Efficient Power Control via Pricing in Wireless Data Networks*

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Abstract

A major challenge in operation of wireless communications systems is the efficient use of radio resources. One important component of radio resource management is power control, which has been studied extensively in the context of voice communications. With increasing demand for wireless data services, it is necessary to establish power control algorithms for information sources other than voice. We present a power control solution for wireless data in the analytical setting of a game theoretic framework. In this context, the quality of service (QoS) a wireless terminal receives is referred to as the *utility* and distributed power control is a *non-cooperative power control game* where users maximize their utility. The outcome of the game results in a *Nash equilibrium* that is inefficient. We introduce pricing of transmit powers in order to obtain Pareto improvement of the non-cooperative power control game, i.e., to obtain improvements in user utilities relative to the case with no pricing. Specifically, we consider a pricing function that is a linear function of the transmit power. The simplicity of the pricing function allows a distributed implementation where the price can be broadcast by the base station to all the terminals. We see that pricing is especially helpful in a heavily loaded system.

Keywords
Power control, wireless data, game theory, pricing, pareto efficiency

1 Introduction

As the demand for wireless services increases, efficient use of resources grows in importance. A fundamental component of radio resource management is transmitter power control. It is well known that minimizing interference using power control increases capacity [1–3] and also extends battery life. Recently, an alternative approach to the power control problem in wireless systems based on an economic model has been

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offered [4–7]. In this model, service preferences for each user are represented by a utility function. As the name implies, the utility function quantifies the level of satisfaction a user gets from using the system resources. Game-theoretic methods are applied to study power control under this new model. Game theory is a powerful tool in modeling interactions between self-interested users and predicting their choice of strategies [8, 9]. Each player in the game maximizes some function of utility in a distributed fashion. The game settles at a Nash equilibrium if one exists. Since users act selfishly, the equilibrium point is not necessarily the best operating point from a social point of view. Pricing the system resources appears to be a powerful tool for achieving a more socially desirable result.

In this work, we are primarily concerned with the impact of pricing the usage of wireless services on QoS. Pricing of services in wireless networks emerges as an effective tool for radio resource management because of its ability to guide user behavior towards a more efficient operating point. To that end, we introduce a model for power control in wireless data networks using concepts from microeconomics. We model utility to reflect the level of satisfaction (QoS) a data user gets from using system resources [4]. We consider the uplink power control problem in a single cell CDMA wireless data system with n users where each user maximizes its own utility. While the resulting noncooperative power control game has a Nash equilibrium, it is inefficient. Therefore, we introduce pricing to improve efficiency. We then show that there exist equilibria in the non-cooperative power control game with pricing and that they are Pareto superior compared to the equilibrium of the game with no pricing. However, the game with pricing is still unable to achieve a socially optimum power solution.

This paper is organized as follows. In Section 2, we discuss the concept of utility and develop a utility function that represents the QoS of data users. In Section 3, we construct the non-cooperative power control game. The equilibrium and its properties are discussed in Section 4. Section 5 is devoted to showing the inefficiency of the Nash equilibrium obtained as a result of the non-cooperative power control game. In Section 6, we introduce pricing as a means of remedying this inefficiency and discuss the game with pricing. In Section 6.1, we define supermodular games and discuss the relevance of this class of games in the context of the present work. Comparisons of results in games with and without pricing and the significance of these results are discussed in Section 7. In Section 8, we define the social optimum and discuss how it relates to solutions for the games discussed in this work. Finally, in Section 9 we present an overview of the results in this paper and conclusions.

2 Utility Function

The concept of utility is commonly used in microeconomics and refers to the level of satisfaction the decision-taker receives as a result of its actions. Formally, a utility function is defined as follows [8, 9].

Definition 1 *A function that assigns a numerical value to the elements of the action set A ($u : A \to \Re^1$) is a utility function, if for all $x, y \in A$, x is at least as preferred compared to y if and only if $u(x) \geq u(y)$.*

The utility function that describes a particular set of preference rules is not unique. Any function that puts the elements of A in the desired order is a candidate for a utility function. We first identify the preference relations that are specific to our problem and then describe a utility function that satisfies this structure.

Users access a wireless system through the air interface which is a common resource and they transmit information expending battery energy. Since the air interface is a shared medium, each user's transmission is a source of interference for others. The signal-to-interference ratio (SIR) is a measure of the quality of signal reception for the wireless user. Typically, a user would like to achieve a high quality of reception (high signal-to-interference ratio, SIR) while at the same time expending a small amount of energy. Thus, it is possible to view both SIR and battery energy (or equivalently transmit power) as commodities that a wireless user desires. There exists a trade-off relationship between obtaining high SIR and low energy consumption. Finding a good balance between the two conflicting objectives is the primary focus of the power control component of radio resource management. This trade-off is illustrated through the conceptual plot in Figure 1. If the transmit power were fixed (fixed battery drain), the terminal would experience lower error rates as the SIR increases which leads to increased satisfaction of the use of the system resources. For large enough SIR values, the error rate approaches zero which results in an asymptotic increase in utility in the high SIR region. If the SIR were to be fixed (fixed error rate), increasing the transmit power expedites the battery drain, which effectively reduces the satisfaction of the mobile terminal.

An optimum power control algorithm for wireless *voice* systems maximizes the number of conversations that can simultaneously achieve a certain quality of service (QoS) objective. Typically, the QoS objective for a voice terminal is to achieve a minimum acceptable SIR. However, this approach is not appropriate for the efficient operation of a wireless data system [4, 10]. This is because the QoS objective for data signals differs from the QoS objective for telephones. In a data system, error-free communication had high priority. The SIR is an important quantity since there is a direct relationship between the SIR and the probability of transmission errors.

Consider a single-cell system where each user transmits L information bits in frames (packets) of $M > L$ bits at a rate R bits/second using p Watts of power[1]. In this work, the term frame and packet have identical meanings. We assume fixed rate R for all terminals. Optimization assuming variable rates is treated in [6, 12]. Let P_c denote the probability of correct reception of a frame at the receiver, i.e., the frame success rate (FSR). P_c is a function of the SIR obtained by the terminal at its base station and depends on the properties of the system such as modulation, radio propagation and receiver structure. The utility function can be expressed as [10]

$$u = \frac{LRP_c}{Mp} \quad \text{bits/Joule} \quad . \tag{1}$$

Utility as defined above is the number of information bits received successfully per Joule of energy expended. Assuming perfect error detection and no error correction, we can express the FSR as $P_c = (1 - P_e)^M$ where P_e is the bit error rate (BER). In the case of an additive white Gaussian noise (AWGN) channel, the BER expressions for various modulation techniques are given in Table 1. In all cases the BER decreases monotonically with SIR, where SIR is denoted by γ. Consequently, P_c is a monotonically increasing function

[1]A multi-cell system is studied in [11].

BPSK	$Q(\sqrt{2\gamma})$
DPSK	$\frac{1}{2}e^{-\gamma}$
Coherent FSK	$Q(\sqrt{\gamma})$
Non-coherent FSK	$\frac{1}{2}e^{-\gamma/2}$

Table 1: The bit-error-rate (BER) as a function of signal-to-interference-ratio (SIR) for various modulation schemes

of the SIR. Therefore, P_c can be expressed as a function of γ and substituted in equation (1) to obtain the utility function for a specific system. However, the utility function given in (1) has a mathematical anomaly in its formulation. In case of transmit power $p = 0$, for all modulation schemes, the best strategy for the receiver is to make a guess for each bit, resulting in $P_c = 2^{-M}$, resulting in infinite utility. This suggests that in order to maximize utility, all users in the system should transmit zero power and just wait for the receiver to guess the correct data. To avoid this degenerate solution, we approximate the FSR, P_c, by an *efficiency function* that closely follows the behavior of the probability of correct reception while producing $P_c = 0$ at $p = 0$[2]. The efficiency function is defined as,

$$f(\gamma) = (1 - 2P_e)^M \quad , \tag{2}$$

to replace P_c in (1). The resulting utility function will be examined in the remainder of this paper. It is given as

$$u = \frac{LRf(\gamma)}{Mp} \quad \text{bits/Joule} \quad . \tag{3}$$

The efficiency function yields the desirable properties $f(0) = 0$ for $p = 0$ and $f(\infty) = 1$. At any other value of the SIR, its shape follows that of P_c. Figure 2 demonstrates how closely the efficiency function follows the FSR in case of BPSK and non-coherent FSK modulation schemes. In the remainder of the paper, we consider power control schemes where each data user tries to maximize its individual utility. A thorough discussion on the efficiency function for different types of modulation techniques can be found in [13].

3 Non-cooperative Power Control Game

Let $G = [\mathcal{N}, \{P_j\}, \{u_j(\cdot)\}]$ denote the noncooperative power control game (NPG) where $\mathcal{N} = \{1, 2, \cdots, N\}$ is the index set for the mobile users currently in the cell, P_j is the strategy set and $u_j(\cdot)$ is the payoff function of user j. Each user selects a power level p_j such that $p_j \in P_j$. Let the power vector $\mathbf{p} = (p_1, \cdots, p_N) \in P$ denote the outcome of the game in terms of the selected power levels of all the users, where P is the set of all power vectors. The resulting utility level for the jth user is $u_j(\mathbf{p})$. We will occasionally use an alternative notation $u_j(p_j, \mathbf{p}_{-j})$ where $\mathbf{p}_{-j}$ denotes the vector consisting of elements of $\mathbf{p}$ other than the jth element. The latter notation emphasizes that the jth user has control over its own power, p_j only. The strategy space of all the users excluding the jth user is denoted by P_{-j}.

[2] An interpretation of this modification is the implicit inclusion of a delay constraint in the utility function.

The utility user j obtains by expending p_j can be expressed more formally as

$$u_j(p_j, \mathbf{p}_{-j}) = \frac{LR}{Mp_j} f(\gamma_j) \quad \text{bits/Joule} \quad , \tag{4}$$

where γ_j is the SIR of user j defined as

$$\gamma_j = \frac{W}{R} \frac{h_j p_j}{\sum_{j \neq i} h_j p_j + \sigma^2} \quad , \tag{5}$$

W is the available spread spectrum bandwidth [Hz], σ^2 is the AWGN power at the receiver [Watt], and $\{h_j\}$ is the set of path gains from the mobile to the base station. It should be noted that the derivation of the SIR expression given in (5) assumes conventional matched filter receivers and pseudo random signature sequences [14, 15]. We assume that the strategy space P_j of each user is a compact, convex set with minimum and maximum power constraints denoted by $\underline{p}_j$ and $\overline{p}_j$, respectively. For NPG, we let $\underline{p}_j = 0$ for all j which results in the strategy space $P_j = [0, \overline{p}_j]$. The utility function takes the generic form given in Figure 3 for fixed interference.

