Transactions Papers

Half-Duplex Relaying in Downlink Cellular Systems

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***Abstract*—We compare the performance of half-duplex relays in downlink cellular system against a baseline system without relays. We simulate the performance of (i) a collaborative power addition scheme, where the relay boosts the received power (P-CPA) at the mobile locations, and (ii) a CPA scheme with power control (PC-CPA) at the base station and relays. Evaluations are done in the context of a 19-cell, 57-sector set-up in which each of the served users must be delivered a message. The user messages are taken to have the same size and 90% of users in the network must be served. Improvements over the baseline due to relay deployments are measured in terms of increase in common rate of users as well as power savings in terms of reduction in peak or average power transmitted by base stations. In the CPA schemes with base stations and relays transmitting at full power, the peak power saving is 1.46 dB, alternately, the throughput improvement over a 1 bit/sec/Hz baseline rate is 21%. In the PC-CPA scheme, the peak power saving is 2.6 dB and the average total power in the system can be reduced by 3 dB.**

***Index Terms*—Resource allocation and interference management, cellular technology, relays in cellular systems.**

I. INTRODUCTION

A. Motivation and background

THE deployment of relays in cellular system is a topic of study in the 3GPP Long Term Evolution (LTE) [1] and has recently been standardized in the WiMAX, IEEE 802.16j standard. Relays are dedicated network elements, placed at certain locations (planned or unplanned) in the cell to help 'forwarding' the message from the base station to the user in the downlink, and from the user to the base station in the uplink. Relays are smarter than ordinary repeaters and could perform some digital base-band signal processing to improve reception at the destination terminals. Since they rely on air interfaces, relays avoid the back-haul costs involving data aggregation and infrastructure costs associated with backbone connectivity. Below, we present some open issues related to relaying in cellular systems.

1) *Throughput gains due to relay deployments*. Relays provide power gains due to reduction of distance attenuation in coverage limited scenarios [2]. These power gains, in turn, could translate to rate improvements for the edge users. In interference-limited settings, however, uncoordinated transmission by relays may increase the overall interference levels in the cell and could be counter-productive by reducing the signal-to-interference plus noise (SINR) levels of users in the system. Coordination of transmissions across the system would require centralized control that may incur significant costs and overhead, especially in the uplink. In this paper, we evaluate throughput gains due to relay deployment in urban macrocellular systems, where the relays collaborate with the base stations to transmit to the users. We see that the increase in common rates is around 30% to 40%. From a system design perspective, the cost of relay deployment may exceed the returns due to throughput improvements of this order. However, that there may exist specific scenarios where relays provide valuable throughput improvements.
2) *The relay placement problem*. Throughput improvements due to relay deployments depend on the relay transmit power, relay antenna pattern and location of the relays in the system. Placing relays closer to the edge user may help the edge user, but may interfere with the reception of edge user in the line of sight of the neighboring cell, when transmissions are uncoordinated. The optimal relay placement depends on the transmission and scheduling strategies, transmit power of the relays etc. In macrocellular environments, propagation characteristics of the base-relay link and the relay-user link could be completely different, depending on whether the relays are mounted on tall poles or on roof tops. These parameters may have an effect on the the system performance due to relay deployments.

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3) *Lack of good system models for relaying in cellular systems.* Multihopping in wireless networks has been studied in the context of ad hoc networks and peer-to-peer networks, e.g., the AODV and DSR routing protocols [3]. The main issue addressed in such networks is the routing problem. Interference constraints are abstracted as combinatorial constraints and many insightful results and good algorithms have been proposed to improve throughput of such networks [4]. Cellular networks are, however, unique in that the traffic is one-to-many in the downlink and many-to-one in the uplink. Direct application of the protocols and algorithms developed for ad hoc networks may not be optimal for macrocellular systems.
4) *Fairness*. Service level agreements (SLA) of cellular operators entail them to conform to certain fairness requirements. Many fairness schemes like proportional fairness [5] are proposed for cellular systems serving voice and data. Present day cellular systems implement schedulers in the MAC layer to provide various degrees of fairness to users. In this paper, we assume a simple fairness model in that the 90% of users are required to be served at a common rate. When relays are present in the system, designing distributed scheduling schemes to provide fairness is an active area of research.

We evaluate the performance of low-cost half-duplex relays (that cannot simultaneously transmit and receive simultaneously in the same band) in the downlink of a cellular system. The deployment scenario we consider is to mount a low-cost (preferably low-powered) device per sector over roof-tops of buildings. Such devices can relay the information from the base station to users in the cell. We restrict our study to urban macrocellular environment because of its practical relevance.

B. Related work

The information theoretic relay channel [6] has been an active area of research for three decades. But for some coding strategies proposed by Cover and El Gamal in [7] for special cases of the single relay channel, the capacity of the general relay channel is still unknown [8]. Though most of the earlier works assume that the relay can transmit and listen over the same band, the half-duplex constraint is taken into account in later works, e.g., [9], [10]. The information theoretic studies reveal that when there are one or two relays, the best strategy is to make use of both the source and relay transmissions at the user location, rather than multihopping from the source to the destination through the relay(s). The intuition is that the user can make use of signals from both the source and the relay to get a better signal strength and hence a better rate. Multihopping on the other hand, ignores the signal from the source, however strong it is.

