} = \text{Trace}\left[\sum_{k=1}^{L}\sum_{j=1}^{L}(\mathbf{S}_k\mathbf{S}_k^\top + \Gamma_k)\alpha_{kj}^2(\mathbf{S}_j\mathbf{S}_j^\top + \Gamma_j)\right] - \sum_{k=1}^{L}\sum_{j=1}^{L}\text{Trace}\left[\Gamma_k\alpha_{kj}^2\Gamma_j\right] + \sum_{i,j=1}^{B}\text{Trace}\left[\mathbf{W}_{ij}^2\right] \tag{31}$$

where

$$\alpha_{kj} = \sum_{i=1}^{B} g_{ik}g_{ij} \qquad \Omega_k = \sum_{i,j=1}^{B} g_{ik}g_{jk}\mathbf{W}_{ij} \tag{32}$$

and $\Gamma_i,\ i = 1,\ldots,L$ is the solution set of the linear system of equations

$$\Omega_k = \sum_{j=1}^{L}\alpha_{kj}^2\Gamma_j\ , \qquad k = 1,\ldots,L \tag{33}$$

A similar theorem can be stated in this case:

Theorem 4 *In a colored noise environment, the TSC of a system with L locations and B basestations is minimized when the set of codewords associated with each location satisfies*

$$\mathbf{S}_i\mathbf{S}_i^\top + \Gamma_i = \frac{M_i + \mathit{Trace}\,[\Gamma_i]}{N}\mathbf{I}_N\ , \qquad i = 1,\ldots,L \tag{34}$$

and the value of the minimum TSC is

$$\mathit{TSC}_{\min} = \frac{1}{N}\sum_{k,j=1}^{L}\alpha_{kj}^2(M_k + \mathit{Trace}\,[\Gamma_k])(M_j + \mathit{Trace}\,[\Gamma_j]) - \sum_{k,j=1}^{L}\mathit{Trace}\left[\Gamma_k\alpha_{kj}^2\Gamma_j\right] + \sum_{i,j=1}^{B}\mathit{Trace}\left[\mathbf{W}_{ij}^2\right] \tag{35}$$

5 A TSC Reduction Algorithm

Interference avoidance algorithms which use iterative codeword adjustment to minimize TSC for single basestation systems are described in [10, 11]. Each user adjusts its codewords to reduce TSC at each step. However, different from previous work, here each user does NOT have access to the whole signal space and in fact can only span at most N signal space dimensions as compared to the NB dimensions of the received vector $\mathbf{r}$. So the usual algorithm of replacing codewords by the minimum eigenvalue eigenvector of the covariance (less the covariance of the signal being replaced) is not possible in general.

So, let us define $\mathbf{a}_i$ as the gain vector associated with codeword $\mathbf{s}_i$. We avoid the previously used $\mathbf{g}$ notation since these were associated with location-to-base gains and we want to emphasize the distributed per-codeword approach of the algorithm. Thus, in terms of each codeword we can write

$$\mathbf{S} = [\mathbf{s}_1\mathbf{s}_2\ldots\mathbf{s}_M] \tag{36}$$

and

$$\mathbf{R} = \sum_{j=1}^{M}\mathbf{a}_j\mathbf{a}_j^\top \otimes \mathbf{s}_j\mathbf{s}_j^\top + \mathbf{W} \tag{37}$$

At each step of the algorithm one codeword is changed to decrease TSC and preserve the codeword norm (assumed 1). The part of the correlation matrix invariant under codeword $\mathbf{s}_i$ replacement is

$$\mathbf{R}_i = \sum_{j=1,\, j\neq i}^{M} \mathbf{a}_j\mathbf{a}_j^\top \otimes \mathbf{s}_j\mathbf{s}_j^\top + \mathbf{W} \tag{38}$$

such that

$$\mathbf{R} = \mathbf{R}_i + \mathbf{a}_i\mathbf{a}_i^\top \otimes \mathbf{s}_i\mathbf{s}_i^\top \tag{39}$$

