Power Control for Wireless Data

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***Abstract* - With cellular phones mass-market consumer items, the next frontier is mobile multimedia communications. This situation raises the question of how to do power control for information sources other than voice. To explore this issue, we use the concepts and mathematics of microeconomics and game theory. In this context, the Quality of Service of a telephone call is referred to as the "utility" and the distributed power control problem for a CDMA telephone is a "non-cooperative game". The power control algorithm corresponds to a strategy that has a locally optimum operating point referred to as a "Nash equilibrium." The telephone power control algorithm is also "Pareto efficient," in the terminology of game theory.**

When we apply the same approach to power control in wireless data transmissions, we find that the corresponding strategy, while locally optimum, is not Pareto efficient. Relative to the telephone algorithm, there are other algorithms that produce higher utility for at least one terminal, without decreasing the utility for any other terminal. This paper presents one such algorithm. The algorithm includes a price function, proportional to transmitter power. The price acts as a tax on the utility of a transmission. When terminals adjust their power levels to maximize the net utility (utility - price), they arrive at lower power levels and higher utility than they achieve when they individually strive to maximize utility.

I. BACKGROUND AND MOTIVATION

The technology and business of cellular communications systems have made spectacular progress since the first systems were introduced fifteen years ago. With new mobile satellites coming on line, business arrangements, technology and spectrum allocations make it possible for people to make and receive telephone calls anytime, anywhere. The cellular telephone success story prompts the wireless communications community to turn its attention to other information services, many of them in the category of "wireless data" communications. To bring high-speed data services to a mobile population, several "third generation" transmission techniques have been devised. These techniques are characterized by user bit rates on the order of hundreds or thousands of kb/s, one or two orders of magnitude higher than the bit rates of digital cellular systems. One lesson of cellular telephone network operation is that effective radio resource management is essential to promote the quality and efficiency of a system. One component of radio resource management is power control, the subject of this paper.

An impressive set of research results published since 1990 documents theoretical insights and practical techniques for assigning power levels to terminals and base stations in voice communications systems [1-4]. The principal purpose of power control is to provide each signal with adequate quality without causing unnecessary interference to other signals. Another goal is to minimize the battery drain in portable terminals. An optimum power control algorithm for wireless telephones maximizes the number of conversations that can simultaneously achieve a certain quality of service (QoS) objective. There are several ways to formulate the QoS objective quantitatively. Two prominent examples refer to a *QoS target.* In one example, the target is the minimum acceptable signal-to-interference ratio and in the other example the target is the maximum acceptable probability of error.

In turning our attention to data transmission, we have discovered that this approach does not lead to optimum results. This is because the QoS objective for data signals differs from the QoS objective for telephones. To formulate the power control problem for data, we have adopted the vocabulary and mathematics of microeconomics in which the QoS objective is referred to as a *utility function*. The utility function for data signals is different from the telephone utility function. Our research indicates that when all data terminals individually adjust their powers to maximize their utility, the transmitter powers converge to levels that are too high. To obtain better results, we introduce a *pricing function* that recognizes explicitly the fact that the signal transmitted by each terminal interferes with the signals transmitted by other terminals. The interference caused by each terminal is proportional to the power the terminal transmits. This leads us to establish a price (measured in the same units as the utility function) to be calculated by terminals in deciding how much power to transmit. Terminals adjust their powers to maximize the difference between utility and price. In doing so, they all achieve higher utilities than when they aim for maximum utility without considering the price.

II. UTILITY FUNCTIONS FOR VOICE AND DATA

A utility function is a measure of the satisfaction experienced by a person using a product or service. In the wireless communications literature the term *Quality of Service* (QoS) is closely related to utility. Two QoS objectives are low delay and low probability of error. In telephone systems low delay is essential and transmission errors are tolerable up to a point. By contrast, data signals can accept some delay but have very low tolerance to errors. In establishing a minimum signal-to-interference ratio for telephone signals, engineers implicitly represent utility as a function of signal-to-interference ratio in the form of Figure 1. We consider systems to be unacceptable (utility=0) when the signal-to-interference ratio (γ) is below a target level, γ_0. When $\gamma >= \gamma_0$, we assume that the utility is constant. Our power control algorithms implicitly assume that there is no benefit to having a signal-to-interference ratio above the target level.