The information theoretic relaying protocols mentioned above often involve complicated multiuser coding and decoding techniques, that are far from being practically feasible. There have been some recent works trying to bridge the gap between the information theoretic and practical multihopping schemes, e.g., [11], [12]. Most of the results in these works correspond to the case of linear network of nodes, where there is a single commodity flow of message from the source node to sink node through a set of relay nodes. The interference is only due to simultaneous transmissions from different relay nodes. This can be completely eliminated, by multiuser coding/decoding techniques. Such analysis does not carry over directly to the cellular systems since there are multiple simultaneous flows and multiuser techniques may incur significant overhead.

Deployment of relays in a cellular system has been proposed to solve the issue of lack of coverage over a large area [13] and for capacity improvement [14]. Viswanathan et. al [15] studied the performance of a centralized throughput-optimal scheduler on a cellular network with relays. In [15], the authors present a centralized downlink scheduling scheme that guarantees the stability of user queues for the largest set of arrival rates into the system. In another related work [16], the author investigates performance enhancements from multihop routing and spectrum reuse policies in the presence of multiuser diversity gains from opportunistic scheduling methods. The work in [16] does not however, capture the interference limits due to multiple tier of cells around the center cell. The common message conveyed by earlier results [15]–[17] is that simultaneous transmissions (due to multihopping) exploiting spatial reuse could lead to cell-wide throughput gains in a cellular network.

In this paper, we consider a *collaborative relaying scheme* with a single relay available per user. We bring an additional dimension to the benefits of relays in a cellular system, by quantifying the power savings due to deployment of relays in a cellular system. Peak power savings in cellular networks are very important elements of amplifier costs in base stations. Significant peak power savings can reduce the cost of amplifiers and hence capital expenses for deploying cellular networks. Also, average power savings while operating cellular networks can save operational expenses such as electricity bills for the cellular operators. We do not consider any multiuser scheduling gains, MIMO gains and any other complex interference mitigation techniques. Thus the gains shown in the network are purely due to the power gains at the user location due to the relay transmissions.

The paper is organized as follows. The simulation set up is explained in Section II. The collaborative power addition scheme is explained in Section III. Sections IV and V elaborate on the different collaborative power addition schemes studied in the paper and and conclude in Section VI.

II. The Set-up

This work aims to evaluate the power savings and improvement in common rate among users due to relay deployments in a cellular system. However, to model and simulate all the dynamics of a cellular system can be too complicated. In order to overcome such difficulties, we make some reasonable simplifying assumptions and take an idealized look at the model and operation of a cellular system in our work. The assumptions are consistent across systems with and without relays to make a fair comparison. We consider a cellular system with idealized hexagonal cells with a base station at the center of each cell. The first two tiers of interferers

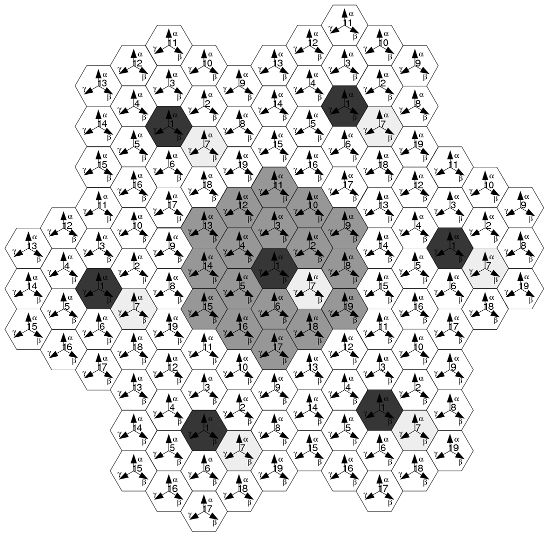


Fig. 1. Wrap-around simulation model. The center ring of 19 cells are used for the simulation. The surrounding cell activity is mirrored in the center ring. The direction of the arrows represent the direction of the main lobe of the sectorized antenna.

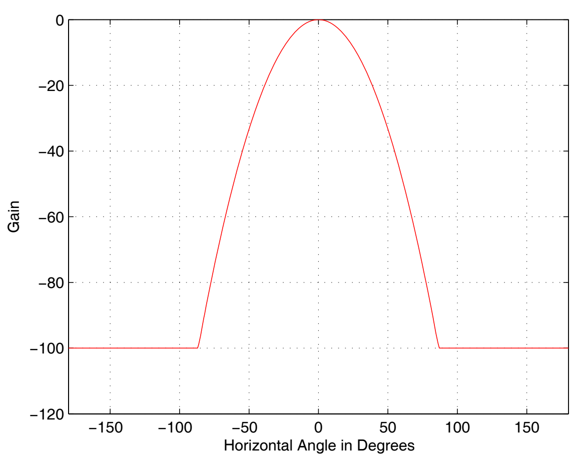


Fig. 2. Antenna gain pattern (from [18]) as a function of the horizontal angle in degrees. The mathematical expression for the gain is given in equation (1).

are considered and the activities of the farther tier of cells are mirrored by the center ring of 19 cells as shown in Figure 1. The site-to-site distance (distance between any two base stations) is taken to be 1 mile. The cells are divided into 120° sectors, each sector illuminated by a base station antenna pattern given by

$$A(\theta) = -\min\left(12\left(\frac{\theta}{\theta_{3dB}}\right)^2, A_{\max}\right), \tag{1}$$

where $A(\theta)$ is the antenna gain in dBi in the direction θ, $-180^\circ \leq \theta \leq 180^\circ$, $\min(.)$ denotes the minimum function, θ_{3dB} is the 3 dB beamwidth and $A_{\max} = 20$ dB is the maximum attenuation. The antenna pattern is shown in Figure 2.