Note that equation (4) demonstrates the strategic interdependence between users. The level of utility each user gets depends on its own power level and also on the choice of other players' strategies, through the SIR of that user. The efficiency function can be chosen to represent any given modulation technique consistent with the approximation rule described in the previous section.

In the power control game, each user maximizes its own utility in a distributed fashion. Formally, the non-cooperative power control game (NPG) is expressed as

$$\textbf{(NPG)} \quad \max_{p_j \in P_j} u_j(p_j, \mathbf{p}_{-j}), \quad \text{for all } j \in \mathcal{N} \tag{6}$$

where u_j is given in (4) and $P_j = [0, \overline{p}_j]$ is the strategy space of user j. The transmit power that optimizes individual utility depends on transmit powers of all the other terminals in the system. It is necessary to characterize a set of powers where the users are satisfied with the utility they receive given the power selections of other users. Such an operating point is called an *equilibrium*.

4 Nash equilibrium in NPG

The solution that is most widely used for game theoretic problems is the *Nash equilibrium* [16].

Definition 2 *A power vector* $\mathbf{p} = (p_1, \cdots, p_N)$ *is a* Nash equilibrium *of the NPG* $G = [\mathcal{N}, \{P_j\}, \{u_j(\cdot)\}]$ *if for every* $j \in \mathcal{N}$, $u_j(p_j, \mathbf{p}_{-j}) \geq u_j(p'_j, \mathbf{p}_{-j})$ *for all* $p'_j \in P_j$.

At a Nash equilibrium, given the power levels of other players, no user can improve its utility level by making individual changes in its power. The power level chosen by a *rational* self-optimizing user constitutes a *best response* to the powers actually chosen by other players. Formally, terminal j's best response $r_j : P_{-j} \rightarrow P_j$

is the correspondence that assigns to each $\mathbf{p}_{-j} \in P_{-j}$ the set

$$r_j(\mathbf{p}_{-j}) = \left\{ p_j \in P_j : u_j(p_j, \mathbf{p}_{-j}) \geq u_j(p'_j, \mathbf{p}_{-j}) \text{ for all } p'_j \in P_j \right\} \quad . \tag{7}$$

With the notion of a terminal's best response correspondence, the Nash equilibrium definition can be restated in a compact form as follows: The power vector $\mathbf{p} = (p_1, \cdots, p_N)$ is a Nash equilibrium of the NPG $G = [\mathcal{N}, \{P_j\}, \{u_j(\cdot)\}]$ if and only if $p_j \in r_j(\mathbf{p}_{-j})$ for all $j \in \mathcal{N}$. Figure 4 illustrates how the Nash equilibrium is determined by using the best response correspondences in a game with 2 players. In this fictitious example, the strategy space for both players is $[0, 1]$ and both r_1 versus p_2 and r_2 versus p_1 are plotted on the same figure. The intersection point of the two plots fulfills the condition $p_j \in r_j(\mathbf{p}_{-j})$ for all $j \in \mathcal{N}$ given above in the definition of a Nash equilibrium. Note that if for every $\mathbf{p}_{-j} \in P_{-j}$, $r_j(\mathbf{p}_{-j})$ is composed of precisely one element, then $r_j(\cdot)$ can be viewed as a function in the usual sense. Thus, *correspondence* is a generalized concept of a *function* [8].

4.1 Existence and Uniqueness of NPG Equilibrium

The Nash equilibrium concept offers a predictable, stable outcome of a game where multiple agents with conflicting interests compete through self-optimization and reach a point where no player wishes to deviate. However, such a point does not necessarily exist. First, we investigate the existence of an equilibrium in NPG.

Theorem 1 *A Nash equilibrium exists in the NPG, $G = [\mathcal{N}, \{P_j\}, \{u_j(\cdot)\}]$.*

The proof of the above theorem can be found in Appendix A. The analysis presented in the proof uses the efficiency function that approximates the probability of correct reception of non-coherent FSK as an example. However, the result that u_j is quasiconcave in p_j applies to a fairly general class of modems. It is shown in [13] that the efficiency functions that correspond to the modulation schemes listed in Table 1 are all quasiconcave in each user's own power. The above theorem establishes the existence of a Nash equilibrium in NPG where the utility function is quasiconcave in transmit power. At this point, it is important to make a note of equilibrium existence statements in general. An equilibrium existence proof states that under certain conditions, an equilibrium is *guaranteed* to exist. Such a statement *does not* imply however, that if the same conditions are *not* met, there exists no equilibrium. Next, we discuss the properties of the equilibrium itself. First, we derive the best-response correspondence of a terminal in NPG.

Proposition 1 *In NPG, terminal j's best response to a given interference vector $\mathbf{p}_{-j}$ is given as*

$$r_j(\mathbf{p}_{-j}) = \min(\overline{p}, \tilde{p}) \quad , \tag{8}$$

where $\tilde{p}_j = \arg\max_{p_j \in \mathcal{R}_+} u_j(p_j, \mathbf{p}_{-j})$ is the unconstrained maximizer of the utility in (4). Furthermore, $\tilde{p}_j$ is unique.

Proof: In the proof of Theorem 1 given in Appendix A, it is shown that the unconstrained maximization of the utility function results in $\tilde{\gamma}$ as the solution for terminal j where $\tilde{\gamma}$ solves $f'(\tilde{\gamma}_j)\tilde{\gamma}_j = f(\tilde{\gamma}_j)$ for all j. For given interference, $\tilde{\gamma}$ corresponds to the transmit power $\tilde{p}_j$ given by

$$\tilde{p}_j = \frac{\tilde{\gamma}(\sum_{k \neq j} h_k p_k + \sigma^2)}{\frac{W}{R} h_j} \quad . \tag{9}$$

Since $\tilde{\gamma}$ is the unique maximizer of the utility and since there is a one-to-one correspondence between the transmit power and the SIR, the transmit power $\tilde{p}_j$ that maximizes utility for fixed interference is also unique. If $\tilde{p}_j \notin P_j$ for some user j, then since it is not a feasible point, $\tilde{p}_j$ can not be a best response to given $\mathbf{p}_{-j}$. In this case, we observe that $\frac{\partial u_j}{\partial p_j} \leq 0$ for any $\gamma_j \leq \tilde{\gamma}$, and therefore for any $p_j \leq \tilde{p}_j$. This implies that the utility function is increasing in that region. Since $\overline{p}$ is the largest power in the strategy space, it yields the highest utility among all $p_j < \overline{p}$ and thus it is the best response to the given $\mathbf{p}_{-j}$. ■

Note that at any equilibrium of the NPG game, a terminal either attains the utility maximizing SIR, $\tilde{\gamma}$, or it fails to do so and transmits at maximum power, $\overline{p}$.

Theorem 2 *The NPG has a unique equilibrium.*

Proof: By Theorem 1, we know that there exists an equilibrium in NPG. Let $\mathbf{p}$ denote the Nash equilibrium in the NPG. By definition, the Nash equilibrium has to satisfy, $\mathbf{p} = \mathbf{r}(\mathbf{p})$ where $\mathbf{r}(\mathbf{p}) = (r_1(\mathbf{p}), r_2(\mathbf{p}), \cdots, r_N(\mathbf{p}))$. Note that $r_j(\mathbf{p})$ and $r_j(\mathbf{p}_{-j})$ are equivalent. The key aspect of the uniqueness proof is to realize that the best-response correspondence $\mathbf{r}(\mathbf{p})$ is a *standard* function [1]. A function is said to be standard if it satisfies the following properties.

- Positivity: $\mathbf{r}(\mathbf{p}) > 0$.
- Monotonicity: If $\mathbf{p} \geq \mathbf{p}'$ then $\mathbf{r}(\mathbf{p}) \geq \mathbf{r}(\mathbf{p}')$.
- Scalability: For all $\mu > 1$, $\mu\mathbf{r}(\mathbf{p}) > \mathbf{r}(\mu\mathbf{p})$.

It is shown in [1] that the fixed point $\mathbf{p} = \mathbf{r}(\mathbf{p})$ is unique for a standard function. Therefore, the Nash equilibrium of NPG is unique. ■

A special case is when the user configuration is such that all terminals are able to achieve the utility maximizing SIR, $\tilde{\gamma}$, at the Nash equilibrium. We already mentioned that the equilibrium SIR, $\tilde{\gamma}$, of NPG is derived from the efficiency function given in equation (2). If all the wireless terminals use the same modulation technique and the same packet length, M, they have the same efficiency function. Therefore, the value of $\tilde{\gamma}$ that each terminal tries to achieve at equilibrium is the same for all terminals. It is worth noting that the power control solution obtained at the equal-SIR NPG equilibrium is similar to the solutions offered by power control algorithms for speech communications [1,2,17]. In fixed-target type power control algorithms for voice systems, users adjust powers in order to satisfy a minimum target SIR constraint. The algorithm terminates at a set of powers where each terminal has exactly the target SIR. The Nash equilibrium SIR of $\tilde{\gamma}$ can be thought of as the target SIR in voice systems with one important distinction:

the common target SIR for voice systems is determined by subjective measures of speech quality. However $\tilde{\gamma}$ is derived from the particular efficiency function and therefore is dictated by system properties such as modulation technique, channel model and packet length.

Notice that at the utility maximizing SIR, $\tilde{\gamma}$, the utility of a user increases with decreasing interference. Such behavior can be observed by substituting (9) in the utility expression. Consequently, one might consider a scheme where the terminals are scheduled to transmit one at a time: only one terminal transmits at the utility maximizing SIR while the others do not transmit. Such a scheme is not the outcome of the power control game and thus it is not a distributed solution. A power control scheme based on this idea will be the topic of future work.

5 Inefficiency of the NPG Equilibrium

The Nash equilibrium discussed in Section 4 offers a solution to the power control problem where no terminal can increase its utility any further through individual effort. Thus, it is an outcome obtained as a result of distributed decision taking which could be expected to be less *efficient* than a possible power allocation obtained through cooperation between terminals and/or as a result of centralized optimization. Indeed, it is well known that in general the Nash equilibria are inefficient [8,18]. A power allocation is said to be more efficient (or Pareto dominant) if it is possible to increase the utility of some of the terminals without hurting any other terminal. A formal definition is as follows.

Definition 3 *A power vector,* $\hat{\mathbf{p}}$ Pareto dominates *another vector,* $\mathbf{p}$ *if for all* $j \in \mathcal{N}$, $u_j(\hat{\mathbf{p}}) \geq u_j(\mathbf{p})$ *and for some* $j \in \mathcal{N}$, $u_j(\hat{\mathbf{p}}) > u_j(\mathbf{p})$. *Furthermore, a power vector,* $\mathbf{p}^*$, *is* Pareto optimal (efficient), *if there exists no other power vector* $\mathbf{p}$ *such that* $u_j(\mathbf{p}) \geq u_j(\mathbf{p}^*)$ *for all* $j \in \mathcal{N}$ *and* $u_j(\mathbf{p}) > u_j(\mathbf{p}^*)$ *for some* $j \in \mathcal{N}$.

