Pricing and Power Control in a Multicell Wireless Data Network*

Cem U. Saraydar†
Lucent Technologies
101 Crawfords Corner Road
Room 3K-311
Holmdel, NJ 07733
saraydar@lucent.com

Narayan B. Mandayam‡
WINLAB, Rutgers University
73 Brett Road
Piscataway, NJ 08854-8060
narayan@winlab.rutgers.edu

David J. Goodman
Dept. of Elect. Eng.
Polytechnic University
6 Metrotech Center
Brooklyn, NY 11201, USA
dgoodman@duke.poly.edu

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Abstract

In this paper we consider distributed power control in a multicell wireless data system and study the effect of pricing transmit power. Drawing on our earlier work in [1], we formulate the QoS of a data user via a utility function measured in bits per Joule. We consider distributed power control, modeled as a non-cooperative game, where users maximize their utilities in a multicell system. Base station assignment based on received signal strength as well as received signal-to-interference ratio (SIR) are considered jointly with power control. Our results indicate that for both assignment schemes, such a procedure results in an inefficient operating point (Nash equilibrium) for the entire system. We introduce pricing of transmit power as a mechanism for influencing data user behavior and our results show that the distributed power control based on maximizing the net utility (utility minus the price) results in improving the Pareto efficiency of the resulting operating point. Variations of pricing based on global and local loading in cells are considered as a means of improving the efficiency of wireless data networks. Finally, we discuss the improvement in utilities through a centralized scheme where each base station (BS) calculates the best SIR to be targeted by the terminals it is assigned.

Keywords

Power control, multicell networks, base station assignment, wireless data, game theory, pricing, pareto efficiency

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1 Introduction

The enormous demand for data services in wireless networks necessitates efficient and effective radio resource management. Among various resource management methods, power control is significant, especially in CDMA systems, due to its implications on interference and battery consumption. Despite a large number of work on power control for voice traffic [2–4], power control for data is relatively unexplored. Recent work studies power control for data networks under a new light, powered by the mathematics and concepts from microeconomics [1, 5–7]. Particularly, [5] studies a mechanism by which each terminal adjusts its transmit power in a single-cell data network in order to maximize its individual satisfaction of the use of network resources in a distributed fashion. The interaction between self-optimizing terminals is called a *non-cooperative power control game*. At the outcome of the power control game, it is shown that there exists an *equilibrium* vector of transmit powers where no terminal can improve its utility individually any further. However, through a mechanism called *pricing*, improvement in the utilities for all terminals is noted.

In this paper, we extend the analysis presented in [5] to a multicell environment where each user experiences interference from terminals outside its cell in addition to the ones within the same cell. When a base station is appointed to establish the connection of a user with the wireless communication system, we say the user is *assigned* to that base station. Previous work [8,9] considers the combined problem of transmit power control and base station assignment in a wireless *voice* system. Specifically, in the above works, the goal is to determine a transmit power and a base station assignment for each user such that the individual target SIRs are provided to all terminals. The total transmit power is minimized over the set of powers *and* base station assignments subject to minimum SIR constraints. While the minimum SIR constraint-based allocation of resources is appropriate for voice, it does not represent data user satisfaction well. The perception of voice quality remains unchanged beyond a certain SIR, since anything better than that is virtually the same for the human ear. However, data services are sensitive to changes in the SIR in a continuous fashion. Therefore, we need a power control approach that is different than conventional voice power control. We formulate the resource allocation problem in a multicell data system in which the individuals maximize their *utility* with respect to both the *transmit power* and the *base station assignment*. While the framework is the same as in [5], in the noncooperative games of this paper, the utility is expressed differently and the maximization problem is now two-dimensional (transmit power and base station).

The outline of the paper is as follows. First, we present the multicell model in Section 2. Then, in Section 3, we introduce a power control game where the base station assignment is fixed. We present a discussion on the equilibrium of the game, with and without pricing. In Section 4, we generalize the power control game to include base station assignment and show that the resulting equilibrium corresponds to a maximum-SIR based assignment policy. Section 5 examines a centralized scheme to improve utilities. In this scheme, each base station computes an SIR to be targeted by all the terminals assigned to that base station. Finally, we present numerical results and a discussion of these results in Section 6.

2 System Model

We consider a wireless CDMA data system with K cells serving N users which are randomly spread throughout the service area. Let the set of base stations be denoted by the set $\mathcal{K} = \{1, \cdots, K\}$. Terminal j is d_{ij} meters away from base station i with a path gain of h_{ij}. We assume the terminals are stationary and therefore the path gains do not change. Each terminal transmits data at the rate of R bits/second over a spectrum of W Hz. Noise is assumed to be additive white Gaussian noise (AWGN) with noise power of σ^2 Watts at the receiver. For terminal j, the power expended in transmission is p_j Watts and the signal-to-interference ratio (SIR) attained with this power level at base station i is denoted by γ_{ij}. The SIR obtained by user j at base station i is given by

$$\gamma_{ij} = \frac{W}{R} \frac{h_{ij} p_j}{\sum_{k=1, k \neq j}^{N} h_{ik} p_k + \sigma^2} \quad . \tag{1}$$

In a multicell system, each terminal is received at each base station with different power levels, leading to a different SIR level and therefore resulting in a different quality-of-service (QoS) level at each base station. We assume there is no base station diversity, i.e., each terminal is connected to a single base station at any given time, although all terminals interfere at all base stations in the system. The QoS received by terminal j at its assigned base station a_j is quantified through a *utility function* which measures performance in bits per Joule. The utility function is defined as the ratio of the expected number of bits received correctly to the energy consumed in the transmission. The utility of terminal j received at its assigned base station a_j is,

$$u_{a_j j}(\mathbf{p}) = \frac{RL}{M} \frac{f(\gamma_{a_j j})}{p_j} \quad \text{bits/Joule} \quad , \tag{2}$$

where $f(\cdot)$ is the efficiency function that approximates the probability of successful reception, L is the number of information bits in a packet of size M bits, $\mathbf{p}$ denotes the power vector $(p_1, \cdots, p_N)$ and $a_j \in \mathcal{K}$ is the base station assigned for user j. The efficiency function is discussed in [1] and it depends on factors such as channel quality, modulation, coding and packet/frame size. Assuming independent bit errors, the frame success rate (FSR) is a function of the bit-error rate (BER) and it is given by $\text{FSR} = (1 - \text{BER})^M$ for a packet size of M bits. The efficiency function is approximated from the FSR to yield zero utility at zero power and it is given as $f(\text{SIR}) = (1 - 2\text{BER}(\text{SIR}))^M$. The base station assignment, denoted by the vector $\mathbf{a} = (a_1, \cdots, a_N)$, can be arbitrary. Notice that there are a total of K^N different base station assignments possible. In this paper, we discuss base station assignment based on Maximum Received Signal Strength (MRSS), and base station assignment based on Maximum SIR (MSIR). In Section 3, we discuss base station assignment based on MRSS of base pilot signals which allows simple implementation of a distributed scheme. In Section 4 we discuss base station assignment based on MSIR which offers increased utilities.

