Paging Area Optimization based on Interval Estimation in Wireless Personal Communication Networks *

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We consider an optimum *personal paging area* configuration problem to improve the paging efficiency in PCS/cellular mobile networks. The approach is to setup the boundaries of a *one-step* paging area that contain the locations of a mobile user with a high probability and to adjust the boundaries to gain a coverage that is matched to the mobile user's time-varying mobility pattern. We formulate the problem as an *interval estimation* problem. The objective is to reduce the paging signaling cost by minimizing the size of the paging area constrained to certain *confidence measure* (probability of locating the user), based on a finite number of available location observations of the mobile user. Modeling user mobility as a Brownian motion with drift stochastic process and by estimating the parameters of the location probability distribution of the mobility process, the effects of the mobility characteristics and the system design parameters on the optimum paging area are investigated. Results show: (1) the optimum paging area expands with the time elapsed after the last known location of the user; (2) it also increases with the length of a prediction interval and the location probability; (3) the relative change in the paging area size decreases with the increase in the number of location observations.

1. Introduction

In PCS/cellular mobile voice/data networks, the network has to deliver an incoming voice call or data call to a mobile user no matter where it is or if it moves or not. Efficient mobility management of mobile voice/data users is of paramount importance from both a quality of service and a network signaling resource allocation perspectives. The mobility of users is generally tracked through paging/registration procedures. A paging area (PA) is a location area (LA) that consists of a group of cells of base stations (BS) where a mobile user is likely to reside currently. If the user moves out of that area it must report to the system via location registration (LR) procedures.

In a previous study by Xie, Goodman & Tabbane [1, 3], the *individual location area* concept was first introduced. The individual location area treats mobile users with different mobility and call characteristics differently to reduce the average signaling cost of mobility management. The study [4, 6] by Rose and Yates established the fundamental theory of optimal sequential paging on a location-by-location basis. Rose in [5] suggests that the study of the mobility management problems be divided into three basic questions: (1) Given a probability distribution of user locations, what is the least amount of effort necessary (number of locations searched) to find a user? (2) Given the time-varying probability distribution of locations known both by the user and the system, what are the optimal paging procedures based on the information available at the mobile? (3) How can these time-varying location probabilities be efficiently estimated based on measurements and/or models of user motion?

Much of the above work in the literature has been concentrated on answering questions (1) and (2) mentioned above. Our work here presents the first attempt (to the best knowledge of the authors) at an analytical formulation and study of the question (3) about the parameter estimation of the time-varying location probability together with a study on the question (2) about an optimal paging strategy based on the mobility measurements. We consider a paging scheme which improves the fixed and static paging area schemes [13, 14, 22]

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currently used in the second generation cellular mobile networks[1] in the following ways: (1) *personal* one-step paging area is considered, which associates with each individual user and changes with its time-varying mobility pattern; (2) the paging area is minimized with a constraint to maintain a certain fixed probability of locating the user; (3) user's mobility parameters are extracted via estimation techniques based on the history of user's locations; (4) closed-form results are achieved for both one and two dimensional mobility scenarios; (5) the approach used applies to any mobility model with an *independent increment* statistical property, although a typical mobility model is assumed here to gain some insights and tangible results.

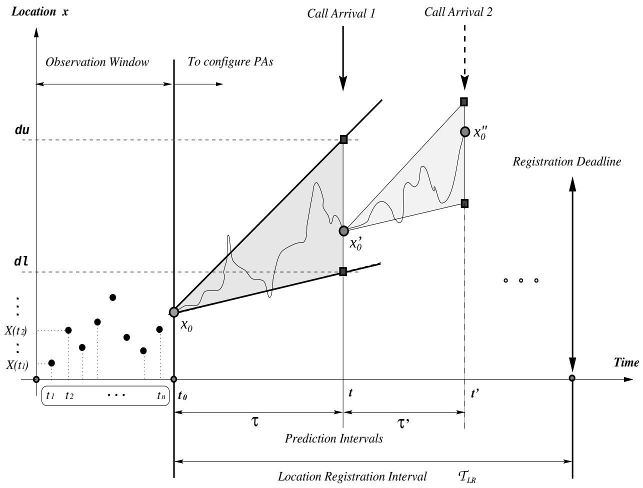

Figure 1. Illustration of the single-step personal paging area (PA) configuration problem. Location observations are collected in observation window, which are used to estimate d_l and d_u — the PA boundaries. τ or τ' is the *prediction interval*. PA is renewed after a successful page. x_0, x'_0 and x''_0 denote the renewal points. $\mathcal{T}_{LR}$ is the location registration (LR) interval.

In this paper, the optimum paging area problem is formulated as an *interval estimation* problem with the upper and lower confidence limits as the boundaries of the paging area for one-dimensional mobility scenario, and a *confidence region estimation* problem in the two-dimensional extension of the study. The objective is to minimize the size of the paging area constrained to a confidence measure which is the location probability of the mobile user over this paging area. Under the assumptions of a one-dimensional and later two-dimensional cellular network environment and a Brownian motion mobility model, the optimum paging area relative to the user's last known location is derived. The effects of the user's mobility characteristics and the system design parameters such as the average speed and the location uncertainty of the mobile user, the sample size of the location observations and the prediction interval as well as the confidence coefficient, on the dimensioning of the optimum paging area are investigated. This study shows that the best paging area for a mobile user should match the underlying mobility pattern assumed for the user for a fixed location performance measure. Further, this work also reveals some new relationships between the user's mobility characteristics, the system design parameters and the optimum paging area configuration.

Figure (1) shows a general picture[2] of this optimum paging area design problem by using the past location information of a particular mobile user. The figure shows only the one-dimensional problem in a 1-D cellular geometry. The locations of base stations are denoted as ordered points along the location x axis. Base station cells are identical and uniformly deployed. In general, we assume a micro-cellular geometry and assume that the mobility parameters of mobile users are on a larger scale than the cell size (see Section 6.3 for details on the justification of these assumptions).

As shown in Figure (1), in the observation window, we collect a set of *location observations* defined as a list of base station locations (base station coordinates) indexed by discrete times, which are measured data, either reported by the mobile user regularly or requested by base stations (see Section 6.2 for a discussion on the methods to collect those location observations). Based on those measured location data, the network can estimate some parameters of the user's mobility and thus design a paging area to send paging messages to cells to cover the user with a high probability, if an incoming call arrives for the user. After a successful page at time t, the the paging area is renewed because the location (base station) of the mobile is exactly known. x_0 and x'_0 denote these renewal points, which are also defined as the last known locations of the user. The time interval τ or τ' is called as a *prediction interval*, which is an important parameter indicating how far into the future

[1] For example, the European GSM systems and their North American counterpart IS-54 systems [22].

[2] The picture is simplified by separating the location sample collection from the paging area configuration. The procedures are actually combined and done sequentially in the analysis.

we are going to predict a future paging area based on our current knowledge about the locations of the mobile user. A timer-based location registration (LR) is assumed, and the LR interval $\mathcal{T}_{LR}$ is the longest time waited if there are no call arrivals [5].

Figure (1) also illustrates a general phenomenon about the mobility of a mobile user: the variance of user location (*location uncertainty*) grows with the time elapsed since the user's last known location x_0 [5]. That is, the longer the time elapsed, the larger the area we have to search for the user assuming the user is always on the move. In other words, our knowledge about the location of a mobile user blurs as the time grows since the time when the location of the user was known for sure (in terms of a base station) to the system. At some time t, we have a pair of the predicted upper and lower boundaries that is an increasing function of time and contain the user location with certain fixed probability.

Since we fix the location probability to be a value close to one, there are some random locations that would be outside the boundaries with a low probability. Those low probability locations represent the *miss* of this one-step paging scheme. Of course we can always use a second paging step which covers the rest of the service area that the first paging step does not cover (see Section 6.1 for discussions). However, the two-step paging scheme is beyond the scope of this paper. We want to focus on the first step of the paging procedure only, since it is usually the most important step of the paging procedures.

We first present a one-dimensional study on the optimum paging area problem with the problem formulation in Section 2, the solution of the problem in Section 3 and the results and discussions in Section 4. In Section 5, we extend the one-dimensional study to a two-dimensional mobility scenario. In Section 6 we discuss some practical issues of the proposed scheme. And Section 7 concludes the paper.

