Chapter 1

A NEW FRAMEWORK FOR POWER CONTROL IN WIRELESS DATA NETWORKS: GAMES, UTILITY, AND PRICING

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Abstract We develop a new framework for distributed power control for wireless data based on the economic principles of utility and pricing. Utility is defined as the measure of satisfaction that a user derives from accessing the wireless data network. Properties of utility functions are introduced and a specific function, based on throughput per terminal battery lifetime including forward error control, is presented and shown to conform to those properties. Users enter into a non-cooperative game to maximize their individual utilities by adjusting their transmitter powers. A unique Nash equilibrium for the above game is shown to exist but is not Pareto efficient. A pricing function is then introduced which leads to Pareto improvements for the non-cooperative game.

Keywords: Radio resource management, power control, pricing, non-cooperative games

1. INTRODUCTION

The technology and business of cellular telephony have made spectacular progress since the first systems were introduced fifteen years ago. With new mobile satellites coming on line, business arrangements, technology and spectrum allocations make it possible for people to make and receive telephone calls anytime anywhere. The cellular telephone success story prompts the wireless communications community to turn its attention to other information services, many of them in the category of "wireless data" communications. One lesson of cellular telephone network operation is that effective radio resource management (power control, channel assignment, and handoffs) is essential to promote quality and efficiency of a system. Radio resource management will be equally, if not more, critical in systems that include high speed data applications. The next generation of wireless systems promise to offer high speed delivery of multimedia information integrated with voice and data. As deployment of these next generation systems draws near, there is an increased urgency to develop and investigate advanced radio resource management techniques centered on wireless data transmission.

This present work focuses on developing a new framework for radio resource management in wireless data networks. The approach proposed here relies on using microeconomic theories that take into account notions of utility and pricing in developing distributed radio resource management algorithms for wireless data services. In particular, a framework for cellular radio power control is developed and investigated, whereby individual users adjust their transmitter powers in order to maximize their utility. It is then shown that system wide increases in utility can be achieved by introducing a penalty, or pricing, function that attempts to police the resource allocations of each user. This type of radio resource management paradigm can be expressed and formulated as a profit maximizing game that attempts to adjust resource levels of each user to maximize the difference between the utility derived and the price exacted.

2. UTILITY AND WIRELESS DATA

In economics, utility is defined as the level of satisfaction that a person (user) derives from consuming a good or undertaking an activity. The physical channel conditions play different roles in affecting the perceived user quality, and hence the user's level of satisfaction, when it comes to voice and data applications. The factors that affect the perceived quality of voice signals include delay and clarity. Weak signals and interference in the wireless channel cause errors to occur in the speech signal and thus degrade the level of clarity and user satisfaction. Further, as telephone conversations are real time applications, information must be delivered in a timely manner. This condition often precludes the reliance on retransmissions to improve voice quality when frames are received in error. This dependence of voice quality on low and consistent delay differentiates it from data type services. For delay tolerant data applications the commodity is information in the form of bits, and there exist no subjective quality metrics for bits as there are for speech signals. This inherent difference in perceived quality and end user satisfaction points to conceptually different utilities for voice and data systems, and warrants new investigation of the radio resource problem for wireless data.

We believe an explicit economic model can serve as a useful guide in developing a good radio resource mechanism for data users by understanding the factors that affect quality of data communications. To achieve this, we develop a model to quantify the level of satisfaction, or utility, experienced by a data user. This measure, as discussed above, differentiates voice from data traffic. For voice communications the acceptable quality of a connection is often specified by some maximum tolerable BER (usually 10^{-3}). This required BER could be mapped directly to some minimum SIR requirement. Above this required SIR, the voice user has an acceptable connection (utility is "one") and below this required SIR value the voice user does not have satisfactory voice quality (utility is "zero"). Because of this zero-one property of voice utility, a voice user is usually indifferent to small changes in its SIR. A data user has more stringent BER requirements (of the order of 10^{-6} or lower). Further, data users generally communicate information in the form of packets. Due to their intolerance of errors, a packet containing errors will have to be retransmitted, since error correction coding can correct only a few errors. A higher SIR will lead to lesser number of retransmissions, and hence reduce the traffic delay experienced by the data user. Thus for a data user, a higher SIR generally implies a better throughput. This implies that the utility function for a data user is a continuous function of the SIR obtained by the user. Figure 1 shows a conceptual plot of the utilities of a voice user and a data user. Our aim is to formulate a meaningful utility function for data users keeping in mind the dependence of the utility on SIR.