If we replace $\mathbf{s}_i$ with a vector $\mathbf{x}$, with $\|\mathbf{x}\| = \|\mathbf{s}_i\|$. The variation in TSC as a result of this replacement is

$$\Delta\text{TSC} = \text{TSC}_{\text{before}} - \text{TSC}_{\text{after}} = \text{Trace}\left[(\mathbf{R}_i + \mathbf{a}_i\mathbf{a}_i^\top \otimes \mathbf{s}_i\mathbf{s}_i^\top)^2\right] - \text{Trace}\left[(\mathbf{R}_i + \mathbf{a}_i\mathbf{a}_i^\top \otimes \mathbf{x}\mathbf{x}^\top)^2\right] \tag{40}$$

Using Kronecker properties one can easily show that

$$\Delta\text{TSC} = 2\left[(\mathbf{a}_i \otimes \mathbf{s}_i)^\top \mathbf{R}_i(\mathbf{a}_i \otimes \mathbf{s}_i) - (\mathbf{a}_i \otimes \mathbf{x})^\top \mathbf{R}_i(\mathbf{a}_i \otimes \mathbf{x})\right] \tag{41}$$

To decrease TSC with each replacement we seek to maximize ΔTSC and therefore seek an $\mathbf{x}$ with $\|\mathbf{x}\ | = \|\mathbf{s}_i\|$ which minimizes $(\mathbf{a}_i \otimes \mathbf{x})^\top \mathbf{R}_i(\mathbf{a}_i \otimes \mathbf{x})$, or formally

$$\min_{\|\mathbf{x}\|=\|\mathbf{s}_i\|} [a_{1i}\mathbf{x}^\top \; a_{2i}\mathbf{x}^\top \; \ldots a_{Bi}\mathbf{x}^\top]\mathbf{R}_i \begin{bmatrix} a_{1i}\mathbf{x} \\ a_{2i}\mathbf{x} \\ \vdots \\ a_{Bi}\mathbf{x} \end{bmatrix} \tag{42}$$

which looks similar to a Rayleigh quotient problem but is not owing to the constrained structure of the vector $\mathbf{a}_i \otimes \mathbf{x}$. However, we *can* reformulate the problem as a Rayleigh quotient after suitable manipulation

$$\min_{\|\mathbf{x}\|=\|\mathbf{s}_i\|} \mathbf{x}^\top \mathbf{K}_i \mathbf{x} \tag{43}$$

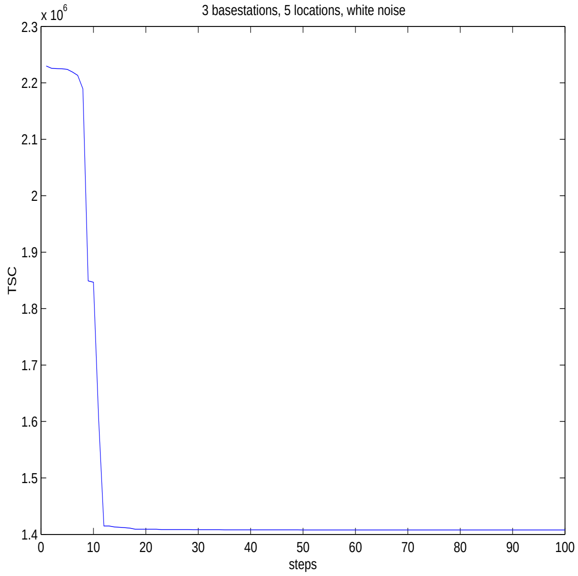
where

$$\mathbf{K}_i = \mathbf{S}[a_{1i}\mathbf{G}_1 + a_{2i}\mathbf{G}_2 + \ldots + a_{Bi}\mathbf{G}_B]^2\mathbf{S}^\top + (a_{1i}^2\mathbf{W}_1 + a_{2i}^2\mathbf{W}_2 + a_{Bi}^2\mathbf{W}_B) - \|\mathbf{a}_i\|^2\mathbf{s}_i\mathbf{s}_i^\top \tag{44}$$