2. Problem Formulation

When an incoming call arrives for a mobile user, instead of paging the user over the whole service area, we want to configure a paging area and then send paging signals to the base stations within the paging area to page the user. This paging area should cover the most likely places (base station cells) where the user resides at the current time in order to improve the performance of locating the user. Our aim is to reduce the paging cost by minimizing this paging area while still maintaining a certain fixed probability of locating the mobile user. As will be seen shortly, this problem can be formulated as *an interval estimation problem.*

The definition of the paging area here is *personal* as mentioned in Section 1 because this paging area actually depends on the mobility characteristics of the mobile user considered. Different mobility patterns would result in different paging areas. To study the effects of a mobility pattern on a paging area, we use a specific stochastic process to model the mobility of the user. Based on the given mobility model and its parameters, the paging area optimization problem has been studied extensively in literature, for example in [1, 3–6, 8–10]. This study is more concerned with the location probability estimation aspect of the problem — the estimation of the mobility parameters of a mobile user, based on a set of location samples observed from the location probability distribution associated with the mobility model.

2.1. Cost Structure of Paging and Location Registration

Assuming that incoming call arrivals are Poisson with mean λ_p (calls/unit time), the overall signaling cost for the mobility management of a mobile user is the sum of the paging signaling cost and the location registration signaling cost [8, 10]:

$$\mathcal{C} = S_p\, \lambda_p\, \mathcal{F}\left\{ \frac{E[A_{PA}]}{A_c} \right\} + S_r \frac{1}{E[\mathcal{T}_{LR}]} \quad \text{[signaling units/time/user]} \qquad (1)$$

where S_p and S_r are cost coefficients for paging and the registration [signaling units/event] respectively, $\mathcal{F}\{\cdot\}$ is a counting function that counts every cell included in a paging area with the average area $E[A_{PA}]$. A_c is the cell area assuming all the cells in the system are identical. $E[\mathcal{T}_{LR}]$ is the average length of the location registration intervals, and its reciprocal is the average rate of location registrations. We approximate $\mathcal{F}\{\cdot\}$ for the convenience of analysis as $\mathcal{F}\left\{\frac{E[A_{PA}]}{A_c}\right\} \approx \frac{E[A_{PA}]}{A_c}$ assuming $E[A_{PA}] \gg A_c$.

In this study, we will focus on the paging aspect of the paging/registration problem by assuming a fixed strategy for location registration, for example, a timer-based location registration strategy [5,8]. Consequently, signaling cost generated from location registrations is a constant here in this study. Joint optimization of paging and registration problem can be found in [5, 8, 10].

Therefore the average cost of paging in a one-dimensional cellular system is proportional approximately to the average rate of incoming call arrivals λ_p, and the size of paging area L_{PA}. as follows [8, 9]:

$$\mathcal{C}_{paging} \;\propto\; \lambda_p \times \left\{ \frac{L_{PA}}{L_c} \right\} \quad \text{[rate} \times \text{quantity of paging messages]} \tag{2}$$

where L_c is the one-dimensional cell size (length of cell).

For fixed λ_p and L_c, the reduction in paging cost is proportional to the reduction in the paging area size L_{PA}. As will be shown in the following, the problem of finding the best paging area for a mobile user with a fixed location probability is a constrained optimization problem of choosing the paging area boundaries to minimize the size of the paging area.

2.2. Interval Estimation and the Paging Area Optimization Problem

2.2.1. Confidence Interval Estimation

Let $X(t_1),\ X(t_2),\ X(t_3),\ \cdots\ X(t_n)$ be a set of n location observations at time instants $t = t_i,\ 1 \le i \le n$ from a sample function or realization of the stochastic process $X(t)$ that describes the mobility of the mobile user. For notational convenience, let X_i denote the corresponding location observation, $X(t_i)$. Assume the actual unknown location of the user at some time $t > t_n$ is a parameter θ, and let θ_l and θ_u be two statistics as follows:

$$\theta_l = \Theta_l\ (X_1, X_2, \cdots, X_n\ ), \tag{3}$$

$$\theta_u = \Theta_u\ (X_1, X_2, \cdots, X_n\ ) \tag{4}$$

where $\Theta_l\ (\cdot)$ and $\Theta_u\ (\cdot)$ are two functions that depend only on the n location samples $\{\, X_i = X(t_i) \,\},\ \ 1 \le i \le n$ drawn from a realization of the mobility process $X(t)$.

Choosing a probability γ close to 1, we determine those two quantities θ_l and θ_u such that the probability that θ_l and θ_u include the unknown value of the parameter θ is equal to the probability γ. Mathematically,

$$P\{\ \theta_l\ <\ \theta\ <\ \theta_u\ \}\ =\ \gamma \tag{5}$$

where θ_l and θ_u are called *lower*, and *upper confidence limits* and $0 \le \gamma \le 1$ is called the *confidence coefficient.* The interval $(\theta_l,\ \theta_u)$ is called a $\gamma-$*confidence interval* or *interval estimate* for the unknown parameter θ [17, 19, 20]. It is obvious that for any fixed γ, Equation (5) itself does not have a *unique* solution for $(\theta_l,\ \theta_u)$ as shown in Figure (2). We have to select θ_l and θ_u so as to minimize the length of the interval $(\theta_l,\ \theta_u)$, subject to the constraint that the confidence coefficient is γ [17].

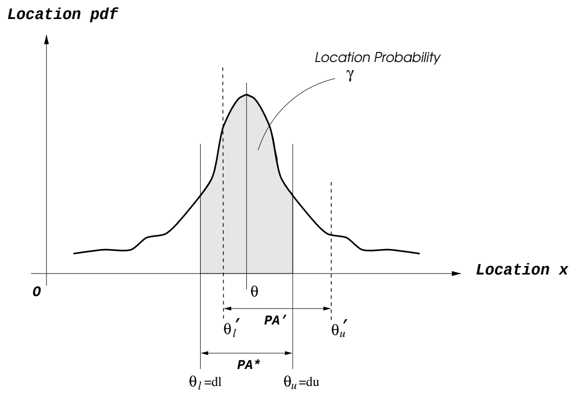

Figure 2. An illustration of the solution of the optimum PA problem with a a location pdf which is *unimodal* and *symmetrical* about its mean θ: for the same confidence γ, the size of the optimum PA interval PA^* is less than that of any other intervals, for example, an arbitrary interval *PA'* as shown.

2.2.2. Paging Area Optimization Problem

The paging area optimization problem can be cast into the interval estimation problem described above as the following:

We wish to select two location quantities θ_l and θ_u based on n available location observations so as to minimize the paging area size $L_{PA} = (\theta_u - \theta_l) > 0$, subject to the constraint that the location probability of the user over the paging area $(\theta_l,\ \theta_u)$ is equal to the prespecified value γ which is close to one. Mathematically we have the following paging area optimization problem:

Find θ_l and θ_u so as to

$$\min_{\{\theta_l,\,\theta_u\}} L_{PA}\ =\ \min_{\{\theta_l,\,\theta_u\}} (\theta_u\ -\ \theta_l) \tag{6}$$

$$\text{subject to} \qquad P\,\{\,\theta_l\ <\ \theta\ <\ \theta_u\,\}\ =\ \gamma \tag{7}$$

where $\theta_l\ =\ \Theta_l\ (\underline{X})$ and $\theta_u\ =\ \Theta_u\ (\underline{X})$ with $\underline{X}\ =\ (X_1, X_2, \cdots, X_n)$ as in Equations (3) and (4). The solution of the above problem will give the optimum paging area denoted by $(d_l,\ d_u)$ (see Figure (2)) with its size as L^*_{PA}, where $(d_l,\ d_u)$ is a particular outcome of the random interval $(\theta_l,\ \ \theta_u)$ that minimizes the paging area size $L_{PA} = \theta_u - \theta_l$. While the above problem in general is difficult to solve analytically, a simple solution exists for the case with any probability density function which is *unimodal* and *symmetric* about its mean θ. In this case the solution of the above problem can be mathematically expressed as a typical interval estimation problem:

$$\arg_{\{\theta_l,\ \theta_u\}} P\{\theta_l < \theta < \theta_u\} = \gamma \quad \text{with} \quad \theta_u - \theta = \theta - \theta_l. \tag{8}$$

Figure (2) above illustrates the solution of the optimum paging area problem with a simple location probability distribution function that is symmetric and unimodal about its mean. For the same level of confidence γ (keeping a fixed size of the shaded area under the pdf curve), the choice $\theta_l = d_l$ and $\theta_u = d_u$ gives the minimum paging area PA^*, which is the *unique* solution of the problem. For any other choice of $(\theta_l,\ \theta_u)$ than $(d_l,\ d_u)$, for example $(\theta'_l,\ \theta'_u)$, there always exists $PA' > PA^*$, i.e., PA^* is the minimum paging area.

The optimum paging area problem can be solved analytically by estimating the two confidence limits d_l and d_u based on the set of previous location observations $\{X_i = X(t_i)\}$, $1 \le i \le n$, if the movement of the user as a function of time $X(t)$ can be described statistically by a timing-varying probability distribution function. Therefore the analysis involves the problem of how to model the time-varying behavior of the mobile user.