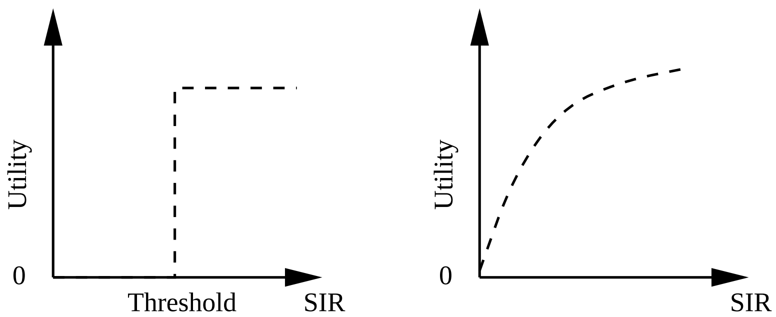

Figure 1.1 Conceptual Utility for Voice and Data

3. DEVELOPING A UTILITY FUNCTION

The definition of utility described above is extremely general and the question is to relate this notion of utility to a wireless data system. In a wireless communication system each user transmits its information over the air using some multiple access system. Since air is a common medium for all the signals, each user's signal acts as an interference to the other users' signals. Further, because of fading, multipath, and other impairments, the radio signal gets distorted by the time it travels from the transmitter to the receiver. A common denominator to account for all these impediments is the SIR of the received signals. Therefore, we can consider the SIR as one of the commodities that a user desires. Further, wireless users typically have a limited battery energy and hence transmitter power is another valuable commodity and users are reluctant to part with it. As the SIR is proportional to the user's transmit power, we see an implicit tradeoff developing. On the one hand, a user desires a strong received signal and the high SIR that can be achieved with a large transmit power. However due to limited battery life a user must also exercise parsimony with its transmitter power. Therefore depending upon the transmit power and the SIR obtained by the user, we would like to formulate an expression to determine the user's satisfaction from using the network.

3.1 PROPERTIES OF A UTILITY FUNCTION

We now stipulate some desirable properties of the utility function with respect to the SIR and transmitter power of a user. We denote the SIR achieved by user j as γ_j, and the transmitter power of user j as P_j. Ideally, a user would prefer to transmit as little power as possible, since battery life of a mobile terminal is limited. At the same time, the user would like its SIR to be as high as possible. Thus we would like the utility function to be a monotonically increasing function of the SIR for a fixed radiated power. However, as in typical communicaiton channels, when the SIR increases beyond large values, the throughput approaches the asymptote of an error free channel. These two facts can be mathematically summarized in the following two properties.

Property 1: The utility function, U_j, is a monotonically increasing function of the user's SIR (γ_j) for a fixed transmitter power (P_j) .

$$\frac{\partial U_j(P_j,\gamma_j)}{\partial \gamma_j} > 0, \quad \forall \gamma_j, P_j > 0. \tag{1.1}$$

Property 2: The utility function, U_j, obeys the law of diminishing marginal utility for high values of γ_j.

$$\lim_{\gamma_j \to \infty} \frac{\partial U_j(P_j,\gamma_j)}{\partial \gamma_j} = 0, \quad P_j > 0. \tag{1.2}$$

We further assume the utility to be a non-negative funciton of SIR, and when $\gamma_j = 0$ we assume that that the utility U_j is zero. In terms of transmit power, the desirable attribute is to conserve battery energy. At the same time, we would like to avoid degenarate situations such as zero transmit power. These attributes can be specified in the form of the following three properties.

Property 3: The utility function, U_j is a monotonically decreasing function of the user's transmitter power, for a fixed SIR γ_j.

$$\frac{\partial U_j(P_j,\gamma_j)}{\partial P_j} < 0, \quad \forall \gamma_j, P_j > 0 \tag{1.3}$$

Property 4: In the limit that the transmitter power tends to zero, the utility value also tends to zero, or:

$$\lim_{P_j \to 0} U_j = 0 \tag{1.4}$$

Property 5: In the limit that the transmitter power tends to infinity, the utility value tends to zero, or:

$$\lim_{P_j \to \infty} U_j = 0 \tag{1.5}$$

The properties that we have specified above will aid us in developing a meaningful utility function.

3.2 DEVELOPING A GENERAL UTILITY FUNCTION

For data communications, information is usually transmitted in the form of packets or frames. The channel introduces errors in the received signal with a probability function that is determined by the received SIR. We will assume that we are able to detect all errors in the received signal through the use of CRC check codes, that we can correct up to and including t errors per frame by employing error correcting codes, and that all data that is unable to be corrected must be retransmitted. The achieved throughput T, in terms of information bits per second, can then be expressed as:

$$T = R_i f(\gamma), \tag{1.6}$$

where the information rate R_i is the rate at which information (total data minus overhead for error correction coding and CRC) is transmitted and $f(\gamma)$ is a measure of the efficiency of the transmission protocol. Note that the efficiency of the protocol should depend on the SIR achieved over the channel and the power of the error correction code t, and its value varies from zero to one (i.e., $f(\gamma) \in [0, 1]$). However, we have explicitly dropped t from the notation of the efficiency function because we assume that the error coding is chosen on a system wide basis and that every user employs the same level of error correction. A higher SIR means a lower BER and hence the efficiency of the protocol should be better, while a greater value of t implies that more errors can be corrected without retransmissions, also increasing the efficiency of the protocol.

