Scheduling Variable Rate Links via a Spectrum Server

Chandrasekharan Raman, Roy D. Yates and Narayan B. Mandayam
WINLAB, Dept of ECE
Rutgers University
Piscataway, NJ
{chandru, ryates, narayan}@winlab.rutgers.edu

***Abstract*— We consider a centralized Spectrum Server that coordinates the transmissions of a group of links sharing a common spectrum. Links employ on-off modulation with fixed transmit power when active. In the on state, a link obtains a data rate determined by the signal-to-interference ratio on the link. By knowing the link gains in the network, the spectrum server finds an optimal schedule that maximizes the average sum rate subject to a minimum average rate constraint for each link. Using a graph theoretic model for the network and a linear programming formulation, the resulting schedules are a collection of transmission modes (sets of active links) that are time shared in a fashion that is reminiscent of spatial reuse patterns in cellular networks. In the special case when there is no minimum rate constraint, the optimal schedule results in a fixed dominant mode with highest sum rate being operated all the time. In order to offset the inherent unfairness in the above solution, we introduce a minimum rate constraint and characterize the resulting loss in sum rate when compared to the case when there is no minimum rate constraint. We also investigate alternate fairness criteria by designing scheduling algorithms that achieve max-min fairness and proportional fairness. It is shown that the max-min fair rate allocation maximizes the minimum common rate among the links. Simulation results are presented and future work is described.**

I. Introduction

Since the earliest days of radio regulation, spectrum management has been driven by improvements in technology, from improved filters and frequency stability that allowed more channels to be created, to sophisticated logic and radio techniques that created the worldwide phenomenon of cellular. More recently, however, a new paradigm has emerged in which regulation has driven technology. A relatively small regulatory experiment in "open spectrum" that began in the ISM (Industrial Scientific, Medical) bands has spawned an impressive variety of important technologies and innovative uses, from cordless phones and wireless LANs to toll takers, meter readers and home entertainment products. This obvious success has further energized an already intense debate about regulatory strategy by introducing a new set of issues and beliefs, and while this debate displays intensely held regulatory and economic viewpoints, it inevitably turns on the old-fashioned fulcrum of technological capability as well. Ultimately, the capacity of the open access bands, and the quality of service they can offer, will depend on the degree to which radios can be designed to adapt to a wide variety of conditions.

As a consequence, radios in future wireless systems are envisaged to be 'smart' and 'interference aware.' Such radios, often referred to as cognitive radios, are expected to have the ability to cooperate and dynamically share spectrum among several interfering radios. In addition to the degree of flexibility and adaptability of these radios, the need for global information regarding signals in space, time and frequency plays a prominent role in successful cooperation and coexistence. In this paper, we introduce the notion of a *Spectrum Server* which can serve as an information aid to enable coexistence of radios in a shared environment. Specifically, these radios could be made to cooperate by the centralized spectrum server which can determine neighborhood and interference information from measurements from the radios and enable efficient coordination. The spectrum server could then 'advise' a set of links, so that spectrum can be used efficiently. There are many different ways in which the spectrum server can coordinate a set of radios in a wireless network [1], [2]. In this work, we consider the problem of scheduling transmissions for a group of links which have a fixed transmission power, under the objective of maximizing the sum rate obtained by the links. We also address issues of fairness by deriving scheduling algorithms that result in max-min fair and proportional fair rate allocations. Max-min fair scheduling of rates have been studied extensively in the context of flow control of sources in a network [3]. Proportional fair scheduling has been studied in the context of multiuser diversity [4] and downlink scheduling for HDR [5]. But to the best of our knowledge, it has not been studied in the context of our framework.

Scheduling transmissions in a wireless network has been studied in various contexts. In [6], a joint scheduling and power control strategy is proposed to maximize network throughput and energy efficiency of the system. Their algorithm selects candidate subsets of concurrently active links, and applies the distributed power control algorithm [7] to find the minimal power vector. Another direction in this problem is addressed in [8], [9], where the authors look at the cross-layer issues of routing, scheduling and power control. In [10], a centralized MAC protocol is proposed but the objective is to maximize a utility function. The authors in [11] introduce the concept of transmission modes and develop a framework for integrated link scheduling and power control policies to

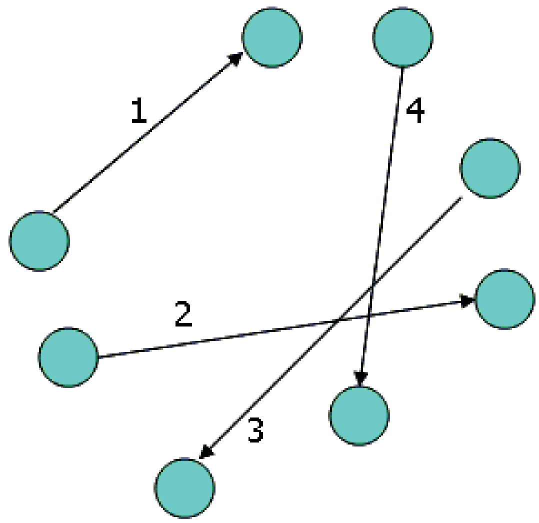

Fig. 1. Graph of network showing the nodes and directed links

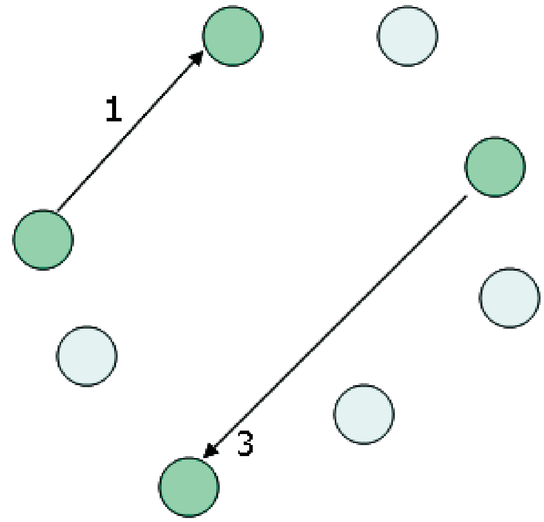

Fig. 2. Graph of network showing transmission mode corresponding to (1 0 1 0)

maximize the average network throughput, when each link is subject to an average power constraint and each node is subject to a peak power constraint. The authors assume a model in which the data rate of a link is a linear function of the signal-to-interference ratio at the receiver.