3 Base Station Assignment Based on MRSS

In this section, we assume the assigned base station of j, denoted by a_j, is determined based on the received signal strength of base station pilot signals. Assuming signal degradation is similar in both the uplink and the downlink direction, we find that the base station (BS) assigned to user j is the one that receives the terminal signal with the highest gain, h_{ij}, among all $i \in \mathcal{K}$. Equivalently, since we assume a path loss model based on distance from the BS, the base station assignment is determined by identifying the base station closest in distance, d_{ij}, to the terminal,

$$a_j = \arg\max_i h_{ij} \equiv \arg\min_i d_{ij}, \quad \text{for all } j \in \mathcal{N} \quad . \tag{3}$$

This rule is referred to as maximum received signal strength, MRSS. Once the base station assignment, $\mathbf{a}$, is determined as described above and fixed, the multicell allocation problem is reduced to finding the *best transmit power*, instead of jointly choosing the best [transmit power, base station] pair. Given the set of interference powers, each terminal computes the transmit power that maximizes its own utility. The group of terminals adjusting transmit powers to maximize individual utilities describes what we call a non-cooperative power control game. In the following, we formally define the power control game, examine the solution concepts resulting from the game and discuss the efficiency of the solution followed by a discussion of a pricing strategy to improve efficiency.

3.1 Power Control Game with no Pricing

Denote the index set for the mobile users currently in the system by $\mathcal{N} = \{1, 2, \cdots, N\}$, the set of transmit powers available to terminal j by P_j, and the utility function of user j by $u_j(\cdot)$. We assume that the strategy space P_j of terminal j is a compact, convex set with minimum and maximum power constraints denoted by $\underline{p}_j$ and $\overline{p}_j$, respectively. Throughout this paper, we assume $\overline{p}_j = \overline{p}$ for all $j \in \mathcal{N}$. Furthermore, in this section we let $\underline{p}_j = 0$ for all j. Given the base station assignment, a_j, under the MRSS rule given in (3), the utility of user j is defined as,

$$u_j(\mathbf{p}) = u_{a_j j}(\mathbf{p}) \quad . \tag{4}$$

In the power control game, each user maximizes its own utility in a distributed fashion by selecting a transmit power. Formally, the multicell power control game (MCPG) can be expressed as

$$\textbf{(MCPG-MRSS)} \quad \max_{p_j \in P_j} u_j(p_j, \mathbf{p}_{-j}) = u_{a_j j}(p_j, \mathbf{p}_{-j}) \quad \text{for all } j \in \mathcal{N} \quad , \tag{5}$$

where a_j is given in (3) and $\mathbf{p}_{-j}$ denotes the vector of transmit powers of all terminals except the transmit power p_j of terminal j. First, we examine equilibrium properties of the MCPG-MRSS. The solution concept that is most widely used for game theoretic problems is the *Nash equilibrium* [10]. At the Nash equilibrium, no user has any incentive to change its power level since no individual improvement in utilities is possible.

In the context of equilibrium analysis, a key property of the utility function in (4) is its quasiconcavity. For a quasiconcave function, either the local maximum of the function is at the same time a global maximum or the function is constant in the neighborhood of a local maximum [11]. It can be shown that the utility function used in this study is quasiconcave [7]. Once quasiconcavity is established, the existence of a Nash equilibrium in MCPG-MRSS follows directly. In fact, the proof is similar to that given in [5] for a single-cell power control game.

For any given interference vector, $\mathbf{p}_{-j}$, the power that maximizes $u_j(p_j, \mathbf{p}_{-j})$ given in (4) is found to satisfy,

$$f'(\gamma_{a_j j})\gamma_{a_j j} - f(\gamma_{a_j j}) = 0 \quad , \tag{6}$$

by the first order necessary optimality condition. Here we let a_j denote the base station assignment with MRSS. Suppose that the SIR value that satisfies equation (6) is $\gamma_{a_j j} = \tilde{\gamma}$ and it is the same for all $j \in \mathcal{N}$. The value of the utility maximizing SIR, $\tilde{\gamma}$, depends on system parameters such as modulation and packet length. The transmit power p_j, that maximizes utility, is the power that achieves $\gamma_{a_j j} = \tilde{\gamma}$. However, some users might not be able to achieve $\tilde{\gamma}$ *at the equilibrium.* If this is the case, the best a terminal can do is to transmit at maximum power, $\overline{p}$. The *best response* of terminal j to some given interference vector, $\mathbf{p}_{-j}$, can be expressed as,

$$r_j(\mathbf{p}_{-j}) = \arg\max_{p_j} u_j(\mathbf{p}) = \min\left(\overline{p}_j, \frac{\tilde{\gamma}(\sum_{k\neq j} h_{a_j k} p_k + \sigma^2)}{\frac{W}{R} h_{a_j j}}\right) \quad . \tag{7}$$

Making use of the best response correspondence, we can show that MCPG-MRSS equilibrium is unique. The key aspect of the uniqueness proof is to realize that the best-response correspondence is a *standard* function [5].

The Nash equilibrium is reached as a result of distributed decision taking which is expected to be less *efficient* than a possible power allocation obtained through centralized control. A power allocation is said to be more efficient (or Pareto dominant) if it is possible to increase the utility of some of the terminals without hurting any other terminal. We can show that the MCPG-MRSS equilibrium is inefficient. The proof is similar to the inefficiency proof in [5] given for the Nash equilibrium of a noncooperative game in a single-cell system. The key idea is to show that there exists a power vector smaller than the equilibrium vector where each user receives higher utilities than they receive at the equilibrium. We seek a way to have each terminal reduce its transmit power in a *distributed fashion* and yet obtain a utility higher than the MCPG-MRSS equilibrium and thus result in a more efficient power allocation. A usage-based pricing scheme can achieve such an outcome. Usage-based refers to the dependence of the scheme on the amount of resources consumed. In [5], it is shown that a linear usage-based pricing scheme, that is proportional to the amount of usage of the radio resources in terms of transmit power, improves utilities of terminals in a single-cell wireless data network. Thus, we seek to improve utilities offered at the equilibrium of MCPG-MRSS by pricing the transmit powers.