In cellular telephone systems, the target, γ_0 is system dependent. For example analog systems aim for γ_0 = 18 dB. In GSM digital systems the target can be as low as 7 dB, and in CDMA it is on the order of 6 dB [5]. In each case γ_0 is selected to provide acceptable subjective speech quality at a telephone receiver.

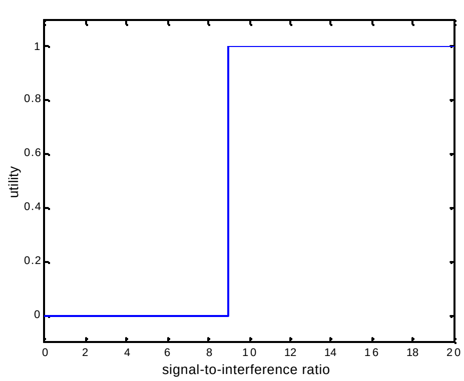

Figure 1. Quality of Service metric for wireless telephones represented as a utility function.

In a data system, the signal-to-interference ratio, γ, is important because it directly influences the probability of transmission errors. When a system contains forward error correction (FEC) coding, we consider a transmission error to be an error that appears at the output of the FEC decoder. Because data systems are intolerant of errors, they employ powerful error detecting schemes. When it detects a transmission error, a system retransmits the affected data. If all transmission errors are detected, a high γ increases the system throughput (rate of reception of correct data), and decreases the delay relative to a system with a low γ. When γ is very low, virtually all transmissions result in errors and the utility is near 0. When γ is very high, the probability of a transmission error approaches 0, and utility rises asymptotically to a constant value. In addition to the speed of data transfer, a factor in the utility of all data systems, power consumption is an important factor in mobile computing. The satisfaction experienced by someone using a portable device depends on how often the person has to replace or recharge the batteries in the device. Battery life is inversely proportional to the power drain on the batteries. Thus, we see that utility depends on both γ and transmitted power. Of course, these quantities are strongly interdependent. With everything else unchanged, γ is directly proportional to transmitted power. In a cellular system, however, many transmissions interfere with one another and an increase in the power of one transmitter reduces the signal-to-interference ratio of many other signals. To formalize these statements, we consider a cellular system in which there are N mutually interfering signals. For signal i, i = 1,2, ..., N, there are two variables that influence utility: the signal-to-interference ratio γ_i and the transmitted power p_i. Because each γ_i depends on $p_1, p_2, ..., p_N$, the utility of each signal is a function of all of the N transmitter powers.

A. The Data Utility Function

The wireless data system transmits packets containing L information bits. With channel coding, the total size of each packet is M>L bits. The transmission rate is R b/s. At the receiver of terminal i, the signal-to-interference ratio is γ_i and the probability of correct reception is q (γ_i), where the function q( ) depends on the details of the data transmission including modulation, coding, interleaving, radio propagation, and receiver structure. The number of transmissions necessary to receive a packet correctly is a random variable, K. If all transmissions are statistically independent, K is a geometric random variable with probability mass function:

$$P_K(k) = \begin{cases} q(\gamma_i)[1- q(\gamma_i)]^{k-1} & k=1,2,3,... \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

The expected value of K is E [K]=1/q(γ_i). The duration of each transmission is M/R seconds and the total transmission time required for correct reception is the random variable KM/R seconds. With the transmitted power p_i watts, the energy expended is the random variable, p_iKM/R joules with expected value $E[K]p_iM/R=p_iM/[R\ q(\gamma_i)]$. The benefit is simply the information content of the signal, L bits. Therefore, our utility measure is

$$\frac{E\,[benefit]}{E\,[energy\ cost]} = \frac{LRq(g_i)}{Mp_i} \quad \text{b/J.} \tag{2}$$

The utility can be interpreted as the number of information bits received per Joule of energy expended. Zorzi and Rao use an objective that combines throughput and power dissipation in a similar manner in a study of retransmission schemes for packet data systems [6].