At the receiving terminal (relay or user), the transmitted power undergoes attenuation due to the distance traveled and shadowing effects around the receiver. The propagation

TABLE I
PARAMETERS USED IN SIMULATIONS

Network Topology	19 cells, 3 sectors per cell with wraparound
Site-to-site distance	1 mile
Bandwidth	5 MHz
Path loss model	COST-231 Hata model
Path loss exponent	$\alpha = 3.8$
Shadowing	Lognormal, with zero mean, 8 dB standard deviation for access and backhaul
Multipath fading	None
Antenna Pattern	Sectorized for base stations Omnidirectional for relays
Antenna gains	15 dB (for base station and relays) -1 dB for users
Other losses	10 dB
Thermal noise power at the receiver	-102 dBm
Outage	10% for baseline and with relays

attenuation between a transmitting terminal (base station or relay) and a receiving terminal (relay or user) consists of the path loss and the shadowing component. The effect of small scale fading is ignored in our simulations. The parameters used in the above mentioned simulation set-up are summarized in Table I. The parameters are specific to urban macro-cellular environments and our results are only applicable to urban environments.

All users share the same band of frequencies and hence simultaneous transmissions interfere with each other. The total interference at each receiving terminal from all transmitters in the system is modeled as Gaussian noise and the achievable rate to a user i at time t is calculated as the Shannon rate $R_i(t) = \log_2(1 + \text{SINR}_i(t))$, where SINR denotes the signal-to-interference plus noise ratio.

A. Placement of users in the system

We simulate a downlink OFDM-like system wherein users in orthogonal time or frequency slots do not interfere with each other. Users in the same resource unit interfere with the other transmissions in the band. We simulate the worst case scenario where the system is fully loaded, i.e., users are present in all available resource units (or time-frequency slots) in all the sectors. The time-frequency slots are reused in each sector. We focus only on a particular time-frequency slot within which we simulate the complete cellular system such that there is one active user per sector at a given time. Hence, in a 19-cell network with 3 sectors per base antenna, at most 57 users can be served at one resource unit, i.e., time-frequency slot. In our simulations, we randomly place users uniformly across the network one-by-one until all 57 base station sectors are occupied. Each user associates with the base station with the highest received signal strength. If the base station sector is

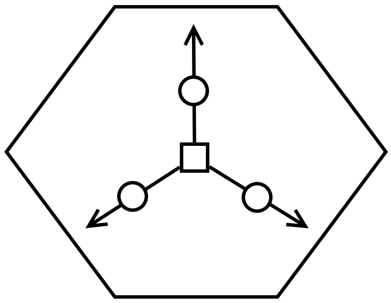


Fig. 3. Position of relay location in a cell. The relays (represented by small circles) are placed at half the cell radius in the direction (given by the arrows) of the main lobe of the sector antenna. The base station at the center of the cell is represented by a square.

already occupied by another user, the user is not allowed into the system and a new user realization is generated. Each user is equipped with an omni-directional antenna.

B. Placement of relays in the system

A relay with an omni-directional antenna is placed at half the cell radius in the direction of the main lobe of each base station sector antenna as shown in Figure 3. The relays are associated with the corresponding base station sector. The relays are not necessarily placed at optimal locations. The power gains and throughput improvements depend on the interference generated by the relays, which in turn, depends on the transmit power, geographic location of the relays and the propagation environment. In this work, we experiment with various relay placements and the simulation results are presented for the relay locations for which the gains are found to be maximum. The relay powers are also varied so as to improve the peak power savings.

III. Collaborative Power Addition Scheme

In the Collaborative Power Addition (CPA) scheme devised in [19], the relay collaborates with the base station to help the message reach to the destination. In our simulation model, each base station sector has a single user to be served and a relay that may help the source to deliver the message to the user associated with the source. In what follows, we focus our attention on an isolated triplet of base station (source), relay and user in a single sector. Gaussian encoding is used across all other sectors, the interference from other sectors is considered as if it were additive Gaussian noise. Suppose the source wants to transmit one of M messages to the destination, under a power constraint P. The source transmits a Gaussian codeword of length $N = (\log M)/R$, where R is the rate of the code. By Shannon's channel coding theorem [20, Chapter 9], if N is large enough, the message can be decoded reliably at the destination provided $R < \log(1 + \rho_{SD})$, where ρ_{SD} is the received SINR at the destination. In our simulations, we are interested in achievable rates and assume that the instantaneous mutual information at the receiver is exactly $R = \log(1 + \rho_{SD})$.