3.2 Power Control Game with Pricing

In the presence of pricing, the net utility user j receives at base station i can be expressed as,

$$u_{ij}(\mathbf{p}) - c_i p_j \quad , \tag{8}$$

where c_i is a constant referred to as the *pricing factor* announced by base station i. The vector of pricing factors $(c_1, c_2, \cdots, c_K)$ is denoted by $\mathbf{c}$. Since in the absence of base station diversity, each terminal is connected to only one base station at any given time, and given that the base station assigned to terminal j is denoted by a_j, the *net* utility function with pricing for terminal j can be expressed as,

$$u_j^c(\mathbf{p}) = u_{a_j j}(\mathbf{p}) - c_{a_j} p_j \quad . \tag{9}$$

Given the base station assignment, $\mathbf{a}$, as determined in (3), the MCPG with pricing (MCPGP) can be stated as,

$$\textbf{(MCPGP-MRSS)} \quad \max_{p_j \in P_j} u_j^c(\mathbf{p}) = u_{a_j j}(\mathbf{p}) - c_{a_j} p_j \text{ for all } j \in N \quad . \tag{10}$$

Notice that in this section the base station assigned to a user is the same at the equilibrium of both the game with pricing and the game without pricing. In the following section, we first explain the special case of equal pricing factors at each base station. We refer to this case as *global pricing*. We then return to the general formulation where each base station computes a pricing factor based on local traffic information. This case is referred to as *local pricing*. Then, we investigate the existence and properties of the Nash equilibria in the game with pricing.

3.2.1 Global Pricing

Suppose the pricing factors are such that $c_i = c$ for all i for some constant c. We refer to this pricing strategy as global pricing. Under a linear global pricing scheme, the MCPGP for the multicell system and the pricing game for a single-cell system discussed in [5] are mathematically equivalent. The only distinction is that the SIR of terminal j in the single cell case is now replaced with a multicell-equivalent SIR expression that is determined by the path gains from all terminals to the assigned base station of terminal j. As in the case of the single cell solution, the system determines a good selection of the global pricing parameter c in order to improve utilities of all terminals in all the cells.

3.2.2 Local Pricing

The global pricing factor of the previous section is a systemwide parameter which is determined regardless of the relative load on each base station and therefore is representative of the cumulative traffic statistics. For pricing in a multicell scenario, it is possible to exploit the fact that each base station has a different degree of congestion and thus should determine a *local pricing factor* based on the amount of traffic that is offered in its service area. Notice that the global pricing strategy is constrained to pricing vectors of the form $\mathbf{c} = (c, c, \cdots, c)$. In local pricing, base station i broadcasts the pricing factor c_i determined locally, leading to the general pricing vector $\mathbf{c} = (c_1, c_2, \cdots, c_K)$. Each terminal then uses the pricing factor corresponding to its base station assignment, to calculate the power that maximizes its net utility.

Note that each possible pricing vector is a point in the $\mathcal{R}_+^K$ space. As opposed to the single-cell setting, it is clearly impractical to search the entire space of pricing factors to determine the best one in a multi-cell system. Therefore, we develop a heuristic pricing strategy where base station i calculates its pricing factor as $c_i = \alpha N_i$ where α is a scalar and N_i is the number of terminals assigned to base i. From our single cell studies, we observe that the pricing factor that benefits the terminals most, tends to increase with the load in the system. Thus, we chose the pricing factor to be proportional to the number of terminals within the cell. Also, such a choice of pricing factors in effect reduces the abundance of alternatives of the pricing factor from the entire space to the particular one described above. For this specific choice, the pricing vector can be varied by changing the value of the scalar α.

3.2.3 Nash Equilibria

Since the net utility function in equation (9) is not quasiconcave, equilibrium existence result for MCPG does not apply to MCPGP. Instead, the existence of equilibrium follows from the supermodularity of the game with pricing. This approach was taken in our previous work in [5] to show equilibrium existence in single cell games. Supermodular games have utilities with non-decreasing differences (NDD). More specifically, in a game with NDD, a terminal has to increase its own power in response to an increase in interference power in order to increase its utility. Equivalently for continuous and twice differentiable utilities, $u_j(p_j, \mathbf{p}_{-j})$ has NDD in $(p_j, \mathbf{p}_{-j})$ if and only if $\frac{\partial^2 u_j(\mathbf{p})}{\partial p_j \partial p_k} \geq 0$ for all $j \neq k$. The second order NDD condition, $\frac{\partial^2 u_j}{\partial p_j \partial p_k} \geq 0$ for all $j, k \in \mathcal{N}$, is satisfied for the utility function of MCPGP if $\gamma_{a_j j} \geq \hat{\gamma}$ for all j where $\hat{\gamma} = 2\log M$. The constraint on SIR of user j can be translated to a lower bound on the strategy space of that user as $\underline{p}_j = \frac{\hat{\gamma}(\sum_{k\neq j} h_{a_j k} p_k + \sigma^2)}{\frac{W}{R} h_{a_j j}}$. Let $\underline{\mathbf{p}}$ denote the power vector corresponding to $\gamma_{a_j j} = \hat{\gamma}$ for all j assuming that it exists. Also let $\overline{\mathbf{p}}$ denote the power vector corresponding to the maximum power constraint. The MCPGP with the described strategy space (which is by assumption nonempty) is supermodular. Supermodular games have Nash equilibria [12]. Furthermore, the set of Nash equilibria in a supermodular game has a smallest and a largest power vector. In MCPG, let E denote the set of Nash equilibria and $\mathbf{p}_{\mathrm{S}}(\mathbf{c})$ and $\mathbf{p}_{\mathrm{L}}(\mathbf{c})$ denote the smallest and the largest power vector, respectively, and all the equilibria $\mathbf{p} \in E$ are located such that $\mathbf{p}_{\mathrm{S}}(\mathbf{c}) \leq \mathbf{p} \leq \mathbf{p}_{\mathrm{L}}(\mathbf{c})$. Note the dependence of the smallest and largest equilibrium vectors on the pricing factors, $\mathbf{c}$. The vectors, $\mathbf{p}_{\mathrm{S}}(\mathbf{c})$ and $\mathbf{p}_{\mathrm{L}}(\mathbf{c})$, can be shown to be nonincreasing in the vector $\mathbf{c}$ in a manner similar to the proof given in [5] for a single-cell system. Thus, we have established existence of equilibria in MCPGP-MRSS and discussed some properties of the set of Nash equilibria. The equilibrium can be implemented by a totally asynchronous algorithm similar to that discussed in [5]. In this algorithm, each user updates its power by maximizing the net utility at that instance. The power vector converges to an equilibrium of the game.

In the following section, we generalize the resource allocation game to include base station assignment as one of the optimization variables.

4 Base Station Assignment Based on MSIR

Section 3 studies the power control game under a fixed base station assignment rule specified by equation (3). In this section, we examine the joint problem of optimally selecting a transmit power and a base station assignment that maximizes the utility of the user, i.e., base station assignment is also a variable of utility maximization.

4.1 Power Control Game with no Pricing

The following formulation generalizes the MCPG-MRSS given in (5) to include base station assignment. The power control game for a multicell system can be expressed as,

$$\textbf{(MCPG)} \quad \max_{p_j, a_j} u_{a_j j}(p_j, \mathbf{p}_{-j}) \quad \text{for all } j \in \mathcal{N} \quad . \tag{11}$$

The major difference between the optimization in (5) and (11) is that in the latter the optimization space is 2 dimensional: transmit power and base station. The technical difficulty in analyzing this game is overcome by recognizing the useful property given next.