As a starting point for deriving a power control algorithm, Equation (2) has some advantages and disadvantages. On the plus side are its physical interpretation (bits per Joule) and its mathematical simplicity. Its disadvantages derive from the simplifying assumption that all packet transmission errors can be detected at the receiver. Data transmission systems contain powerful error detecting codes that make this assumption true, "for all practical purposes". However, it causes problems mathematically because the probability of a packet arriving correctly is not zero with zero power transmitted. In a binary transmission system with M bits per packet and p_i=0, a receiver simply guesses the values of the M bits that were transmitted. The probability of correct guesses for all M bits is 2^{-M}. Therefore with p_i=0, the numerator of Equation (2) is positive and the function is infinite. This suggests that the best approach to power control is to turn off all transmitters and wait, for the receiver to produce a correct guess. This strategy has two flaws. One is that the waiting time for a correct packet could be months, and the other is that there will be other guesses (ignored in our analysis) that are incorrect but undetectable by the error detecting code.

To retain the advantages of Equation (2) and eliminate the degenerate solution, p_i=0, from the optimization process, we modify the utility function by replacing $q(\gamma_i)$ with another function $f(\gamma_i)$ with the properties $f(\infty)=1$ and $f(\gamma_i)/p_i=0$ for p_i=0. Thus we seek a power control algorithm that maximizes the following utility function:

$$U_i = \frac{LRf(g_i)}{Mp_i} \quad \text{b/J} \tag{3}$$

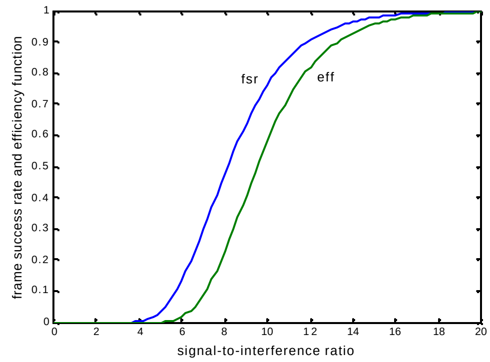

Figure 2. Relationship of frame success rate to the efficiency function $f(\gamma_i)$: non-coherent FSK modem, 80 bits per packet.

In the numerical examples of this paper, we have assumed a system with no error correcting code and γ_i constant over the duration of each packet. In these examples,

$$q(g_i) = (1 - BER_i)^M \tag{4}$$

where BER_i is the binary error rate of transmitter-receiver pair i. To work with a well-behaved utility function, we introduce the following “efficiency” function

$$f(g_i) = (1 - 2BER_i)^M \tag{5}$$

in our definition of utility. This function has the desirable properties stated above at the limiting points $\gamma_i = 0$ and $\gamma_i = \infty$, and its shape follows that of q( ) at intermediate points. For example, Figure 2 shows $f(\gamma_i)$ and $q(\gamma_i)$ for M=80 and $BER_i = 0.5\exp(-\gamma_i/2)$, the binary error rate of a non-coherent frequency shift keying modem. The similar shapes of the two curves leads us to expect that a set of transmitter powers that maximizes U_i in Equation (3) will be close to the powers that maximize the utility measure in Equation (2).

B. Power Control For Maximum Utility

Our aim is to derive a distributed power control algorithm that maximizes the utility derived by all of the users of the data system. In a distributed algorithm, each transmitter-receiver pair adjusts its transmitter power p_i in an attempt to maximize its utility U_i. For each i, the maximum utility occurs at a power level for which the partial derivative of U_i with respect to p_i is zero:

$$\frac{\partial U_i}{\partial p_i} = 0 \tag{6}$$

We observe in Equation (3) that in order to differentiate Equation (6) with respect to p_i, we need to know the derivative of γ_i with respect to p_i. A general formula for signal-to-interference ratio is

$$g_i = \frac{p_i h_{ii}}{I_i + N_i} = \frac{p_i h_{ii}}{\sum_{\substack{k=1 \\ k \neq i}}^{N} p_k h_{ik} + s_i^2} \qquad (7)$$