2.3. Mobility Modeling

As mentioned previously, the knowledge about the location of a mobile user blurs as the time grows after a user-network interaction where the user location was known for sure (in terms of base stations). This is a general phenomenon about the mobility of an individual user that the motion variance (*location uncertainty*) grows with time after the user was last seen. Therefore, the longer the time elapsed, the larger the area we have to search for the user. The optimum paging area should grow with time correspondingly to maintain a fixed probability of locating the user.

The best candidate model to describe this time-varying behavior is *Brownian motion with drift process*

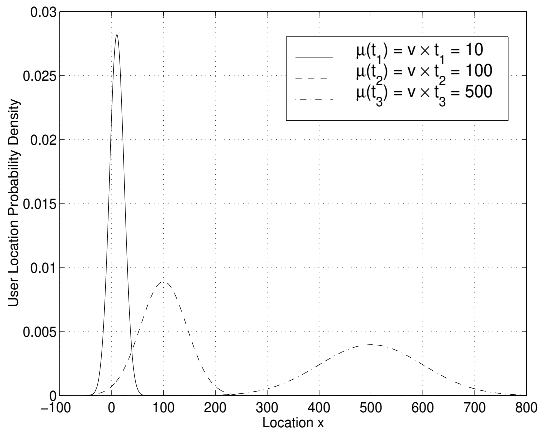

Figure 3. Drifting and spreading — the time-varying user location probability density function modeled by Brownian motion with drift process at three time instants: $t_1 < t_2 < t_3$ ($\mathcal{D} = 200$, $v = 10$).

which is hence chosen to model the user mobility in this study, also particularly for the purpose of detailing the results. The one-dimensional Brownian motion with drift process starting at position $X(t_0) = x_0$ at time $t = t_0$ can be described by the distribution [18–20]:

$$p_x(x|x_0, t) = \frac{1}{\sqrt{2\pi\mathcal{D}(t-t_0)}} \exp\left\{\frac{-(x - x_0 - v(t-t_0))^2}{2\mathcal{D}(t-t_0)}\right\},\ \ t \ge t_0 \tag{9}$$

where the mean and variance of the process are linear functions of the length of the interval $t - t_0$, i.e.,

$$E[x(t)] = \mu(t) = x_0 + v(t - t_0) \tag{10}$$

$$Var[x(t)] = \sigma^2(t) = \mathcal{D}(t - t_0) \tag{11}$$

$\mathcal{D}$ is the *diffusion parameter* (also called *variance parameter*) with units of (length^2/time). It represents the average *location uncertainty* of the motion [5], and v the drift velocity (length/time) or average velocity of a moving user.

The distribution of Brownian motion with drift process is simply a Gaussian pdf parameterized by the length of the time interval $(t - t_0)$. Statistically, the location X of the Brownian motion with drift process is drifting away from its starting point $x_0 = X(t_0)$ and spreading out from its mean as time goes on, as shown in Figure (3). $\mathcal{D}$ and v are important parameters in characterizing the mobility of $X(t)$. High $\mathcal{D}$ and v indicate a very active movement, while low $\mathcal{D}$ and v suggest very little change in the location as time elapses. However, it is worth noting that large v and small $\mathcal{D}$ imply low *location uncertainty* even though the user might move rapidly [5].

In our analysis, $x_0 = x(t_0)$ represents the last known location of the user where the mobile user and the network contacted each other. Whenever there is a network-user interaction, the attachment point of the user in terms of base stations is known. Hence x_0 is a renewal point where the whole estimation process repeats itself.

Another statistical property about Brownian motion with drift process is that it is an *independent stationary incremental process* [18]. The disjoint increments such as $(X_1 - X_0,\ t_1 - t_0),\ (X_2 - X_1,\ t_2 - t_1),\ \cdots,\ (X_n - X_{n-1},\ t_n - t_{n-1})$ are independent. The joint distribution of the observations $\{\, X_i = X(t_i)\,\}$, $1 \le i \le n$ can be expressed in terms of the joint distribution of the disjoint increments [18].

3. Mobility Parameter Estimation and the Optimum Paging Area

The current location of a mobile user is estimated using its history. This estimate can be a single number or an interval. In the context of the problem we are examining, it is more appropriate to speak of a certain region, or interval that the mobile is likely to be found with a desired degree of certainty. Equation (10) gives the expected location of the mobile at time t. We wish to formulate bounds for this quantity at time t based on location history at time instances $t_1 < t_2 < \cdots < t_n$ where $t > t_n$. Since $\mu(t) = x_0 + v(t - t_0)$, bounding the expected location is equivalent to bounding the value of the unknown velocity parameter, v. We consider two cases: the case where the diffusion parameter $\mathcal{D}$ is given, hence the variance of the motion is known and the case where it is not known.

3.1. Interval Estimation of Mean $\mu(t)$ with Known Diffusion Parameter

Let $X(t_1), X(t_2), X(t_3), \cdots X(t_n)$ be a set of n location samples observed at time instants $t = t_i$ from the random process $X(t)$ defined by the Brownian motion with drift mobility model. Each $X_i = X(t_i)$ has a Gaussian distribution with mean $x_0 + v(t_i - t_0)$ and variance $\mathcal{D}(t_i - t_0)$ at time instants $t = t_i,\ 1 \le i \le n$. The random variables X_i are not independent. However, since Brownian motion is an independent increment process as mentioned previously, the displacements of the mobile locations from one observation instant to another are independent random variables.

If we define the displacement in space and time respectively as

$$U_i = X_i - X_{i-1}, \quad i = 1, \cdots, n, \tag{12}$$

$$T_i = t_i - t_{i-1}, \quad i = 1, \cdots, n, \tag{13}$$

then each U_i has Gaussian distribution with $\mathcal{N}(vT_i, \mathcal{D}T_i)$ [18]. For simplicity, we make use of $\{U_i\}$ instead of $\{X_i\}$ to derive bounds for v. Define a quantity Q as the following:

Definition 1: *Let us define Q as*

$$Q \triangleq \frac{\overline{U} - \frac{v}{n}\sum_i T_i}{\frac{1}{n}\sqrt{\mathcal{D}\sum_i T_i}} \tag{14}$$

where

$$\overline{U} = \frac{1}{n}\sum_{i=1}^{n} U_i. \tag{15}$$

Notice that $\overline{U}$ is a Gaussian random variable with mean $E[\overline{U}] = v\frac{\sum_i T_i}{n}$ and variance $Var[\overline{U}] = \frac{\mathcal{D}\sum_i T_i}{n^2}$. Hence Q is a $\mathcal{N}(0,\ 1)$ Gaussian random variable and therefore independent of the parameter we are trying to estimate. It should be emphasized however, that in this section we assume the diffusion parameter $\mathcal{D}$ is known. The probability constraint expression in Equation (7) then becomes:

$$P\{a < Q < b\} = P\{a' < v < b'\} = \gamma \tag{16}$$

where

$$a' = \frac{\overline{U} - \frac{b}{n}\sqrt{\mathcal{D}\sum_i T_i}}{\frac{1}{n}\sum_i T_i} \tag{17}$$

$$b' = \frac{\overline{U} - \frac{a}{n}\sqrt{\mathcal{D}\sum_i T_i}}{\frac{1}{n}\sum_i T_i} \tag{18}$$

and

$$a = \mathcal{G}^{-1}\left(\frac{1-\gamma}{2}\right) \tag{19}$$

$$b = \mathcal{G}^{-1}\left(\frac{1+\gamma}{2}\right) \tag{20}$$

with $\mathcal{G}(\cdot)$ being the cumulative distribution function (cdf) of the standard Gaussian density function.

Since the probability density function involved here is *symmetric* and *unimodal* (standard Gaussian pdf), this choice of a and b achieves the minimum confidence interval or the minimum paging area L_{PA} as in Equation (6). Further, due to symmetry, the problem in Equation (6) can be simplified by $a = -b$.

The calculation of the boundaries of the optimum paging area d_l and d_u can be derived from the interval estimate of the velocity v given the diffusion parameter $\mathcal{D}$,

$$\begin{aligned} & a' < v < b' \\ & \implies x_0 + a'(t - t_0) < x_0 + v(t - t_0) < x_0 + b'(t - t_0) \quad (21) \\ & \implies x_0 + a'(t - t_0) < \mu(t) < x_0 + b'(t - t_0) \quad (22) \\ & \implies d_l(t) < \mu(t) < d_u(t). \quad (23) \end{aligned}$$

Therefore

$$d_l(t) = x_0 + a'(t - t_0) \quad (24)$$

and

$$d_u(t) = x_0 + b'(t - t_0). \quad (25)$$

3.2. Interval Estimation of $\mu(t)$ with Unknown Diffusion Parameter

Generally, the network would have no information about any of the mobility characteristics of a mobile user if it has not seen the appearance of the mobile user in any locations of the system. In the context of the mobility model assumed in this study, the mobility parameters such as v and $\mathcal{D}$ are not known to the network at the beginning and must be estimated based on available location observations.

