Pricing and Power Control for Joint Network-Centric and User-Centric Radio Resource Management

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Abstract

Objectives of most radio resource management schemes can be classified as either user centric or network centric. Schemes like minimizing outage probability for a given user may be thought of as user-centric in that they try to maximize the interests of individual users. On the other hand, schemes like maximizing aggregate throughput or minimizing aggregate power may be viewed as network-centric in that they optimize collective metrics for all users. These two types of resource management tend to result in qualitatively different resource allocations (with, sometimes, very different degrees of fairness). This raises interesting questions regarding the interactions between these two kinds of resource management. In this paper, we consider the joint optimization of both user-centric and network-centric metrics. Specifically, we use a utility function (measured in units of bits per Joule) as the user-centric metric and for the network-centric counterpart, we consider a function of the sum of the throughputs of users in the network. The user-centric measure reflects individual user's throughput as well as the battery energy (transmit power) consumed to achieve it. The network-centric measure reflects the total revenue derived by the usage of network resources. We introduce an explicit pricing mechanism to mediate between the user-centric and network-centric resource management problems. Users adjust their powers in a distributed fashion to maximize the difference between their utilities and their payments (measured as a product of the unit price and throughput). The network adjusts the unit price in order to maximize its revenue (measured as the sum of the individual payments). We show that the distributed user-centric power control results in a unique Nash equilibrium. Our numerical results indicate that there exists a unique unit price that maximizes the revenue of the network. We also derive a semi-analytical, computationally simple and highly accurate approximation to the optimal solution. Our results show that while users with better channels receive better qualities of service as usual (e.g., as in waterfilling), they also make proportionally higher contributions to the network revenue.

Index Terms

Power Control, Pricing, Utility, Radio Resource Management, Revenue Maximization

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I. INTRODUCTION

The importance of radio resource management (RRM) increases as the demand for higher data rates over radio links increases; this is especially true for the emerging wireless data applications. Most radio resource management can be classified as either user-centric or network-centric. User-centric RRMs attempt to maximize the interests of individual users, while network-centric ones optimize network interests. Distributed power control [1, 2] and minimization of outage probability [3, 4] can be thought of as examples of user-centric resource management. Maximization of objectives like the sum of network information capacity [5, 6] or the sum of throughputs [7] falls into the network-centric category. User-centric and network-centric RRMs are motivated by different interests and hence ought to result in dissimilar resource allocations. One remarkable anecdotal difference between the two kinds is that user-centric management tends to distribute qualities of services (QoS) more evenly to users than network-centric ones. In [5–7], network maximum solutions tend to provide most if not all of the radio resources to the few users with the best channels (this is true even in the classic water-filling solution), a property that tends not to appear in solutions of user-centric management

In the context of a *wired* network, the system optimization decomposed into a user problem and a network problem [8–10] has been applied for elastic traffic management. Both of their system objectives are to maximize the aggregate user utilities over flow rate subject to capacity constraints. In order to eliminate the coupling of users through shared links, system optimization is decomposed into subsidiary optimization problems for users and networks respectively, by using price per unit flow as a Lagrange multiplier that mediates between these two subproblems. The main difference between Kelly's work [8,9] and Low's work [10] is that they propose different mechanisms. Kelly allows the users to decide their payments and the network allocates the rate, while in Low's approach, users decide the rate and pay what the network charges. In a wireless system, the nature of the interactions between users is quite different from that encountered in wired networks. Users in a wireless network are coupled very strongly due to mutual interference and further the energy constraints play a very important role in wireless communications. Therefore we have to design our user and network problems in the context of a wireless system from scratch.

In our previous work [11–14], a utility function was specifically defined in the context of a wireless data system as the total number of useful information bits successfully transmitted per Joule of battery energy expended for each user. The main objective is for each individual user to maximize its QoS quantified through the utility function by unilaterally adjusting its transmitter power. It was shown that such user-centric management can be analyzed in the framework of a non-cooperative game and further the resulting equilibrium is Pareto inefficient [11]. Heuristic pricing schemes that hold users accountable for the level of their transmitter powers (and hence the interference levels) have been shown to enable Pareto improvements [12–15]. All of the above work have focused on user-centric resource management and have not addressed the network interests.