In Equation (7), h_{ik} is the path gain from terminal i to the base station of terminal k, I_i is the interference received at the base station of terminal i, and σ_i^2 is the noise in the receiver of the signal transmitted by terminal i. I_i and σ_i^2 are independent of p_i. Therefore

$$\frac{\partial g_i}{\partial p_i} = \frac{h_{ii}}{I_i + N_i} = \frac{g_i}{p_i}. \qquad (8)$$

Referring to Equations (3) and (8), we can express the derivative of utility with respect to power as

$$\frac{\partial U_i}{\partial p_i} = \frac{LR}{Mp_i^2}\left(g_i \frac{df(g_i)}{dg_i} - f(g_i) \right), \qquad (9)$$

Therefore, with $p_i>0$, the necessary condition for terminal i to maximize its utility is

$$g_i \frac{df(g_i)}{dg_i} - f(g_i) = 0. \qquad (10)$$

This states that to operate at maximum utility a base station receiver has to have a signal-to-interference ratio, γ^*, that satisfies Equation (10).

C. Properties Of The Maximum-Utility Solution

The signal-to-interference ratio, γ^*, that maximizes the utility of user i, is a property only of the efficiency function f( ), defined in Equation (5). If all of the interfering terminals use the same type of modem and the same packet length, M, they operate with the same efficiency function. Therefore, *the signal-to-interference ratio g^*, for maximum efficiency, is the same for all terminals.* This is an important observation because earlier work on speech communications derives an algorithm [2-4] that allows all terminals to operate at a common signal-to-interference ratio. This algorithm directs each terminal to determine the interference periodically and adjust its power to achieve its target signal-to-interference ratio. After each adjustment, the other terminals adjust their powers in the same way. Provided the number of terminals is not too high[1], all power levels will converge to values that produce the target signal-to-interference ratio at all receivers. In speech communications, the target is determined by considerations of subjective speech quality. Our mathematical analysis tells us that in data communications the modem and the packet length dictate the target.

In speech, the distributed power control system, leads to a *globally optimum* solution. There is no set of powers that produces a better result than the set that results from the algorithm described in the previous paragraph. This is not the case in a data system. In a data system, we can show that if all terminals operate with the power levels that satisfy Equation (10), *they can all increase their utilities by simultaneously reducing their power by a small (infinitesimal) amount.* This implies that the distributed power control algorithm for data signals is locally optimum but not globally optimum. As a consequence, we must extend our study to find power control schemes that do a better job than the signal-to-interference ratio balancing technique implied by Equation (10). To do so, we introduce concepts of microeconomics that do not play a role in traditional communications systems engineering, *games* and *prices.*

III. GAME THEORY FORMULATION OF POWER CONTROL

In the context of game theory, we say that in adjusting its transmitter power, each terminal pursues a strategy that aims to maximize the utility obtained by the terminal. In doing so, the action of one terminal influences the utilities of other terminals and causes them to adjust their powers. The distributed power control algorithms we have described are referred to as *non-cooperative games* because each terminal pursues a strategy based on locally available information. By contrast, a centralized power control algorithm uses information about the state of all terminals to determine all the power levels. A centralized algorithm corresponds to a *cooperative game*. In game theory terminology, The convergence of the distributed power control algorithm to a set of powers that maximize the utility of each terminal corresponds to the existence of a *Nash equilibrium* for the non-cooperative game. However, the algorithm is not *Pareto efficient*. Note that in optimization problems regarding radio resource management, globally optimal usually refers to a single unique operating point. However, Pareto efficiency usually may refer to several points (which form the Pareto frontier) some of which may produce higher utilities than others. From a practical point of view, finding solutions that

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[1] The literature on power control algorithms for voice systems states a feasibility condition, which depends on the number of terminals and their locations relative to base stations. If this condition is not satisfied it is impossible to meet the signal-to-interference ratio requirements for all terminals simultaneously.

offer Pareto improvements may sometimes be sufficient rather than searching for Pareto efficient points.