In contrast, we consider transmitters with a fixed power on-off modulation and devise schedules that maximize the system throughput. We assume that we obtain a non-zero rate in the links for any non-zero signal-to-interference ratio (SIR). The optimization problem, subject to minimum rate constraints in the individual links, is posed as a linear program. If the link gains are known to the spectrum server, it can schedule the transmissions among the links to maximize the system throughput. It is shown that when there is no minimum rate constraint, a fixed set of links (called the dominant mode) which maximizes the sum rate is operated all the time. In order to offset the inherent unfairness in the above solution, we introduce a minimum rate constraint and characterize the resulting loss in sum rate when compared to the case when there is no minimum rate constraint. We also investigate alternate fairness criteria by designing scheduling algorithms that achieve max-min fairness and proportional fairness. We show that the max-min fair rate allocation can be obtained in one step by solving a linear program which maximizes the minimum common rate among the links. The proportional fair schedule is obtained by solving a non-linear convex optimization program. The paper is organized as follows. In section II, we describe the system model. The problem formulation and analytical results are described in section III. We present the max-min fair schedule in section IV and the proportional fair schedule in section V. The simulation results are presented in section VI. We conclude in section VII with pointers to future work.

Before we explain the system model, we comment on the notation of this paper. We use boldface lowercase characters for vectors and boldface uppercase for matrices. If $\mathbf{a}$ is a vector, $\mathbf{a}^T$ denotes its transpose and $\mathbf{a}^T\mathbf{b} = \sum_i a_i b_i$ represents the inner product of the vectors $\mathbf{a}$ and $\mathbf{b}$. The vector of all zeros and all ones are represented by $\mathbf{0}$ and $\mathbf{1}$ respectively.

II. System Model

Consider a wireless network with N nodes forming L logical links sharing a common spectrum. The network can be represented as a directed graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where the nodes in the network are represented by the set of vertices $\mathcal{V}$ of the graph and the links are represented by a set of directed edges $\mathcal{E}$. Therefore the cardinalities $|\mathcal{V}| = N$ and $|\mathcal{E}| = L$. A directed edge from a node m to node n implies that n wishes to communicate data to node m. We consider the scenario where the spectrum server coordinates the activity of the set of L links to share the spectrum efficiently.

Define the set of *transmission modes* $\mathcal{T} = \{0, 1, \ldots, M-1\}$, where $M = M - 1$ denotes the number of possible transmission modes. Then the *mode activity vector* t_i of mode i is a binary vector, indicating the on-off activity of the links. If $t_i = (t_{1i}, t_{2i}, \ldots, t_{Li})$ is a mode activity vector, then

$$t_{li} = \begin{cases} 1, & \text{link } l \text{ is active under transmission mode } i, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Note that there are M possible transmission modes including the mode in which all links are off. Figure 1 shows a representative network and Figure 2 shows particular transmission mode for the set of links.

Let the transmitter power on a link l be P_l. If G_{lk} is the link gain from the transmitter of link k to the receiver of link l and σ_l^2 is the noise power at the receiver of link l, the SIR γ_{li} at the receiver of link l in transmission mode i is given by

$$\gamma_{li} = \frac{t_{li} G_{ll} P_l}{\sum_{k\in\mathcal{E}, k\neq l} t_{ki} G_{lk} P_k + \sigma_l^2}. \tag{2}$$

The link gain between a transmitter and receiver takes into account the path loss and attenuation due to shadow fading. We assume that the link gains between each transmitter and receiver are known to the spectrum server. The data rate in each link depends on the SIR in that link. We assume that the transmitter can vary its data rate, possibly through a combination of adaptive modulation and coding. In particular, for a given mode, the transmitter and receiver on a link employ the highest rate that permits reliable communication given the link SIR in that mode. For purposes of this study, we assume that the transmission of other links are treated as Gaussian noise and that a transmission on link l is reliable in a given mode i with a data rate

$$c_{li} = \log(1 + \gamma_{li}). \tag{3}$$

We emphasize here that we do not consider any minimum SIR threshold required at each receiver, i.e., associated with each transmission mode i, a non-zero γ_{li} defines some rate on the link l. Let x_i be the fraction of time that transmission mode i is active and r_l be the average data rate of link l. Each link has a minimum average data rate requirement $r_l^{\min}$. The average data rate in link l is the time average of the data rates of all the transmission modes that include link l. Thus,

$$r_l = \sum_i c_{li} x_i, \tag{4}$$

or in vector form,

$$\mathbf{r} = \mathbf{C}\mathbf{x}, \tag{5}$$

where $\mathbf{C} = [c_{li}]$ is an $L \times M$ matrix with non-negative entries, such that column i indicates the rate obtained by each link in mode i.

III. Maximum Sum Rate Scheduling

We are interested in maximizing the sum of the average data rates over all links $l = 1, 2, \ldots, L$, subject to constraints on the minimum rate for each link. The optimization problem can be posed as the linear program (LP):

$$\begin{aligned} \max \quad & \mathbf{1}^T \mathbf{r} && (6) \\ \text{subject to} \quad & \mathbf{r} = \mathbf{C}\mathbf{x}, && (6a) \\ & \mathbf{r} \geq \mathbf{r}_{\min}, && (6b) \\ & \mathbf{1}^T \mathbf{x} = 1, && (6c) \\ & \mathbf{x} \geq \mathbf{0}. && (6d) \end{aligned}$$

The objective function $\mathbf{1}^T \mathbf{r} = \sum_i r_i$ is the sum of average rates of the individual links. The constraint (6b) represents the minimum rate constraint and (6c) is the normalization for the schedule.

The variables in the LP (6) are $\mathbf{r}$ and $\mathbf{x}$. Rewriting the LP in terms of the variable $\mathbf{x}$ only, we get

$$\begin{aligned} c_{\text{opt}}(\mathbf{r}_{\min}) \ = \ \max \quad & \mathbf{1}^T \mathbf{C}\mathbf{x} && (7) \\ \text{subject to} \quad & \mathbf{C}\mathbf{x} \geq \mathbf{r}_{\min}, && (7a) \\ & \mathbf{1}^T \mathbf{x} \leq 1, && (7b) \\ & \mathbf{x} \geq \mathbf{0}. && (7c) \end{aligned}$$

Since $\mathbf{C}$ is a matrix with non-negative entries, the constraint $\mathbf{1}^T \mathbf{x} = 1$ can be replaced by the constraint $\mathbf{1}^T \mathbf{x} \leq 1$ since the optimum $\mathbf{x}$, say $\mathbf{x}_{\text{opt}}$, will satisfy $\mathbf{1}^T \mathbf{x}_{\text{opt}} = 1$. Otherwise, we could scale $\mathbf{x}_{\text{opt}}$ up so that the objective function is increased. We denote the optimal value $\mathbf{1}^T \mathbf{C}\mathbf{x}_{\text{opt}}$ as $c_{\text{opt}}(\mathbf{0})$.