Figure 5 explains the concept of Pareto dominance and Pareto optimality on a generic utility possibility set. In the example in Figure 5, there are two terminals in the game and their strategy sets are mapped to the utility possibility set shown as the shaded area. Any power vector that provides a Pareto improvement with respect to $\mathbf{y}$ results in non-decreasing changes in individual utilities, $u_j(\mathbf{y})$, and therefore would lie in the area labeled *Region of Pareto improvement*. From the figure, we can observe that $\mathbf{x}$ is such a point. We can also refer to $\mathbf{x}$ as the *Pareto-preferred* power vector when compared to $\mathbf{y}$. The concepts of Pareto-dominance and Pareto-optimality should not be confused: Pareto-optimal power allocations *do not* necessarily Pareto-dominate *all* other power vectors. For example, compare the utilities obtained by $\mathbf{y}$ and $\mathbf{z}$ in Figure 5. Notice that $\mathbf{z}$ is a Pareto optimal power allocation. However, since $u_2(\mathbf{z}) < u_2(\mathbf{y})$, $\mathbf{z}$ does not Pareto dominate $\mathbf{y}$, regardless of the fact that $u_1(\mathbf{z}) > u_1(\mathbf{y})$.

We now seek improvements to the outcome obtained as a result of the NPG. In this paper, the power vectors that improve utilities (in the Pareto sense) with respect to the Nash equilibrium are referred to as *NPG-dominant*. The focus of this section and the section on pricing is to seek NPG-dominant power allocations.

Theorem 3 *The NPG equilibrium is inefficient.*

Proof: Recall that at the equilibrium of the NPG, there are two types of terminals: those that achieve the utility maximizer $\tilde{\gamma}$ and those that transmit at maximum power $\overline{p}$ while attaining less than $\tilde{\gamma}$. Let H denote the index set of terminals that are able to reach $\tilde{\gamma}$ and $\overline{H}$ denote index set for the rest of the terminals. Suppose that at NPG, all $j \in H$ reduce their powers by a factor of μ where $0 < \mu \leq 1$, while all $j \in \overline{H}$ keep their powers at $\overline{p}$. The utility of user $j \in H$ with these reduced powers is

$$u_j(\mu) = \frac{LR}{M\mu p_j} f(\gamma_j^\mu) \tag{10}$$

where

$$\gamma_j^\mu = \frac{W}{R} \frac{\mu h_j p_j}{\sum_{k\in H, k\neq j} \mu h_k p_k + \sum_{k\in\overline{H}} h_k \overline{p} + \sigma^2} \quad \text{for all } j \in H. \tag{11}$$

Similarly, the utility of user $j \in \overline{H}$ with these reduced powers is

$$u_j(\mu) = \frac{LR}{M\overline{p}} f(\gamma_j^\mu) \tag{12}$$

where

$$\gamma_j^\mu = \frac{W}{R} \frac{h_j \overline{p}}{\sum_{k\in\overline{H}, k\neq j} h_k \overline{p} + \sum_{k\in H} \mu h_k p_k + \sigma^2} \quad \text{for all } j \in \overline{H}. \tag{13}$$

We need to examine how the utility value changes for all terminals as the value of μ changes. As the value of μ goes from 1 to 0, the terminals in the set H dissipate a power lower than equilibrium powers. If this decrease in μ results in nondecreasing utilities for *all* terminals, we have a proof that there exists an NPG-dominant power vector. First, let us focus on only those terminals in the set H, i.e., the terminals that achieve $\tilde{\gamma}$ at the NPG equilibrium. Taking the first order derivative of utility in (10) with respect to μ and evaluating the resulting expression at $\mu = 1$ we get,

$$\frac{\partial u_j(\mu)}{\partial \mu}|_{\mu=1} = \frac{LR}{Mp_j} \left(\frac{f'(\gamma_j)\gamma_j(\sum_{k\in\overline{H}} h_k\overline{p} + \sigma^2)}{\sum_{k\in H,k\neq j} h_k p_k + \sum_{k\in\overline{H}} h_k \overline{p} + \sigma^2} - f(\gamma_j) \right) \quad . \tag{14}$$

For terminal $j \in H$, since the FONOC $f'(\gamma)\gamma = f(\gamma)$ is satisfied, (14) can be simplified to get

$$\frac{\partial u_j(\mu)}{\partial \mu}|_{\mu=1} = \frac{LR}{Mp_j} f(\gamma_j) \left(\frac{\sum_{k\in\overline{H}} h_k\overline{p} + \sigma^2}{\sum_{k\in H,k\neq j} h_k p_k + \sum_{k\in\overline{H}} h_k \overline{p} + \sigma^2} - 1 \right) \quad . \tag{15}$$

Notice that the above expression has a negative value, i.e., $\frac{\partial u_j(\mu)}{\partial \mu}|_{\mu=1} < 0$. Therefore, as μ tends from unity, utilities of the terminals in set H have a tendency to increase. Although this proves the utilities for all $j \in H$ increases, we still need to show that the terminals in set $\overline{H}$ also received increased utilities as a result of scaling of powers by μ by the users in the set H. Recall the utility of user $j \in \overline{H}$ with reduced powers is given in (12) and the SIR is given in (13). Observe that when terminals in H reduce their powers by μ, the denominator term in (13) decreases. Since the numerator of this term remains the same (namely, the terminals in $\overline{H}$ do not change their equilibrium power of $\overline{p}$), the SIR increases for a terminal in $\overline{H}$. With an increased SIR, the utility of terminal $j \in \overline{H}$ given in (12) increases since the efficiency function is a monotonic increasing function of the SIR and the denominator remains the same. Thus, we conclude that there exists a $\mu < 1$ where utilities of *all* terminals increase. Since at $\mu < 1$, the utilities of all the users increase, by definition the Nash equilibrium of the NPG is not a Pareto optimum. The scalar μ was taken to have the same value for all users for purposes of convenient illustration of the non-Pareto optimality property of the Nash equilibrium. However, when seeking Pareto efficient (Pareto optimal) power vectors, we are not constrained to power vectors that are scalar multiples of the equilibrium power vector. ■

In the rest of this paper, we seek to improve the utilities obtained at the Nash equilibrium of NPG. We should note in [19] the authors take a centralized approach to improve the NPG equilibrium. The improvement is searched over the solution space constrained to equal-SIR power vectors. The value of the best equal-SIR solution is derived and shown that it is an NPG-dominant power allocation. With the same motivation of improving the NPG utilities, we examine a more decentralized method.

6 Non-cooperative Power Control with Pricing

In the NPG, each terminal aims to maximize its own utility by adjusting its own power, but it ignores the cost (or harm) it imposes on other terminals by the interference it generates. The self-optimizing behavior of an individual terminal is said to create an *externality* when it degrades the quality for every other terminal in the system. Among the many ways to deal with externalities, *pricing* (or taxation) has been used as an effective tool both by economists and researchers in the field of computer networks. Typically, pricing is motivated by two different objectives: (1) it generates revenue for the system, (2) it encourages players to use system resources more efficiently. In this work, pricing does not refer to monetary incentives, but rather refers to a control signal to motivate users to adopt a *social* behavior. An efficient pricing mechanism makes decentralized decisions compatible with overall system efficiency by encouraging efficient sharing of resources rather than the aggressive competition of the purely noncooperative game. A pricing policy is called *incentive compatible* if pricing enforces a Nash equilibrium that improves social welfare. Roughly speaking, social welfare is defined as the sum of utilities.

It is possible to use various pricing policies, such as flat-rate, access-based, usage-based, priority-based, etc. This situation raises the question of which pricing policy is appropriate. The service provider determines both the pricing policy and the specific prices for the use of resources based on the system, the kind of resources it offers and the type of the demand for these services. An efficient price will reflect accurately the costs of usage of a resource and must take into account the nature of the demand for the offered service. Usage-based pricing is an approach commonly encountered in the literature. In usage-based pricing, the price a terminal pays for using the resources is proportional to the amount of resources consumed by the user.

In order to improve the equilibrium utilities of NPG in the Pareto sense, we resort to usage-based pricing schemes. Through pricing, we can increase system performance by implicitly inducing cooperation and yet we maintain the noncooperative nature of the resulting power control solution. An efficient pricing scheme should be tailored for the problem at hand. Within the context of a resource allocation problem for a wireless system, the resource being shared is the radio environment and the resource usage is determined by terminal's transmit power. Furthermore, in Section 5, we show that the decentralized power control game has an equilibrium that is inefficient. We argue that efficiency in power control can be promoted by a usage-based pricing strategy where each user pays a penalty proportional to its transmit power.

Keeping the above guidelines for a pricing strategy in mind, we develop a non-cooperative game with pricing. Let $G_c = [\mathcal{N}, \{P_j\}, \{u_j^c(\cdot)\}]$ denote an N-player noncooperative power control game with pricing (NPGP). Utilities for NPGP are

$$u_j^c(\mathbf{p}) = u_j(\mathbf{p}) - c_j(p_j, \mathbf{p}_{-j}) \tag{16}$$

where $c_j : P \rightarrow \Re^1_+$ is the pricing function for terminal $j \in \mathcal{N}$. The multi-objective optimization problem that NPGP solves can be expressed as,

$$\textbf{(NPGP)} \quad \max_{p_j \in P_j} u_j^c(p_j, \mathbf{p}_{-j}) = u_j(\mathbf{p}) - c_j(p_j, \mathbf{p}_{-j}), \quad \text{for all } j \in \mathcal{N} \quad . \tag{17}$$

The above formulation does not assume any particular form for the pricing function, $c_j(\cdot)$. However, motivated by the discussion in the preceding sections, we impose a price that increases monotonically with the transmit power of the user. Particularly, we restrict our attention to linear pricing schemes (see also [4]) of the form

$$c_j(p_j, \mathbf{p}_{-j}) = c\alpha_j p_j, \tag{18}$$

where c and $\{\alpha_j\}$ are positive scalars. The pricing factor, c, can be considered to have units bits/sec/Watt^2 so that it is consistent with the units of the *net utility* u_j^c in bits/Joule.