Since the diffusion parameter $\mathcal{D}$ of the motion process is unknown (i.e., the variance of the motion is unknown), the methods in previous subsection will not work. It can be observed that in Equation (14), the value of $\mathcal{D}$ is used directly, but now we assume this parameter is no longer available. We must find a way to construct the confidence interval in the absence of the diffusion parameter $\mathcal{D}$ and just by making use of the observations. The observations U_i are distributed normally with $\mathcal{N}(vT_i, \mathcal{D}T_i)$. It is difficult to derive the interval in general, however it is straightforward for the case when the time increments are fixed and equal, i.e.:

$$T_i = \delta. \quad (26)$$

Under this assumption, all the observations U_i are equal-mean, equal-variance Gaussian random variables with $\mathcal{N}(v\delta, \mathcal{D}\delta)$. Fixing time increments implies a system where the location information of the mobile is updated on a regular basis.

Definition 2: *Let us define H as*

$$H \triangleq \frac{\overline{U} - v\,\delta}{\sqrt{S^2/n}} \quad (27)$$

where $\overline{U}$ is the sample mean given by

$$\overline{U} = \frac{1}{n}\sum_{i=1}^{n} U_i \quad (28)$$

and S^2 is the sample variance given by

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n} [U_i - \overline{U}]^2. \quad (29)$$

The confidence interval expression is similar to Equation (16):

$$P\{-c < H < c\} = P\{a' < v < b'\} = \gamma \quad (30)$$

where

$$a' = \frac{\overline{U} - c\sqrt{S^2/n}}{\delta} \quad (31)$$

$$b' = \frac{\overline{U} + c\sqrt{S^2/n}}{\delta}. \quad (32)$$

Note that H is a random variable with a **Student *t*-distribution** with $(n-1)$ degrees of freedom [21] (see **Appendix** for the derivation). Let $\mathcal{T}_{(n-1)}(\cdot)$ denote a Student t-distribution with $(n-1)$ degrees of freedom. Then c can be calculated as,

$$c = \mathcal{T}^{-1}_{(n-1)}\left(\frac{1+\gamma}{2}\right). \quad (33)$$

Because the probability density function involved here is the Student t-distribution which is also *symmetric* and *unimodal*, the choice of $-c$ and c achieves the minimum confidence interval or the minimum paging area L_{PA} in Equation (6). H approaches a standard Gaussian random variable for large n, therefore we can use Equation (20) to approximate c as the sample size gets large [17].

Finally, the lower and upper bounds are given by,

$$
\begin{aligned}
a' &< v < b' \\
&\Longrightarrow\ x_0 + a'(t-t_0) < x_0 + v(t-t_0) < x_0 + b'(t-t_0) && (34)\\
&\Longrightarrow\ x_0 + a'(t-t_0) < \mu(t) < x_0 + b'(t-t_0) && (35)\\
&\Longrightarrow\ D_l(t) < \mu(t) < D_u(t) && (36)
\end{aligned}
$$

where

$$D_l(t) \;=\; x_0 \;+\; \frac{\overline{U} \;-\; c\sqrt{S^2/n}}{\delta}\,(t \;-\; t_0) \qquad (37)$$

and

$$D_u(t) \;=\; x_0 \;+\; \frac{\overline{U} \;+\; c\sqrt{S^2/n}}{\delta}\,(t \;-\; t_0)\,. \qquad (38)$$

3.3. Dimensioning of the Optimum Paging Area

3.3.1. Case of Known Diffusion Parameter

If the diffusion parameter $\mathcal{D}$ is given for the Brownian motion mobility model, the boundaries of the optimum paging area can be obtained by the interval estimation equation (16). The lower and upper limits of the confidence interval (d_l, d_u) are given by Equations (24) and (25). Using these expressions we can obtain the size of the paging area as,

$$
\begin{aligned}
L_{PA}(t) &= d_u(t) - d_l(t) && (39)\\
&= (b-a)\sqrt{\frac{\mathcal{D}}{\sum_i T_i}}\;(t-t_0) && (40)\\
&= 2\,\mathcal{G}^{-1}\Big(\frac{1+\gamma}{2}\Big)\sqrt{\frac{\mathcal{D}}{\sum_i T_i}}\;(t-t_0). && (41)
\end{aligned}
$$

If we use the equal time increments assumption that we used in Section 3.2, Equation (41) can be expressed as:

$$L_{PA}(t) \;=\; 2\,\mathcal{G}^{-1}\Big(\frac{1+\gamma}{2}\Big)\sqrt{\frac{\mathcal{D}\,\delta}{n}}\;\Big(n+\frac{\tau}{\delta}\Big) \qquad (42)$$

where $\tau = t - t_n > 0$ is the prediction interval defined earlier.

3.3.2. Case of Unknown Diffusion Parameter

If the mobility parameter v and $\mathcal{D}$ are unknown for the Brownian motion mobility model, the boundaries of the optimum paging area can also be obtained by the interval estimation equation (30). Using the lower and upper limits of the confidence interval (D_l, D_u) given by Equations (37) and (38), respectively, we can find the size of the paging area as,

$$
\begin{aligned}
L_{PA}(t) &= D_u(t) - D_l(t) && (43)\\
&= 2\,\mathcal{T}^{-1}_{(n-1)}\Big(\frac{1+\gamma}{2}\Big)\,\frac{\sqrt{S^2/n}}{\delta}\;(t-t_0) && (44)\\
&= 2\,\mathcal{T}^{-1}_{(n-1)}\Big(\frac{1+\gamma}{2}\Big)\sqrt{\frac{S^2}{n}}\;\Big(n+\frac{\tau}{\delta}\Big). && (45)
\end{aligned}
$$

The size of the optimum paging area from the known and the unknown diffusion parameter cases differs basically in the probability distributions involved (Standard Gaussian vs. Student t-distribution) and in the computation of the variance using the exact value or an estimate (the sample variance).

4. Results and Discussions

We present the results and their implications derived from both the analysis and the numerical calculations of the optimum one-step paging area with respect to various parameters. The mobility parameters are the average drift speed v (*length/time*) and the average location uncertainty $\mathcal{D}$ (*length*^{2}*/time*) from the Brownian motion with drift mobility model [3]. The system design parameters for the dimensioning of the optimum paging area are the confidence coefficient γ and the maximum sample size $n = n_{max}$ which represents the limitation of the system processing power. For a given sample size $1 \le n \le n_{max}$ and a desired value of the confidence coefficient γ, we can design the optimum paging area for any value of the prediction interval τ. We assume location observations are taken at regular time intervals.

4.1. Optimum Paging Area Matched to Mobility Pattern

Figure (4) shows an experiment that plots a sample path of the Brownian motion with drift process starting at $x_0 = 0$ and the desired paging area boundaries based on the previous analytical calculations in case of known diffusion parameter $\mathcal{D}$. For different values of confidence coefficient γ, e.g., 0.9 and 0.99, the size of the paging

[3] The relationship of v to the actual moving speed of a user, and the physical units of v and $\mathcal{D}$ in the context of a specific Brownian motion model, are discussed in detail in [6, 9, 10].

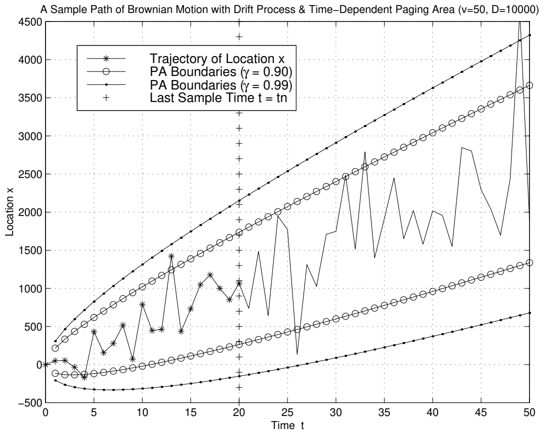

Figure 4. A sample path of a mobile user modeled by Brownian motion with drift process ($v = 50$ and $\mathcal{D} = 10^4$) with the last known location $x_0 = 0$ at time $t_0 = 0$, and the optimum paging area designed using the known diffusion parameter method for two different values of confidence coefficient γ (0.9 and 0.99).