Because we know that the strategy of maximizing utility leads everyone to transmit at a power that is too high, we seek a means to encourage terminals to transmit at lower power. To derive such a technique, we examine the effect of each terminal's power adjustment on the utility of all other terminals. We define the effect on terminal j of a power adjustment at terminal i
as the *cost coefficient*,

$$C_{ij} = -\frac{\partial U_j}{\partial p_i} p_i \quad \text{b/J. } (i \neq j) \qquad (11)$$

Each cost coefficient is positive because any increase in the power of one terminal reduces the signal-to-interference ratio of every other terminal, and hence decreases the utility. The total cost, imposed on all terminals by terminal i transmitting at a power level p_i is:

$$C_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} C_{ij} \quad \text{b/J} \qquad (12)$$

In the systems we have studied, we have discovered that at equilibrium, the cost imposed by each terminal is a monotonic increasing function of the distance[2] of the terminal from its base station. Examining terminals with increasing distances from their base stations, we find: (a) increasing power necessary to achieve the equilibrium signal-to-interference ratio, (b) lower equilibrium utility, and (c) higher cost imposed on the other terminals. Thus if we index the N terminals in the system in order of increasing distance from the serving base station, where the distance of terminal i is d_i meters, we have

$$\begin{aligned} &d_1 < d_2 < \ldots < d_N \quad \text{and, at equilibrium} \\ &U^*_1 > U^*_2 > \ldots > U^*_N \\ &p^*_1 < p^*_2 < \ldots < p^*_N \quad \text{and} \\ &C^*_1 < C^*_2 < \ldots < C^*_N. \end{aligned} \qquad (13)$$

In these inequalities, the asterisks denote equilibrium values of power, utility, and cost.

To find an improved power control algorithm, we take these observations into account by imposing a price on each transmission. The price is a tax, measured in the units of utility, bits per Joule, that reduces the utility. The inequalities in Equation (13) suggest that the price should be monotonic increasing with power. Moreover, by combining Equations (11) and (12) with the definition of utility in Equation (3), we find that under all conditions, not just at equilibrium, the cost imposed by terminal j on the other terminals is proportional to p_j:

$$C_j = \frac{LR}{M} t_j p_j \quad \text{b/J.} \qquad (14)$$

Although it would be intuitively pleasing to penalize each terminal by the value of C_j in Equation (14), this is not feasible in a distributed power control system. The value of t_j depends on the current transmitter powers of all terminals in the system, and on all the path gains, h_{ij}. Therefore to determine t_j, each terminal would need detailed information about conditions at all the other terminals.

To derive a distributed algorithm that takes the costs into account, we have adopted a price function that is proportional to the power transmitted at each terminal, where the proportionality constant is the same for all terminals:

$$V_j = \frac{LR}{M} t p_j \quad \text{b/J.} \qquad (15)$$

Then, we adopt a power control algorithm in which each terminal maximizes its *net utility*

$$U_i' = U_i - V_i \qquad \text{b/J} \qquad (16)$$

IV. THE NET UTILITY FUNCTION

At first glance it appears that our task in deriving a power control algorithm is not very different from the task we started with. We began by deriving an algorithm in which each terminal adjusts its power to maximize the utility function in Equation (3). Now we ask for an algorithm in which the function to be maximized is the net utility in Equation (14), which is simply the difference between Equation (3) and a term proportional to power. However, this price term changes the nature of the algorithm considerably. For one thing, U', the function to be maximized, can have negative values. More importantly, when each terminal seeks to maximize its own net utility, it does not aim for the same equilibrium signal-to-interference ratio as all the other terminals. That is because when we differentiate the net utility function for each terminal, the condition corresponding to Equation (10) contains a term that depends explicitly on the power of each terminal.

$$\gamma_i \frac{df(\gamma_i)}{d\gamma_i} - f(\gamma_i) - tp_i^2 = 0 \qquad (17)$$

In contrast to Equation (10) The value of γ_i that satisfies this equation is different for each terminal. It depends on all the path gains h_{ik} in Equation 7 and on σ_i^2, the noise in the receiver of terminal i.

[2] The dependence of various quantities on distance is a property of the radio propagation conditions of a system. The monotonic dependence of power to distance relates to a simple propagation model. Mathematically, the powers, utilities and costs depend on the path gains, h_{ij} between transmitters and receivers.