The pricing factor, c, needs to be tuned such that user self-interest leads to best possible improvement in overall network performance. The NPGP with linear pricing is as follows.

$$\textbf{(NPGP)} \quad \max_{p_j \in P_j} u_j(\mathbf{p}) - c\alpha_j p_j, \quad \text{for all } j \in \mathcal{N} \quad . \tag{19}$$

Notice that the NPGP is practically the same game as the NPG with different payoff functions. We seek a Nash equilibrium point that solves NPGP, if one exists. In game $G = [\mathcal{N}, \{P_j\}, \{u_j(\cdot)\}]$, each utility function is quasiconcave in its own strategy. We established that in a game with such utility functions there exists a unique equilibrium. The NPGP, however, does not have quasiconcave utility functions. Analytical techniques used to prove Nash existence under strong assumptions of convexity and differentiability are no longer applicable. Thus, we turn to *supermodularity theory* to show existence of equilibria.

We now present the theory of supermodular games which we will use to investigate Nash equilibria in the NPGP.

6.1 Supermodular Games and NPGP

Supermodularity was introduced into the game theory literature by Topkis [20] in 1979. In a supermodular power control game, each player's desire to increase its power increases with an increase in other players' powers, i.e., the best response of a terminal is monotone increasing in interferers' strategy. Supermodular games are of particular interest since they have Nash equilibria. Furthermore, it is possible to identify a set of Nash equilibria defined by two Nash equilibria that constitute a lower bound and a higher bound on the Nash set. The simplicity of supermodular games makes convexity and differentiability assumptions unnecessary. A formal definition of a supermodular game can be found in [9, page 491]. For the special case of single dimensional user strategy sets which are of interest in this work, the definition simplifies to the following.

Definition 4 *Consider a generic game $G = [\mathcal{N}, \{P_j\}, \{u_j(\cdot)\}]$ with strategy spaces $P_j \subset \mathcal{R}$ for all j. G is* supermodular *if for each j, $u_j(p_j, \mathbf{p}_{-j})$ has non-decreasing differences (NDD) in $(p_j, \mathbf{p}_{-j})$.*

If the utility of user j has NDD in $(p_j, \mathbf{p}_{-j})$, then user j's marginal utility is nondecreasing in the transmit powers of interferers, i.e., in response to an increase in the power level of another user, terminal j increases its transmit power level in order to increase its utility. NDD property is formally defined as follows.

Definition 5 *$u_j(p_j, \mathbf{p}_{-j})$ has NDD in $(p_j, \mathbf{p}_{-j})$ if for all $\mathbf{p}_{-j} \geq \mathbf{p}'_{-j}$ the quantity $u_j(p_j, \mathbf{p}_{-j}) - u_j(p_j, \mathbf{p}'_{-j})$ is nondecreasing in p_j. Equivalently for continuous and twice differentiable utilities, $u_j(p_j, \mathbf{p}_{-j})$ has NDD in $(p_j, \mathbf{p}_{-j})$ if and only if $\frac{\partial^2 u_j(\mathbf{p})}{\partial p_j \partial p_j} \geq 0$ for all $j \neq i$.*

The significance of this property is the fact that such utilities lead to a system of best response correspondences that have a *fixed point* [21, p. 180]. Recall that a fixed point in best response correspondences implies a Nash equilibrium. Finally, we can state the fundamental result by Topkis.

Theorem 4 *[21] The set of Nash equilibria of a supermodular game is non-empty. Furthermore, the Nash set has a largest element and a smallest element.*

A proof of the theorem can be found in [21]. Let the set of Nash equilibria be denoted by E and the largest and the smallest elements of E be denoted by $\mathbf{p}_{\mathrm{L}}$ and $\mathbf{p}_{\mathrm{S}}$, respectively. The largest and smallest vector in a set of vectors refer to the component-wise comparison between vectors in that set. For example, for two vectors $\mathbf{x}, \mathbf{y} \in \mathcal{R}^m$, $\mathbf{x} < \mathbf{y}$ if and only if $x_j < y_j$ for all $j = 1, \cdots, m$. The theorem states that all the equilibria $\mathbf{p} \in E$ are located such that $\mathbf{p}_{\mathrm{S}} \leq \mathbf{p} \leq \mathbf{p}_{\mathrm{L}}$, however it *does not* say that *all* points in that interval are equilibrium points.

If the utilities of the game under consideration are such that there is a parameter that none of the users have control over, we call that parameter an *exogenous* one. Consider a game with exogenous parameter, ϵ, $G_\epsilon = [\mathcal{N}, \{P_j\}, \{u_j^\epsilon(\cdot)\}]$ with utilities $u_j(p_j, \mathbf{p}_{-j}, \epsilon)$. The supermodularity definition for a generic game G given earlier (corresponding to $\epsilon = 0$) can be readily extended to the game G_ϵ with an exogenous parameter ϵ by imposing an additional NDD condition regarding the parameter.

Definition 6 *A game G_ϵ with an exogenous parameter ϵ is said to be* supermodular*, or it is a* parameterized game with complementarities *if $u_j(p_j, \mathbf{p}_{-j}, \epsilon)$ has NDD in $(p_j, \mathbf{p}_{-j})$ and in (p_j, ϵ) for all j.*

The following important result and its proof can be found in [21].

Theorem 5 *In a parameterized supermodular game, both $\mathbf{p}_S(\epsilon)$ and $\mathbf{p}_L(\epsilon)$ are nondecreasing in ϵ.*

It should be observed that in fact the pricing game G_c as given in (19) is a game with an exogenous parameter, the pricing factor, c. NPGP $G_c = [\mathcal{N}, \{P_j\}, \{u_j^c(\cdot)\}]$ is not a supermodular game by Definition 6. However, if the strategy spaces of users are modified appropriately, we can show that the resulting game is supermodular. The modified strategy space for user j denoted by $\hat{P}_j$ is a compact set defined by $\hat{P}_j = [\underline{p}_j, \overline{p}_j]$ where the smallest power in the strategy set $\underline{p}_j$ is derived from $\underline{\gamma_j} \geq 2\ln M$. Note that $\gamma = 2\ln M$ corresponds to the point of maximum rate of change of the efficiency with increasing SIR, i.e., $\frac{\partial^2 f(\gamma)}{\partial \gamma^2} = 0$. We find this SIR requirement using the condition given in Definition 5, i.e., $\frac{\partial^2 u_j(\mathbf{p})}{\partial p_i \partial p_j} \geq 0$ for all $j \neq i$. The largest power $\overline{p}_j$ is the maximum power constraint of the system. In this work, we assume the modified strategy space $\hat{P}$ is non-empty, i.e., there exists a $\underline{p}_j$ such that $0 < \underline{p}_j \leq \overline{p}_j$ for all j. Note that in the modified strategy space of NPGP described above, the power levels that yield $\gamma_j \leq 2\ln M$ are no longer available to the terminal. In NPG, the terminals can use any nonnegative power as long as it is below the maximum power limit. Thus, the users in NPGP operate in a smaller feasibility region as compared to the terminals in NPG. By the following result existence of Nash equilibria in the pricing game is established.

Theorem 6 *Modified NPGP $\hat{G}_c = [\mathcal{N}, \{\hat{P}_j\}, \{u_j^c(\cdot)\}]$ with exogenous parameter c is a supermodular game.*

Proof: We test whether the conditions in Definition 6 are satisfied. $\hat{G}_c$ has NDD in $(p_j, \mathbf{p}_{-j})$ since the condition given in Definition 5 yields the same expression for $\hat{G}_c$ as $\hat{G}$. We need only check whether the utility $u_j(p_j, \mathbf{p}_{-j}, c)$ has NDD in (p_j, c). First, perform a change of variables from c to ϵ where $\epsilon = -c$. When we take the partial derivative with respect to both p_j and ϵ, we get $\frac{\partial u_j}{\partial p_j \partial \epsilon} = \alpha_j \geq 0$ for all j. Thus, $\hat{G}_c$ is supermodular. ■

Using Theorems 4 and 5, we have all the Nash equilibria of $\hat{G}_c$ within the set $E_c = \{\mathbf{p} \in \hat{P} : \hat{\mathbf{p}}_{\mathrm{S}}(c) \leq \mathbf{p} \leq \hat{\mathbf{p}}_{\mathrm{L}}(c)\}$, and both $\hat{\mathbf{p}}_{\mathrm{S}}(c)$ and $\hat{\mathbf{p}}_{\mathrm{L}}(c)$ are nonincreasing in c. It is worth remembering a comment we made earlier about equilibirum existence results. Equilibrium existence results do not imply an equilibrium does not exist if the conditions of the proof are not met. Therefore, the NPGP with original strategy spaces is not supermodular, but we do not know for certain that it does not have an equilibrium. In fact, from some experimental results, we gather that it does have an equilibrium in some instances of the problem. We discuss these results in Section 7.

We discuss a totally asynchronous algorithm that generates a sequence of powers that converges to the smallest Nash equilibrium, $\mathbf{p}_{\mathrm{S}}(c)$. Suppose that terminal j updates its power at time instances given by the set $T_j = \{t_{j1}, t_{j2}, t_{j3}, \cdots\}$ where $t_{jk} < t_{j(k+1)}$ and $t_{j0} = 0$ for all j. Define $T = \{\tau_1, \tau_2, \cdots\}$ as the set of update instances $T_1 \cup T_2 \cup \cdots \cup T_N$ sorted in increasing order. Assume that no two time instances in set T are exactly the same. Let $\underline{\mathbf{p}}$ and $\overline{\mathbf{p}}$ denote the smallest and the largest vectors in modified strategy space $\hat{P}$, respectively.

Algorithm 1 (Terminal) *Consider the noncooperative power control game with pricing (NPGP) as given in (19). Generate a sequence of powers as follows.*

1. *Set the initial power vector at time $t = 0$: $\mathbf{p}(0) = \underline{\mathbf{p}}$. Also let $k = 1$.*
2. *For all k such that $\tau_k \in T$*
   (a) *For all terminals $j \in N$ such that $\tau_k \in T_j$*
      i. *Given $\mathbf{p}(\tau_{k-1})$, compute $r_j(\tau_k) = \arg\max_{p_j \in \hat{P}_j} \quad u_j^c(p_j, \mathbf{p}_{-j}(\tau_{k-1}))$.*
      ii. *Assign the transmit power as $p_j(\tau_k) = \min(r_j(\tau_k))$.*

We refer to $r_j(\tau_k)$ as the *set* of best transmit powers for terminal j at time instance k in response to the interference vector $\mathbf{p}_{-j}(\tau_{k-1})$. It is important to note that the terminal j optimizes the net utility over the modified strategy space of the NPGP, $\hat{P}_j$, where $\hat{P}_j$ is bounded by $\gamma_i \leq 2\ln M$. Implementation of this lower bound in the algorithm assumes that the instantaneous SIR at the base station is known by the terminal. The terminal then uses this information to derive the lower bound on its transmit power.