A. No minimum rate constraint

We now consider the special case when $\mathbf{r}_{\min} = \mathbf{0}$, i.e., when there is no minimum rate requirement for any of the links.

Theorem 1: When $\mathbf{r}_{\min} = \mathbf{0}$, the solution to problem (7) is $\mathbf{x}_{\text{opt}} = [0\ 0 \ldots 1 \ldots 0\ 0]^T$, where the position of 1 corresponds to the transmission mode with the maximum sum rate. The optimal objective value is the maximum column sum of the rate matrix $\mathbf{C}$. Hence, the optimal strategy is to always operate the transmission mode with the maximum sum rate.

Proof: The proof of the theorem is straightforward. Since $\mathbf{r}_{\min} = \mathbf{0}$, any $\mathbf{x}$ satisfying (7b) and (7c) is feasible, as (7a) is trivially satisfied. Since $\mathbf{1}^T \mathbf{C}$ represents the row-vector of column sums of $\mathbf{C}$, the objective function $\mathbf{1}^T \mathbf{C}\mathbf{x}$ is some convex combination of column sums of the matrix $\mathbf{C}$. Thus,

$$\begin{aligned} \mathbf{1}^T \mathbf{C}\mathbf{x} &= \sum_{l=1}^{L} \sum_{i=1}^{M} c_{li} x_i && (8) \\ &= \sum_{i=1}^{M} x_i \sum_{l=1}^{L} c_{li} && (9) \\ &\leq \sum_{i=1}^{M} x_i \ \max_i \sum_{l=1}^{L} c_{li} && (10) \\ &= \max_i \sum_{l=1}^{L} c_{li} && (11) \end{aligned}$$

where the equality in (11) is true since $\sum_i x_i = 1$. Equality holds in (10) when $\mathbf{x} = \mathbf{x}_{\text{opt}} = [0\ \ 0 \ldots 1 \ldots 0\ \ 0]^T$ where the position of 1 in $\mathbf{x}_{\text{opt}}$ is $\hat{i} = \arg\max_i \sum_{l=1}^{L} c_{li}$. Hence the proof. ∎

Depending on the geometry of the links, the dominant transmission mode can be a single active link or a collection of geographically separated links. However, one implication of the above theorem is that the links that are not a part of the dominant transmission mode are starved. So, the system is not fair in terms of providing non-zero data rates to all the links.

B. Non-zero minimum rate constraint

In the case when $\mathbf{r}_{\min}$ is non-zero, any $\mathbf{x}$ satisfying (7b) and (7c) may not be feasible. There is an additional constraint in (7a) which has to be met. Hence the optimal objective value cannot exceed $c_{\text{opt}}(\mathbf{0})$. We now characterize the loss in sum rate due to the minimum rate constraint. We begin by writing the dual problem for the LP.

The Lagrangian for the LP (7) is

$$L(\mathbf{x}, \mathbf{u}, v) = \mathbf{1}^T \mathbf{C}\mathbf{x} + \mathbf{u}^T (\mathbf{C}\mathbf{x} - \mathbf{r}_{\min}) + v(1 - \mathbf{1^T x}), \tag{12}$$

where $\mathbf{u} \in \mathcal{R}^L$ and $v \in \mathcal{R}$ are the dual variables. The Lagrange dual is

$$\begin{aligned} g(\mathbf{u}, v) &= \sup_{\mathbf{x} \geq \mathbf{0}} L(\mathbf{x}, \mathbf{u}, v) && (13) \\ &= -\mathbf{u}^T \mathbf{r}_{\min} + v \\ &\quad + \sup_{\mathbf{x} \geq \mathbf{0}} (\mathbf{1^T C} + \mathbf{u}^T \mathbf{C} - v \mathbf{1}^T) \mathbf{x} && (14) \\ &= \begin{cases} -\mathbf{u}^T \mathbf{r}_{\min} + v, & \mathbf{1}^T \mathbf{C} + \mathbf{u}^T \mathbf{C} - v \mathbf{1}^T \leq \mathbf{0} \\ \infty, & \text{otherwise} \end{cases} && (15) \end{aligned}$$

Thus the dual problem for the LP (7) is

$$\begin{aligned} \text{minimize} \quad & -\mathbf{r}_{\min}^T \mathbf{u} + v && (16) \\ \text{subject to} \quad & \mathbf{C}^T (\mathbf{1} + \mathbf{u}) \leq v \mathbf{1}, && (16a) \\ & \mathbf{u} \geq \mathbf{0}, v \geq 0. && (16b) \end{aligned}$$

By strong duality [12, Chapter 5], the optimal value of the dual problem in (16) is equal to $c_{\text{opt}}(\mathbf{r}_{\min})$. Let $(\mathbf{u}^*, v^*)$ be the solution of (16). Since by Theorem 1, $c_{\text{opt}}(\mathbf{0})$ is the maximum column sum of $\mathbf{C}$ and $\mathbf{u} \geq \mathbf{0}$, we have according to (16a), $v^* \geq c_{\text{opt}}(\mathbf{0})$. Therefore, the optimal value of (16)

$$\begin{aligned} c_{\text{opt}}(\mathbf{r}_{\min}) &= -\mathbf{r}_{\min}^T \mathbf{u}^* + v^* \\ &\geq -\mathbf{r}_{\min}^T \mathbf{u}^* + c_{\text{opt}}(\mathbf{0}). \end{aligned}$$

Since $c_{\text{opt}}(\mathbf{0}) - c_{\text{opt}}(\mathbf{r}_{\min}) \leq \mathbf{r}_{\min}^T \mathbf{u}^*$, the loss in sum rate is at most $\mathbf{r}_{\min}^T \mathbf{u}^*$. An interpretation of the dual variable $\mathbf{u}^*$ is that it can be viewed as the amount of rate loss for a unit increase in $\mathbf{r}_{\min}^T$. This is analogous to the dual prices interpretation, in which the dual variables are interpreted as the price paid for using the limited resources (primal variables), the constraints of which are specified in the primal problem.

C. Maximum sum rate schedule with high SNR links

We can examine the special case of high SNR links when each link transmits with a large power P. Let us define a set of modes

$$\hat{\mathcal{T}} = \{ i_l : t_{l i_l} = 1, \, t_{k i_l} = 0 \text{ for all } k \neq l \}.$$

In mode i_l, link l transmits in isolation and thus we call $\hat{\mathcal{T}} = \{i_1, i_2, \ldots, i_L\}$ the set of isolation modes.

When the transmit power P is high, all links have high SNR and a link l achieves a high rate when transmitting in the isolation mode i_l. However, in a shared (non-isolation) mode $j \notin \hat{\mathcal{T}}$, links will have interference-limited SIRs and relatively low data rates. These observations lead to the following theorem.