This property of the data power control algorithm takes us away from a signal-to-interference-ratio balancing algorithm cor-responding to optimum power control for voice signals. In addition, we have to find a numerical value for the proportionality constant t. This too is a departure from our original situation in which the function that we maximize depends only on observable properties of the communications system: L, R, M, p_I, the modulation technique (which determines the function f( )), and the operating environment (which determines h_{ij}). To find a good value for t, we have resorted to experiments in which we calculate transmitter powers for specific system models and then examine the effects of adopting a range of values for t, the price coefficient. The following Section describes these experiments.

V. NUMERICAL EXAMPLES

To shed light on the salient properties of the power control algorithms derived for wireless data transmission, we have considered a simple model based on a generic single-cell CDMA system with no coding for forward error correction and a fixed packet size. This analysis has provided us with insights into the differences between power control for data signals and voice signals. Armed with this basic understanding, we have expanded the analysis to consider forward error correction, variable transmission rates, and variable packet sizes. The simple system examined in this paper has the following design parameters:

- Number of information bits per packet: L=64
- Total number of bits per packet M= 80 (with no forward error correction, the difference M-L=16 is the number of bits in the cyclic redundancy check error-detecting code)
- Chip rate: 10^6 chips/s
- Bit rate: 10^4 b/s
- Modulation technique: non-coherent frequency-shift keying with binary error rate $0.5e^{-0.5\gamma}$. (This assumes that each signal encounters a non-fading channel in which the interference appears as white gaussian noise.)
- Receiver noise power spectral density: 5 x 10^{-21} W/Hz, which produces a noise power of σ_i^2 = 5 x 10^{-15} W in a receiver with 1 MHz bandwidth.

For this system, the efficiency function is

$$f(\gamma_i)=[(1-\exp(-0.5\gamma_i)]^{80} \qquad (18)$$

and the utility function is

$$U_i = 64 \times 10^4 \; [(1-\exp(-0.5\gamma_i)]^{80}/80p_i \;\; \text{b/J.} \qquad (19)$$

For this efficiency function, the equilibrium signal-to-noise ratio, found by solving Equation (10) is γ^* = 12.4 = 10.9 dB. This is the target signal-to-interference ratio that all terminals aim for when each one seeks to maximize its utility. For this CDMA system, the feasibility condition for this target is given by the following bound on the number of terminals [2]:

$$N <= 1+(W/R)/\gamma^* = 9.05 \text{ terminals} \qquad (20)$$

If the number of terminals transmitting to the base station is less than or equal to 9, all terminals can operate with $\gamma=\gamma^*$. Moreover, when all links operate with $\gamma=\gamma^*$, all of the signals arrive at the base station with the same power:

$$p^*_{receive} = \frac{\gamma^* \sigma^2}{(W/R)-(N-1)\gamma^*} \; Watts \qquad (21)$$

The remaining quantities that determine the properties of this system are the number of terminals, N, and the N path gains[3], h_1, h_2, ...,h_N. In the calculations reported here, we use a simple propagation model in which all of the path gains are deterministic functions, with propagation exponent 3.6, of the distance between a terminal and the base station

$$h_i=const / d_i^{3.6}, \qquad (22)$$

where d_i meters is the distance between terminal i and the base station. In our calculations, the proportionality constant in Equation (22) is 7.75 x 10^{-3}. We chose this value to establish a transmit power of 10 W for a terminal operating at 1000 meters from the base station in a system with N=9 terminals, all operating with $\gamma=\gamma^*$=12.4. Figure 3 shows the transmitter power as a function of terminal-to-base station distance for this system. Reflecting Equation (22), the transmitter power in each curve varies as $d^{3.6}$.

To demonstrate that the power control algorithm operating with a target of γ^* is not globally optimum, consider a system with N=9 terminals, all operating with $\gamma_i=\gamma^*$. Let all of the terminals reduce their power levels by a factor of 10. By working with Equation (21), we find that they arrive at the same signal-to-interference ratio, 11.7. With γ=11.7, the efficiency decreases from f(12.4)=0.85 to f(11.6)= 0.80, a factor of 0.93. However this negative effect on utility is far outweighed by the positive effect of a 10:1 power reduction. While the new power control algorithm, based on a target of γ=11.7 is more efficient (in the Pareto sense) than the algorithm with a target of γ^*, it is not an equilibrium point of a non-cooperative game.