Proposition 1 *Given an interference vector, $\mathbf{p}_{-j}$, the following is true for MCPG:*

$$\max_{p_j, a_j} u_{a_j j}(p_j, \mathbf{p}_{-j}) \equiv \max_{p_j} \quad \max_{a_j} u_{a_j j}(p_j, \mathbf{p}_{-j}) \quad . \tag{12}$$

Proof: For fixed interference, each terminal solves

$$\max_{p_j, a_j} \quad u_{a_j j}(p_j, \mathbf{p}_{-j}) = \max_{a_j} \quad \max_{p_j} \quad \frac{RLf(\gamma_{a_j j})}{M p_j} \quad , \tag{13}$$

where

$$\gamma_{a_j j} = \frac{\frac{W}{R} h_{a_j j} p_j}{\sum_{k \neq j} h_{a_j k} p_k + \sigma^2} \quad . \tag{14}$$

Recall that a terminal maximizes its utility when the SIR is equal to $\tilde{\gamma}$. With $\gamma_{a_j j} = \tilde{\gamma}$, the transmit power is $p_j = \frac{\tilde{\gamma} I_{a_j j}}{\frac{W}{R} h_{a_j j}}$, where $I_{a_j j} = \sum_{k \neq j} h_{a_j k} p_k + \sigma^2$. Substituting the utility maximizing SIR and the transmit power in (13), we obtain,

$$\max_{a_j} \quad \frac{RLf(\tilde{\gamma})}{M\tilde{\gamma}} \frac{\frac{W}{R} h_{a_j j}}{I_{a_j j}} = \frac{RLf(\tilde{\gamma})}{M\tilde{\gamma}} \quad \max_{a_j} \quad \frac{\frac{W}{R} h_{a_j j}}{I_{a_j j}} \quad . \tag{15}$$

Suppose the base station assignment that solves (15) for terminal j is denoted by a'_j which is given by

$$a'_j = \arg\max_{a_j} \quad \frac{\frac{W}{R} h_{a_j j}}{I_{a_j j}} \quad . \tag{16}$$

Equivalently,

$$\frac{\frac{W}{R} h_{a'_j j}}{I_{a'_j j}} \geq \frac{\frac{W}{R} h_{a_j j}}{I_{a_j j}} \quad \text{for all} \quad a_j \in \mathcal{K} \quad . \tag{17}$$

Multiplying both sides with an arbitrary transmit power, p_j, we obtain

$$\gamma_{a'_j j} = \frac{\frac{W}{R} h_{a'_j j} p_j}{I_{a'_j j}} \geq \frac{\frac{W}{R} h_{a_j j} p_j}{I_{a_j j}} = \gamma_{a_j j} \quad \text{for all} \quad a_j \in \mathcal{K} \quad , \tag{18}$$

which implies that

$$a'_j = \arg\max_{a_j} \gamma_{a_j j} \quad . \tag{19}$$

Therefore, a'_j is the *maximum SIR base station assignment*. So far, we have omitted the maximum power constraint in our discussion. The arguments hold even if the maximum power constraint is active at any base station assignment, since any such assignment is strictly dominated (in terms of utility) by any base assignment where the terminal can achieve $\tilde{\gamma}$.

Note that *regardless* of the value of the transmit power of terminal j, the base station assignment that solves (15) is the base station that yields maximum SIR. This statement corresponds to determining the base station assignment as $\arg\max_{a_j} \frac{RLf(\gamma_{a_j j})}{Mp_j}$. We can then maximize over the possible transmit powers which results in the following equivalence,

$$\max_{p_j, a_j} \quad u_{a_j j}(p_j, \mathbf{p}_{-j}) \equiv \max_{p_j} \quad \max_{a_j} \quad u_{a_j j}(p_j, \mathbf{p}_{-j}) \quad . \tag{20}$$

■

Although we start out with the general problem of joint power and base station assignment, first, we simplify the problem by identifying the base station assignment as a maximum-SIR assignment and then we select the transmit power that maximizes user utility.

Defining the utility of user j as

$$u_j(\mathbf{p}) = \max_{i \in \mathcal{K}} \quad u_{ij}(\mathbf{p}) \quad , \tag{21}$$

we can formulate the optimization problem that is equivalent to (11) as follows,

$$\textbf{(MCPG-MSIR)} \quad \max_{p_j \in P_j} u_j(p_j, \mathbf{p}_{-j}) \quad \text{for all } j \in \mathcal{N} \quad . \tag{22}$$

At the equilibrium of the MCPG as formalized above

$$a_j = \arg\max_i \quad u_{ij}(\mathbf{p}) = \arg\max_i \quad \gamma_{ij} \quad , \tag{23}$$

which is the maximum SIR-based base station assignment. Thus, we refer to this problem as MCPG under the maximum SIR base station assignment rule, or in short MCPG-MSIR.

Theorem 1 *The utility function given in (21) for game MCPG-MSIR is quasiconcave in the transmit power of the terminal.*

Proof: Given the interference vector, $\mathbf{p}_{-j}$ for terminal j, let the transmit power be $p_j \in P_j$ such that

$$i' = \arg\max_i \quad u_{ij}(p_j, \mathbf{p}_{-j}) \quad . \tag{24}$$

Therefore,

$$u_{i'j}(p_j, \mathbf{p}_{-j}) = \frac{RL}{M}\frac{f(\gamma_{i'j})}{p_j} \geq \frac{RL}{M}\frac{f(\gamma_{ij})}{p_j} = u_{ij}(p_j, \mathbf{p}_{-j}) \quad , \tag{25}$$

for all $i \neq i'$, where $\gamma_{ij} = \frac{W}{R}\frac{h_{ij}p_j}{\sum_{k=1,k\neq j}^{N} h_{ik}p_k + \sigma^2}$. Notice that since the efficiency function is monotonically increasing in SIR, (25) is true if and only if

$$\gamma_{i'j} = \frac{W}{R}\frac{h_{i'j}p_j}{\sum_{k=1,k\neq j}^{N} h_{i'k}p_k + \sigma^2} \geq \frac{W}{R}\frac{h_{ij}p_j}{\sum_{k=1,k\neq j}^{N} h_{ik}p_k + \sigma^2} = \gamma_{ij} \quad , \tag{26}$$

for all $i \neq i'$. However, inequality (26) holds regardless of the transmit power p_j. Therefore, given $\mathbf{p}_{-j}$

$$u_{i'j}(p_j, \mathbf{p}_{-j}) \geq u_{ij}(p_j, \mathbf{p}_{-j}) \quad , \tag{27}$$

for all $p_j \in P_j$. Since $u_{i'j}(p_j, \mathbf{p}_{-j})$ dominates all utilities at other base stations for terminal j for a given interference vector $\mathbf{p}_{-i}$, we can write

$$u_j(\mathbf{p}) = u_{i'j}(\mathbf{p}) \quad . \tag{28}$$

where i' is given by equation (24). We know that the utility function $u_{ij}(\mathbf{p})$ as given in (2) is quasiconcave in p_j. Hence follows the quasiconcavity of $u_j(\mathbf{p})$.