Theorem 2: If the interference gains G_{lk} are all non-zero, then for sufficiently large transmit power P, the solution to (7) is time sharing among the transmission modes in $\hat{\mathcal{T}}$.

Proof: If P is the transmit power in all links $l \in \mathcal{E}$, from (2) the SIR γ_{lj} of link l in transmission mode j is given by

$$\gamma_{lj} = \frac{t_{lj} G_{ll} P}{\sum_{j \in \mathcal{E}, k \neq l} t_{kj} G_{lk} P + \sigma_l^2}. \tag{17}$$

For all modes $j \notin \hat{\mathcal{T}}$, the nonzero interference gains G_{lk} and the monotonicity of the fraction $P/(cP + \sigma^2)$ imply that

$$\gamma_{lj} < \bar{\gamma}_{lj} = \frac{G_{ll}}{\sum_{j \in \mathcal{E}, k \neq l} t_{kj} G_{lk}}. \tag{18}$$

We can thus upper bound the SIR γ_{lj} of any link l in any transmission mode $j \notin \hat{\mathcal{T}}$ as

$$\gamma_{lj} < \bar{\gamma} = \max_{j \notin \hat{\mathcal{T}}} \max_{l} \bar{\gamma}_{lj}. \tag{19}$$

It follows from (3) that

$$c_{lj} \leq \bar{c} = \log(1 + \bar{\gamma}), \qquad j \notin \hat{\mathcal{T}}. \tag{20}$$

Note that $\bar{c}$ serves as an upper bound for the rate that can be obtained by any link l in a shared mode $j \notin \hat{\mathcal{T}}$. However, in a mode $i_l \in \hat{\mathcal{T}}$ in which only link l is active,

$$\gamma_{l i_l} = \frac{G_{ll} P}{\sigma_l^2} = \gamma_l(P), \tag{21}$$

a monotone increasing function of P. Let us define

$$c_l(P) = \log(1 + \gamma_l(P)). \tag{22}$$

as the data rate obtained when link l transmits with power P in the isolation mode i_l. Since $c_l(P)$ is a monotone increasing function of P, there exists a transmit power P^*, such that $P > P^*$ implies $c_l(P) > L\bar{c}$ for all links l.

Now, let us suppose that $P > P^*$, but $\mathbf{x}$ is an optimal schedule for problem (7) with $x_j > 0$ for a shared mode $j \notin \hat{\mathcal{T}}$. Consider a new schedule $\mathbf{x}'$ given by

$$x_i' = \begin{cases} 0 & i = j \\ x_i + x_j/L & i \in \hat{\mathcal{T}} \\ x_i & \text{otherwise} \end{cases} \tag{23}$$

The schedule $\mathbf{x}'$ reallocates the time x_j in mode j equally to the isolation modes i_l in $\hat{\mathcal{T}}$. In particular, an isolation mode $i_l \in \hat{\mathcal{T}}$ will now be active for time

$$x_{i_l}' = x_{i_l} + \frac{x_j}{L}. \tag{24}$$

We now show that every link l receives a positive rate increase by switching to schedule $\mathbf{x}'$. Under schedule $\mathbf{x}$, a link l obtains rate

$$r_l = \sum_i c_{li} x_i = c_{lj} x_j + c_{l i_l} x_{i_l} + \sum_{i \notin \{j, i_l\}} c_{li} x_i. \tag{25}$$

Under schedule $\mathbf{x}'$, link l obtains rate

$$r_l' = \sum_i c_{li} x_i' = c_{l i_l} x_{i_l}' + \sum_{i \notin \{j, i_l\}} c_{li} x_i. \tag{26}$$

For link l, the difference in rates is

$$\begin{aligned} r_l' - r_l &= c_{l i_l} (x_{i_l}' - x_{i_l}) - c_{lj} x_j && (27) \\ &= \left(\frac{c_{l i_l}}{L} - c_{lj} \right) x_j. && (28) \end{aligned}$$

However, $P > P^*$ implies that in the isolation mode i_l, link l obtains rate

$$c_{l i_l} = c_l(P) > L\bar{c}. \tag{29}$$

It follows that $r'_l - r_l > 0$ for all links l. This contradicts the optimality of schedule $\mathbf{x}$ in that every link achieves a strictly higher rate under schedule $\mathbf{x}'$. ■

IV. Max-min fair rate scheduling

The maximum sum rate scheduling is biased towards links that have the best quality (i.e., least interference) and is unfair to the other links that are not a part of the dominant transmission mode. To address this, we will consider two other fairness criteria in deriving scheduling strategies - max-min fair and proportional fair. In this section, we present the max-min fair [3] schedule.

Definition 1: A vector of rates $\mathbf{r}$ is said to be *max-min fair* if it is feasible and for each $l \in \mathcal{E}$, r_l cannot be increased while maintaining feasibility without decreasing $r_{l'}$ for some link l' for which $r_{l'} \leq r_l$. Formally, for any other feasible allocation $\tilde{\mathbf{r}}$, with $\tilde{r}_l > r_l$, there must exist some l' such that $\tilde{r}_{l'} < r_{l'} \leq r_l$.

In the context of flow control of sources in a communication network, iterative algorithms for computing max-min fair rate vectors exist [3]. Such iterative algorithms use a 'progressive filling' technique that starts with all rates equal to zero and increases the rates until one or several link capacity limits are reached. In order to obtain the max-min fair schedule in our setting, we begin by formulating the LP to maximize the minimum common rate in all the links. We will then show that the solution to this LP results in the max-min fair solution. The LP which maximizes the minimum common rate among the links is

$$
\begin{aligned}
r^* = \max \quad & r_{\min} && (30)\\
\text{subject to} \quad & \mathbf{r} = \mathbf{C}\mathbf{x}, && (30a)\\
& \mathbf{r} \geq r_{\min}\mathbf{1}, && (30b)\\
& \mathbf{1}^T\mathbf{x} = 1, && (30c)\\
& \mathbf{x} \geq \mathbf{0}. && (30d)
\end{aligned}
$$

Before proving that the above LP results in the max-min fair schedule, we state the following theorem:

Theorem 3: If the link gains G_{lj} are all non-zero, then the LP (30) which maximizes the minimum common rate among the links results in all links getting the same rate r^*, i.e., $\mathbf{r}^* = r^*\mathbf{1}$.

The proof of Theorem 3 appears in the appendix. We now show that the schedule obtained by solving (30) is max-min fair.

Theorem 4: The solution $\mathbf{x}^*$ obtained by solving the LP (30) results in the max-min fair solution for the maximum sum rate problem (7).