The measure of satisfaction derived from using a mobile data system clearly is related to the amount of information that a user can transfer in the lifetime of its battery. Therefore, we choose the throughput per battery life as the utility function, as this brings a strong practical and meaningful metric to serve as the definition of utility. If we denote the energy content of user j's battery as E Joules, we can express the utility a user derives from the network as the total number of information bits that it can transmit correctly in the lifetime of its battery, as below:

$$U_j(P_j, \gamma_j) = \frac{E}{P_j} R_i f(\gamma_j). \tag{1.7}$$

Note that the unit of the above utility function is bits. Since $f(\gamma)$ is an increasing nonnegative function of the SIR, it can be shown that the utility function satisfies property (1) through (3). This formulation is very general and applicable to a variety of communication systems. In what follows, however, we illustrate the radio resource allocation concepts with a specific example.

3.3 EXAMPLE UTILITY FUNCTION

In this section we detail a specific utility function that corresponds to a particular CDMA transmission system that employs BCH error correcting codes and interleaving. The specific efficiency function for such a system can be derived as follows. Users are distributed at different distances throughout the cell and access the system through a battery powered device. All users share the same bandwidth and chip rate of W chips per second, and encoded data rate of R (information plus overhead). The SIR of user j can then be written as:

$$\gamma_j = \frac{W}{R} \frac{h_{jk} P_j}{\sum_{i \neq j} h_{ik} P_i + \sigma_k^2} \tag{1.8}$$

where W/R is the processing gain of the spread spectrum system, σ_k^2 is the background noise at the receiver at base station k, and h_{jk} is the path gain of user j to base station k. The path gains, and consequently the location of each user, become significant in determining that user's SIR and therefore its utility.

The SIR value determines the bit error rate seen by user j. Consider the simple case of a Gaussian channel, in which all users share the same frequency band. The bit error rate P_e for a Gaussian channel in terms of the SIR is given as:

$$P_e = Q(\sqrt{\gamma}) \approx \frac{1}{2} e^{-\frac{\gamma}{2}}. \tag{1.9}$$

We incorporate the effects of forward error control into the efficiency function by encoding all information into data frames of length L encoded bits. Forward error control is achieved through the use of error correcting codes that are capable of correcting up to and including t bit errors per frame, and a C bit CRC check is included in each frame for error detection.

The benefits of forward error control come at the expense of a reduction in the amount of information bits present in each frame. In order to provide error correction it is necessary to insert a number of parity bits into each data frame, increasing the overhead. A upper bound for the number of parity bits, m, necessary to correct t corrupted bits per frame can be found, as in [Lin and Costello, 1983], to be $m = t \log_2 (L - C + 1)$. Including the C CRC check bits we can express the ratio of information bits to encoded bits, or code rate $\Gamma(t)$, of the system by

$$\Gamma(t) = \frac{L - C - m}{L} = \frac{L - C - t \log_2 (L - C + 1)}{L}. \tag{1.10}$$

Therefore we can represent the information rate of the system R_i as the total data rate R multiplied by the code rate of the system as follows:

$$R_i = R\Gamma(t). \tag{1.11}$$

By assuming sufficient interleaving and hard decision decoding [Proakis, 1989], we can express the frame-success-rate (FSR) experienced by user j of the CDMA system as,

$$FSR_j(\gamma_j, t) = \sum_{i=0}^{t} \binom{L}{i} \left(\frac{e^{-\frac{\gamma_j}{2}}}{2}\right)^i \left(1 - \frac{e^{-\frac{\gamma_j}{2}}}{2}\right)^{L-i}. \tag{1.12}$$

It is very tempting to use the FSR as the efficiency function $f(\gamma)$, however this approach leads to some degenerate solutions as will be shown now. The function does not satisfy Property (4), since at zero SIR the FSR function is not zero, instead it attains a value of $\sum_{i=0}^{t} \binom{L}{i} 2^{-L}$. Therefore, theoretically at zero power a user can obtain infinite utility since it obtains a fixed value of $f(\gamma)$, but is using zero power. This case arises from the fact that when the channel is extremely poor, the worst the receiver can do is to randomly guess at the transmitted bits causing the worst bit error performance to be $P_e = 1/2$. As such there is an extremely small, but non-zero probability that the receiver will guess the right frame. Therefore given enough time the user can get an infinite amount of information through the system at no cost to its battery life. To prevent this degenerate case we would like the efficiency function to approach zero as the power is reduced to zero. One way to do this is to use a probability of bit error function of the form:

$$P_e = e^{-\frac{\gamma}{2}}. \tag{1.13}$$

This will yield a modified frame-success-rate that can be used as the efficiency function and satisfy Properties (1) through (5). So that the utility now becomes

$$U_j(P_j, \gamma_j) = \frac{ER}{P_j} \cdot \frac{L - C - \log_2(L - C + 1)}{L} \cdot \sum_{i=0}^{t} \binom{L}{i} \left(e^{-\frac{\gamma_j}{2}}\right)^i \left(1 - e^{-\frac{\gamma_j}{2}}\right)^{L-i} \tag{1.14}$$

$$U_j(P_j, \gamma_j) = \frac{ER\Gamma(t)}{P_j} \cdot f(\gamma_j). \tag{1.15}$$

A plot of this utility function is shown in Figure (1.2). In this figure the level of interference is kept constant, and the user's utility is plotted versus the transmitter power. This utility function still has practical resonance for a wireless communications system, in that it serves as an underestimate of the number of information bits that can be transmitted during the lifetime of a terminal's battery. It also is an unbiased modification because the modified function applies equally well to all users of the system regardless of distance, power, or SIR. We feel that although this function does not have an exact practical definition, valuable insights can be derived from its approximate physical significance[1]. We now describe the proposed resource allocation method based on maximizing this measure of utility.

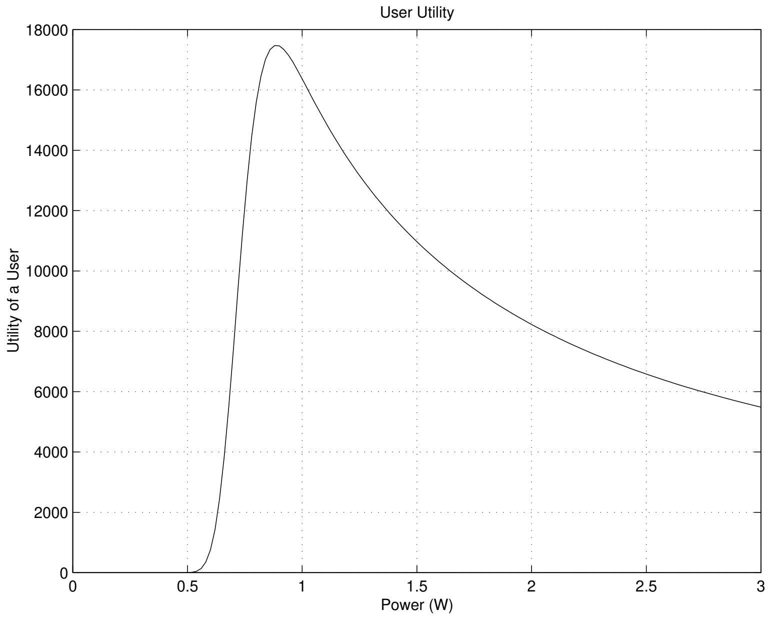

Figure 1.2 Utility function of a data user for fixed interference

4. DISTRIBUTED POWER CONTROL BASED ON UTILITY

In this section we present a distributed power control scheme based on the economic framework that we developed in the previous section. We formulate the power control scheme as a distributed utility maximization problem in which each user updates its power to maximize its utility. This is an example of a non-cooperative game, since each user behaves in a selfish way to maximize its own utility. Mathematically, we can formulate this non-cooperative power control game as:

$$\max_{P_i} U_i(P_1, P_2, \ldots, P_i, \ldots, P_K) \quad P_i \in S_i, \quad \forall i = 1, \ldots, K \tag{1.16}$$

where S_i is the strategy space of user i and is the set of values that the user i's power is restricted to. We will assume that the strategy space S_i of each user i is continuous in its power P_i. The joint strategy space $S = S_1 \times S_2 \times \ldots \times S_K$, is the cartesian product of the individual strategy sets for the K users. We will restrict the limits of the strategy space of each user to $S_i = [0, P_{\max}], \forall i = 1, \ldots, K$, where $P_{\max}$ is some maximum power constraint. Equation (1.16) implies that each user maximizes its utility by assuming the power of the other users as exogenously given. The iterations will converge to an equilibrium point (if one exists) where all the (K) users are satisfied with the utility that they are getting. Mathematically we can represent the necessary condition for equilibrium as:

$$\frac{\partial U_j}{\partial P_j} = 0 \quad j = 1, \ldots, K \tag{1.17}$$

which reduces to the following:

$$f(\gamma^*) = \gamma^* \cdot f'(\gamma^*). \tag{1.18}$$

Observe that in the final equation we have dropped the subscript purposely to point to the fact that the first order condition is the same for each user and is only a function of the user's SIR. Each user's utility reaches an (equilibrium) extreme point at some value of SIR given by γ^*. The solution to the non-cooperative equilibrium reduces to finding a power vector $\mathbf{P}^* = [\mathbf{P_1^*}, \mathbf{P_2^*}, \ldots, \mathbf{P_K^*}]$ that satisfies:

$$\left[\mathbf{I} - \gamma^* \frac{R}{W}\mathbf{F}\right]\mathbf{P} = \boldsymbol{\mu}, \quad \mathbf{P} \geq \mathbf{0}, \tag{1.19}$$

where $\boldsymbol{I}$ is the $K \times K$ identity matrix, and $\boldsymbol{F}$ is the following nonnegative matrix:

$$F_{ij} = \begin{cases} 0 & i = j \\ h_{ji}/h_{ii} & i \neq j \end{cases} \tag{1.20}$$

and $\boldsymbol{\mu}$ is the vector with elements $\mu_i = \frac{\gamma^* R\sigma_i^2}{Wh_{ii}}$. Note that there may be more than one SIR or no SIR value that satisfies the first order equilibrium condition (1.18). We will investigate this issue shortly. For now, assume that there is only one such γ^*.

Theorem 1 *There exists a power vector* $\mathbf{P}^*$ *that is feasible (i.e., it is a solution to equation (1.19)) if the Perron-Frobenius eigenvalue of the matrix* F *satisfies:*

$$e_{\mathbf{F}} < \frac{W}{R}\frac{1}{\gamma^*}, \tag{1.21}$$

where $e_{\mathbf{F}}$ *is the Perron-Frobenius eigenvalue defined as the largest eigenvalue of matrix* F.

The proof of the above theorem on the feasibility of power control follows using standard arguments based on the Perron-Frobenius theorem such as those used for fixed target type power control schemes [Yates, 1995]. While equation (1.18) gives only the necessary conditions for the existence of an equilibrium, we will now prove the existence and uniqueness of such an equilibrium.

5. EXISTENCE AND UNIQUENESS OF THE EQUILIBRIUM

In this section we describe the solution to the power control problem. We begin with a definition of the equilibrium for the non-cooperative power control game.

Definition 1: *A Nash equilibrium [Nash, 1950] for the non-cooperative power control game is a power vector* $\mathbf{P}^*$ *such that no single user can improve its utility by a unilateral change in its power.*

Mathematically we can represent a Nash equilibrium as a power vector $\mathbf{P}^*$ such that $U_i(\mathbf{P}^*) \geq \mathbf{U_i}(\mathbf{P_i'}, \mathbf{P_{-i}^*})$, $\forall P_i' \in S_i, i \in 1, 2, \ldots, K$, where $\mathbf{P_{-i}}$ is a power vector that contains the powers of all the users except the i^{th} user.

Theorem 2 *There exists a unique Nash equilibrium to the non-cooperative power control game in (1.16).*

The proof of the above theorem follows from the fact that the utility function in equation (1.15) is quasi-concave [Roberts and Varberg, 1973] and the use of Debreu's theorem [Debreu, 1952]. This theorem is also equally credited to Fan [Fan, 1952] and Glicksberg [Glicksberg, 1952]. We can see that the Nash equilibrium dictates that every user achieve the same received SIR at the base station. It is interesting to note that the utility maximization approach to power control yields a solution similar to the fixed target type of power control. However, the difference here is that the target, γ^*, is determined by the efficiency function $f(\gamma)$ and the solution to equation (1.18). Further, it can also be shown that a distributed power control scheme using SIR balancing (as in [Yates, 1995]) would also converge to the same unique Nash equilibrium. This result is easily shown by using Yates' framework (see [Yates, 1995]) for power control that is based on the notions of positivity, monotonicity, and scalability of interference functions.

5.1 PROPERTIES OF THE EQUILIBRIUM

Equal Received Powers at the Base Station. It can be shown, as in [Shah et al., 1998], that in a single cell system the solution to the non-cooperative power control game with forward error correction achieves equal received powers at the base station. That is, at the equilibrium solution all user's have chosen transmitter powers such that their received signals at the base station are all equal. This implies that users who are further from the base station must transmit more power to achieve the same SIR target than those that are closer to the base station.

Pareto Efficiency. A classical measure of the efficiency of an equilibrium solution is Pareto efficiency [Varian, 1996], which is defined for our wireless system below.

Definition 2: *A power vector $\boldsymbol{P^*}$ is Pareto efficient if and only if there exists no such vector $\boldsymbol{P'}$ such that at least one user achieves higher utility while the other users' utilities remain the same or improve. In other words there exists no power vector $\mathbf{P}'$ such that $U_j(\mathbf{P}')> U_i(\mathbf{P}^*)$, $\forall i = 1,\ldots,K, i \neq j$, where K is the number of users in the cell.*

Alternatively, we can say that if a power vector $\mathbf{P}^*$ is Pareto efficient then for any other allocation $\mathbf{P}'$, if one user improves its utility, then there is at least one other user whose utility is decreased, i.e., for any $U_j' > U_j^*$ there exists at least one i such that $U_i' < U_i^*$. Further, if no such Pareto improvement can be found, then $\mathbf{P}^*$ is Pareto efficient.