On a final note, the proof is not constrained to the specific function we use in our work, but rather holds for the class of utility functions with an efficiency function that increases in SIR monotonically. ■

Theorem 2 *There exists a unique Nash equilibrium in MCPG-MSIR.*

Proof: Recall that if the utility function of each player is quasiconcave in the transmit power of the player and also if the strategy space of each terminal is nonempty, convex and compact, then the game has a Nash equilibrium. By Theorem 1, we know the utilities in MCPG-MSIR are quasiconcave. Also, the strategy space of terminal j is $P_j = [\underline{p}_j, \overline{p}_j]$ where $\underline{p}_j = 0$ and $0 < \overline{p}_j < \infty$ for all j which is a nonempty, convex and compact subset of $\mathcal{R}_+$. Therefore, a Nash equilibrium exists in MCPG with base station assignment based on MSIR.

For proof of uniqueness, we make use of the *best response correspondence* of a terminal in MCPG-MSIR. If the base assignment of terminal j were fixed to base station i, the best transmit power for terminal j in response to interference powers, $\mathbf{p}_{-j}$, is expressed as,

$$r_{ij}(\mathbf{p}_{-j}) = \min\left(\overline{p}, \frac{\tilde{\gamma}(\sum_{k=1,k\neq j}^{N} h_{ik}p_k + \sigma^2)}{\frac{W}{R}h_{ij}}\right) \quad , \tag{29}$$

where $\tilde{\gamma}$ is the SIR that maximizes utility for all terminals. The best response correspondence of terminal j over all possible base station assignments is given by,

$$r_j(\mathbf{p}_{-j}) = \min_{i\in\mathcal{K}}(r_{ij}) \quad , \tag{30}$$

which is equivalently expressed as,

$$r_j(\mathbf{p}_{-j}) = \left(\overline{p}, \min_{i\in\mathcal{K}} \frac{\tilde{\gamma}(\sum_{k=1,k\neq j}^{N} h_{ik}p_k + \sigma^2)}{\frac{W}{R}h_{ij}}\right) \quad . \tag{31}$$

Note that the base station assignment corresponding to the best response is $a_j = \arg\min_{i\in\mathcal{K}}(r_{ij})$. A power vector $\mathbf{p}$ is a Nash equilibrium if it is a fixed point of the best response correspondence, $\mathbf{p} = \mathbf{r}(\mathbf{p})$, where $\mathbf{r} = (r_1, \cdots, r_N)$. Using ideas from [13], we can easily show that $\mathbf{r}(\mathbf{p})$ is *standard*. If a standard function has a fixed point, then the fixed point is unique. Since by the first part of the proof, we have shown that there exists an equilibrium, we conclude that it must be a unique equilibrium. ■

The equilibrium powers are as follows. Recall that $\gamma_{a_j j} = \tilde{\gamma}$ is the utility maximizer. Suppose that for all base station assignments, $\gamma_{a_j j} = \tilde{\gamma}$ for all j, is feasible. Since at the equilibrium, all the terminals will attain the same SIR, the BS assignment that maximizes the utility is the one that results in minimum transmit powers and such a power vector exists as shown in [8, 9]. In the case $\gamma_{a_j j} = \tilde{\gamma}$ for all j is *not* feasible, some terminals will not be able to reach the utility maximizer and therefore transmit with maximum power, $p_j = \overline{p}$ at the equilibrium. If at equilibrium, the terminal transmits at maximum power, it follows that the equilibrium BS assignment is the base station with the highest SIR.

Once again, we resort to pricing of transmit powers to increase the efficiency of the equilibrium power allocation.

4.2 Power Control Game with Pricing

The game with pricing in its general form is formulated similar to MCPG given in (11),

$$\textbf{(MCPGP)} \quad \max_{p_j, a_j} u_{a_j j}(p_j, \mathbf{p}_{-j}) - c_{a_j} p_j \quad \text{for all } j \in \mathcal{N} \quad . \tag{32}$$

However, due to practical considerations, we simplify the problem by using maximum-SIR based base assignment. We define the utility function of terminal j with pricing as,

$$u_j^c(\mathbf{p}) = u_j(\mathbf{p}) - c_{a_j} p_j \quad , \tag{33}$$

where $u_j(\mathbf{p})$ is given by equation (21) and a_j denotes the base station given by equation (23). The multicell power control game with pricing under maximum SIR base-station assignment can be formally expressed as,

$$\textbf{(MCPGP-MSIR)} \quad \max_{p_j \in P_j^c} u_j^c(p_j, \mathbf{p}_{-j}) \quad \text{for all } j \in \mathcal{N} \quad . \tag{34}$$

4.2.1 Global versus Local Pricing

Recall that under global pricing, all the base stations use the same value of the pricing factor, $c_i = c$ for all $i = \{1, \cdots, K\}$, where c is some constant. The utility of terminal j is $u_j^c(\mathbf{p}) = u_j(\mathbf{p}) - cp_j$, and the maximization problem is,

$$\max_{p_j} \quad u_j(\mathbf{p}) - cp_j \quad \text{s.t. } p_j \in P_j \quad . \tag{35}$$

Just as in the game with no pricing, the maximum SIR base station assignment and maximum utility base station assignment are equivalent under global pricing. However, the actual base station assignment at the equilibrium of the game with global pricing is possibly different from the equilibrium BS assignment of the game with no pricing. With no pricing, the utility is maximized at $\tilde{\gamma}$ for all terminals in the system. With pricing, each terminal targets a different SIR to maximize its net utility, depending on the value of the pricing factor and the location of the terminal within the system. As a result, the equilibrium BS assignment could be different than MCPG.

It is potentially possible to improve performance even further by allowing each base station to determine a pricing factor based on local traffic information. We study pricing factor values that are proportional to the number of terminals in the coverage area of each base station. This heuristic method was discussed earlier in Section 3.2.2.

Finally, note that MCPGP and MCPGP-MSIR yield the same base assignment at the equilibrium when the pricing policy is global, however this property is not guaranteed under a local pricing strategy.

5 Best Equal-SIR Solution

The power control games discussed so far result in equilibria where the equilibrium utility for a terminal is higher than another terminal if it is closer to the base station – or, equivalently if it has a higher path gain. We observe that distribution of resources are more favorable to users with better channels. At the equilibrium of the game with no pricing (MCPG), all the terminals achieve the same SIR but the utility received depends on the user location. We should note that in this section, the game refers to the game with MRSS. Pricing the transmit powers (MCPGP) results in higher utilities for every terminal, but the ordering in terms of channel quality is preserved. Furthermore, the terminal with the better channel receives higher SIR at the equilibrium of the MCPGP as opposed to the equal-SIR equilibrium of the MCPG. A more equitable distribution is possible by enforcing every terminal to achieve the same signal-to-interference ratio (SIR). The question then is to find a desirable SIR value that would maximally benefit all the terminals.