In this work, we motivate and define a radio resource management problem that allows both network- and user-centric objectives to compete. Specifically, we apply the utility function (measured in the unit of bits per Joule) defined in our previous work [12–14] as the QoS for the user-centric problem. For the network-centric objective, we consider the revenue, defined as the sum of payments from all the users. Unlike our earlier work on user-centric management [13–15] where pricing was used as a policing mechanism to improve user behavior and system efficiency, we propose here a pricing mechanism that acts as a mediator between the possibly conflicting user and network objectives. The payment of any user is defined as the product of the unit price (or pricing factor) and its throughput. According to the network-broadcasted unit price, users adjust their transmitter powers to maximize their net utilities (defined as the difference between their utilities and their corresponding payments). The network, on the other hand, adjusts its unit price to maximize its revenue. The user-centric and the network centric problems, though solved in a distributed fashion, are coupled with each other. Given any unit price, the output of the user-centric resource management is a set of equilibrium powers which is the input back to the network-centric problem. The net result is a trade-off between the two seemingly conflicting user-centric and network-centric objectives.

This paper is organized as follows. In Section II, we define our user metric (utility function) and network metric (revenue) as well as the pricing (or payment) function that mediates between the user objectives and the network objective. We present in Section III our joint user-centric and network-centric optimization problems. For the user-centric optimization, we propose a distributed power control for the net utility maximization. Formulating such power control as a non-cooperative game, we analyze the existence and uniqueness of the Nash equilibrium for this game. Further, we derive an iterative power adjustment algorithm and prove its convergence to the equilibrium. For the network centric optimization, we show the existence of a unit price that maximizes the revenue. Our numerical results in Section IV suggest that such a unit price is unique. The user and network metrics under optimum unit price are demonstrated with our numerical experiments in Section IV. We also present in Section IV the derivation of a computationally efficient approximation to the optimum solution. We verify the accuracy of the approximation with numerical experiments.

II. SYSTEM MODEL

Consider the up-link of a single-cell CDMA system with a fixed number of users. The system consists of wireless users whose QoS is mapped to a utility function and a fixed network whose self interest is measured as a function of total throughput called revenue. The user objective and the network objective are interconnected by a pricing mechanism which we will discuss in the following.

A. User Metric: Utility Function

While the varied QoS requirement of data communications are understood qualitatively, how they can be translated into a quantifiable metric is not, especially in the wireless context. In the recent literature [12–14, 16–19], several notions of utility have been proposed for a wireless system. The utility function in [16] is generic and orthogonal codes are assumed in the paper so that there is no coupling among users. The utility in [17] is defined as a non-decreasing concave function of data rate. Both utility functions in [16, 17] are applied on the down-link of a CDMA system instead of the up-link. In this work, we adopt as a user metric the utility function for a wireless data user which is defined as the average number of information bits transmitted correctly per Joule of battery energy [12–14]. We choose this utility function because it combines the two main criteria of wireless transmission: throughput denoted as T and transmitter power denoted as p. The utility of the i^{th} user U_i is generally given by:

$$U_i \triangleq \frac{T_i}{p_i}(\mathrm{bits/Joule})\ . \tag{1}$$

This specific definition of utility function is also adopted in [18, 19]. We consider the up-link of a single-cell CDMA system with N mutually interfering users. Data bits are packed into frames of M bits containing $L < M$ information bits per frame, where $M-L$ bits are used for error detection. The signal of user i is transmitted at the rate of R_i bits per second. The received signal to interference plus noise ratio (SINR) is γ_i. We assume perfect error detection and automatic retransmission request (ARQ): a frame with error is retransmitted until received correctly. If we further assume that bit errors are independent of each other, the throughput of user i can be rewritten as:

$$T_i \ = \ \frac{L}{M} R_i f(\gamma_i)\ , \tag{2}$$

where the efficiency function $f(\gamma_i) \ = \ [1 - 2\,\mathrm{BER}(\gamma_i)]^M$, with $\mathrm{BER}(\cdot)$ being the bit error rate. Note that the efficiency function is an approximation to the frame success rate (FSR), which is given by FSR $= [1 - \text{BER}(\gamma_i)]^M$. As pointed out in [12, 13], the above approximation renders $f(\gamma) \in [0, 1]$ and also yields the following properties for the utility function:

1) $U_i \to 0$ as $p_i \to 0$: zero utility when no usage.
2) $U_i \to 0$ as $p_i \to \infty$: zero utility when power consumption is excessive.
3) U_i is quasi concave in p_i when $\text{BER}(\gamma)$ decays exponentially in γ.