Proof: Our objective is to seek a max-min fair solution in the set (7a)-(7c). Denote the set by $\mathcal{S}$. Let us consider the solution $\mathbf{x}^*$ of (30) which results in the rate allocation $\mathbf{r}^* = r^*\mathbf{1}$. Now, for any feasible $\mathbf{x} \in \mathcal{S}$, there can be only three different possibilities:

1) $\mathbf{r}$ such that the rates in all links $r_l \leq r^*$, $l \in \mathcal{E}$.
2) $\mathbf{r}$ such that for some links $r_l < r^*$ and some links $r_{l'} \geq r^*$ for $l, l' \in \mathcal{E}$.
3) $\mathbf{r}$ such that the rates in all links $r_l > r^*$, $l \in \mathcal{E}$.

The third possibility can be ruled out since it contradicts the optimality of (30). From Definition 1 of max-min fairness, it follows that $r^*\mathbf{1}$ is the max-min fair rate vector when the first two possibilities hold. ■

V. Proportional fair scheduling

The max-min fair schedule derived in the previous section leads to global fairness. In this section, we discuss a fairness criteria which leads to fairness of individual links.

Definition 2: A vector of rates $\mathbf{r}$ is *proportional fair* if it is feasible, i.e., $\mathbf{C}\mathbf{x} = \mathbf{r}$ for $\mathbf{x}$ such that $\mathbf{1}^T\mathbf{x} = 1$ and $\mathbf{x} \geq \mathbf{0}$, and if for any other feasible vector $\mathbf{r}'$, the aggregate of proportional change is negative.

$$\sum_i \frac{r'_i - r_i}{r_i} \leq 0. \tag{31}$$

In [13], Kelly proposed proportional fairness in the context of rate control for elastic traffic. It can be shown that the proportionally fair vector is the one that maximizes the sum of logarithms of the utility functions. Hence, to obtain the proportional fair rates, we solve the following non-linear optimization problem with linear constraints

$$
\begin{aligned}
\max \quad & \sum_l \log r_l && (32)\\
\text{subject to} \quad & \mathbf{r} = \mathbf{C}\mathbf{x}, && (32a)\\
& \mathbf{1}^T\mathbf{x} = 1, && (32b)\\
& \mathbf{x} \geq \mathbf{0}. && (32c)
\end{aligned}
$$

The objective function of the above non-linear optimization problem is increasing and strictly concave. The constraint set is linear and hence the problem is a convex optimization problem [12]. This implies that the problem has a unique global maximum over the constraint set. The solution for such problems can be found out by gradient search algorithms [12].

VI. Simulation results

We present some simulation results in this section. Though the analytical results are true for more general cases, we present simulation results for some specific cases to illustrate our findings. The simulation set-up is a 50×50 grid. The links are of fixed lengths and placed at random locations in the grid. The interference gain G_{lj} between the transmitter of link j and the receiver of a link l is given by $G_{lj} = d_{lj}^{-4}$, where d_{lj} is the separation distance between the transmitter and receiver. The transmit powers are fixed for all transmissions and the link geometries are characterized through the signal-to-noise ratio (SNR) at the receiver for that link (in the absence of interference).

In the case of maximum sum rate scheduling with no minimum rate constraint, the transmission mode with the highest sum rate is chosen. The links which are not a part of this transmission mode are not operated at all. In the special case when the noise at the receiver is high, the denominator in the SIR expression is dominated by the receiver noise. This approximates the case when there is no interference from the

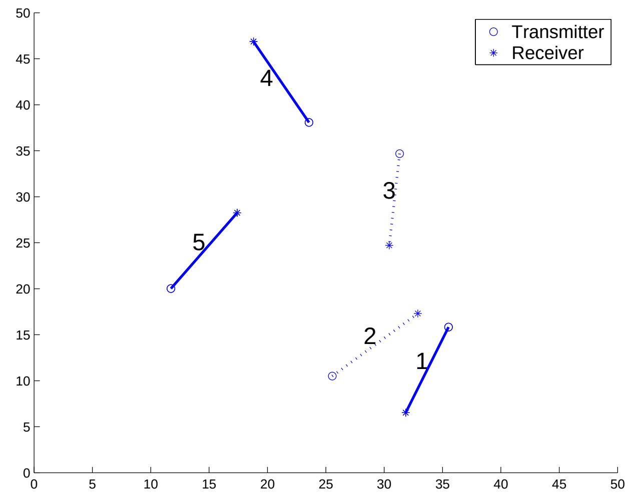

Fig. 3. Set of five links each of length $d = 10$. The dominant transmission mode at SNR=10 dB is shown in solid lines.

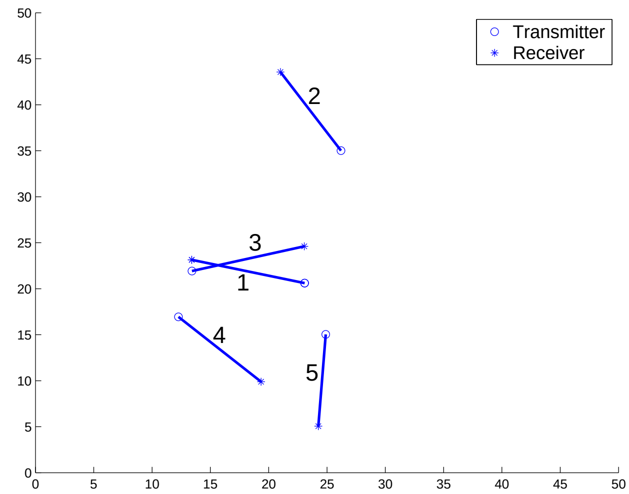

Fig. 4. A set of source-destination pairs

neighboring links. Hence the best policy would be to turn on all the links in order to maximize the sum rate in all the links.

As the SNR in each link increases, the interference from neighbors also increases. Then the best transmission mode is that which has the highest sum rate among all the other transmission modes. The set of links chosen follows spatial reuse patterns that are reminiscent of those used in cellular networks. Figure 3 shows a set of links and the dominant transmission mode at SNR = 10 dB. The links in the dominant mode are shown in solid lines.

In the case of maximum sum rate scheduling with non-zero minimum rate constraint, we see that more than one transmission mode is operated since there is a minimum rate requirement for each link. The best mode is selected for most of the time and the mode which includes the poorer quality links are turned on for a fraction of time just enough to satisfy their minimum rate requirement. Most of the transmission modes are thus not used at all since it is best to use the dominant mode during all other times except when a minimum rate should be guaranteed to some poor quality links.