In the game with pricing, more than one transmit power might constitute a best response to a given interference vector. In this case, the algorithm determines the transmit power of a terminal by selecting the smallest power among all possibilities as dictated by the algorithm.

Theorem 7 *Algorithm 1 converges to a Nash equilibrium of NPGP. Furthermore, it is the smallest equilibrium, $\mathbf{p}_S(c)$, in the set of Nash equilibria.*

The proof can be found in Appendix B. Experiments suggest $\hat{\mathbf{p}}_{\mathrm{S}}(c) = \hat{\mathbf{p}}_{\mathrm{L}}(c)$ for our problem. If this is indeed true analytically, it implies that the Nash equilibrium in the modified NPGP is unique and can be reached from either the *top* or the *bottom* of the strategy space by implementing Algorithm 1. Since we do not know if there is a unique equilibrium, we compare the equilibria in the Nash set E_c to determine if there exists a single equilibrium that dominates all other equilibria. Indeed, we can show that $\hat{\mathbf{p}}_{\mathrm{S}}(c)$ is the *best* equilibrium in the set E_c.

Theorem 8 *If $\mathbf{x}, \mathbf{y} \in E_c$ are two Nash equilibria in modified NPGP where $\mathbf{x} \geq \mathbf{y}$, then $u_j^c(\mathbf{x}) \leq u_j^c(\mathbf{y})$ for all j.*

Proof: Notice that for fixed p_j and c, utility $u_j^c = \frac{LR}{Mp_j} f(\gamma_j) - c\alpha_j p_j$ decreases with increasing $\mathbf{p}_{-j}$ for all j. Therefore, since $\mathbf{x}_{-j} \geq \mathbf{y}_{-j}$ we have,

$$u_j^c(x_j, \mathbf{x}_{-j}) \leq u_j^c(x_j, \mathbf{y}_{-j}). \tag{20}$$

Also, by definition of Nash equilibrium and since $\mathbf{y}$ is a Nash equilibrium of NPGP, we have

$$u_j^c(x_j, \mathbf{y}_{-j}) \leq u_j^c(y_j, \mathbf{y}_{-j}). \tag{21}$$

M, total number of bits per frame	80
L, number of information bits per frame	64
W, spread spectrum bandwidth	10^6 Hz
R, bit rate	10^4 bits/second
σ^2, AWGN power at the receiver	5×10^{-15} Watts
modulation technique	non-coherent FSK
$\overline{p}$, maximum power constraint	2 Watts

Table 2: The list of parameters for the single-cell CDMA system used in the experiments.

By the above equations,

$$u_j^c(\mathbf{x}) \leq u_j^c(\mathbf{y}). \tag{22}$$

■

Corollary 1 *For modified NPGP, $\hat{\mathbf{p}}_S(c) \in E_c$ is the Pareto-dominant equilibrium, i.e., $u_j^c(\hat{\mathbf{p}}_S(c)) \geq u_j^c(\mathbf{p}^c)$ for all j, for all $\mathbf{p}^c \in E_c$*

Proof: By Theorem 8 we know that componentwise smaller equilibrium results in higher utilities for all users than a larger equilibrium. Since $\hat{\mathbf{p}}_S(c) \leq \mathbf{p}^c$ for all $\mathbf{p}^c \in E_c$, we conclude that for all $\mathbf{p}^c \in E_c$,

$$u_j^c(\hat{\mathbf{p}}_S(c)) \geq u_j^c(\mathbf{p}) \quad \text{for all } j \quad . \tag{23}$$

■

Note that this result implies that in case NPGP has Nash equilibria, the one that yields highest net utilities is the Nash equilibrium with the minimum total transmit powers.

7 Numerical Results

We demonstrate the improvement in performance obtained as a result of the NPGP outcome on a single-cell CDMA system with stationary users, fixed frame size and no forward error correction. The system we examine has the design parameters listed in Table 2. Also, the system we consider has 9 terminals that are located at $\mathbf{d} = [310, 460, 570, 660, 740, 810, 880, 940, 1000]$ meters from the base station. Path gains are obtained using the simple path loss model $h_j = K/d_j^4$ where $K = 0.097$ is a constant.

For our numerical examples, we use the efficiency function,

$$f(\gamma_j) = (1 - e^{-0.5\gamma_j})^M \quad , \tag{24}$$

which approximates P_c for non-coherent FSK. A comparison of the difference between P_c and $f(\gamma)$ as a function of the SIR for $M = 80$ can be found in Figure 2. Using the efficiency function given in equation (24) and the linear pricing regime with $\alpha_j = 1$ for all j, the equilibrium powers that solve the NPGP given in (19) are obtained by use of Algorithm 1. We first get the equilibrium powers in NPGP with no pricing

($c = 0$), which is equivalent to playing the NPG given in (6). Recall that the equilibrium powers in NPG are obtained by solving $\gamma_j = \tilde{\gamma}$ for all j if it is feasible. The utility-maximizing SIR for the specific system under examination is found to be $\tilde{\gamma} = 12.4$ by solving $f'(\gamma)\gamma = f(\gamma)$ or (33). For this example, we compute that an equal-SIR equilibrium is feasible if $N \leq 9$. Once the equilibrium with no pricing is obtained, the NPGP is played again after incrementing the pricing factor, c, by a positive value, Δc. Algorithm 1 returns a set of powers at equilibrium with this value of the pricing factor. If the utilities at this new equilibrium with some positive price c improve with respect to the previous instance, the pricing factor is incremented and the procedure is repeated. We continue until an increase in c results in utility levels worse than the previous equilibrium values for at least one user. We declare the last value of c with Pareto improvement to be the best pricing factor, c_{BEST}. The way c_{BEST} is determined by the network can be summarized in algorithmic format as follows.

Algorithm 2 (Network)

1. *Set* $c = 0$ *and announce* c *to all terminals.*
2. *Get* u_j *for all* $j \in \mathcal{N}$ *at equilibrium, increment* $c := c + \Delta c$ *and announce to all terminals*
3. *If* $u_j^c \leq u_j^{c+\Delta c}$ *for all* $j \in \mathcal{N}$ *then go to step 2, else stop and declare* $c_{BEST} = c$.

Figure 6 is constructed by letting Algorithm 1 reach Nash equilibrium at each value of c. We terminate incrementing the pricing factor if at least one user receives worse payoff than the previous equilibrium utility. It can be observed that solution by NPGP with $c = c_{\text{BEST}}$ offers a significant improvement in total utilities with respect to the solution offered by NPG. Increase in individual utilities can be examined in Figure 7. The corresponding equilibrium powers are displayed in Figure 8. The terminals that are closer to the base station receive much higher utilities while expending smaller power as compared to terminals further away from the base station in both NPG and NPGP equilibria. Yet, we observe that utilities improve significantly *for all terminals* as a result of pricing and that the powers decrease from values at equilibrium with no pricing. The numerical results also reveal that although the equilibrium SIRs for the game with zero pricing are equal for all terminals ($\gamma_j = \tilde{\gamma}$ for all $j \in \mathcal{N}$), the SIRs at equilibrium in NPGP with $c = c_{\text{BEST}}$ are higher for terminals closer to the base station ($\gamma_j \geq \gamma_i$ if $d_j \leq d_i$). We should note that in many of our experiments where we find c_{BEST}, the first terminal to experience a decrease in utility as the pricing factor is increased is generally the one with the worst path gain. Therefore, the terminal that triggers the choice of c_{BEST} is usually the one that already receives lower utility. In such cases, we note that the choice of c_{BEST} has implications of a *max-min fair* outcome. The max-min fairness concept is commonly used in computer networks in the context of flow control of sources within the network. It refers to a flow rate allocation where it is not possible to increase the flow from a source without having to decrease the flow of a source that is already receiving a smaller portion of the allocation [22]. Notice that if increasing the pricing factor beyond c_{BEST} results in the farthest terminal receiving decreased utility, then c_{BEST} is a max-min fair pricing factor for the (Network) algorithm described in Algorithm 2.

The equilibrium results presented for NPGP are guaranteed by the equilibrium existence results of Section 6.1. However, remember that if the conditions of the existence theorem are not met, it does not automatically imply there is no equilibrium in the game. Actually, even if the strategy space of the modified NPGP is relaxed to the original space of NPG, our experiments show that an equilibrium exists. Furthermore, an equilibrium can be reached starting from $\mathbf{p} = \mathbf{0}$, the all-zero power vector, and updating the transmit powers iteratively. If the modified NPGP equilibrium SIRs are such that $\gamma_j > 2 \ln M$ for all $j \in \mathcal{N}$, then modified NPGP and the original NPGP equilibria are identical. Otherwise, when the SIR constraint is active at the equilibrium of the modified NPGP for at least one terminal, the original NPGP equilibrium yields zero equilibrium powers for some of the weaker terminals. It should be observed that zero transmit power can never be an equilibrium value for modified NPGP due to the lower bound on SIR. Although the original NPGP equilibria typically return higher sum of utilities than the modified NPGP for the same value of the pricing factor, some of the terminals receive utilities strictly equal to zero. Thus, it might be reasonable to interpret pricing for the original NPGP as an admission control mechanism. However, since in this work the motivation of pricing is to improve the NPG equilibrium utilities *for all the terminals*, we are more interested in the results from the modified NPGP.

8 NPGP and the Social Optimum

In NPGP, we choose the value of $c = c_{\text{BEST}}$ that brings maximal Pareto improvement to the solution from NPG. However, the power vector obtained as a result is not necessarily a social optimum. In this section, we discuss the connection between a social optimum and a general pricing function. The pricing function is not restricted to have a linear form.

Theorem 9 *A power vector $\mathbf{p}^*(\boldsymbol{\beta})$ that solves the social problem ($S_{\boldsymbol{\beta}}$) is Pareto optimal where ($S_{\boldsymbol{\beta}}$) is defined as*

$$(S_{\boldsymbol{\beta}}) \qquad \max_{\mathbf{p}} \boldsymbol{\beta} \cdot \mathbf{u} = \max_{\mathbf{p}} \sum_{i=1}^{n} \beta_i u_i \tag{25}$$

with $\boldsymbol{\beta}$ a vector of positive scalars.