However, when all terminals operate with γ =11.7, any terminal can unilaterally improve its utility by raising its power. For example, an increase in power by one terminal by a factor of 1.1, will increase the signal-to-interference ratio

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[3] In general we have the notation h_{ik} for the path gain of terminal i to base station k. In our simple example, there is only one base station. Therefore, we simplify the notation to a use single subscript so that h_i is the path gain from terminal i to the system base station.

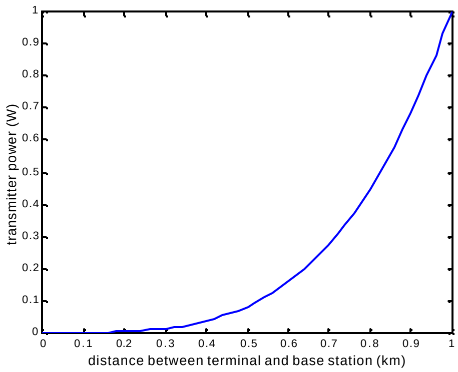

Figure 3. Transmitter power in a system with N=9 terminals all operating with signal-to-interference-ratio $\gamma=\gamma^*=12.4$

of that terminal to 11.7 x 1.1= 12.9 and increase the efficiency to f(12.9)=0.88. This benefit to the utility (0.88/0.80 =1.11) slightly outweighs the negative impact of a 10% increase in power. However, this action by one terminal will cause the utility of the other terminals to decrease, which in turn will stimulate the other terminals to increase their power levels. The chain reaction will bring all terminals to the equilibrium signal-to-interference ratio of $\gamma^*=12.4$.

This situation motivates us to introduce the price function to create a non-cooperative game that causes terminals to transmit at reduced powers relative to those in Figure 3. In this game each terminal unilaterally maximizes its net utility in Equation (16). To find the power transmitted by each terminal, we solve the N simultaneous equations corresponding to Equation (17) with i=1,2,...,N. To do so, we start with initial values of the N transmitter powers and find a numerical solution of Equation (17) with i=1 and p_j held at the initial values for the other values of j. We do the same thing in turn for i=2,3,…,N and repeat the process until the N power levels converge to their equilibrium values. The results differ from the results of the non-cooperative game that maximizes U_i in that the equilibrium signal-to-interference ratios are not equal. Terminals nearer the base station have higher values of γ_i at equilibrium than terminals further away. With unequal signal-to-interference ratios, the received powers are unequal and the power transmitted by each terminal depends not only on the distance of that terminal from the base station, but also on the distances of all other terminals from the base station.

These properties of the game with a price function are documented in Figures 4 and 5. The numerical results apply to nine terminals transmitting data from distances listed in Table 2 in which d_i is proportional to i. The price parameter in Equation (15) is t=50. Figures 4 and 5, which reproduce the results for the game of maximizing utility without a price function, demonstrate that incorporating the price function equilibrium reduces all of the equilibrium powers. The equilibrium signal-to-interference ratios are also lower, but the combined effect on utility is positive for all terminals, as indicated in Figure 5.

Terminal	Dist (km)	Path gain 10^{-10} x	Utility (b/J) pri=0 10^5 x	Utility (b/J) pri=50p 10^5 x	Net util (b/J) 10^5 x
1	0.31	6.16	4.30	34. 7	34.7
2	0.46	1.59	1.11	8.96	8.92
3	0.57	0.74	0.52	4.17	4.10
4	0.66	0.43	0.30	2.44	2.37
5	0.74	0.29	0.20	1.61	1.45
6	0.81	0.21	0.14	1.14	0.92
7	0.88	0.15	0.11	0.85	0.56
8	0.94	0.12	0.08	0.66	0.29
9	1.00	0.10	0.07	0.51	0.08