Assume that the source picks a rate R code $\mathcal{C}_1$ and sends one of M equally probable messages to the destination, using a codeword of length N. Let the received SINR ρ_{SR} between the source and relay be greater than the received SINR ρ_{SD} at the destination. Then, there exists some $\beta > 1$ such that $\log(1 + \rho_{SR}) = \beta \log(1 + \rho_{SD})$, i.e., the capacity of the channel from source to relay is β times greater than the channel from source to destination. We can now construct codebook $\mathcal{C}_2$ derived from $\mathcal{C}_1$ by observing only the first $\lceil N/\beta \rceil$ symbols of every codeword. The relay can then reliably decode the received message since the rate of $\mathcal{C}_2$ is

$$R' = \frac{\log M}{\lceil N/\beta \rceil} < \log(1 + \rho_{SD}) \tag{2}$$

In [21, Appendix F], the authors discuss the coding interpretation of a similar collaborative strategy. The authors also discuss the connection of such a coding setting with coding for arbitrary varying channel (AVC), which was first dealt with in [22] and then subsequently studied in [23]. We simulate a similar collaborative coding strategy wherein before the relay decodes the message, the received power at the destination node is only due to the base station transmission. After the relay decodes the message, the relay joins the base station to help the base station in delivering the message to the destination. At this point, if we assume that transmit symbol time slots at the relay and base station are synchronized and the code books are shared, the system can be viewed as a 2×1 MISO (Multiple-Input Single-Output) system without channel information at the transmitter. There is an effective power addition of the base station and relay transmissions at the destination [24, Chapter 3]. A similar scheme was proposed in the literature as dynamic decode and forward (DDF) scheme [25].

We simulate this collaborative relaying strategy in two ways:

1) Base station and relay transmit at their respective peak powers. In this case, the transmit power is fixed and the users get variable rates depending on SINR at the user locations. When a target rate is obtained by a user, the user leaves the system and the corresponding base station sector is turned off, thus reducing the amount of interference in the system. We term this the peak collaborative power addition (P-CPA) scheme.
2) Base station and relay operate with power control so that the users obtain a target desired rate. In the baseline case, for a given desired rate requirement r_0 bps/Hz, a feasible set of powers are found to better satisfy the rate requirement, allowing for a certain users to be in outage. When the relays decode the message in the collaborative scheme, the optimal powers are recalculated to find another feasible set of powers to satisfy the rate requirement at the same outage level. We term this the power control collaborative power addition (PC-CPA) scheme.

IV. CPA with Peak Power Transmissions (P-CPA)

A. Principle of operation: P-CPA Baseline

In the baseline of the P-CPA scheme, each base station sector transmits at its peak power to its own intended user. Since all users share the same band of frequencies, they observe interference from all the base station sectors in the system. If at time t, $p_i(t)$ is the peak power of the transmitting base station sector corresponding to the ith user and h_{ij} is the channel gain, including path loss and shadowing, from the jth interfering base to the ith user and σ^2 is the variance of the noise power at the receiver, the instantaneous received SINR for user i is given by

$$\rho_i(t) = \frac{h_{ii}p_i(t)}{\sum_{j\neq i} h_{ij}p_j(t) + \sigma^2}. \quad (3)$$

Since we assume Gaussian signaling, the MI (mutual information) or the instantaneous "rate" to each user is given as

$$R_i(t) = \log_2(1 + \rho_i(t)) \text{ bits/symbol}. \quad (4)$$

At time $t = 0$, all base stations simultaneously transmit to their associated user. As time progresses, for any given time interval $[t, t + \Delta t]$, user i accumulates MI $I_i(\Delta t) = R_i(t)\Delta t$. The MI for user i at time t is given by,

$$I_i(t) = \int_0^t \log_2(1 + \rho_i(\xi))\ d\xi. \quad (5)$$

If user i accumulates MI corresponding to the required amount L of data before the deadline T, i.e.,

$$\tau_i = \min_{0\leq t\leq T}\{t : I_i(t) = L\}. \quad (6)$$

then the user leaves the system and his associated base station sector is turned off at time τ_i, reducing the overall interference levels in the system. Hence,

$$p_i(t) = \begin{cases} P, & t < \min(\tau_i, T), \\ 0, & t \geq \min(\tau_i, T), \end{cases} \quad (7)$$

where, P is the peak power of the base station transmission. Note that the $\rho_i(t)$ of user i and the rate $R_i(t)$ are time varying quantities. At time $t = T$, the users that remain in the system are those users that did not get the complete file. It is these remaining users that are ascribed to be in *outage*.

B. P-CPA system with Relays

The operation of the P-CPA system with relays is as follows. The requirement is the same as the baseline case: to deliver a file of size L to as many users within the time T. At time $t = 0$, the base stations transmit at peak power to users associated with them. The relay node placed in the sector also receives the data sent to the user by the base station. If the relay gets the complete file before the user gets it, the relay can potentially be useful to the user by helping it get the message faster. On the flip side, the relay transmission can create additional interference for the other users in the system. In our simulations, we follow a myopic[1] policy on whether to turn on the relay or not: the relay transmits at peak power to help its user only if the instantaneous sum-rate of the whole system increases by turning the relay on. The sum-rate of the system is calculated as the sum total of the instantaneous rates of the existing users in the system and is a natural system-wide metric to use in order to decide whether the relays should transmit or not. At every epoch, a relay gets the message, among the set of all relays that are eligible to be turned on, the myopic sum-rate metric is applied and those relays that increase the sum-rate are turned on to help the users in the system. The information about the sum-rate could be made available through a central controller.