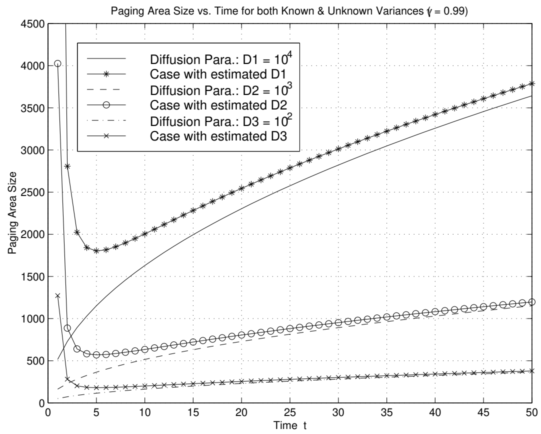

Figure 5. The size of the optimum paging area as a function of the time interval (t_0, t) ($t_0 = 0$, $x_0 = 0$) for the cases of known and unknown diffusion parameter $\mathcal{D}$, given $\gamma = 0.99$ and $\tau = 1$, and with perfect estimation of $\mathcal{D}$ for the case of unknown $\mathcal{D}$. Three different values of $\mathcal{D}$ (10^2, 10^3 and 10^4) and $v = 50$ are used.

area is different. Higher value of γ results in a larger paging area and thus higher paging cost, however better coverage of the mobile's locations.

Figure (4) also shows that the boundaries of the optimum paging area are functions of the time difference $t - t_0$. The mean value of upper and lower boundaries of the optimum paging area at a given time instant follows the actual mean of the non-stationary stochastic process mobility model. Hence generally, the optimum paging area for the mobile user should match the underlying mobility pattern of the user, such that the user can be located with a high probability and incurring the least paging cost (the least number of base stations to send paging messages).

4.2. Optimum Paging Area as Function of Time

The optimum paging area given a user's last known location is an increasing function of the time elapsed as shown in Figure (5) for the scenario of the known diffusion parameter $\mathcal{D}$. This result clearly demonstrates that the optimum paging area should capture the essential characteristics of user mobility — the location uncertainty grows with time elapsed. Therefore a mobile user who has not contacted the system a longer time is more difficult to locate and a larger paging area has to be used to find the user. For users who have just talked to the system, it is relatively easy to find them with a small paging area, since they just cannot move too far away from the last known location. Physical limitations set an upper bound on how fast people can move with any carriers. In Figure (5), higher the variance parameter $\mathcal{D}$, the larger the paging area. This means that users with higher *location uncertainty* have to be paged with a larger paging area. The size of paging area has nothing to do with the mobility parameter v, which means that a deterministic motion speed has no effect on the sizing of the paging area. The plots for the unknown variance scenario are under the perfect estimation of the variance assumption. The convergence of the unknown variance case to the known variance case as the sample size n increases indicates the convergence of Student t-distribution to Gaussian distribution, which differs for different values of the variance parameter $\mathcal{D}$. Figure (5) also shows the settling process of the estimation algorithm. The large paging area sizes at the beginning of the curves under the unknown variance scenario are due to lack of observations.

Figure (6) shows a practical example that when the estimation of the variance is not perfect. The curves of the optimum paging area size are not smooth any more. They are fluctuating around the curves with perfect estimation since they depend on the actual location samples which is random.

4.3. Optimum Paging Area vs. Sample Size and Prediction Interval

Plotted in Figure (7) is the optimum size of the paging area versus the number of location observations — the sample size n, for different values of the prediction interval τ, and for the case of known diffusion param-

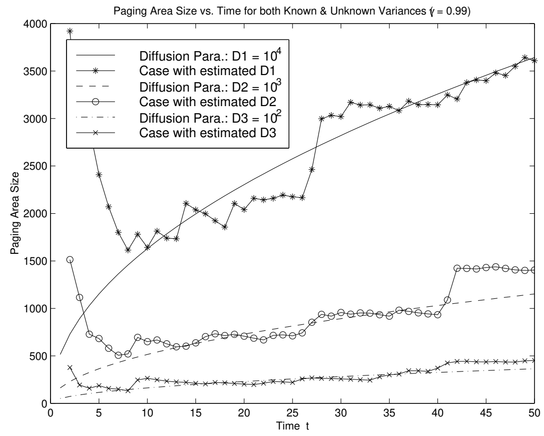

Figure 6. Three examples to show the effect of the actual sample variance estimation of the unknown $\mathcal{D}$ on the size of the optimum paging area, compared to the cases with three known values of $\mathcal{D}$ (10^2, 10^3 and 10^4). $\gamma = 0.99$, $v = 50$, $t_0 = 0$ and $x_0 = 0$.

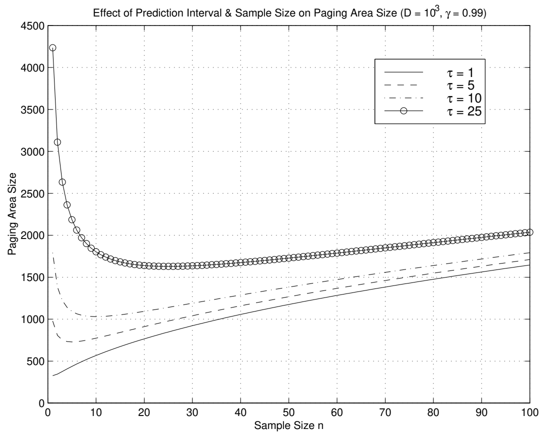

Figure 7. The effects of the sample size n and the prediction interval $\tau = t - t_n$ on the size of the optimum paging area for the case of known diffusion parameter. ($\mathcal{D} = 10^3$, $v = 50$, $\gamma = 0.99$, $t_0 = 0$ and $x_0 = 0$).

eter. An obvious result is that the longer the τ, the larger the paging area, i.e., predicting further into the future expands the paging area size by an extra amount. The increment in paging area size results in an increment in the paging cost. Thus predicting too far demands a huge paging area and incurs a huge paging cost. The implication of this result is the following: being far away in positive time axis from the available observations would render those past location samples useless (too old!). Hence this result demonstrates how the value of the location information depreciates with time.

The observation window is defined by $n_{min} \leq n \leq n_{max}$ where n_{max} is the maximum number of location samples limited by system processing capability, and $n_{min} = 1$ (only one location sample available). As shown in the figure, the worst case happens at the place with the smallest sample size $n = n_{min}$, which has the largest paging area and thus causes the highest paging cost to the system. The increase of the sample size — accumulating more location observations, helps to reduce the relative change in the size of the paging area. That is, as n tends to a large value, the mobile user gets the paging areas with almost the same size for any τ (no matter when the incoming call arrives after the last location observation). Again this result demonstrates the actual process of the estimation technique through accumulating location samples.

Another interesting point to observe in Figure (7) is that for a given τ ($\tau > 0$), there exists a sample size n^* for which the paging area size is smallest, and therefore the paging cost is minimal. Deriving from Equations (42) and (45), this optimum sample size is found to be $n^* = \tau/\delta$. This implies that for the same level of confidence and given a prediction interval $\tau > 0$, a smallest paging area corresponding to that τ can be obtained if only the latest n^* of the location observations are used in the design of the paging area, discarding the remaining $n - n^*$ older observations, for the scenario that the number of location observations accumulated is more than n^* ($n > n^*$). On the other hand, if $n < n^*$, a paging area larger than the smallest has to be designed according to Figure (7).

4.4. Optimum Paging Area and Confidence Coefficient

What value of γ (confidence measure or location probability) should we choose? This is not a mathematical question, but one that depends on the actual application. We have to take into account the affordable risk of making a false decision [20]. The introduction of this γ parameter into the paging scheme offers one more degree of freedom in the paging area design process. A high γ close to 1 is always desirable since the paging scheme has almost perfect reliability for paging coverage. However the problem is that the paging area becomes too large as shown in Figure (8), approaching the blanket paging scheme that covers the whole service area and thus reaching the maximum cost of the paging scheme.