Theorem 3 *There exists some $0 < \alpha < 1$ such that for all i, $U_i(\alpha\mathbf{P}^*) > U_i(\mathbf{P}^*)$.*

The proof follows from the strict negativity of the derivative of the utility function with respect to α at the equilibrium solution. Details of the proof for the above theorem can be found in [Famolari, 1999]. The implication of the above theorem is that using utility maximization (or equivalently fixed target type power control) for data users (characterized by non 0-1 utility functions) results in an operating point that is inefficient in power usage.

6. DISTRIBUTED POWER CONTROL BASED ON PRICING

The equilibrium analysis for a non-cooperative game involving only the utility function showed that the equilibrium is Pareto inefficient. In order to improve upon this inefficient equilibrium we would like to influence user behaviour by imposing upon the users a pricing function which we believe will lead to a better equilibrium point. We will first analyze the equilibrium for the non-cooperative power control game that we defined in the last section. This will help us develop a pricing mechanism for the wireless network. A comparison of equilibrium parameters for different users, with and without pricing will give more insight into the effect of the pricing function.

6.1 DEVELOPING A PRICING FUNCTION

In a typical wireless communication system, there may be many users and each will have its own utility, which is dependent on SIR at the base station and its transmitter power. It is evident that as a user tries to increase its power in order to increase its SIR, it causes more interefernce to other users thereby degrading their performance. One approach would be to charge a user some price for using the network, and in order to maintain a healthy wireless environment, to charge it for causing harm to other users. We would like to have a pricing function that is a monotonically increasing function of transmitted power. We will now give a more rigorous explanation to support this formulation of the pricing function.

It should be noted that our main intention of introducing pricing is to bring about a Pareto improvement in the utilities of the users and not necessarily as a way of generating revenue for the system. We will now analyze the non-cooperative equilibrium point from the previous section to find out which user is creating the maximum harm in a single cell system with K users. We will denote the base station as y. We define the harm (in terms of the degradation in utility of other users) caused by user j by acting as an interference to user i as follows:

$$C_{ij} = -\frac{\partial U_i}{\partial P_j} P_j. \tag{1.22}$$

Note that C_{ij} has the same units as U_i; in this case bits. The differential term in equation (1.22) is similar to the shadow price concept used in [Mackie-Mason and Varian, 1995] for pricing wireline networks. The total harm done by user j to the rest of the users in cell y is:

$$C_j = \sum_{i \neq j, i=1}^{K} C_{ij}. \tag{1.23}$$

We define $C_{jj} = 0$, i.e., the user does not cause any harm to itself, since it can not act as its own source of interference. Based on the above notion of harm caused by a user we state the following proposition regarding the equilibrium of the non-cooperative power control game.

Proposition 1: *If the path gains of the K users in a cell are in the order $h_{iy} > h_{2y} > \ldots, > h_{Ky}$, then it can be shown that the following is true:*

$$P_1^* < P_2* < \ldots < P_K^* \tag{1.24}$$

$$U_1^* > U_2^* > \ldots > U_K^* \quad (1.25)$$
$$C_1^* < C_2^* < \ldots < C_K^*. \quad (1.26)$$

The proof of the above proposition follows from the fact that all users are received at the same power level at the base station and the definition in equation (1.23).

From Proposition 1, we can draw a number of conclusions which will help us in developing a pricing function. Firstly, a user x with the worst path gain (h_{xy}) causes the most amount of harm to other users (since its C_x is the highest) and vice-versa. Further, the same user transmits the highest power. We would like to price the users according to the amount of harm they are causing to others. Hence we need to develop a pricing function which increases as the transmit power of the user increases. Note that there are a myriad of different functions that have this property. However, as a first step to understanding the effects of pricing, we choose a simple linear pricing function which is proportional to the transmit power of the user. We define the price that we will charge the user as:

$$F_j = \beta P_j, \quad (1.27)$$

where β is a positive constant, and P_j is the power radiated by the mobile. To incorporate the fact that the user is transmitting at a raw bit rate of R bits per second, we would like to factor the constant β in equation (1.27) as $\beta = qR$, where q is a positive constant. The unit of β is bits per watt since we want the units of F_j to be the same as that of U_j. It should be noted that the pricing function that we have formulated is by no means the optimum for our problem. Note that the pricing function her is not in terms of money but rather in terms of abstract credits that are redeemable for network services [Peha, 1997]. Each user would like to have as large a utility as possible, but at the same time would want to pay the smallest possible price for it. This implies that each user would like to maximize the difference between its utility and pricing function as a function of its own power. Since the pricing function is a monotonically increasing function of the user's power, we would expect that if the user maximizes the difference between its utility and price, the equilibrium point would shift to a lower power value than before. Conceptually we illustrate this in Figure 1.3. This figure is somewhat misleading since we consider the effect of the pricing function on only one user, assuming the other users transmit at a constant power level. However, all the users should be using lower powers as well in the presence of the pricing function. Recall that from the Pareto optimality discussion for the non-cooperative power control of the utility function we observed that all users would be better off if they reduced their power by some fixed factor α. We can now formulate a distributed power control game based on utility and pricing, where a user's goal is to maximize the difference between utility achieved and price charged as below:

$$\max_{P_i} U_i - F_i \quad P_i \in S_i, \quad \forall i = 1, \ldots, K. \quad (1.28)$$