If the relay increases the sum-rate of the system, the relay is turned on and helps the user with a transmission reinforcing the same message as the base station using the code described in Section III. If $q_i(t)$ is the power transmitted from the relay i at time t and g_{ij} is the channel gain from the user i to the relay j, the effective SINR at i^{th} user location when the relay is active is given by

$$\rho_i^{relay}(t) = \frac{h_{ii}p_i(t) + g_{ii}q_i(t)}{\sum_{j\neq i} h_{ij}p_j(t) + g_{ij}q_j(t) + \sigma^2}. \quad (8)$$

The instantaneous rate and the mutual information for user i at time t are given by

$$R_i^{relay}(t) = \log_2(1 + \rho_i^{relay}(t)) \quad (9)$$

$$I_i^{relay}(t) = \int_0^t R_i^{relay}(\xi))\ d\xi. \quad (10)$$

If H_{ij} denote the channel gain from the jth base station to the ith relay,

$$J_i(t) = \int_0^t \log_2\left(1 + \frac{H_{ii}p_i(\xi)}{\sum_{j\neq i} H_{ij}p_j(\xi) + \sigma^2}\right) d\xi \quad (11)$$

represents the cumulative MI at the relay at time t.

Suppose the relay i becomes eligible to transmit at time t, i.e., $J_i(t) > L$, then denote the sum-rate of the system at time t as a function of $q_i(t)$ as

$$SR(t, q_i(t)) = \sum_i \log_2\left(1 + \frac{h_{ii}p_i(t) + g_{ii}q_i(t)}{\sum_{j\neq i} h_{ij}p_j(t) + g_{ij}q_j(t) + \sigma^2}\right). \quad (12)$$

Then, the relay power at time t is given by

$$q_i(t) = \begin{cases} Q, & \text{if } J_i(t) > L, SR(t, Q) > SR(t, 0) \text{ and } t < T, \\ 0, & \text{otherwise}, \end{cases} \quad (13)$$

where Q is the peak power constraint of the relays. Each user sees a time-varying SINR and the time-varying rate given by $R_i(t) = \log_2(1+\rho_i^{relay}(t))$. As with the baseline case, for any interval of time $[t, t + \Delta t]$, user i accumulates MI amounting to $I_i(\Delta t) = R_i(t)\Delta t$ and the MI for user i at time t is

$$I_i(t) = \int_0^t \log_2(1 + \rho_i^{relay}(\xi))\ d\xi. \quad (14)$$

Similar to the baseline case, if the user accumulates MI amounting to the full file size L within the stipulated time T, the user leaves the system and the associated base station and relay are switched off. Thus the effective interference in the system is reduced. At time $t = T$, the users that remain in the system are those users that did not get the complete file.

C. Network Operation and Simulation Aspects

Our objective is to obtain power savings and throughput improvement benefits due to deployment of relays in cellular system. To compare systems with and without relays in the

[1]The policy is myopic since, at the time when the relay gets the message, the global optimal decision whether the relay should transmit or not is unknown.

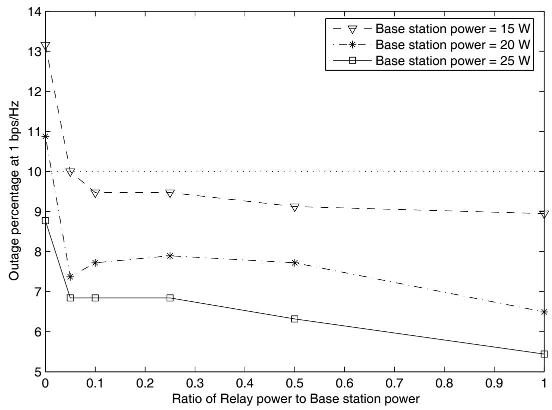


Fig. 4. Variation of outage with relay powers and base station powers. As we increase the base station powers with no relays in the system (ratio = 0), the outage decreases and saturates at around 5%, due to the inteference limit. The interference limit sets in very quickly even for smaller values of relay powers.

CPA based relaying scheme, we simplify the operation of a cellular downlink system such that 90% of the users in the system are guaranteed to be delivered a file of fixed size L, within a fixed period of time T. The file could be different for all users but the file sizes are fixed. Such an operation brings in the notion of a common rate for the users in the system. In order that the system benefits from the users that get the message within the fixed time T, the satisfied users leave the system, thus no longer causing interference to the remaining users. The remaining 10% of the users that are not guaranteed of the file of size L are ascribed to be in *outage*. In our simulations, for the sake of simplicity, all base stations are assumed to have the same peak power threshold values. We run $K = 200$ (amounting to 11400 user instantiations) different user instantiations in the system. The common rate requirement is set as 1 bit/sec/Hz. This common rate requirement translate to 0 dB common target SINR requirement. We divide the total time T into 1000 mini-slots and at the end of each mini-slot, we keep track of the cumulative MI $I_i(t)$ of each user i. If at the end of a mini-slot, a particular user's cumulative MI exceeds the file size L, the base station corresponding to that user is turned off.