Proof: Assume $\mathbf{p}^*(\boldsymbol{\beta})$ solves ($\mathrm{S}_{\boldsymbol{\beta}}$) and suppose $\mathbf{p}^*(\boldsymbol{\beta})$ is not Pareto optimal. Then, there exists some power vector $\mathbf{p}'$ such that $u_i(\mathbf{p}') > u_i(\mathbf{p}^*)$ for some i and $u_i(\mathbf{p}') \geq u_i(\mathbf{p}^*)$ for all i. This implies that $\sum_{i=1}^{n} \beta_i u_i(\mathbf{p}') > \sum_{i=1}^{n} \beta_i u_i(\mathbf{p}^*)$. Therefore $\mathbf{p}^*(\boldsymbol{\beta})$ cannot be a solution to ($\mathrm{S}_{\boldsymbol{\beta}}$) which is a contradiction to the original assumption. Therefore, it has to be a Pareto optimal point. ■

In fact, in our experiments, we observe that the sum of utilities continue to increase beyond $c = c_{\text{BEST}}$. However, such improvement in total utilities result in degraded QoS for at least one user, beginning with the user that is farthest from the base station. The solution to ($\mathrm{S}_{\boldsymbol{\beta}}$) is not even guaranteed to Pareto dominate the Nash solution of the NPG.

Solving ($\mathrm{S}_{\boldsymbol{\beta}}$) with a particular choice of $\boldsymbol{\beta}$ results in one of the points in the Pareto-optimal frontier, which consists of the points on the north-east boundary of the utility possibility set as shown in Figure 5.

By solving ($S_{\boldsymbol{\beta}}$) for all $\boldsymbol{\beta} \in \Re^n_+$ we can construct the Pareto-optimal frontier. What we obtain by the NPGP is a Pareto dominant power vector with respect to the solution offered by the NPG. NPGP solution lies in the space labeled *Region of Pareto improvement* in Figure 5. Note that unless the utility possibility set is a convex set, solution of the social optimum is not guaranteed to yield a point that is Pareto dominant with respect to the NPG.

Nevertheless, an optimal pricing function that has the solution of the social problem as a Nash equilibrium does exist for each user.

Theorem 10 *Let $\mathbf{p}^*(\boldsymbol{\beta})$ solve the social problem ($S_{\boldsymbol{\beta}}$). $\mathbf{p}^*$ is also a Nash equilibrium for the NPGP given in (17) with pricing function $c_i(\mathbf{p}) = -\frac{1}{\beta_i}\sum_{j=1, j\neq i}^{n} \beta_j u_j(\mathbf{p})$.*

Proof: $\mathbf{p}^*$ is a Nash equilibrium of the NPGP if $u_i^c(\mathbf{p}_i^*, \mathbf{p}_{-i}^*) \geq u_i^c(p_i', \mathbf{p}_{-i}^*)$ for all $i \in N$, for all $p_i' \in P_i$. Since $\mathbf{p}^*$ solves the social problem,

$$\sum_{j=1}^{n} \beta_j u_j(\mathbf{p}^*) \quad \geq \quad \sum_{j=1}^{n} \beta_j u_j(p_i', \mathbf{p}_{-i}^*) \tag{26}$$

for all $(p_i', \mathbf{p}_{-i}^*) \in P$. Rearranging terms on both sides and dividing both sides by β_i, we get,

$$u_i(\mathbf{p}^*) + \frac{1}{\beta_i}\sum_{j=1, j\neq i}^{n} \beta_j u_j(\mathbf{p}^*) \quad \geq \quad u_i(p_i', \mathbf{p}_{-i}^*) + \frac{1}{\beta_i}\sum_{j=1, j\neq i}^{n} \beta_j u_j(p_i', \mathbf{p}_{-i}^*) \tag{27}$$

Consider the second term on either side as the cost function and let $c_i(\mathbf{p}) = \frac{1}{\beta_i}\sum_{j=1, j\neq i}^{n} \beta_j u_j(\mathbf{p})$. Expressing in terms of the cost function we get,

$$u_i(\mathbf{p}^*) - c_i(\mathbf{p}^*) \quad \geq \quad u_i(p_i', \mathbf{p}_{-i}^*) - c_i(p_i', \mathbf{p}_{-i}^*) \tag{28}$$

which is true for all $p_i' \in P_i$ and true for all i. This is the definition of a Nash equilibrium. Thus, by definition, $\mathbf{p}^*$ is a Nash equilibrium for the NPGP game. Since the Nash equilibrium of the NPGP game with

$$c_i(\mathbf{p}) = -\frac{1}{\beta_i}\sum_{j=1, j\neq i}^{n} \beta_j u_j(\mathbf{p}) \tag{29}$$

is the point where the social problem is solved, we refer to this pricing function as the Pareto optimal pricing function. ■

Notice that with pricing function (29), each user is trying to maximize the same objective function, individually. The optimal pricing function given here is in the most general form a pricing function can take and does not have the linear form we produced results for in the preceding sections. This pricing function is not practical since we need an algorithm that would guarantee the solution of (25) to emerge as an equilibrium of the social game. However, accomplishing this is as difficult as (if not harder than) the central authority solving the social problem and imposing it on all users. Instead, the implementation proposed in this work has a single pricing factor, c, to be announced by the base station. Thus, users can still implement their distributed power control schemes that unilaterally maximize the utility function $u_i^c(\mathbf{p})$ in equation (16).

9 Summary and Conclusions

We have presented a distributed power control algorithm for wireless data systems. The QoS a wireless terminal receives is referred to as the *utility* and distributed power control where users maximize their utilities is a *non-cooperative power control game* (NPG). The resulting operating point (Nash equilibrium) of such a distributed power control is inefficient in power usage. Therefore, we introduce pricing to improve the NPG result. In the *non-cooperative power control game with pricing* (NPGP), each terminal maximizes its net utility given by the difference between the utility function and a pricing function. The class of pricing functions studied is linear in transmit power, where the pricing function is simply the product of a pricing factor and the transmit power. Such a pricing function allows easy implementation: The power control algorithm is realized by the base station announcing the pricing factor to all the users, which is followed by each terminal choosing the transmit power from its strategy space that maximizes its net utility. For positive values of the pricing factor, we show that there exist Nash equilibria that are not necessarily unique. However, we have proved that the minimum power vector in the set of Nash equilibria yields higher net utilities than any other equilibrium power vector. Such a power vector is said to *Pareto dominate* other equilibrium power vectors. We have also presented an algorithm that reaches the Pareto dominant equilibrium starting from the smallest power vector in the strategy space.

Under zero pricing, the utility is maximized at the same SIR, $\tilde{\gamma}$, for all terminals. The value of $\tilde{\gamma}$ is determined by the system characteristics such as modulation technique, channel model and packet length. As the pricing factor is increased from zero to positive values, the equilibrium begins to shift towards a point where users attain lower SIR, expend lower power and attain higher utilities. At the equilibrium of NPGP, SIRs are no longer equal for all users[3]. In fact, the equilibrium SIR for a user closer to the base station is higher than a user farther away, while all of the SIRs are smaller than the no-pricing equilibrium of $\tilde{\gamma}$. As a special case of an appropriate choice of the pricing factor, we define c_{BEST} as the value of the pricing factor where the utility of at least one terminal begins to decrease with increasing values of c. Using $c = c_{\text{BEST}}$, it is possible to get significant improvement in utility for all terminals. Finally, we have discussed how the utilities obtained using the pricing factor, c_{BEST}, compare with the social optimum which is the power vector that maximizes the sum of utilities of all the terminals in the system. Our results indicate that linear pricing while yielding Pareto improvements (over the case of no pricing) is still unable to achieve the social optimum. The desirable attributes of the linear pricing scheme studied in the present work are that it imposes a fair, usage-based penalty for the use of radio resources.

[3]In [23], it was found that SIR-balancing is not the optimal solution to maximize sum of utilities.

A Proof of Theorem 1

The following result is obtained from [24–26].

Theorem 11 *A Nash equilibrium exists in game $G = [\mathcal{N}, \{P_j\}, \{u_j(\cdot)\}]$ if for all $j = 1, \cdots, N$,*

1. *P_j is a nonempty, convex, and compact subset of some Euclidean space $\Re^N$.*
2. *$u_j(\mathbf{p})$ is continuous in $\mathbf{p}$ and quasiconcave in p_j.*

The set of maximizers of the continuous function $u_j(\cdot, \mathbf{p}_{-i})$ on the compact set P_j in NPG is called the *best-response correspondence* and is denoted by $r_j(\mathbf{p}_{-i})$. It is the mapping $r_j : P_{-i} \to P_j$ and defined as

$$r_j(\mathbf{p}_{-i}) = \{p_j \in P_j : u_j(p_j, \mathbf{p}_{-i}) \geq u_j(p'_j, \mathbf{p}_{-i}) \quad \forall p'_j \in P_j\} \quad . \tag{30}$$

An alternative definition for the Nash equilibrium can be stated using the set of best responses. A power vector $\mathbf{p}$ is a Nash equilibrium of NPG if and only if $p_j \in r_j(\mathbf{p}_{-i})$ for all $j \in N$. When the conditions in Theorem 11 are satisfied, the correspondence $r_j(\cdot)$ is nonempty, convex-valued and upper semicontinuous for all j [24–26]. Thus, there exists a fixed point $\mathbf{p}$ such that $p_j \in r_j(\mathbf{p}_{-i})$ for all $j \in N$. This fixed point is by definition a Nash equilibrium. The proof of the theorem is completed by showing the conditions given in the theorem are met in NPG. Each user has a strategy space that is defined by a minimum power, a maximum power and all the power values in between. We also assume the maximum power is larger than or equal to the minimum power. Thus, the first condition is satisfied. It remains to show that the utility function $u_j(\mathbf{p})$ is quasiconcave in p_j for all j in NPG. First, we define quasiconcavity.

Definition 7 *The function $u_j : P_j \to \Re^1_+$ defined on the convex set P_j is quasiconcave in p_j if and only if*

$$u_j(\lambda p_j + (1-\lambda)p'_j, \mathbf{p}_{-i}) \geq \min(u_j(p_j, \mathbf{p}_{-i}), u_j(p'_j, \mathbf{p}_{-i})) \tag{31}$$

for all $p_j, p'_j \in P_j$ and $\lambda \in [0, 1]$.

Alternatively, either the local maximum of the quasiconcave function is at the same time a global maximum or the quasiconcave function is constant in the neighborhood of a local maximum [27, 28]. We can show that the first part of this condition is true for the utility function used in this study.