From Figure (8), it is obvious that there exists a tradeoff between the location probability γ and the pre-

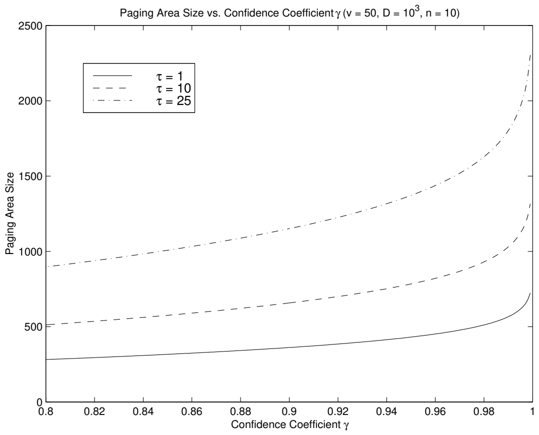

Figure 8. The optimum size of the paging area vs. the confidence coefficient γ ($0.8 \sim 0.999$) for different values of the prediction interval τ for the case of known diffusion parameter. ($\mathcal{D} = 10^3$, $v = 50$, $n = 10$, $t_0 = 0$ and $x_0 = 0$).

diction interval τ, which says that smaller prediction interval τ gives better coverage of locations (higher γ) or vice versa, for the same-sized paging area. Therefore we can sacrifice some paging accuracy to a longer prediction range of the paging scheme, or the other way around. For the case with high τ and high γ, the size of the optimum paging area increases drastically, which is the most costly scenario of the scheme.

5. Extension to Two-Dimensional Mobility Scenario

To understand the optimum personal paging area problem, the developments thus far have focused on a simple one-dimensional scenario of mobility and cellular structure. In this section, we will address the problem of extending the above study to a two-dimensional mobility scenario.

In this section, the structure of cellular system is assumed to consist of identical square cells tessellated over the service coverage area. Each pair of coordinates (x, y) represent the location of a base station or a cell. A micro-cellular environment is assumed, hence the distance between any two neighbor cells is small relative to the distance covered by a moving mobile in unit time.

In the previous one-dimensional study, a mobile user can only have two possible directions of moving: forward or backward. Only the value of the speed needs to be estimated. In 2-D scenario, however, the movement of a user can be in any direction. Not only the value of speed, but also the direction of speed needs to be estimated (see Figure (9)). Further, it required only two limits (upper and lower) to define a 1-D paging area, but in 2-D scenario we have to define the *shape* of a 2-D paging area besides the area size of the paging area.

We first formulate and solve the optimum paging area problem in a 2-D mobility scenario. Assuming a 2-D Brownian motion mobility model for a mobile user, we then estimate the mobility parameters and use them to design a 2-D paging area for the mobile user.

5.1. 2-D Problem Formulation and Its Solution

Similar to the formulation of the one-dimensional problem (see equations (6) and (7)), the problem of finding the optimal two-dimensional paging area $D^*_{PA}(t)$ with an area $A_{PA}(t)$ can be formulated as the following constrained optimization problem:

$$\min_{\{D(t)\}} \quad \mathcal{A}\{D(t)\} = \min_{\{D(t)\}} \int\int_{D(t)} dx\, dy \tag{46}$$

$$\text{s.t.} \quad P\{D(t)\} = \int\int_{D(t)} p_{xy}(x, y \,|\, x_0, y_0, t)\, dx\, dy = \gamma \tag{47}$$

where $\mathcal{A}\{D(t)\}$ represents the area of the region $D(t)$, and $P\{D(t)\}$ represents the probability over the region $D(t)$. The region $D(t)$ is a function of time since we assume a time-varying mobility pattern described by a

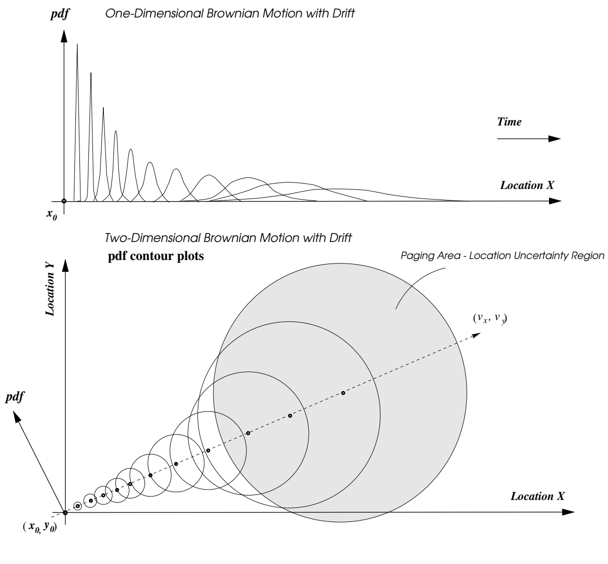

Figure 9. The time-varying pdf of a one-dimensional Brownian motion process, and the corresponding pdf contour plots of a two-dimensional Brownian motion process with equal location probability. Circular shape of the contour lines is due to the isotropic diffusion assumption for the 2-D Brownian motion.

2-D location pdf $p_{xy}(x, y \mid x_0, y_0, t)$. (x_0, y_0) is the initial position of a mobile (or the last known location). The above problem can be thought of as a two-dimensional confidence interval estimation problem, or a **"confidence region estimation"** problem, which is solved by following theorem [8, 10]:

Theorem 1. (**Optimal Paging Area Theorem**) A smallest size region $D^*(t)$ which still meets a confidence measure $P\{D^*(t)\} = \gamma$ can be constructed by choosing the elements $\{x, y\}$ with largest $p_{xy}(x, y|x_0, y_0, t)$ until their collective probability meets γ.

We omit the proof of the theorem here which can be found in [10].

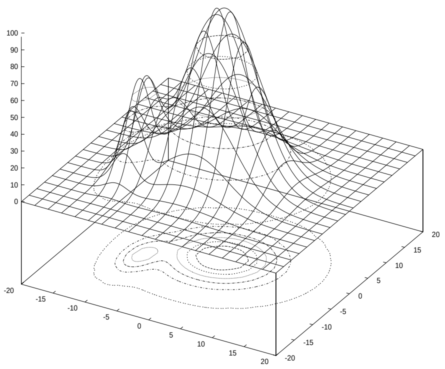

Figure 10. Different contour plots of an arbitrary 2-D location pdf for different values of γ, and their corresponding paging areas (on the bottom plane) with boundaries defined by the projections of the contour lines on to x-y plane.

For our assumption with two-dimensional location pdf, Theorem 1 indicates that the optimal paging area on x-y plane with the location probability γ is the region which has the minimum area and meanwhile attains the probability γ. It consists of a set of locations with a boundary defined simply by the isocline of user location pdf formed by letting $p(x, y|x_0, y_0, t) = \phi^*$, where ϕ^* is a constant that depends on γ and the location pdf $p(x, y|x_0, y_0, t)$ [10]. Figure 10 serves to conceptually illustrate this result.

5.2. 2-D Mobility Model

Although the problem formulation of the optimal paging area problem is applicable to arbitrary mobility models and their location pdfs, it is useful to illustrate with a concrete example.

We use a two-dimensional Brownian motion model with diffusion constants $\mathcal{D}_x$, $\mathcal{D}_y$, and drift velocities v_x, v_y as our example. We assume that the correlation coefficient $\rho_{xy} = 0$, that is the 2-D Brownian motion consists of two *independent* Brownian motion processes along x and y axes. This simplifies our discussion without loss of generality, since the general Brownian motion pdf expression with $\rho_{xy} \neq 0$ can always be reduced to an independent x and y form by proper rotation of coordinates [18].

Let $x(t)$ and $y(t)$ be two independent Brownian motion processes with constant drift velocities $v_x \geq 0$, $v_y \geq 0$, and variance parameters $\mathcal{D}_x$, $\mathcal{D}_y$. They are described by two time-varying conditional Gaussian pdfs:

$$p_x(x|x_0, t) \sim \mathcal{N}\{x_0 + v_x(t - t_0), \mathcal{D}_x(t - t_0)\} \tag{48}$$

$$p_y(y|y_0, t) \sim \mathcal{N}\{y_0 + v_y(t - t_0), \mathcal{D}_y(t - t_0)\} \tag{49}$$

We then have their conditional joint pdf for the location distribution given the starting point (or the last known location) (x_0, y_0) as:

$$p_{xy}(x, y|x_0, y_0, t) = \frac{1}{2\pi\sqrt{\mathcal{D}_x\,\mathcal{D}_y}(t - t_0)} \times \exp\left\{\frac{-[x - x_0 - v_x(t - t_0)]^2}{2\mathcal{D}_x(t - t_0)} + \frac{-[y - y_0 - v_y(t - t_0)]^2}{2\mathcal{D}_y(t - t_0)}\right\} \tag{50}$$

For the convenience of analysis, let us consider the *isotropic* Brownian motion by assuming $\mathcal{D}_x = \mathcal{D}_y = \mathcal{D}$. After rewriting Equation (50), we then have a simplified 2-D location pdf as:

$$p_{xy}[x, y|\mu_x(t), \mu_y(t), t] = \frac{1}{2\pi\mathcal{D}(t - t_0)} \times \exp\left\{\frac{-[x - \mu_x(t)]^2 + [y - \mu_y(t)]^2}{2\mathcal{D}(t - t_0)}\right\} \tag{51}$$

where

$$\mu_x(t) = x_0 + v_x(t - t_0) \tag{52}$$

$$\mu_y(t) = y_0 + v_y(t - t_0) \tag{53}$$

With this two-dimensional isotropic Brownian motion with drift mobility model, the optimum paging area forms a *circular* region as shown in Figure (9), which is a typical case of the contour plots of the paging areas shown in Figure (10).