We run the baseline for different peak power values of the base station (5 W to 30 W in increments of 5 W). For each peak power value, the relay powers are varied as a factor of the base station power. Figure 4 shows the variation of outage probability for various base station powers and various relay powers. For the case when there are no relays in the system (ratio of relay power to base station power is zero), increasing the peak powers of the base station decreases the outage. The percentage of outage saturates below a certain threshold as the interference limit sets in. As we increase the relay powers by increasing the ratio of relay power to base station power, the outage reduces but quickly saturates to a certain threshold outage value, because of the interference limit. From the Figure 4 it is clear that interference limit is quickly reached and limits the performance of system with relays. This is because we do not control the interference and peak power transmissions from the base stations and relays lead to a highly interference limited scenario.

D. Simulation results

1) Power savings: The base station peak power required to guarantee 90% of the users (after the 10% users in outage have been removed) a rate of 1 bit/sec/Hz is 21 W in the baseline case and it requires 15 W for a system with relays. The relays transmit 1 W of peak power. Hence the peak power savings at the base station locations in this case is 1.46 dB.

2) Rate gains: In order to evaluate the throughput improvement, we find how much the common rate of 90% of users can be improved with the peak power of the base stations being fixed. For the baseline, we fix the power of the base stations to 21 W, so that 90% of the users are guaranteed to get 1 bit/sec/Hz (as obtained in the previous section). For the P-CPA system with relays, the peak power threshold of the base stations are fixed to 21 W (the same value as in the base line case). For the same peak power for the base stations and with relays present in the system, we expect the common rate to be better than 1 bits/sec/Hz. To find the improvement in common rate, we fix a desired common rate $r' > 1$ bit/sec/Hz and run the sytem with relays. If this desired common rate is feasible[2], we double the desired common rate and run the simulations again. Else, if the desired common rate is infeasible, we fix the new desired common rate at half the difference between the highest feasible common rate and the lowest infeasible common rate and rerun the simulations. In this manner, we converge to the achievable common rate in the presence of relays. In our simulations, we find that the common rate can be improved to 1.21 bits/sec/Hz in the CPA based relaying scheme. Hence the common rate improvement is 21%.

V. Power Control Based Collaborative Power Addition Relaying

In the previous section, we observed that the interference from the other relays and base station sectors was limiting the peak power savings in the system with relays. The reason for that is when the relays transmit to help the users, they transmit with peak powers and hence increase the interference levels in the system. If we could find the optimal set of powers to transmit for the base station and the relays, we could reduce the overall interference levels in the system. This may improve the gains in the system.

A. Operating the baseline system

In the baseline case, each base station sector instead of transmitting at its peak power, powers down its transmit power subject to a peak power constraint so that 90% of users are guaranteed 1 bit/sec/Hz. The set of feasible powers for the 90% of users could be obtained by solving a feasibility problem of linear constraints. We eliminate 10% of users in the following way. We first discard the user who caused the power constraint go active. We repeat this process until all

[2]The common rate is feasible if all the users present in the system are able to get the desired rate.

users are guaranteed a rate of 1 bit/sec/Hz, meeting the peak power constraints.

In reality, each base station increases its power autonomously in small increments, until it hits the peak power or when the user associated with it attains the desired rate of 1 bit/sec/Hz. As mentioned, those users that make the power constraint to go active before attaining the desired rate are discarded.

B. Operating the system with relays

When relays are present in the system, 10% of users are discarded as in the case of the baseline system. The system starts out as it does for the baseline case. The base stations increase their powers autonomously in small increments targetting the users rates to increase. As and when the relay in the sector gets the message, the relay and base station jointly adjust their powers so that the user gets the desired rate, thus making sure that the base station and relay transmit just enough power to the user to obtain the desired rate.

Power control in cellular system with relays gives us more degrees of freedom to optimize over the sum total of powers in the system, to reduce the maximum peak power transmission in the system, reduce total energy in the system etc. In what follows, we assume that a central controller has the knowledge of the all the channel gains between the base stations as well as relays and the users. We explain ways to achieve various aforementioned objectives using linear program formulations.

C. Minimizing the total power in the system

In this section, we are interested in evaluating the benefits of relays in minimizing the total sum power in the system while delivering a target common rate with 10% of the users being omitted from the system. The desired common rate for existing 90% users both with and without relays are fixed at 1 bits/symbol. Our aim is to find how much of the total power in the system can be reduced thereby reducing the operating electricity expenses. Hence, we minimize the sum total of transmit powers in the system. For brevity, we drop the argument t and denote the power transmitted by the base station i as p_i and the power transmitted by the relay as q_i. We define the set of all active relays, i.e., the set of relays that have the message and are ready to help the base station as $\mathcal{A}$. Define

$$\rho_i = \frac{h_{ii}p_i + g_{ii}q_i}{\sum_{j\neq i} h_{ij}p_j + g_{ij}q_j + \sigma^2}, \tag{15}$$

for $i = 1, \ldots, N$.