B. Network Metric: Revenue

With the existence of a pricing scheme, a natural metric of the network satisfaction is its revenue. Revenue is the product of price per unit service and the amount of service provided. The amount of service provided by the network to any user is the amount of useful data bits that the user sends to the network over a fixed time frame, which is proportional to the user' throughput in that time frame. We assume a fixed time frame for our system, and in this time frame, the network charges each user in proportion to its throughput. We assume that the network broadcasts a common unit price λ to all the users[1]. Hence, the payment by each user is:

$$\rho_i \triangleq \lambda T_i \,, \quad \forall i \,, \tag{3}$$

resulting in the network revenue of

$$\rho \;\triangleq\; \sum_{i=1}^{N} \rho_i \;=\; \sum_{i=1}^{N} \lambda T_i \,. \tag{4}$$

C. Pricing: Mediator Between User and Network

Pricing used in our previous work [13–15] aims to accomplish user-centric goals. In the context of this paper, the pricing scheme not only explicitly characterizes the network-centric objective (Revenue $= \sum_i$ Payment), but also mediates between the user and the network objectives. Each user optimizes its net utility, defined as its utility minus its payment. Therefore, the user and the network objectives interact with each other through the pricing mechanism, as shown in Figure 1.

After adjusting for its *payment* ρ_i, the *net* satisfaction or the net utility, U_i^{net}, of user i is $U_i - \lambda T_i$. The conversion factor from price to bits per Joule is absorbed into λ, and thus the payment is measured in bits per Joule rather than in explicit monetary units[2].

[1] In this paper, without loss of generality, we use a common unit price for all the users. If the users belong to different priority classes, different unit prices can be applied.

[2] Note that all quantities representing user and network objectives could be translated into monetary units. The actual translation (value of conversion between units) that enables this is a topic of future study.

III. JOINT USER AND NETWORK OPTIMIZATION

A. User Optimization: A Non-Cooperative Power Control Game

With the network broadcasted unit price λ, the user objective is for *each* user to maximize its net utility, defined as the difference between its utility and its payment:

$$[\mathbf{User\ \ Problem}]\ \max_{p_i \in S_i} U_i^{\rm net} \Leftrightarrow \max_{p_i \in S_i} U_i - \lambda T_i, \quad \forall i\ ; \tag{5}$$

where $S_i \triangleq \left\{p_i \mid p_i^{\min} \le p_i \le p_i^{\max}\right\}$ is the strategy space of user i, with $p_i^{\min}$ and $p_i^{\max}$ denoting the minimum and maximum transmitter power respectively.

We assume that there is no communication/cooperation between users and each individual user does its optimization independently. The **User Problem** can be formulated as a non-cooperative game. We now discuss issues of existence and uniqueness of the Nash equilibrium as well as an iterative algorithm to achieve it.

1) Existence of Nash Equilibrium: If *all* the independent users' optimization attempts settle down, the game achieves an equilibrium called the *Nash equilibrium* with the equilibrium power vector $\mathbf{p}^* = (p_1^*,\, p_2^*,\, ...,\, p_N^*)$. The Nash equilibrium is an equilibrium such that no single user can be better off by unilaterally changing its power. Formally, we can define the Nash equilibrium for our non-cooperative power control game as:

Definition III.1: The Nash equilibrium power vector is the power vector at which no single user can improve its net utility by unilaterally changing its power to any other value. Mathematically,

$$p_i^*(\lambda) \triangleq \arg\max_{\xi_i \in S_i} U_i^{\rm net}\left(\xi_i; \mathbf{p}_{-i}^*; \lambda\right), \ \ \forall i\ , \tag{6}$$

where $\mathbf{p}_{-i} \triangleq (p_1,\, p_2,\, ...,\, p_{i-1},\, p_{i+1},\, ...,\, p_N)$ is the transmitter power vector without user i's power.

Theorem III.1: A Nash equilibrium exists for the non-cooperative game defined in the **User Problem** if BER(γ) decays exponentially in SINR denoted by γ.