For a differentiable function, the first order necessary optimality condition is given as $\frac{\partial u_j(p_j, \mathbf{p}_{-i})}{\partial p_j} = 0$. The partial derivative of $u_j(\cdot)$ with respect to p_j is

$$\frac{\partial u_j(p_j, \mathbf{p}_{-i})}{\partial p_j} = \frac{LR}{Mp_j^2}\left(f'(\gamma_j)\gamma_j - f(\gamma_j)\right) \tag{32}$$

where $f'(\gamma_j) = df(\gamma_j)/d\gamma_j$. Since $p_j \geq 0$ for NPG, we examine only positive real numbers. Evaluating equation (32) at $p_j = 0$, we get $\frac{\partial u_j(p_j, \mathbf{p}_{-i})}{\partial p_j} = 0$. Therefore, $p_j = 0$ is a stationary point and the value of utility at this point is $u_j(0, \mathbf{p}_{-i}) = 0$. If we evaluate utility in the ϵ-neighborhood of $p_j = 0$, where ϵ

is a small positive number, we notice that utility is positive which implies utility is increasing at $p_j = 0$. Therefore, we conclude zero can not be a local maximum. For nonzero values of the power, we examine the values of γ_j that make $f'(\gamma_j)\gamma_j - f(\gamma_j) = 0$. Suppose the modulation format used is non-coherent FSK for which the BER expression is given in Table 1. The efficiency function corresponding to non-coherent FSK can be derived using (2). Expressing $f'(\gamma_j)$ in terms of $f(\gamma_j)$ and rearranging terms, we get $\frac{M}{2}\gamma_j e^{-\gamma_j/2} - (1 - e^{-\gamma_j/2}) = 0$ or

$$\frac{M}{2}\gamma_j + 1 = e^{\gamma_j/2} \quad . \tag{33}$$

We observe that the right hand side of the above equation is convex in γ_j, the left hand side is monotonously increasing in γ_j, and the equation is satisfied at $\gamma_j = 0$. Therefore, there is a single value that satisfies the given expression for $\gamma_j > 0$. Let this value be $\gamma_j = \tilde{\gamma}$ where $\tilde{\gamma}$ is derived numerically from equation (33) and it is the same value for all users assuming each user operates with the same efficiency function. The second order partial derivative of the utility with respect to the power reveals that this point is a local maximum and therefore a global maximum. Hence, the utility function of user j is quasiconcave in p_j for all j. This completes the proof of the theorem.

Finally, it should be emphasized that although we used non-coherent FSK as an example in the proof, the results apply to a fairly broad class of modulation schemes [13].

B Proof of Theorem 7

If the smallest power vector in the strategy space, $\underline{\mathbf{p}}$, is already an equilibrium of NPGP, following the definition of an equilibrium, the power updates will result in the same power vector. If $\underline{\mathbf{p}}$ is not an equilibrium point, then we need to demonstrate how the power vector evolves in time. Remember that we defined $T = \{\tau_1, \tau_2, \cdots\}$ as the set of update instances $T_1 \cup T_2 \cup \cdots \cup T_N$ sorted in increasing order. Note that $\tau_1 = \min_{j \in \mathcal{N}} t_{j1}$. Since $\mathbf{p}(0)$ is not an equilibrium point, $p_{j'}(0) \leq p_{j'}(\tau_1) = \min(r_{j'}(\tau_1))$ where $j' = \arg\min_{j \in \mathcal{N}} t_{j1}$. Since $p_{j'}$ is the only component of the power vector to be updated at $t = \tau_1$, we also have $\mathbf{p}(\tau_0 = 0) \leq \mathbf{p}(\tau_1)$. Suppose that for some k', $\mathbf{p}(\tau_k) \leq \mathbf{p}(\tau_{k+1})$ for all $k = 0, 1, \cdots, k'-1$. Since we have already established that this assumption is true for $k' = 1$, it is sufficient to show that $\mathbf{p}(\tau_{k'}) \leq \mathbf{p}(\tau_{k'+1})$. Consider user j such that $\tau_{k'+1} \in T_j$. Suppose the previous update of user j took place at $t = \tau_{k''}$ where k'' is some time index in $\{0, \cdots, k'\}$. By the induction assumption, we have

$$\mathbf{p}(\tau_{k''}) \leq \mathbf{p}(\tau_{k''+1}) \leq \cdots \leq \mathbf{p}(\tau_{k'}) \quad . \tag{34}$$

Since $p_j(\tau_{k''}) = \min(r_j(\tau_{k''}))$, the power vector at $t = \tau_{k''}$ is $\mathbf{p}(\tau_{k''}) = (p_j(\tau_{k''}), \mathbf{p}_{-j}(\tau_{k''-1}))$. Since terminal j has the same power value for all update instances $\tau_{k''} \leq t \leq \tau_{k'}$, the power vector at $t = \tau_{k'}$ is $\mathbf{p}(\tau_{k'}) = (p_j(\tau_{k''}), \mathbf{p}_{-j}(\tau_{k'}))$. The power ordering in (34) is,

$$\mathbf{p}(\tau_{k''}) = (p_j(\tau_{k''}), \mathbf{p}_{-j}(\tau_{k''-1})) \leq (p_j(\tau_{k''}), \mathbf{p}_{-j}(\tau_{k'})) = \mathbf{p}(\tau_{k'}) \quad . \tag{35}$$

Above inequality implies $\mathbf{p}_{-j}(\tau_{k''-1}) \leq \mathbf{p}_{-j}(\tau_{k'})$. Since $p_j(\tau_{k''}) = \min(r_j(\tau_{k''}))$ and $p_j(\tau_{k'+1}) = \min(r_j(\tau_{k'+1}))$ and since the best response correspondence $r_j(\cdot)$ is increasing in $\mathbf{p}_{-j}$, we have $p_j(\tau_{k''}) \leq p_j(\tau_{k'+1})$ and hence,

$$\mathbf{p}(\tau_{k'}) \leq \mathbf{p}(\tau_{k'+1}) \quad . \tag{36}$$

Thus we confirm that $\mathbf{p}(t)$ is a nondecreasing sequence of powers in time. Furthermore, the convergence of the power vector such that $\lim_{t\to\infty} \mathbf{p}(t) = \mathbf{p}'$ where $\mathbf{p}' \in P$ follows because $\mathbf{p}(t) \in P$ for all t and P is a compact set and since a nondecreasing sequence has a limit point in a compact set [29].

Although, $\mathbf{p}'$ has been shown to be a limit point of the sequence of powers generated by the algorithm, it is yet to be verified that $\mathbf{p}'$ is also an equilibrium of NPGP. For any user j, for all update instances $\tau_k \in T_j$, $\lim_{k\to\infty} p_j(\tau_k) = p'_j$ and at any instant $p_j(\tau_k) \in P_j(\mathbf{p}_{-j}(\tau_{k-1}))$. It is possible to find some sequence $p''_j(\tau_k) \in P_j(\mathbf{p}_{-j}(\tau_{k-1}))$ for all $\tau_k \in T_j$ such that $\lim_{k\to\infty} p''_j(\tau_k) = p''_j$. Since $p_j(\tau_k)$ belongs to the best response correspondence of terminal j at that instance, we have,

$$u_j(p''_j(\tau_k), \mathbf{p}'_{-j}(\tau_k)) \leq u_j(p'_j(\tau_k), \mathbf{p}'_{-j}(\tau_k)) \quad , \tag{37}$$

which results in,

$$u_j(p''_j, \mathbf{p}'_{-j}) = \lim_{k\to\infty} u_j(p''_j(\tau_k), \mathbf{p}'_{-j}(\tau_k)) \leq \lim_{k\to\infty} u_j(p'_j(\tau_k), \mathbf{p}'_{-j}(\tau_k)) = u_j(p'_j, \mathbf{p}'_{-j}) \quad . \tag{38}$$

The argument applies to all the terminals. Hence by definition of Nash equilibrium $\mathbf{p}'$ is an equilibrium point of NPGP.

We seek to prove by induction that the algorithm results in the smallest Nash equilibrium. Let $\mathbf{p}'$ be any equilibrium of the game. We know that the initial vector of the algorithm is the smallest point in the strategy space and hence $\mathbf{p}(\tau_0) = \underline{\mathbf{p}} \leq \mathbf{p}'$. Suppose that for some $k > 0$, $\mathbf{p}(\tau_k) \leq \mathbf{p}'$. At time τ_{k+1}, suppose that terminal j updates its power. By construction of the algorithm, $p_j(\tau_{k+1})$ is the smallest value in its best correspondence, $r_j(\tau_{k+1})$. Also p'_j has to belong to its best response correspondence since it is an equilibrium point. Recalling that the best response correspondence is a nondecreasing function of the interference and that we assume $\mathbf{p}(\tau_k) \leq \mathbf{p}'$, we conclude that

$$(p_j(\tau_{k+1}), \mathbf{p}_{-j}(\tau_k)) = \mathbf{p}(\tau_{k+1}) \leq \mathbf{p}' \quad . \tag{39}$$

Since by induction, $\mathbf{p}(\tau_{k+1}) \leq \mathbf{p}'$ for all k which implies $\lim_{k\to\infty} \mathbf{p}(\tau_k) \leq \mathbf{p}'$, we prove that the limit point of the sequence generated by the algorithm is the smallest equilibrium in the Nash set.

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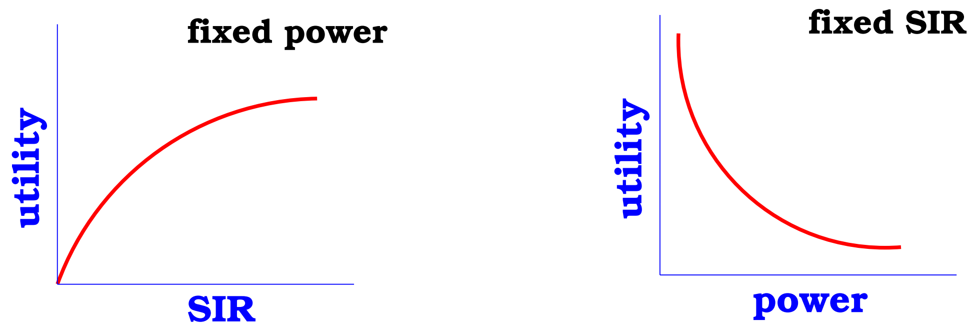

Figure 1: The behavior of the data user satisfaction (utility) as a function of SIR for fixed power and as a function of power for fixed SIR.

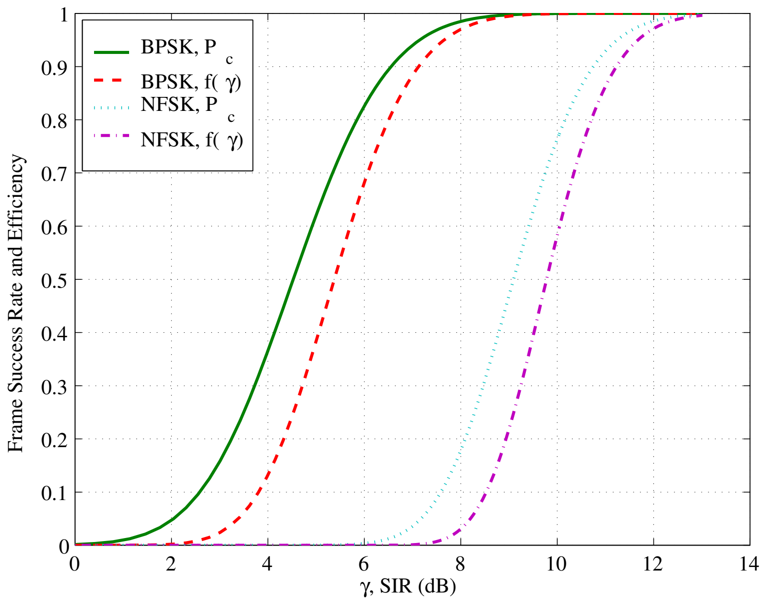

Figure 2: The frame success rate (FSR) and efficiency as a function of terminal SIR for BPSK and non-coherent FSK modulation schemes. Efficiency is an approximation to FSR.