At a given time t, the central controller solves the following optimization problem:

$$\begin{aligned}
\min_{\substack{p_1,\ldots,p_N\\ q_1,\ldots,q_N}} \quad & \sum_i p_i + q_i && \text{(16a)}\\
\text{subject to} \quad & \log_2(1+\rho_i) \geq r_0,\ i = 1,\ldots,N, && \text{(16b)}\\
& 0 \leq p_i \leq p_{i,\max},\ i = 1,\ldots,N, && \text{(16c)}\\
& 0 \leq q_i \leq q_{i,\max},\ i \in \mathcal{A}, && \text{(16d)}\\
& q_i = 0,\ i \in \mathcal{A}^c. && \text{(16e)}
\end{aligned}$$

The above optimization problem (16) is a linear program, since we can write the constraint (16b) as

$$\frac{1}{2^{r_0}-1}(h_{ii}p_i+g_{ii}q_i)-\sum_{j\neq i}(h_{ij}p_j+g_{ij}q_j) \geq \sigma^2,\ i = 1,\ldots,N, \tag{17}$$

a set of linear inequalities in p_i and q_i.

D. Minimizing the peak transmit power

Minimizing the peak transmit power amounts to minimizing the peak power transmission in the system. This leads to peak power savings in the system. To this end, the central controller solves the following optimization problem:

$$\begin{aligned}
\min_{p_1,\ldots,p_N} \quad & \max_i p_i && \text{(18a)}\\
\text{subject to} \quad & \log_2(1+\rho_i) \geq r_0, && \text{(18b)}\\
& 0 \leq p_i \leq p_{i,\max},\ i = 1,\ldots,N, && \text{(18c)}\\
& 0 \leq q_i \leq q_{i,\max},\ i \in \mathcal{A}, && \text{(18d)}\\
& q_i = 0,\ i \in \mathcal{A}^c. && \text{(18e)}
\end{aligned}$$

We refer the reader to [26], [27] for further details on the formulation of the PC-CPA scheme and user discarding methodology.

E. Simulation results

We operate the baseline system as well as the system with relays such that, over a large number of user loading iterations, 90% of users obtain a common average rate. We follow an iterative heuristic approach to discard users in the system. For the baseline case, at the outset, we solve (18) with $q_{i,\max} = 0,\ i = 1,\ldots,N$. We discard users that cause the power constraint in (18c) one-by-one. We stop when there exists a feasible set of powers $p_1,\ldots,p_N$. This way, for a given user loading in the network, we retain those users that can obtain the desired common rate. The optimal value of (18) is the peak base transmit power required to deliver the common rate for the remaining 90% users in each instantiation of the user placements. We calculate the minimum peak base power required to deliver the desired rate at 10%-outage as the maximum of the peak power required among the large number of user loadings. By choosing appropriate values of $p_{i,\max} = p_{\max}$, we make sure that 10% of users are discarded over large number of drops. The peak base power is limited by the propagation characteristics of the worst user that has not been discarded.

In the case when there are relays in the system, we again follow a similar iterative heuristic approach to discard users. We start by solving (18) and discard users that cause the power constraint in (18c) one-by-one. Note that we there may be some user such that (18d) goes active, but we do not discard that user. We stop when there exists a feasible set of powers $p_1,\ldots,p_N,q_1,\ldots,q_N$, to deliver the desired common rate for users not in outage in each drop. As with the baseline case, the minimum peak base power required to deliver the desired rate at 10%-outage is the maximum of the peak powers required among user loadings.

We observed that the peak power savings in the downlink when power control is employed were around 2.6 dB, while

TABLE II
SUMMARY OF PEAK POWER SAVINGS DUE TO DEPLOYMENT OF RELAYS IN CELLULAR SYSTEMS

Scenario 1: 57 sectors, one relay per sector, one user per sector. Base station and relays transmit at peak power.		
Peak power required to guarantee 1 bit/sec/Hz at 10% outage Base station only (No relays)	Peak power required to guarantee 1 bit/sec/Hz at 10% outage Base station with relays	Savings in dB
21 W	15 W	1.46
Common rate for 90% users Base station only (No relays)	Common rate for 90% users Base station with relays	Percentage rate increase
1 bps/Hz	1.21 bps/Hz	21 %
Scenario 2: 57 sectors, one relay per sector, one user per sector. Base station and relays transmit just enough power to maintain uniform rate using power control.		
Peak power required to guarantee 1 bit/sec/Hz at 10% outage Base station only (No relays)	Peak power required to guarantee 1 bit/sec/Hz at 10% outage Base station with relays	Savings in dB
10 W	5.5 W	2.6
Common rate for 90% users Base station only (No relays)	Common rate for 90% users Base station with relays	Percentage rate increase
1 bps/Hz	1.34 bps/Hz	34%

the average power savings were 3 dB. We also performed experiments in which the rate improvements was the objective, and we observed 34% improvement in the throughput for 90% users in the system, with the baseline system being served at 1 bit/sec/Hz.