The proof of this theorem is given in Appendix A. Please note that the exponentially decaying BER(γ) captures the performances of a wide class of modulations for communications over AWGN channels.

2) Iterative Algorithm and Its Convergence: We now present an iterative algorithm to reach the Nash equilibrium described above. In terms of the power vector at the $t^{\rm th}$ iteration denoted by $\mathbf{p}(t) = (p_1(t), p_2(t), ..., p_N(t))$, the power update rule for the next iteration is:

$$p_i(t+1) \ = \ \arg\max_{\xi_i \in S_i} U_i^{\rm net}\left(\xi_i; \mathbf{p}(t)\right) \quad \forall i \ = \ 1,\ 2,\ ...,\ N. \tag{7}$$

The update rule for all the users can be expressed in general as:

$$\mathbf{p}(t+1) \;=\; \mathbf{X}(\mathbf{p}(t)), \tag{8}$$

where $\mathbf{p}(t) \in \Re^N$ and $\mathbf{X}(\cdot) \in \Re^N$ is the mapping corresponding to the update rule.

Theorem III.2: Given any unit price λ, starting from any initial point $\mathbf{p}^0 \in S \triangleq S_1 \times S_2 \times \ldots \times S_N$, the iteration specified by $\mathbf{p}(t+1) \;=\; \mathbf{X}(\mathbf{p}(t))$ always converges to the unique Nash equilibrium. We can show (see Appendix B for proofs) that for all $\mathbf{p} \in S$, the update rule function $\mathbf{X}(\cdot)$ has the following properties:

1) Positivity $\mathbf{X}(\mathbf{p}) > 0$;
2) Monotonicity If $\mathbf{p} > \mathbf{p}'$, then $\mathbf{X}(\mathbf{p}) \geq \mathbf{X}(\mathbf{p}')$;
3) Scalability $\forall \alpha > 1$, $\alpha \mathbf{X}(\mathbf{p}) > \mathbf{X}(\alpha \mathbf{p})$.

Any update rule $\mathbf{X}(\cdot)$ satisfying these properties is called a *standard* interference function for distributed power control defined in [2]. We can apply the results in [2] directly and thereby prove Theorem III.2.

To summarize the user-centric optimization, for any given unit price λ, the distributed net utility maximization (power control) yields a unique equilibrium transmitter power vector $\mathbf{p}^*(\lambda)$.

B. Network Optimization

The output of the user optimization $\mathbf{p}^*(\lambda)$ is applied as the input to the network optimization. The network aims to find its highest revenue by searching over $\lambda \geq 0$:

$$[\textbf{Network Problem}] \;\; \max_{\lambda \geq 0} \; \rho(\lambda) \triangleq \max_{\lambda \geq 0} \sum_{i=1}^{N} \lambda T_i(\mathbf{p}^*(\lambda)) \, . \tag{9}$$

Theorem III.3: The revenue $\rho(\lambda) \;=\; \sum_{i=1}^{N} \lambda T_i(\mathbf{p}^*(\lambda))$ has the following properties:

1) $\rho(\lambda) \geq 0$.
2) $\rho(\lambda) \;=\; 0$ when $\lambda \;=\; 0$.
3) $\rho(\lambda) \rightarrow 0$ as $\lambda \rightarrow \infty$.
4) $\rho(\lambda) < \infty$ when the number of users, N, is finite.

The formal proof of Theorem III.3 is shown in Appendix C. The above theorem yields the following interpretations: (1) Revenue is always non-negative because throughputs are always non-negative. (2) By the definition of payments and the finiteness of throughputs, zero unit price leads to zero payments and zero revenue. (3) The network can not earn arbitrarily high revenue by greedily increasing the unit price. This is because users can not afford the service when the unit price

TABLE I

MAIN PARAMETERS OF A SINGLE-CELL CDMA SYSTEM USED IN THE EXPERIMENTS

Parameter	Value
Channel Bandwidth W	1MHz
Transmission Rate R	10 kbps
Total Bits per packet M	96 bits/frame
Information Bits per packet L	80 bits/frame
AWGN Power at receiver σ^2	5×10^{-15}Watts

becomes too high; users will lower their usages towards zero in reaction to the high price. (4) The maximum revenue that the network can obtain is always finite. The revenue has these desirable properties, indicating that its definition is sensible. These properties together with the continuity of revenue yields:

Corollary III.1: There exists an optimum unit price $\lambda = \lambda^{\text{opt}}$ which maximizes the revenue $\rho(\lambda)$. Further, both λ^{opt} and $\rho(\lambda^{\text{opt}})$ are finite.