Table 2: Simulation data

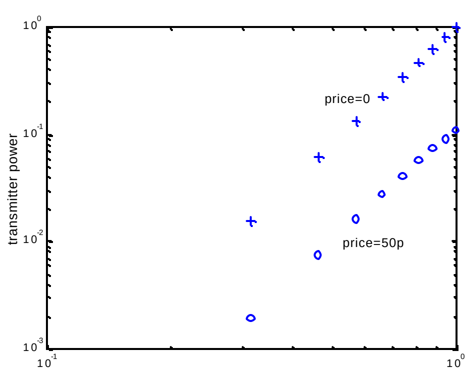

Figure 4. Transmitter power in a system with N= 9 terminals: comparison of equilibrium powers with and without a pricing function.

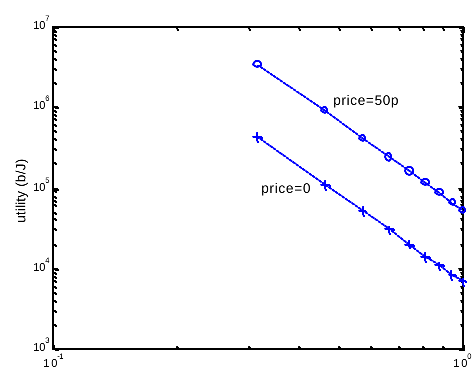

Figure 5. Utility in a system with N= 9 terminals: comparison of equilibrium utility with and without a pricing function.

VI. DISCUSSION OF RESULTS

The numerical experiments demonstrate that when each terminal operates independently to maximize its utility, the set of transmitter powers converges to a locally optimum result, in which all terminals obtain the same signal-to-interference ratio, $\gamma^*=12.4$, the solution to Equation (10). However, we also find that this result is not globally optimum. By reducing their powers by the same factor, all terminals achieve higher utility. To work within the context of a non-cooperative game (terminals operating independently to achieve best performance), we have introduced a pricing function that causes each terminal to maximize its net utility, defined as the difference between utility and price. In contrast to the original algorithm with zero price, the algorithm with a positive pricing function converges to an equilibrium point with unequal signal-to-interference ratios at different terminals. All terminals operate with lower power, lower signal-to-interference ratio, lower efficiency, and higher utility than they do when the price is zero. Because utility is the ratio of efficiency to power, this implies that the benefit achieved by introducing pricing is entirely due to reduced power.

While all terminals achieve higher utility when they maximize net utility, rather than the utility itself, the benefits are highest for terminals near the base station. Using an algorithm with a positive price function, terminals closer to the base station operate with higher signal-to-interference ratios than terminals further away. This property of the power control scheme conforms to the properties of advanced practical wireless systems in which Quality of Service is location-dependent. This dependency is introduced in rate adaptation schemes, such as those incorporated in EDGE (Enhanced Data Rates for GSM Evolution) [11] and W-CDMA (wideband code division multiple access) [12], and in incremental redundancy techniques for responding to transmission errors [13].

One drawback of power control based on pricing is that we do not have a convenient algorithm for implementing it in practice. By following the definition of the algorithm, each terminal has to solve Equation (17) periodically and then adjust its power accordingly. The new power is a complicated function of the present signal-to-interference ratio. By contrast, the adjustments required to converge to the solution to Equation (10) (corresponding to Equation (17) with t=0) are simple. The new power of terminal i is simply the old power multiplied γ^*/γ_i, the ratio of the target signal-to-interference ratio to the present signal-to-interference ratio.

Most of the work reported here appears in the Master of Science dissertation of Viral Shah [7,8]. The dissertation introduces the utility function used in this paper and proves formally many of the statements in this paper. Extensions of the work here to include the effects of error-correcting coding can be found in [9]. Joint transmitter power and transmission rate control based on utility maximization as well as the effect of packet size can be found in [10]. Investigation of Pareto efficient pricing policies for transmit power control can be found in [14].

While all of the above work pertains to circuit-switched wireless data communications, extensions are currently underway at WINLAB to introduce such a microeconomics framework to packet-data wireless communication scenarios. Another related effort at WINLAB includes the study of dynamic utility maximization algorithms that take into account mobility, channel variations and residual battery life.

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