VI. CONCLUSIONS

We simulated the downlink of cellular system and evaluated the peak power savings in base station when a common rate of 1 bps/Hz is required by 90% users in the system. We observe that when the system is interference limited the peak power savings are hard to come by. We propose a framework for power control in a downlink cellular system when relays are present in the system. The framework can be posed as a linear program formulation when the relays help the user by power addition at the user locations. This formulation can be used to evaluate the average and peak power savings in the system. The peak power savings and the rate gains improve when power control is employed. The results are summarized in the Table II.

REFERENCES

[1] 3GPP TR 36.814 v. 1.2.1, "Further Advancements for E-UTRA: Physical Layer Aspects," June 2009.

[2] L. Le and E. Hossain, "Multihop cellular networks: potential gains, research challenges, and a resource allocation framework," *IEEE Commun. Mag.*, vol. 45, no. 9, pp. 66-73, Sep. 2007.

[3] C. E. Perkins, *Ad Hoc Networking*. Addison-Wesley, 2001.

[4] A. Kumar, D. Manjunath, and J. Kuri, *Wireless Networking*. Morgan Kaufman Publishers, 2008.

[5] P. Bender, P. Black, M. Grob, R. Padovani, N. Sindhushayana, and A. Viterbi, "CDMA/HDR: a bandwidth-efficient high speed wireless data service for nomadic users," *IEEE Commun. Mag.*, vol. 38, no. 7, pp. 70-77, July 2000.

[6] E. C. van der Meulen, "Transmission of information in a T-terminal discrete memoryless channel," Ph.D. dissertation, Univ. of California, Berkeley, CA, 1968.

[7] T. M. Cover and A. A. E. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, no. 5, pp. 572-584, Sep. 1979.

[8] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3037-3063, Sep. 2005.

[9] M. A. Khojastepour, A. Sabharwal, and B. Aazhang, "On the capacity of 'cheap' relay networks," in *Proc. 37th CISS*, 2003.

[10] G. Kramer, "Models and theory for relay channels with receive constraints," in *Proc. Allerton Conf. Commun., Control Comput.*, 2004.

[11] D. Chen, M. Haenggi, and J. N. Laneman, "Distributed spectrum-efficient routing algorithms in wireless networks," *IEEE Trans. Wireless Commun.*, vol. 7, no. 12, pp. 5297-5305, Dec. 2008.

[12] M. Sikora, J. N. Laneman, M. Haenggi, J. D. J. Costello, and T. E. Fuja, "Bandwidth- and power-efficient routing in linear wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2624-2633, June 2006.

[13] "Harmonized Contribution on 802.16j (Mobile Multihop Relay) Usage Models." Available: http://ieee802.org/16.

[14] R. Pabst, B. H. Walke, D. C. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. D. Falconer, and G. P. Fettweis, "Relay-based deployment concepts for wireless broadband radio," *IEEE Commun. Mag.*, vol. 42, no. 9, pp. 80-89, Sep. 2004.

[15] H. Viswanathan and S. Mukherjee, "Performance of cellular networks with relays and centralized scheduling," *IEEE Trans. Wireless Commun.*, vol. 4, no. 5, pp. 2318-2328, Sep. 2005.

[16] O. Oyman, "Opportunistic scheduling and spectrum reuse in relay-based cellular OFDMA networks," in *Proc. Globecom*, 2007.

[17] O. Oyman, J. N. Laneman, and S. Sandhu, "Multihop relaying for broadband wireless mesh networks: from theory to practice," *IEEE Commun. Mag.*, vol. 45, no. 11, pp. 116-122, Nov. 2007.

[18] "Multi-hop relay system evaluation methodology." Available: http://ieee802.org/16.

[19] G. Foschini, A. Tulino, and R. Valenzuela, "Performance comparison for basic relay systems," tech. memo, Bell Labs NJ, 2008.

[20] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. Wiley, 2006.

[21] M. Katz and S. Shamai, "Transmitting to colocated users in wireless ad hoc and sensor networks," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3540-3563, Oct. 2005.

[22] D. Blackwell, L. Breiman, and A. J. Thomasian, "Capacity of a class of channels," *Annals Mathematical Statistics*, vol. 30, pp. 1229-1241, 1968.

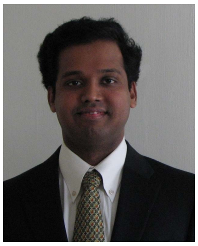
[23] I. Csiszar and P. Narayan, "Capacity of the Gaussain arbitrarily varying channel," *IEEE Trans. Inf. Theory*, vol. 37, pp. 18-26, Jan. 1991.

[24] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press, 2005.

[25] K. Azarian, H. E. Gamal, and P. Schniter, "On the achievable diversity-vs-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inf. Theory*, vol. 51, pp. 4152-4172, Dec. 2005.

[26] C. Raman, G. Foschini, R. Valenzuela, R. Yates, and N. Mandayam, "Power savings from half-duplex relaying in downlink cellular systems," in *Proc. Allerton Conf. Commun., Control Comput.*, Sep. 2009.

[27] ——, "Relaying in downlink cellular systems," in *Proc. CISS*, 2010.



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