Practical significance of this mobility model is threefold: (1) the average speed and direction of a mobile user are modeled by the speed vector $\mathbf{v} = (v_x,\ v_y)$;

(2) effect of growing location uncertainty region as time elapses after the last known location (x_0, y_0) are modeled by $\mathcal{D}(t-t_0)$; (3) the model is particularly useful in studying the dynamic search strategies for an individual (such as paging strategies) with knowledge about the time length and a previous location known to the system.

5.3. 2-D Mobility Parameter Estimation and the Optimum Paging Area

Let $[X(t_1), Y(t_1)]$, $[X(t_2), Y(t_2)]$, $[X(t_3), Y(t_3)]$, $\cdots$, $[X(t_n), Y(t_n)]$ be a set of n two-dimensional location observations at time instants $t = t_i$, $1 \le i \le n$. Based on those n observations, we have to estimate the parameters of the 2-D mobility model of a mobile user and design a 2-D circular paging area to efficiently cover the mobile user. At some time t, what we need for configuring this 2-D paging area are the radius and the central location of the circular paging area.

5.3.1. The Size of the Optimum 2-D Paging area

From the simplified location pdf in Equation (51) and the location probability constraint of Equation (47), a radius R_c within which the probability of location is equal to γ can be derived as,

$$R_c(t) \;=\; \sqrt{2\,\mathcal{D}\,(t-t_0)\,\ln\!\left(\frac{1}{1-\gamma}\right)}\,. \tag{54}$$

Then the size of the optimum paging area that attains probability γ given the diffusion parameter $\mathcal{D}$ can be computed by:

$$A_{PA}(t) \;=\; 2\pi\,\ln\!\left(\frac{1}{1-\gamma}\right)\,\mathcal{D}\,(t-t_0)\,. \tag{55}$$

For the case of unknown $\mathcal{D}$, we can use the sample variance S^2 of Equation (29) to replace $\mathcal{D}$. The assumptions of independence of motions in x and y and $\mathcal{D}_x = \mathcal{D}_y = \mathcal{D}$ justify the use of the $\mathcal{D}_x$ or $\mathcal{D}_y$ estimate in one-dimensional study as the estimate of $\mathcal{D}$ here in two-dimensional study.

5.3.2. The Position of the Optimum 2-D Paging Area

Generally, to determine the center of the optimum paging area, we have to estimate both the average moving direction and the average moving velocity of a mobile user, or to estimate the speed vector $\mathbf{v} = (v_x,\ v_y)$.

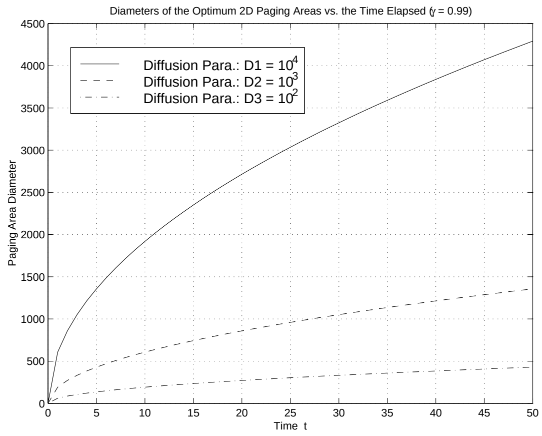

Figure 11. The diameters of the optimum *two-dimensional* paging areas as a function of the time interval (t_0, t) for three values of the known diffusion parameter $\mathcal{D}$ (10^2, 10^3 and 10^4), given a fixed confidence coefficient $\gamma = 0.99$. ($t_0 = 0$, $x_0 = 0$).

However under the independence assumption of the motions in x and y, the estimation results in previous one-dimensional study can be readily applied.

Because of the independence, we can estimate the confidence interval of v_x based on a set of location observations in x separately from the estimation of the confidence interval of v_y. Due to the *unimodal* and *symmetrical* property of the location pdf, the coordinates of the 2-D paging area center can be determined simply by the central values (mean values) of the confidence intervals along x and y. Denote this coordinates as $[\mu_x(t),\ \mu_y(t)]$ as in Equations (52) and (53). Obviously $\mu_x(t)$ is the average location of the mobile user along x and $\mu_y(t)$ is the average location along y.

Assuming the diffusion parameter $\mathcal{D}$ is known and based on the results of the one-dimensional study in Equations (24) and (25), $\mu_x(t)$ and $\mu_y(t)$ can be estimated by:

$$\hat{\mu}_x(t) = \frac{d_{l\,x}(t) + d_{u\,x}(t)}{2} \tag{56}$$

$$\hat{\mu}_y(t) = \frac{d_{l\,y}(t) + d_{u\,y}(t)}{2} \tag{57}$$

If we assume the diffusion parameter $\mathcal{D}$ is unknown, using results in Equations (37) and (38), then $\mu_x(t)$ and $\mu_y(t)$ can be estimated similarly as:

$$\hat{\mu}_x(t) = \frac{D_{l\,x}(t) + D_{u\,x}(t)}{2} \tag{58}$$

$$\hat{\mu}_y(t) = \frac{D_{l\,y}(t) + D_{u\,y}(t)}{2} \tag{59}$$

Furthermore from Equation (10), the estimated average velocities of the mobile user in x and y directions can be found by:

$$\hat{v}_x = \frac{\hat{\mu}_x(t) - x_0}{t - t_0} \tag{60}$$

$$\hat{v}_y = \frac{\hat{\mu}_y(t) - y_0}{t - t_0} \tag{61}$$

From above analysis, we have the radius $R_c(t)$ of and the origin coordinates $[\mu_x(t),\ \mu_y(t)]$ of the circular optimum paging area, the optimum paging area at any time t is therefore completely determined. We have checked and verified that the results in Equations (56), (57), (58), (59), (60) and (61) are the *maximum likelihood* (ML) estimates of the mean location $[\mu_x(t),\ \mu_y(t)]$ and mean velocity $\mathbf{v} = (v_x,\ v_y)$.

Figure (11) shows one of the results of the two-dimensional study. It plots the diameters of the optimum 2-D paging areas as a function of the time elapsed $(t - t_0)$ for the known diffusion parameter $\mathcal{D}$ case. Similar to the result of the 1-D study shown in Figure (5), this figure shows that the 2-D optimum paging areas are increasing functions of time, and the higher the variability of the motion (higher $\mathcal{D}$), the larger the size of the paging area.

Notice that although the two-dimensional study of the optimum paging area problem has a higher practical value, the one-dimensional study is *essential* since it is a fundamental basis for the 2-D study.

6. Implementation Issues

6.1. One-Step Paging or Multiple-Step Paging

In this work, we assume that there is only one paging step for the paging procedures of the system, and the study only focuses on the optimum configuration of this one-step paging area to cover the most likely places where a user would be. The above scheme has only a partial coverage. With some low probability $1-\gamma$, users would be out of the paging area designed. Then the system encounters a paging failure due to the partial coverage of the scheme. However we can always use a high confidence coefficient γ to reduce the paging failure caused by this scheme [8, 10]. There is a tradeoff here, that is, a high γ and thus high paging cost (more locations need to be searched) with little paging failure **or** a reasonable γ and thus reasonable cost of paging with some tolerable paging failure. To search the rest of the locations (that occur with low probability $1 - \gamma$) takes effort which may not be worth doing. Therefore the one-step paging procedure can be useful and practical, especially for paging the mobile users who have more deterministic mobility patterns — people who have routine movement behaviors or move among several fixed locations [15].

On the other hand, some systems may use a multiple-step sequential paging procedure to overcome the drawback of the single-step paging scheme. For example, a system using a two-step sequential paging procedure may use the first paging step to cover some most likely locations with a high probability, and use the second paging step to cover the rest of the locations not covered by the first step [16]. The $M-$step sequential paging procedure may divide the service area into M regions and page them one by one. Still the first step of paging is the most important step because it is more valuable if we can find people in the first try. Those are complete paging schemes since they cover the whole service area of the system. In practical systems, we normally have a paging delay constraint which limits the number of steps for paging [2, 7, 12]. After trying a number of times without any response, the system has to quit and declare a paging failure to save system resources and to avoid system congestion due to paging traffic overload. In general, the one-step paging scheme is a flexible (by adjusting the γ) and cost-efficient (covering the most important places in one-shot) alternative in the mobility management of a cellular system.