We now discuss an illustrative example of this case. Figure 4 shows a set of source-destination pairs. When $\mathbf{r}_{\min}$ is zero at 20 dB received SNR, the mode consisting of links $\{2, 5\}$ is always operated. But as the common minimum rate $\mathbf{r}_{\min}$ increases from zero, an additional set of modes is operated to satisfy the minimum rate requirement for each link. The schedules of the individual transmission modes as shown in Figure 6 varies with the minimum rate so that the minimum rate constraint in each of links is maintained. Notice that only five different modes are active. When $\mathbf{r}_{\min}$ is increased in steps, we observe that the same set of modes is operated. After a certain $\mathbf{r}_{\min}$ value, say $\mathbf{r}'$, a different set of modes has to be operated in order to obtain a feasible schedule. Until then the rate of all the modes falls linearly with increase in $\mathbf{r}_{\min}$. The break point in the sum rate curve occurs at $\mathbf{r}'$. We zoom in to the graph for $\mathbf{r}_{\min}$ values ranging from 1.4 to 1.58

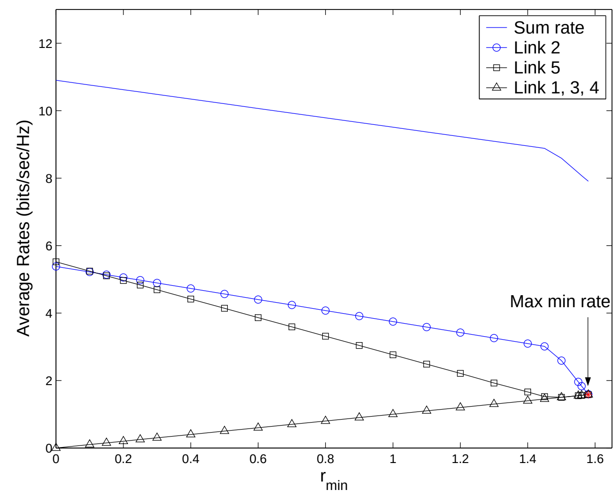

Fig. 5. Variation of sum rates and individual rate as a function of $\mathbf{r}_{\min}$

in Figure 7. We observe that mode $\{1\}$ and mode $\{3\}$ are active for equal amounts of time since link 1 and 3 transmit in isolation in this mode, and they require the same minimum rate to transmit. The fraction of time mode $\{4\}$ is active increases as the fraction of time mode $\{2, 4\}$ transmits decreases to compensate for the increase in rate in link 4. Finally, the rates in all the links are same at $r^* \approx 1.58$.

The rates corresponding to maximum of the minimum common rate r^* is shown in the Figure 5. All the links end up getting the same rates in this case. The schedule and the rate allocation vector the same as those obtained when we solve (7) with $\mathbf{r}_{\min} = r^*\mathbf{1}$. The comparison of scheduling schemes under different optimization settings is shown in Figure 8. From the Figure, we notice that only in the case of maximum sum rate with no minimum rate constraint, there exist links with zero obtained rate. In the case of the max-min fair solution, all the links end up getting the same rate. Table I shows a numerical comparison of the sum rate and individual

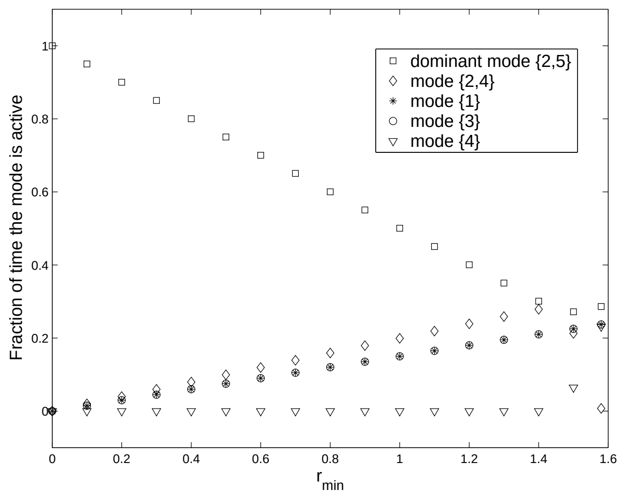

Fig. 6. Schedule of different transmission modes at various $\mathbf{r}_{\min}$ values when maximizing sum rate of links

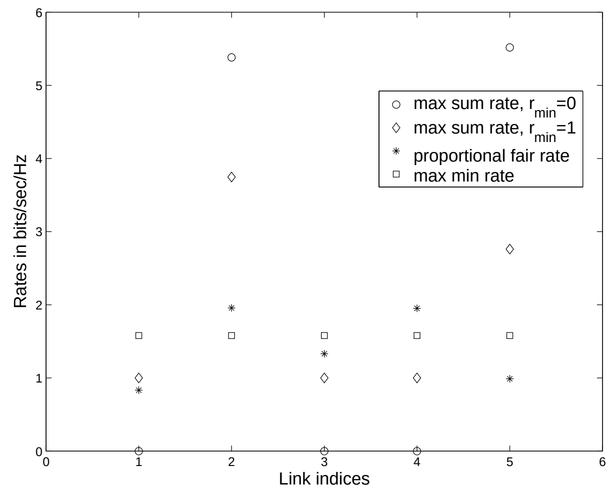

Fig. 8. Comparison between rates of the links under different settings

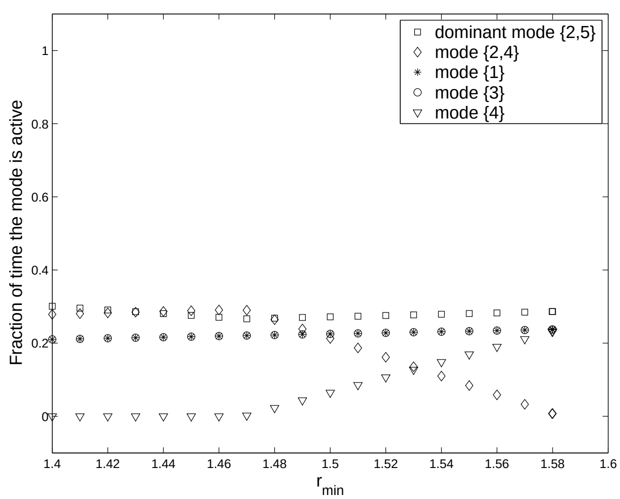

Fig. 7. Schedule of different modes - zoomed in for higher $\mathbf{r}_{\min}$ values

link rates among the various links in the network.

VII. Discussion and Conclusion

We introduced the notion of a Spectrum Server, which allocates a schedule for a set of links in a wireless network, which is modelled as a directed graph. We observe that this problem formulation can also be applied to the case where the links operate in non-overlapping frequency ranges. In this case, some of the link gains G_{lj} may be zero. The model can be easily extended to the case when there are bidirectional links between two nodes (if we assume there are separate transmitters and receivers), in which case the number of transmission modes will be 2^{2L}. In this case, interference at the receiver which is colocated with the transmitter in another link is very high. This may result in schedules in which one of the bidirectional links are active at any given time. If there is a restriction that only one of the bidirectional links can be active at any given time (or when each node has a single transceiver). In this case, the number of transmission modes will be only 3^L.