We show that a pricing function encourages the use of lower transmit powers and thus can lead to Pareto improvements over the non-cooperative game based solely on utility. If the utility-price difference objective function is quasi-concave we can argue as before and show that there exists a Nash equilibrium, but showing the uniqueness of such a solution may be analytically difficult.

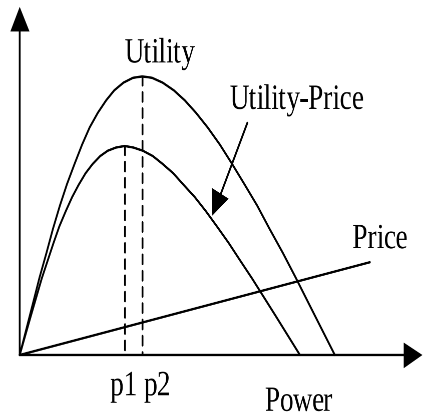

Figure 1.3 Effect of a linear pricing function on the non-cooperative power control game

We do, however, present numerical results that show that an equilibrium does exist and that it does provide Pareto improvements with respect to the the utility only power control game. In the next section we provide some numerical results for the non-cooperative power control game based both on utility and that based on utility and pricing. We then go on to further elaborate the differences in the achieved equilibrium of both games.

7. NUMERICAL RESULTS

The non-cooperative power control game was played for a cell containing users at random distances from the base station. The channel gains of the users was determined by the PCS extension of the Hata propagation model for an urban environment [Rappaport, 1996]. The resulting utilities and powers for a fixed level of error correction equal to four correctable errors per frame are shown in Figures 1.4 and 1.5 respectively. The improvements due to pricing, in terms of increased user utility and reduced transmitter power are clearly evident.

Pricing can be used to achieve a more Pareto efficient equilibrium solution for a non-cooperative power control game for a fixed level of error correction. We can also notice the effects that the non-cooperative game based on pricing has on users located at different distances. When no pricing is applied, we know that an equilibrium solution must provide equal received powers at the base station. This means that users that are located further away must transmit at higher powers to achieve the same SIR, and consequently are expending more battery energy for the same quality channel than the users who are closer to the base station. Thus we see that utility is inversely proportional to user distance.

When pricing is applied to the game the situation for all users improves because pricing provides a more Pareto efficient solution. However the improvements are more pronounced for those users who are closer to the base station. This is due to the fact that the distant users are charged a very high price and reduce their powers more than the closer users. This effect can be seen in Figure 1.6 where after pricing is applied, the SIR's of the users are no longer equal. Those users at the edge of the cell are experiencing much poorer channel conditions then they had enjoyed previously. However, this is compensated for by the drastic reduction in their expended battery energy and the net result is a slightly higher utility. For the users that are closer to the base station, this reduction in interference translates into a conservation of battery power without a subsequent degradation in channel quality. Therefore more information can be sent in the lifetime of the battery and the utility increases.

We also investigate the effect that a moving user has on the non-cooperative game. Figure 1.7 shows a plot of equilibrium utility values of stationary users plotted against the distance of the moving user "1" from the base and distance of the stationary users from the base respectively. In this figure there is no level of error correction, a frame length of 80 bits, and a total of 9 users in the system. As user "1" moves closer to the base station, the stationary users transmit more power and their SIR decreases. As a result, their utility is reduced. The utility of user "1" on the other hand is monotonically increasing as it moves towards the base station. The two effects that are responsible for the utility increase of the moving user are a reduction in transmitter power and an improvement in SIR. The implication of the above behavior is that system concepts like INFOSTATIONS [Goodman et al., 1997] may be a natural solution for providing wireless data services.

8. CONCLUSION

We have presented a distributed radio resource management scheme for wireless data networks that is based on microeconomic theories. A utility function based on throughput per battery lifetime including forward error control was developed and shown to yield a unique solution to the non-cooperative power control game. This solution, was shown to be equivalent to a fixed-target type power control solution. However, it was shown that the equilibrium solution was Pareto inefficient. A pricing function was developed to achieve Pareto improvements. The effect of pricing applied to the non-cooperative power control game for wireless data networks was shown to yield favorable utility solutions at reduced transmitter powers.