It should be emphasized that the best equal-SIR policy is a centralized scheme. The terminals no longer employ internal algorithms to seek the power that maximizes their utility. Instead the implementation simply involves the announcement of the best SIR by the base station to the terminals and then the terminals employing an SIR balancing algorithm to reach the given target. SIR balancing is a well-known concept where each terminal iteratively matches its power to the current interference such that the target SIR is achieved [2]. Terminals converge to the target SIR under some conditions of feasibility.

The idea of best equal-SIR selection was proposed and explored in [14] for a single cell. In this section, we use the idea to derive the best target-SIR for a multicell system. Suppose that each base station i selects the SIR value, γ_i to be targeted by the terminals under its coverage. Remember that a group of terminals that have the same SIR are received at the base station with the same power. Thus, the SIR expression for terminals in cell i is,

$$\gamma_i = \frac{\frac{W}{R} q_i}{(N_i - 1)q_i + O_i + \sigma^2} \quad , \tag{36}$$

where q_i is the power received at base station i, N_i is number of terminals assigned to base i and O_i is the out-of-cell interference power at base i which is expressed as,

$$O_i = \sum_{k \in \mathcal{N} \backslash \mathcal{N}_i} h_{ik} p_k \quad . \tag{37}$$

Note that $\mathcal{N}$ is the set of all the terminals in the system, $\mathcal{N}_i$ is the set of the terminals assigned to base station i, and $\mathcal{N} \backslash \mathcal{N}_i$ denotes the set remaining after the elements of the set $\mathcal{N}_i$ are removed from the set $\mathcal{N}$. Also notice, with a slight abuse of notation, that $N_i = |\mathcal{N}_i|$ for all i. Rearranging terms in equation (36), we get

$$q_i = \frac{\gamma_i(O_i + \sigma^2)}{\frac{W}{R} - (N_i - 1)\gamma_i} \quad . \tag{38}$$

In order to achieve SIR γ_i, the terminal j transmits at power $p_j = \frac{q_i}{h_{ij}}$. The resulting utility expression for terminal j in cell i is,

$$u_j(\mathbf{p}) = \frac{RLh_{ij}}{M} \frac{(\frac{W}{R} - (N_i - 1)\gamma_i)}{\gamma_i(O_i + \sigma^2)} f(\gamma_i) \quad . \tag{39}$$

We observe that finding the utility maximizing transmit power for terminal j in cell i is equivalent to finding a utility maximizing SIR,

$$\max_{p_j} u_j(\mathbf{p}) = \max_{\gamma_i} \frac{RLh_{ij}}{M} \frac{(\frac{W}{R} - (N_i - 1)\gamma_i)}{\gamma_i(O_i + \sigma^2)} f(\gamma_i) \quad \text{for all } j \in N_i \quad . \tag{40}$$

The dependence of the utility on the SIR is the same for all the terminals sharing the same cell. It is possible for each base station to find a local best equal-SIR depending only on the number of terminals in its own cell by deriving the first order conditions of optimality with respect to SIR. Taking the derivative of (39) with respect to γ_i and making it equal to zero, we obtain the best equal-SIR, γ_i for base station i as the SIR that solves,

$$f'(\gamma_i)\gamma_i = f(\gamma_i) \frac{G}{G - \gamma_i(N_i - 1)} \quad , \tag{41}$$

where $N_i \geq 1$ is the number of terminals in cell i. The value of γ_i that solves (41) does not depend on O_i. The out-of-cell interference term O_i drops out of the equation since it is the sum of received powers (which are independent variables) from terminals outside cell i and it does not depend on the SIR for that cell. Once γ_i is found for each i, we need to see whether, given the physical distribution of terminals within the system, it is possible for all the terminals to achieve the target-SIRs in their assigned cells (feasibility). A close look at the expression for the received power given in (38) reveals a set of feasibility conditions. Since the received power has to be a positive number, the denominator must be greater than zero, or equivalently

$$N_i < 1 + \frac{W}{R\gamma_i} \quad \forall i \in \mathcal{K} \quad . \tag{42}$$

The satisfaction of the above set of constraints does not by itself guarantee feasibility of the best equal-SIR operating point. We still need the feasibility test similar to the one given in Section 3. It should be noted that in this section we assumed there is no maximum transmit power constraint.

6 Results and Discussion

We consider a multicell CDMA data system with $N = 28$ terminals distributed uniformly in an area covered by $K = 4$ base stations. The system we study has the design parameters listed in Table 1. We assume that the terminals are stationary, the frame size is fixed and there is no forward error correction. The path gains are obtained using the simple path loss model $h_{ij} = \mathcal{C}/d_{ij}^4$ where $\mathcal{C}$ is a constant and d_{ij} is the distance between terminal j and base station i. The locations of

K	number of base stations	4
N	number of terminals	28
M	total number of bits / frame	80
L	number of info. bits / frame	64
W	spread spectrum bandwidth	10^6 Hz
R	bit rate	10^4 bits/second
σ^2	AWGN power at the receiver	5×10^{-15} Watts
	modulation technique	non-coherent FSK
$\overline{p}$	maximum power constraint	1 Watt

Table 1: The list of parameters for the multicell CDMA system used in the experiments.

the terminals for a randomly generated set are as shown in Figure 1. Compare the assignments in Figure 1 (a) and (b) which display the base station assignments at equilibrium of the multicell game with no pricing under the MRSS and the MSIR rules, respectively. Since MRSS is a fixed assignment scheme, the base station assignments remain the same at the equilibrium of the game with pricing. Observe that in this particular example, going from the base station assignments at equilibrium of MCPG under MRSS to the base station assignments at equilibrium of MCPG under MSIR, two of the terminals previously assigned to base station number 2 end up in neighboring cells. Notice that the major difference in MCPG-MSIR is that the terminal maximizes its utility over the set of transmit powers *and* over the set of base station assignments.

We also compare the equilibrium utilities and the equilibrium transmit powers, with and without pricing, for the same system of terminals. In general, we observe that the equilibrium utilities for MCPGP increase as compared to the equilibrium utilities for MCPG, regardless of the base station assignment rule. Similarly, with pricing, equilibrium transmit powers decrease from their levels with no pricing. Although we do not have an equilibrium existence result for MCPGP-MSIR, our experimental results suggest there exists one. The utilities and the transmit powers for MCPG and MCPGP under BS assignment based on MSIR with local pricing (LP), where the pricing factor for base station i is $c_i = RN_i$, are compared in Figures 2 and 3. The improvement in utility as a result of pricing is higher than 25% for MCPGP-MSIR(LP) while the savings in power as a result of pricing is at least 25% for MCPGP-MSIR(LP). Each circle in the plots represents a terminal that resides at the given distance from its serving base station. The fact that the results presented identifies the terminal only by its distance to its base station should not be misleading since this is done for ease of illustration. We should note that a different set of terminal locations with the exact same distances to serving base stations will lead to different results.