In our work, the problem of maximizing the sum rate in all the links subject to minimum rate constraint is posed as a linear program. The solution to the linear program gives the optimum schedule for the transmission modes in the network. But for a network involving L links, there is an exponential number of transmission modes and thus the LP we solve involves an exponential number of variables. However, we conjecture that almost always there are only very few active transmission modes as corroborated by our simulation results.

In the present work, the whole process is centralized since the spectrum server just solves the linear program to compute the schedules for each of the links. A heuristic algorithm to find the best schedule with less measurement overhead would be an interesting future work to address. This may involve links reporting the interference seen by them from all the other links. This would also be a first step to finding a completely distributed scheduling algorithm. The centralized approach proposed in this paper would then serve as an upper bound to the performance of such distributed algorithms.

Throughout this paper, we have assumed that the spectrum server has knowledge of the link gains. This involves measurement of link gains by the spectrum server. An interesting issue is how coarse the measurement can be and how it affects the scheduling algorithm. If the link gains are modelled by a time-varying fading process, then finer measurements would be very expensive.

The problem formulation in this work yields itself to many optimization problems. One such example is to minimize the sum fraction of times the links are on so that the total transmit power in all the links is minimized. Another example is to maximize the sum rate in all the links subject to the condition that all the links are active for equal amount of time.

In this work, we assume that there is a single hop com-

TABLE I

COMPARISON OF SUM RATE AND INDIVIDUAL RATES

	$r_{\min} = 0$	$r_{\min} = 0.5$	$r_{\min} = 1$	$r_{\min} = 1.58$ (Max-min fair solution)	Proportional fair solution
Sum rate	10.8987	10.2035	9.5082	7.9	7.0632
Link 1	0	0.5	1.0	1.58	0.8324
Link 2	5.3814	4.5640	3.7466	1.58	1.9581
Link 3	0	0.5	1.0	1.58	1.3314
Link 4	0	0.5	1.0	1.58	1.9516
Link 5	5.5173	4.1394	1.0	1.58	0.9896

munication between the source destination pairs. Yet another interesting issue would be to find the solution for the maximum sum rate problem if we have a schedule with multiple hops between the source and destination.

While we have primarily considered links of equal length for the purposes of numerical illustration, it is of interest to study the performance of the various scheduling algorithms for the case of links of unequal lengths as well.

VIII. ACKNOWLEDGEMENT

This work is supported in part by the NSF under grant number NeTS-0434854 and by the Defense Spectrum Office (DSO) of the Defense Information Systems Agency. The first author thanks Jasvinder Singh for useful discussions on the material in this paper.

APPENDIX

Proof of Theorem 3: The LP which maximizes the minimum common rate is

$$\begin{aligned} \max \quad & r_{\min} && (33)\\ \text{subject to} \quad & \mathbf{r} = \mathbf{C}\mathbf{x}, && (33a)\\ & \mathbf{r} \geq r_{\min}\mathbf{1}, && (33b)\\ & \mathbf{1}^T\mathbf{x} = 1, && (33c)\\ & \mathbf{x} \geq \mathbf{0}. && (33d) \end{aligned}$$

Let r^* be the optimal value of (33), corresponding to a schedule $\mathbf{x}^*$ and a set of active transmission modes $\mathcal{T}^* = \{i \in \mathcal{T} : x_i^* > 0\}$. Note that the idle transmission mode with the all zero activity vector would never be a part of $\mathcal{T}^*$ because, if it were, we can improve the rates of links in $\mathcal{L}_2$ and this contradicts that r^* is the optimal solution of (33). It is required to prove that at optima, the rate vector $\mathbf{C}\mathbf{x} = r^*\mathbf{1}$. We assume the contrary that the solution to (33) leads to unequal rates over the set of L links. We can then partition the sets of links $\mathcal{E}$ into two disjoint non-empty sets

$$\mathcal{L}_1 = \{l \in \mathcal{E} : r_l > r^*\}$$

and

$$\mathcal{L}_2 = \{l \in \mathcal{E} : r_l = r^*\}.$$

This in turn induces a partition on the set $\mathcal{T}^*$ of all active transmission modes for the optimal solution into three disjoint sets $\mathcal{T}_1^*$, $\mathcal{T}_2^*$ and $\mathcal{T}_3^*$ such that

$$\begin{aligned} \mathcal{T}_1^* &= \{i \in \mathcal{T}^* : t_{il} = 0, \text{ for all } l \in \mathcal{L}_2\}, && (34)\\ \mathcal{T}_2^* &= \{i \in \mathcal{T}^* : t_{il} = 0, \text{ for all } l \in \mathcal{L}_1\}, && (35)\\ \mathcal{T}_3^* &= \mathcal{T}^* \backslash \{\mathcal{T}_1^* \cup \mathcal{T}_2^*\}. \end{aligned}$$

$\mathcal{T}_1^*$ and $\mathcal{T}_2^*$ contain active transmission modes which consist of links only from $\mathcal{L}_1$ and $\mathcal{L}_2$ respectively, and $\mathcal{T}_3^*$ contains transmission modes which consist of links in both $\mathcal{L}_1$ and $\mathcal{L}_2$. We consider two cases below.

A. Case (i): $\mathcal{T}_1^*$ is non-empty

There exists an active transmission mode $i \in \mathcal{T}_1^*$ consisting of links only from $\mathcal{L}_1$. Consider the mode i' with activity vector $t_{i'}$ given by

$$t_{li'} = \begin{cases} 1, & \text{for all } l \in \mathcal{L}_2, \\ 0, & \text{otherwise.} \end{cases} \quad (36)$$

Mode i' consists of all links from $\mathcal{L}_2$. Therefore, $c_{li'} > 0$ for $l \in \mathcal{L}_2$. In the optimal schedule $\mathbf{x}^*$, we know that $x_i^* > 0$ but $x_{i'}^*$ may be zero. The rate in link l under schedule $\mathbf{x}^*$ is $r_l = \sum_k c_{lk}x_k^*$. Define for some fixed $\epsilon_1 > 0$, the feasible schedule

$$\hat{\mathbf{x}} = [x_1^* \ldots x_i^* - \epsilon_1 \ldots x_{i'}^* + \epsilon_1 \ldots x_{M-1}^*]^T.$$

For sufficiently small ϵ_1, the schedule $\hat{\mathbf{x}}$ will be feasible. Now, for $l \in \mathcal{L}_2$, the rate $\hat{r}_l$ due to schedule $\hat{\mathbf{x}}$ is

$$\hat{r}_l = \sum_k c_{lk}\hat{x}_k = r^* - c_{li}\epsilon_1 + c_{li'}\epsilon_1 \quad (37)$$

Since $c_{li'} > 0$ for $l \in \mathcal{L}_2$,

$$\hat{r}_l = r^* + c_{li'}\epsilon_1. \quad (38)$$

Thus, we conclude that $\hat{r}_l > r^*,\ l \in \mathcal{L}_2$. Note that ϵ_1 needs to be chosen such that for all $l \in \mathcal{L}_1$, $\hat{r}_l > r^*$. The choice of ϵ_1 such that $c_{li}\epsilon_1 < \min_{l\in\mathcal{L}_1} r_l - r^*$ ensures that $\hat{r}_l > r^*$ for $l \in \mathcal{L}_1$. We can thus improve the rates in all links in $\mathcal{L}_2$. This contradicts the optimality of r^*. We denote this step as *Increase*(1).