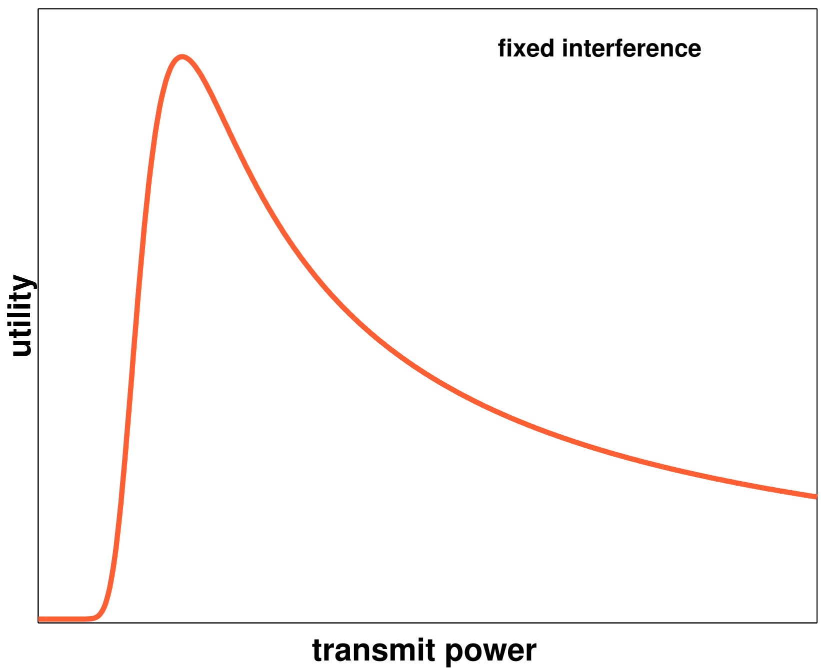

Figure 3: Shape of the utility as a function of the user transmit power for fixed interference.

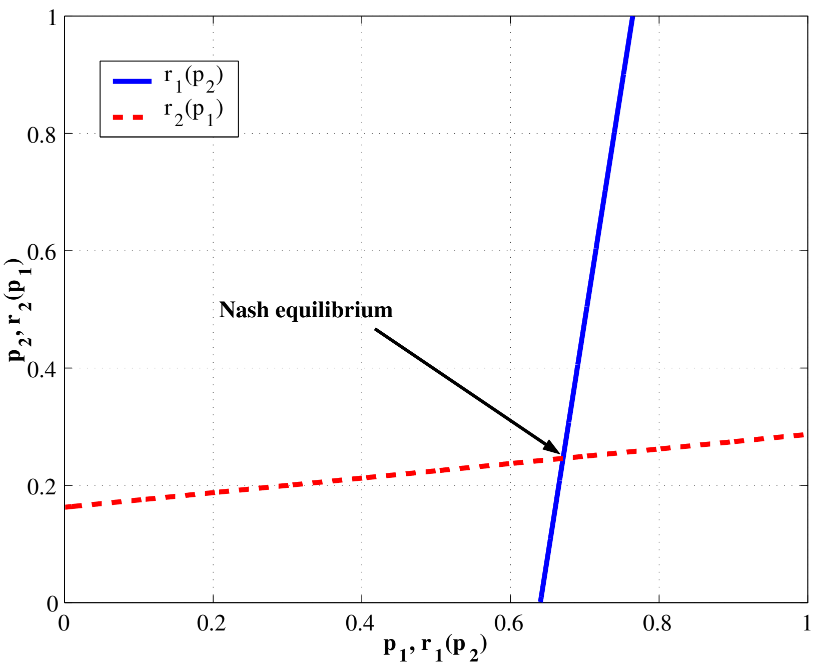

Figure 4: Best response correspondences in a game with 2 players. The point of intersection is a Nash equilibrium.

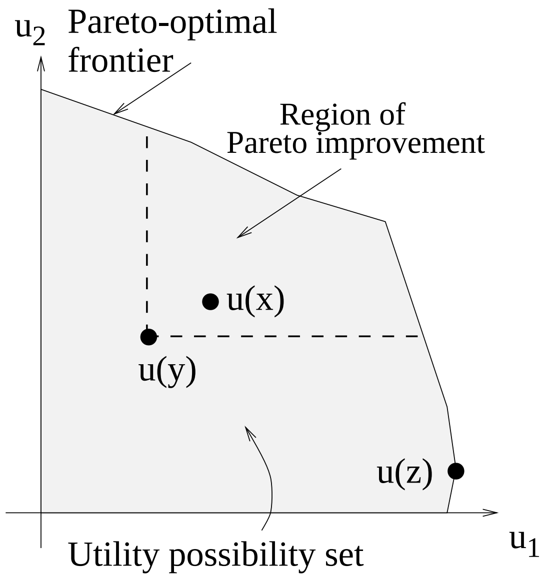

Figure 5: Power vector $\mathbf{x}$ Pareto dominates power vector $\mathbf{y}$ and $\mathbf{z}$ is Pareto optimal.

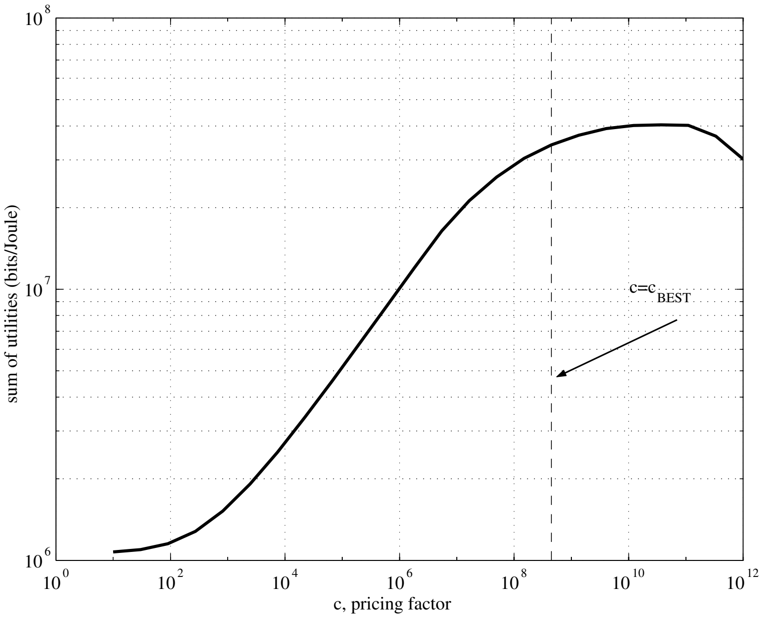

Figure 6: Sum of equilibrium utilities in a game with 9 terminals as a function of the pricing factor, c.

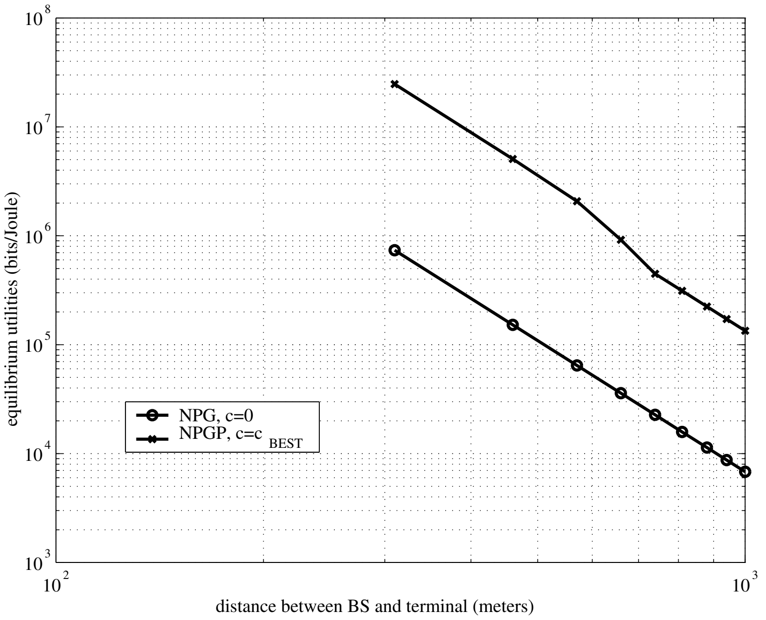

Figure 7: Utilities at equilibrium of NPG and NPGP with $c = c_{\mathrm{BEST}}$.

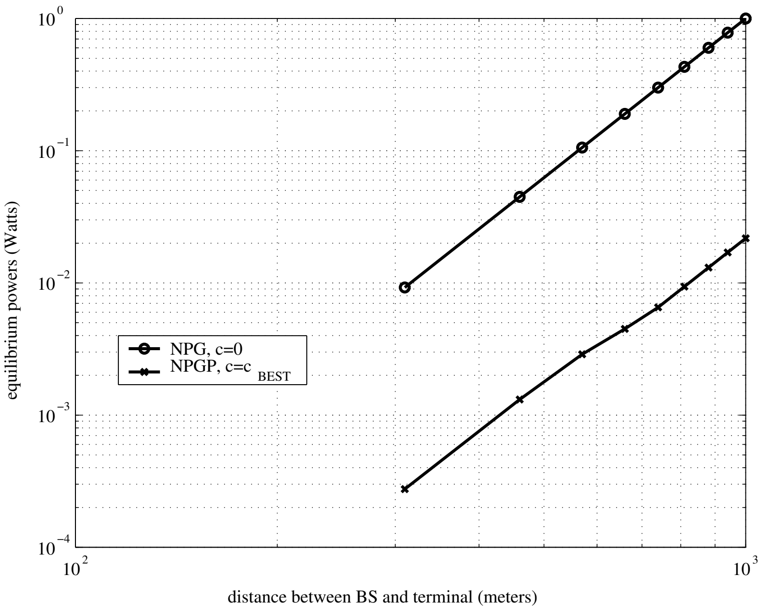

Figure 8: Powers at equilibrium of NPG and NPGP with $c = c_{\mathrm{BEST}}$.