Through an example with a different set of terminal locations, we present how local pricing in MCPGP under the MSIR based BS assignment rule can be used by the system to divert traffic from highly loaded cells into less congested neighboring cells. Figure 4 (a) shows the location of terminals and their base station assignments at the equilibrium of MCPG under MSIR. Figure 4 (b) shows the same set of terminals at the equilibrium of MCPGP under MSIR where the pricing factor announced by base station i is $c_i = RN_i$. Note that two of the terminals change their base station assignments as a result of the higher pricing factor in the cell with the highest number of terminals (BS number 2). Specifically, in the example presented, one of the users is reassigned to BS 1 and the other one to BS 3. In both plots, the terminals for which the base assignment changes is identified by a '*' following the number that specifies its base station assignment. Since the overall interference is decreased as a result, local pricing benefits both the terminals individually, and the system load as a whole.

Another configuration of terminals is used to show how the best equal-SIR helps improve utilities with respect to the utilities obtained at the equilibrium of MCPG for an MRSS based base station assignment. We note a substantial improvement in utilities as shown in Figure 5. Similarly, the savings in transmit powers can be seen in Figure 6. The number of terminals that reside in cells 1–4 are 7, 10, 5, 6, and the corresponding best equal-SIR values are $10.1dB$, $9.4dB$, $10.4dB$, $10.3dB$, respectively. The SIR values were calculated using equation (41).

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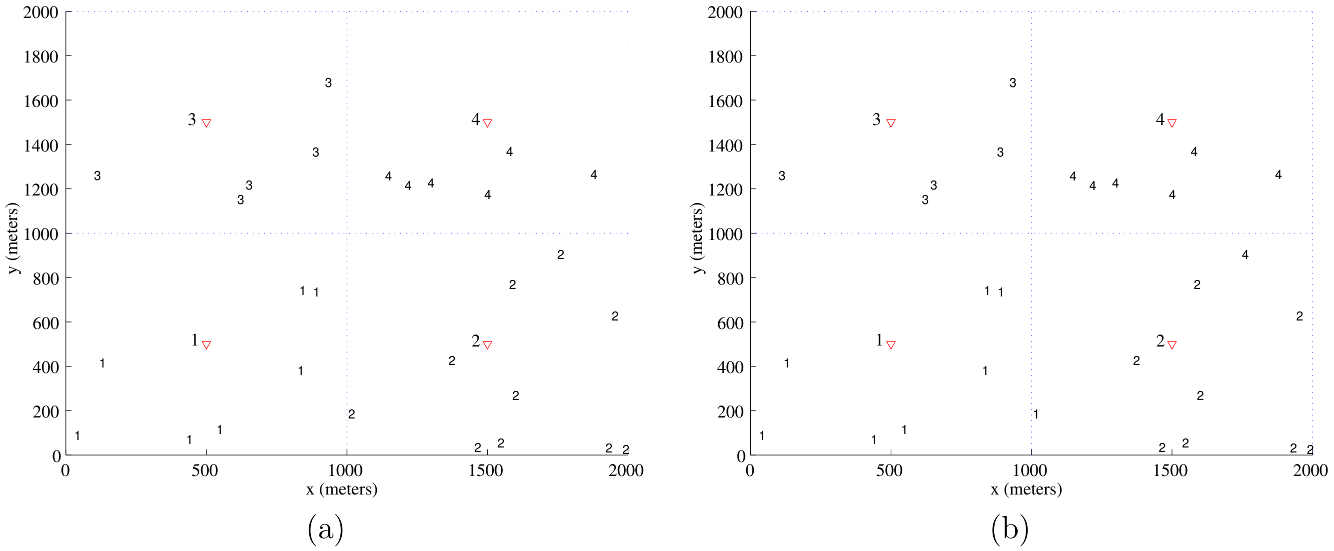

Figure 1: Terminal locations and base station assignments based on (a) maximum received signal strength (MRSS) and (b) maximum SIR (MSIR) without pricing. A terminal is represented by the base station number it is assigned. Base stations are represented by a triangle with its number next to it.

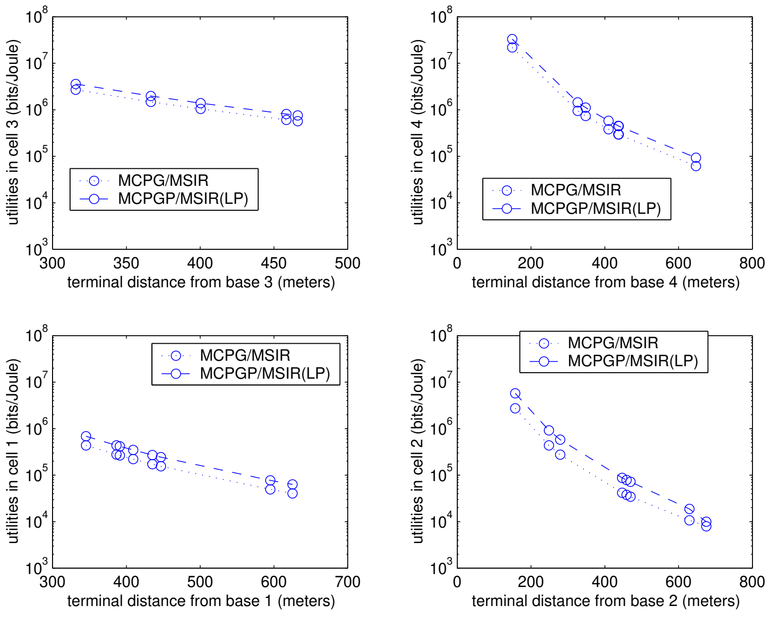

Figure 2: (Utilities of terminals at the equilibrium of MCPG and MCPGP under base station assignment based on MSIR with local pricing (LP). Each plot displays the utility of a terminal against its location with respect to its assigned base.

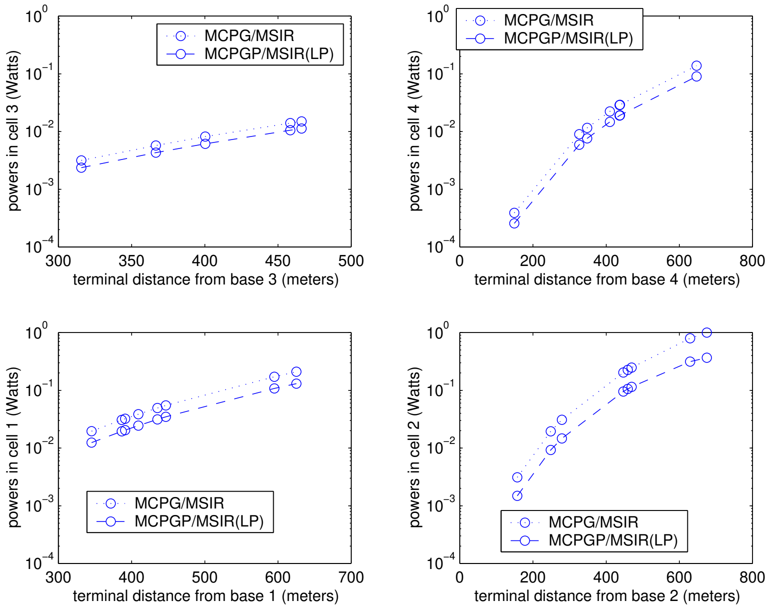

Figure 3: Transmit powers of terminals at the equilibrium of MCPG and MCPGP under base station assignment based on MSIR with local pricing (LP). Each plot displays the utility of a terminal against its location with respect to its assigned base.