Notes

1. For a general formulation of efficiency and utility functions for BPSK, QPSK, DPSK, and FSK modulation techniques see [Fang et al., 1999]

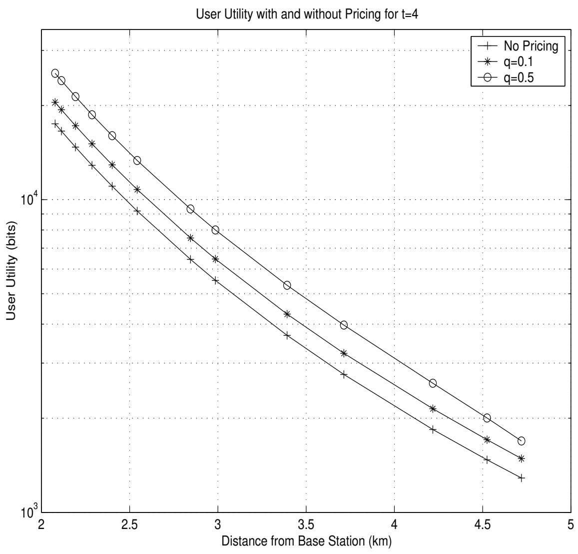

Figure 1.4 User Utility versus Distance with and without Pricing for $t = 4$ Correctable Errors

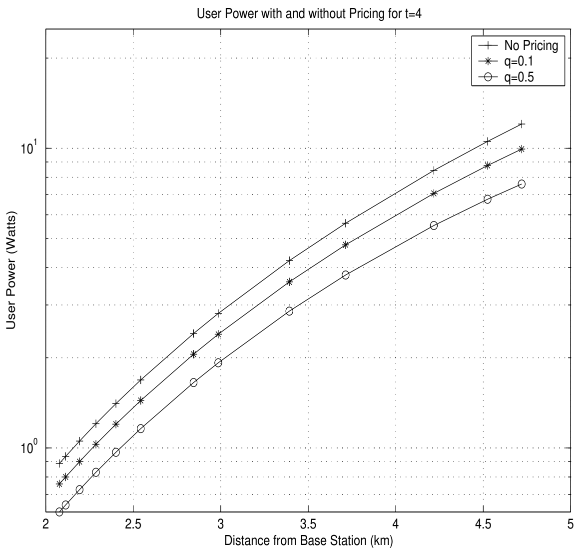

Figure 1.5 User Power versus Distance with and without Pricing for $t = 4$ Correctable Errors

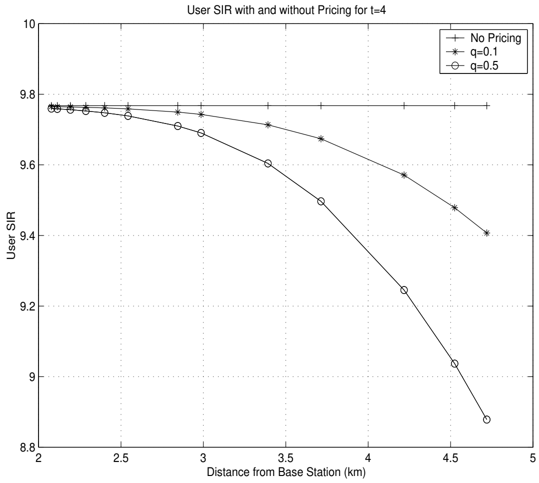

Figure 1.6 User SIR with and without Pricing for $t = 4$ Correctable Errors

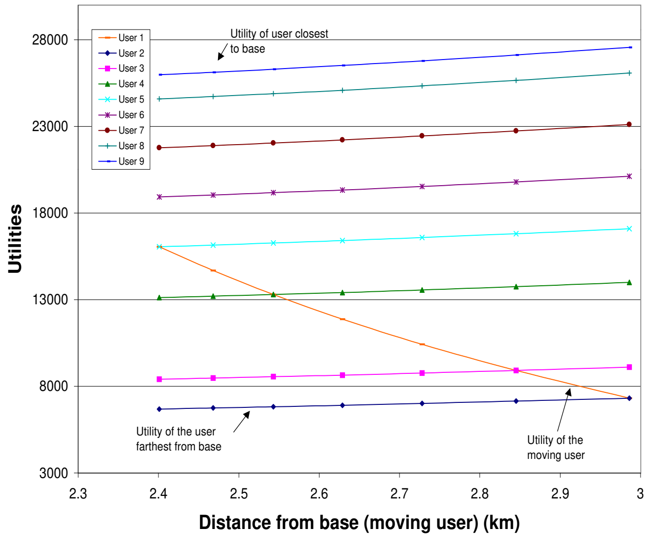

Figure 1.7 Effect a moving user has on the equilibrium utility values

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