While we do not have a formal proof for the uniqueness of the optimum unit price, all our numerical results seem to support such a hypothesis.

IV. NUMERICAL EXPERIMENTS

We consider the uplink of a single cell CDMA system. The main system parameters are listed in Table I. We use a distance based path gain formula, i.e., a user at a distance d from the base station has a channel path gain $h = \text{constant}/d^4$. We use non-coherent FSK as the modulation method with BER$(\gamma) = \frac{1}{2}e^{-\gamma/2}$, which is appropriately reflected in the utility function via equation (1) and equation (2).

We want to demonstrate by numerical experiments how the network metrics and the user metrics behave under the joint optimization. We also aim to illustrate the interactions between users and between users and network under our present joint network and user optimization.

Consider a wireless system with nine users and assume that $h_1 \geq h_2 \geq ... \geq h_9$. We consider four scenarios; in each case user 1 is assumed to have the same channel, i.e., h_1 is fixed. Scenario (a) has only user 1 in the system and serves as a benchmark for performance. Scenarios (b)-(d) allow all the nine users to compete with each other ($N = 9$). Scenario (b) is an extreme example where all the nine users are located on a circle such that their distances to the base station are the same and so are their path gains. Scenario (c) is the one where all the nine users are located such that $\Delta h = h_i - h_{i+1} = 1$dB, $i = 1, 2, ..., N-1$. Scenario (d) is similar to scenario (c) except that

$\Delta h = h_i - h_{i+1} = 3$dB, $i = 1, 2, ..., N-1$. We now discuss in detail the resulting network metrics and the user metrics under the joint optimization for each scenario.

A. Network Metrics

Since the equilibrium power vector $\mathbf{p}^*(\lambda)$, which solves the **User Problem**, is a function of the unit price λ, so are the users' payments and the network metric revenue. Figure 2 shows the plots of the revenue and also the individual user's payment as a function of the unit price λ for the scenarios (c) and (d). When $\lambda = \lambda^{\text{opt}}$, the network achieves its maximum revenue. We can observe the contributions from each user to the network revenue when the network revenue is maximized. We perform the network maximization for all the four scenarios. The trivial conclusions (not shown in Figure 2) are that the best user makes 100% contribution to the revenue when $N = 1$ and each user contributes $1/9$ of total revenue when they have same path gains ($N = 9$, $\Delta h = 0$dB). When the users have different path gains, we can see that the users with better channels make higher contributions to the revenue, as shown in Figure 2. The difference between Scenario (c) and (d) is in the distribution of individual payments. The system with a wider path gain interval ($\Delta h = 3$ dB) collects its revenue mainly from the few (two) users with best channels; while more users make non-negligible contributions to the revenue in the system with a narrower path gain interval ($\Delta h = 1$ dB).

B. User Metrics at λ^{opt}

Figure 3 shows the user metrics as a function of distance from the base station when the revenue is maximized at λ^{opt}. The user metrics we use here are frame success rate, transmitter power, utility and payment. The four scenarios are plotted for each user metric in Figure 3. Our results indicate that users with better channels use lower powers, obtain higher utilities and higher throughputs, but make higher payments. We also examine the relationship between QoS (utility) and payment in Figure 4. We observe that the proportionality holds between the user metrics and its payment: users with better channels get better services (higher utilities), but they pay proportionally more. This proportionality shows that our model is more sensible compared to a flat rate model, where the users are charged the same rate despite their uneven QoS.

C. Approximate Solution

Our proposed joint user and network optimization is a computationally intensive problem. For each unit price λ , the resulting power vector $\mathbf{p}^*(\lambda)$ is obtained as the Nash equilibrium of a non-cooperative power control game. If the network needs to choose the optimum unit price that maximizes the revenue among K discrete candidate unit prices, the non-cooperative game has to be repeated for each of the candidate unit prices. We propose a semi-analytical approximation to the optimum unit price in the following that effectively requires only one instantiation of the non-cooperative power control game.