6.2. Location Observations: Centralized or Distributed

Observations of the locations of a user can be taken by base stations or by a mobile terminal itself, which results in two different schemes: centralized or distributed.

If the observations are done by base stations, every base station has to poll each mobile user to check the appearance of the user in its cell and report the result to mobile switching center (MSC). Due to the large number of users in the system, the system resources (both downlink and uplink control channels in a cell) used for the polling may easily get overloaded, resulting in congestion or even blocking of the control channels. Therefore the centralized location measurements place a heavy burden on the system and is hence prone to congestion.

If the observations are done by mobile users themselves, each user has to monitor the beacon signals of base stations that it travels across, which is the way that the current systems operate. Then what we need is a small memory space in a mobile terminal to store the list of base stations detected and accumulated by the mobile user (a list of BS coordinates). The size of this memory space determines the maximum window size (i.e., the maximum sample size n_{max}) of the most recent location samples observed. Additionally, the user has to report its stored base station list to the network at times to update the list kept at the network side for the user. Relatively this signal processing load is lower than that of the polling of mobile users by base stations. Another advantage of the distributed location measurements is that the processing load is spread among all the mobile terminals in the system, hence the congestion of system resources may be avoided. Therefore the distributed location observations can be a better choice for future system implementation.

In addition, there is some extra signaling traffic generated by a mobile user's reporting of the collected location data to its serving base station at the end of the observation window. This increased signaling traffic is the price to pay to integrate some *learning intelligence* into the system. If the network can learn automatically about the mobility behavior of a mobile user by examining the collected location data of the user, it can always do better to minimize the paging signaling traffic by minimizing the number of base stations to send page messages. Therefore, the overall signaling traffic may not be necessarily increased. On the other hand, as FCC's E911 service requirement is becoming a necessity for cellular network service providers, the location measurement data of mobile users will become readily available, which will make this paging scheme more practical.

6.3. Mobility Model and Paging Area: Continuous or Discrete

In this paper, we assume that the 1-D and 2-D mobility models are stochastic processes described by continuous pdf functions. Because the location observations are discrete random variables selected from the set of BS coordinates, it seems better to use mobility models that work on those discrete random variables. For example, we can use 1-D and 2-D *random walk* models with the step size as the distance between regular cells. However, with random walk mobility models the optimal paging area problem is analytically intractable. Instead in this study, we use 1-D and 2-D Brownian motion with drift mobility models which are the continuous versions of the random walks (random walk process converge to Brownian motion with drift process as the step size tends to zero) [18–20]. The assumption we made in Section 1 about the micro-cellular geometry and that the mobility parameters of mobile users are on a larger scale than the cell size, justifies the use of the continuous Brownian motion models to replace the discrete random walk models. By doing so, the closed-form results of the optimal paging area problem are obtained.

Also in this work, we assume that there is no fundamental indivisible unit of area in the domain of interest. In the real world of a cellular system, however, the fundamental elements are cells of certain area which cannot be further divided. Then the size of a paging area is measured in terms of the number of cell areas contained in the paging area, and thus the paging area is actually discrete. The location probability associated with the paging area is a sum of the cell probabilities which is also discrete. In this case, we can look at the problem as a result of the granulation of cell area relative to paging area. When the granulation becomes finer, the problem converges to the results of the above analysis. If the granulation is rough, then it will affect the above results in the following ways: (1) the boundary of a paging area is actually formed by the perimeter of the group of cells included in the paging area; (2) the location probability over a cell will help to determine whether to include that cell into a paging area. When adding a cell with the largest probability among the remaining cells to a region makes the total probability over that region meet or exceed the probability threshold γ, the cell should be included in that region. This paging area region may be larger than the exact optimum paging area associated with a probability threshold γ in the continuous case. The last cell has to be included as a whole piece and the area of the cell can never be cut to make the total probability over the region just satisfy the γ probability requirement [10].

7. Conclusion

In general, this study provides insights into the mobility management problem on how to configure optimal single-step paging areas based on the parameter estimation of a mobile user's mobility process. It also analyzes the effects of the user's mobility characteristics and the system design parameters on the dimensioning of this optimal paging area. In particular, the conclusions of the study are summarized in the following:

1. The position of the optimum paging area depends on the time difference $t - t_0$, the average motion velocity vector (v_x, v_y) and the last known location of the mobile user (x_0, y_0). When a mobile user moves as time elapses, the optimum paging area moves accordingly along with the user, following the statistical mean of its movement.

2. The size of the optimum paging area depends on the time difference $t - t_0$, *location uncertainty* (or motion variability $\mathcal{D}(t - t_0)$) and n, the number of location samples taken within the observation window. The size of the paging area expands with the increase of the location uncertainty, and also increases with the *prediction interval* τ. But the relative change in the size of the paging area decreases with the increase of the *sample size* n. Obviously more observations help locate the mobile user with a higher certainty.

3. The *confidence coefficient* γ is another system design parameter. Paging area increases drastically as γ increases close to 1, but the paging failure due to the partial coverage of the one-step paging procedure reduces. The study reveals a tradeoff between γ and the prediction interval τ, i.e., higher γ can be compensated by smaller τ. This result indicates a fact that we can never expect a paging scheme using past location data to predict further into the future and still maintain the same accuracy without incurring extra cost. The actual choice of γ depends on the particular performance requirements of the paging scheme.

4. The study results demonstrate a general phenomenon: the value of the location information (location observations) to a paging scheme depreciates with the increase of time.

5. This work suggests that while a mobile user is moving in a given direction the probability mass of the user location is also moving along in the same direction. Therefore the optimum paging area for a mobile user should match the time-varying mobility pattern of the user to achieve the best paging efficiency.

6. The *confidence interval* or *confidence region* based formulation of the optimum paging area problem is a favorable approach to combine the study of the optimum paging area problem with the mobility parameter estimation problem. As is shown in this work, this approach exploits the past location information available to the system and a mobile user, and establishes the optimum single-step time-varying paging areas for the mobile user.

Future work will address the performance evaluation of the proposed paging scheme by simulation study. The regular sampling assumption in unknown diffusion parameter case can be further relaxed. More generally, the performance of the paging scheme should be investigated based on the actual mobility data collected from mobile users.

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Appendix

Student t–Distribution for Interval Estimation in Unknown Diffusion Parameter Case

Let $U_1, U_2, \cdots, U_n$ be a set of independent Gaussian random variables with $\mathcal{N}(\mu,\ \sigma^2)$. Let the sample mean be defined as,

$$\overline{U} \ = \ \frac{1}{n}\sum_{i=1}^{n} U_i \tag{62}$$

Notice that $\overline{U}$ is also a Gaussian random variable with $\mathcal{N}(\mu,\ \sigma^2/n)$. It follows that,

$$W \ = \ \frac{\overline{U} - \mu}{\sigma/\sqrt{n}} \tag{63}$$

is a standard Gaussian random variable.

It can be shown that if $U_1, U_2, \cdots, U_n$ are independent Gaussian random variables with $\mathcal{N}(\mu,\ \sigma^2)$, then

$$Z \ = \ \frac{1}{\sigma^2}\sum_{i=1}^{n} [U_i - \overline{U}]^2 \tag{64}$$

has a chi-square distribution with $(n-1)$ degrees of freedom [21]. The probability density function $p(x)$ of a chi-square distribution with n degrees of freedom (denoted by $\mathcal{X}^2(n)$) is given by

$$p(x) \ = \ \frac{1}{2^{n/2}\,\Gamma(n/2)}\ x^{n/2-1}\ e^{-x/2}, \qquad x \geq 0 \tag{65}$$

where $\Gamma(\cdot)$ is the Gamma function and is defined as,

$$\Gamma(m+1) \ = \ \int_0^{\infty} x^m\, e^{-x} dx, \qquad m > -1. \tag{66}$$

We can define a random variable as

$$\beta \ = \ \frac{U}{\sqrt{Z/(n-1)}} \tag{67}$$

which by definition has a Student t-distribution with $(n-1)$ degrees of freedom and whose distribution is expressed by

$$p(\beta)_{(n-1)} = \frac{\Gamma(n/2)}{\sqrt{(n-1)\pi}\,\Gamma[(n-1)/2]} \left(1 + \frac{\beta^2}{n-1}\right)^{-n/2} \tag{68}$$

Note that Student t-distribution with $(n-1)$ degrees of freedom converges toward a standard Gaussian distribution as n tends to infinity.

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