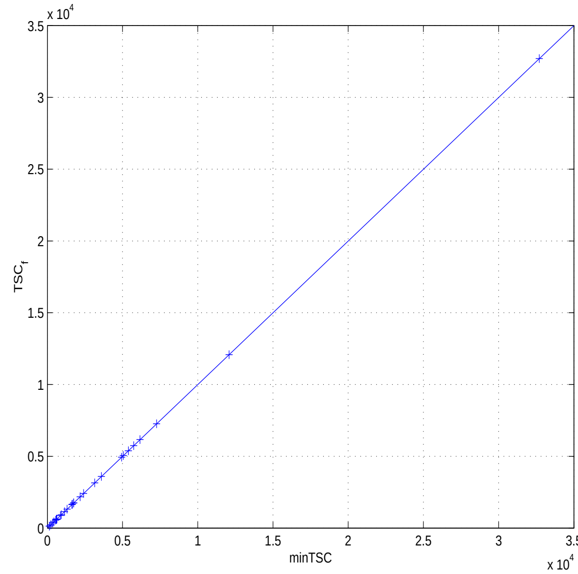
and the $\mathbf{G}_i$ are the gain matrices defined in (3). Obviously $\mathbf{x}$ should be the minimum eigenvalue eigenvector of $\mathbf{K}_i$ [12]. This leads to the TSC reduction algorithm:

TSC Reduction Algorithm
repeat until TSC within tolerance of minimum
    for each codeword $\mathbf{s}_i$, $i = 1, \ldots, M$
        compute $\mathbf{K}_i$
        replace $\mathbf{s}_i$ *with minimum eigenvalue-eigenvector of* $\mathbf{K}_i$
    end
end

Since TSC is bounded from below and the algorithm does not increase TSC, the algorithm must converge in TSC. However, previous experience shows that TSC can have local suboptimal minima [10,11,13] so we are unsure whether this algorithm always converges to a minimum TSC solution. Nevertheless, as was usually the case for interference



(a)



(b)

Figure 2: (a) TSC plot obtained using the minimum TSC algorithm for 3 basestations and 5 locations (each of them using 3 codewords) with white noise; (b) Final TSC from minimum TSC algorithm versus minimum TSC bound for colored noise

avoidance algorithms we have considered in the past, this one too has never converged to anything but the minimum in empirical studies.

That is, the final TSC computed from the algorithm always matches the minimum TSC bound derived analytically. For white noise cases final codewords yielded by the algorithm always satisfy $\mathbf{S}_k\mathbf{S}_k^\top = \frac{M_k}{N}\mathbf{I}_N$ for all k. For colored noise, the final $\mathbf{S}_i\mathbf{S}_i^\top + \Gamma_i$ are not always scaled identity matrices – sometimes they are only diagonal matrices. Regardless, the minimum TSC bound (35) was always achieved. A TSC trajectory plot versus iterations is provided for a white noise case in figure 2.a. This plot is typical for all the simulations using the TSC reduction algorithm for both white and colored noise. TSC is nearly minimized after one cycle through the codewords.

For colored noise in figure 2.b we plotted the final TSC from the algorithm versus minimum TSC derived analytically for a variety of situations (dimensions, noise covariances, gains). It can be seen that the algorithm always reaches the minimum TSC bound (assuming that the energy in the system is enough to reach the minimum).

6 Conclusions

We have found conditions that the received signal covariance matrix must obey in order for TSC to be minimized in multiple base station systems where all information available in the system is made available for decoding. We have also derived conditions that such minimum TSC codeword sets should satisfy to achieve the minimum. Furthermore, we have provided an algorithm that monotonically decreases TSC and that empirically converges to the minimum TSC in all cases we have considered.

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