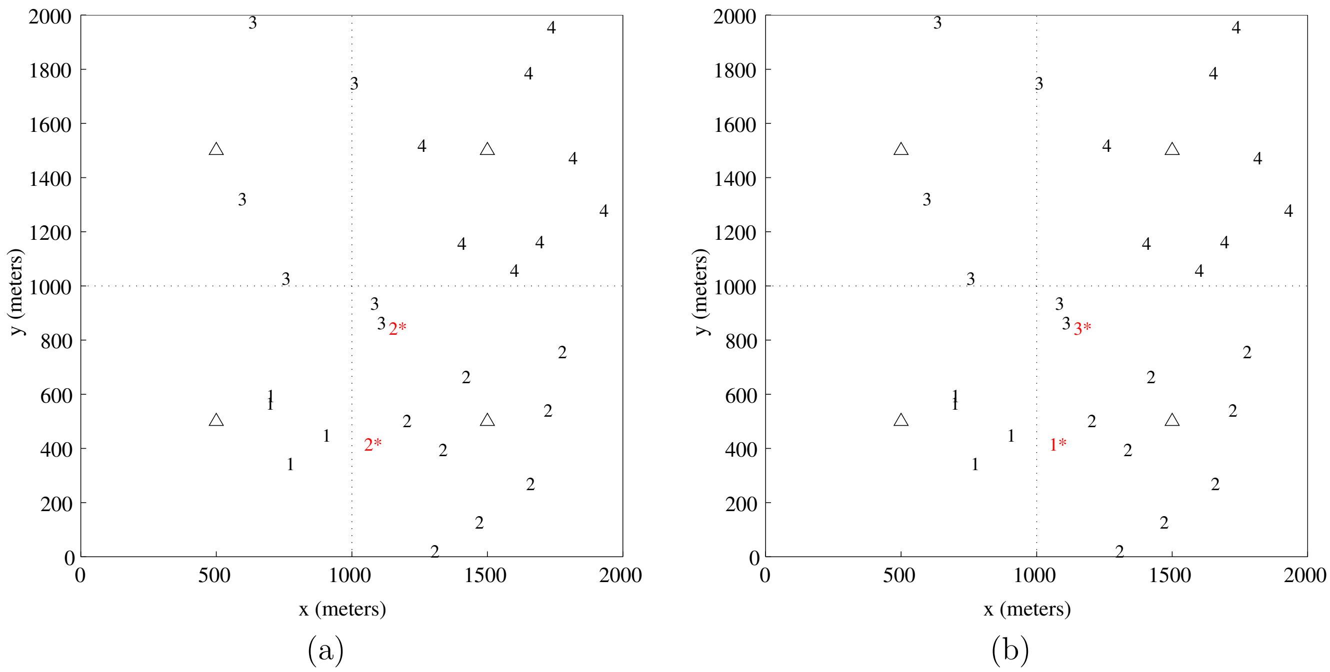

Figure 4: Terminal locations and base station assignments based on maximum SIR (MSIR) at equilibrium (a) with no pricing and (b) with local pricing (LP). A terminal is represented by the base station number it is assigned.

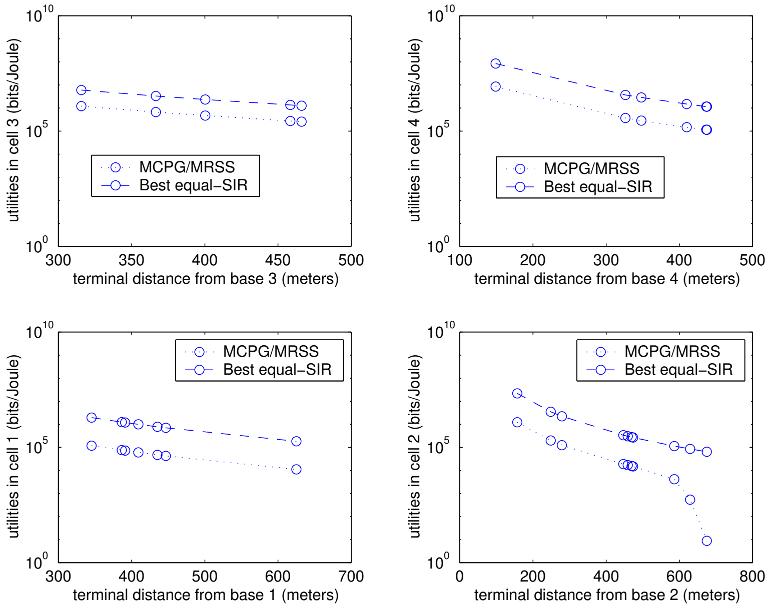

Figure 5: Utilities at the equilibrium of MCPG under base station assignment based on MRSS and utilities with best equal-SIR. Each plot displays the utility of a terminal against its location with respect to its assigned base.

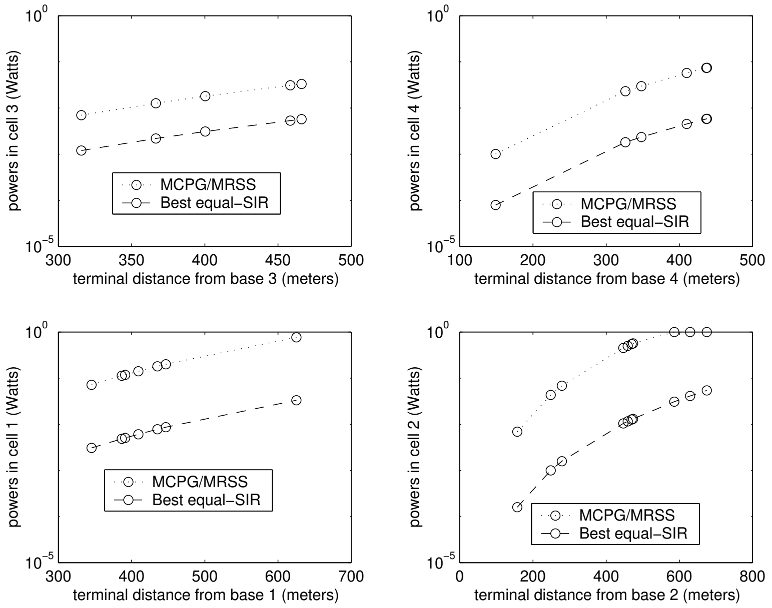

Figure 6: Transmit powers at the equilibrium of MCPG under base station assignment based on MRSS and utilities with best equal-SIR. Each plot displays the utility of a terminal against its location with respect to its assigned base.

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