B. Case (ii): $\mathcal{T}_1^*$ is empty

In this case, if $\mathcal{T}_3^*$ is empty, then $\mathcal{T}^* = \mathcal{T}_2^*$ and hence all rates are equal, and the proof is complete. Thus we consider only the case of $\mathcal{T}_3^*$ being non-empty. For an active mode

$j \in \mathcal{T}_3^*$, there exist non-empty subsets of $\mathcal{L}_1$ and $\mathcal{L}_2$, namely $\mathcal{M}_1$ and $\mathcal{M}_2$ such that the activity vector t_j is given by

$$t_{lj} = \begin{cases} 1, & l \in \mathcal{M}_1 \subseteq \mathcal{L}_1, \\ 1, & l \in \mathcal{M}_2 \subseteq \mathcal{L}_2, \\ 0, & \text{otherwise.} \end{cases} \tag{39}$$

Consider the mode j' for which the activity vector $t_{j'}$ is given by

$$t_{lj'} = \begin{cases} 1, & \text{if } l \in \mathcal{M}_2, \\ 0, & \text{otherwise.} \end{cases} \tag{40}$$

We have assumed that all link gains G_{lj} are non-zero, that is there is lesser interference for links in $\mathcal{M}_2$ in mode j' than in mode j due to a lesser number of active links in mode j'. Thus for links $l \in \mathcal{M}_2$,

$$\begin{aligned} c_{lj} &= \frac{G_{ll}P_l}{\sum_{k \in \mathcal{M}_1 \cup \mathcal{M}_2, k \neq l} t_{kj} G_{lk} P_k + \sigma_l^2} && (41) \\ &> \frac{G_{ll}P_l}{\sum_{k \in \mathcal{M}_2, k \neq l} t_{kj'} G_{lk} P_k + \sigma_l^2} = c_{lj'}. && (42) \end{aligned}$$

Since $j \in \mathcal{T}_3^*$, $x_j^* > 0$. For some $\epsilon_2 > 0$, we define a feasible schedule

$$\hat{\mathbf{x}} = [x_1^* \dots x_j^* - \epsilon_2 \dots x_{j'}^* + \epsilon_2 \dots x_{M-1}^*]^T.$$

Under schedule $\mathbf{x}^*$ and $\hat{\mathbf{x}}$, link l obtains rate

$$r_l = \sum_k c_{lk} x_k^*$$

and

$$\hat{r}_l = \sum_k c_{lk} \hat{x}_k$$

respectively. Thus,

$$\begin{aligned} \hat{r}_l - r_l &= (\hat{x}_j - x_{j'}^*) c_{lj'} + (\hat{x}_j - x_j^*) c_{lj} && (43) \\ &= \epsilon_2 (c_{lj'} - c_{lj}). && (44) \end{aligned}$$

It follows from (41) that $\hat{r}_1 - r_l > 0$. Let us call this step *Increase*(2).

Since $\mathcal{L}_2$ is a finite set, repeatedly applying *Increase*(1) or *Increase*(2) on $\mathcal{L}_2 \backslash M_2$, we can increase the rates of all the links in $\mathcal{L}_2$. This contradicts the optimality of r^*. The proof is complete since both cases contradict the fact that the optimal solution leads to unequal rates in the links.

References

[1] N. Mandayam, “Cognitive algorithms and architectures for open access to spectrum,” *Conf. on the Economics, Technology and Policy of Unlicensed Spectrum, East Lansing, MI*, May 2005. http://quello.msu.edu/conferences/spectrum/program.htm.

[2] M. Buddhikot, P. Kolodzy, S. Miller, K. Ryan, and J. Evans, “DIMSUMNet: New directions in wireless networking using coordinated dynamic spectrum access,” *Conf. on the Economics, Technology and Policy of Unlicensed Spectrum, East Lansing, MI*, May 2005. http://quello.msu.edu/conferences/spectrum/program.htm.

[3] D. Bertsekas and R. Gallager, “Data networks,” *Prentice-Hall*, 1992.

[4] P. Viswanath, D. Tse, and R. Laroia, “Opportunistic beamforming using dumb antennas,” *IEEE Trans. Info. Theory*, vol. 48, pp. 1277–1294, Jun 2002.

[5] S. Shakkottai and A. Stolyar, “Scheduling for multiple flows sharing a time-varying channel: The exponential rule,” *American Mathematical Society Translations*, vol. 207, 2002.

[6] T. Elbatt and A. Ephremides, “Joint scheduling and power control for for wireless ad-hoc networks,” *IEEE Trans. Wireless Commun.*, vol. 3, pp. 74–85, Jan 2004.

[7] G. Foschini and Z. Miljanic, “A simple distributed autonomous power control algorithm and its convergence,” *IEEE Trans. on Vehic. Tech.*, vol. 42, pp. 641–646, Nov 1993.

[8] R. L. Cruz and A. V. Santhanam, “Optimal routing, link scheduling and power control in multi-hop wireless networks,” *IEEE Infocom*, 2003.

[9] R. Bhatia and M. Kodialam, “On power efficient communication over multi-hop wireless networks: Joint routing, scheduling and power control,” *IEEE Infocom*, 2004.

[10] J. Lee, M. Chiang, and R. Calderbank, “Efficient and fair MAC for wireless network: Optimization framework, optimal algorithms performance comparison,” *IEEE Trans. Mobile Computing*, submitted, April 2005.

[11] R. L. Cruz and A. V. Santhanam, “Optimal link scheduling and power control in CDMA multihop wireless networks,” *IEEE Globecom*, pp. 52–56, 2002.

[12] S. Boyd and L. Vandenberghe, “Convex optimization,” *Cambridge University Press*, 2004.

[13] F.Kelly, “Charging and rate control for elastic traffic,” *Euro. Trans. Telecommun.*, vol. 8, pp. 33–37, Jan/Feb 1997.