Our approximation to the optimum unit price is based on the asymptotic behavior of the equilibrium power vector and the corresponding payment at low and high unit prices.

Proposition IV.1: Each individual user's asymptotic behavior can be mathematically expressed as follows:

$$\begin{array}{llll} \gamma_i^*(\lambda) \approx \tilde{\gamma}, & \rho_i \propto \lambda, & \text{as } \lambda \to 0 & (\text{Low Unit Price Asymptote})\ ; \\ p_i^*(\lambda) \approx \left(1 - \frac{1}{M}\right) \frac{1}{\lambda}, & \rho_i(\lambda) \propto \lambda^{-(M-1)}, & \text{as } \lambda \to \infty & (\text{High Unit Price Asymptote})\ ; \end{array} \tag{10}$$

where $\tilde{\gamma} = \gamma^*(0)$ is the equilibrium SINR when $\lambda = 0$ [3].

The proof of Proposition IV.1 is shown in the Appendix D. The low unit price asymptote means that the user behaviors are hardly influenced by pricing. We can see that each user keeps its SINR level flat for a small unit price. Therefore its throughput is constant at low unit prices and its corresponding payment increases linearly with the unit price. The high unit price asymptote implies that the equilibrium transmitter power is identical among all the users $\left(p_i^*(\lambda) \approx \left(1 - \frac{1}{M}\right) \frac{1}{\lambda}\right)$, and vanishes inversely with the unit price. The corresponding payment at high unit price drops at a rate of $\lambda^{-(M-1)}$. Since we know that the payments increase linearly with unit price at the low unit price asymptote and drop at the high unit price asymptote, we can simply draw the conclusion that the user pays its highest payment between the low unit price and the high unit price regimes. We extend the user i's low unit price and high unit price asymptotes towards each other such that they meet at a corner unit price λ_i^{a}. That is, we assume that for the unit price smaller than this corner unit price, the user behaves approximately like the low unit price asymptote; otherwise it behaves approximately like its high unit price asymptote. Mathematically, we express this approximation as follows:

$$\begin{array}{lll} \gamma_i^*(\lambda) \approx \tilde{\gamma}, & \rho_i \propto \lambda, & \text{as } \lambda < \lambda_i^{\mathrm{a}}; \\ p_i^*(\lambda) \approx \left(1 - \frac{1}{M}\right) \frac{1}{\lambda}, & \rho_i(\lambda) \propto \lambda^{-(M-1)}, & \text{as } \lambda > \lambda_i^{\mathrm{a}}\,. \end{array} \tag{11}$$

With such an approximation, any user i pays its highest payment at the corner price λ_i^{a} because it is the transition point from the rise to the fall of the payment. From our numerical experiments (Figure 2), we observe that the unit prices where users pay their highest payment, denoted as λ_i

[3] $\tilde{\gamma}$ is only a function of the efficiency function $f(\gamma)$. It satisfies the equation $f'(\gamma)\gamma = f(\gamma)$ (see for example [14]) .

for user i, are in an increasing order of their channel states, i.e., $\lambda_i \leq \lambda_j, \text{ if } h_i \leq h_j$. Further, the optimum unit price where the total revenue is maximized is very close to the unit price that maximizes the payment from the best user, i.e., $\lambda^{\text{opt}} \approx \max_i \lambda_i$. Since the corner price λ_i^{a} is the approximation to the unit price λ_i, the approximate optimum unit price, denoted as λ^{a}, is the largest corner price. By matching the high and low unit price asymptotes of the transmitter power and assume that at λ^{a} the received powers of all the users except the user with the best channel are negligible[4], we have

$$\lambda^{\text{a}} \triangleq \max_i \lambda_i^{\text{a}} \approx \left(1 - \frac{1}{M}\right) \frac{G}{\sigma^2 \tilde{\gamma}} \max_i h_i, \tag{12}$$

where G is the processing gain.

With the analytical approximation to the optimum unit price, we can numerically obtain other user and network metrics under this approximation. Then we can compare these metrics under λ^{a} with those under λ^{opt} and verify how accurate the approximation is.

For a two-user system, we fix the path gain of one user and vary that of the other. The ratio of path gains ranges from 0.001 to 1000. Figure 5 shows both the ratio of the approximate unit price to the optimum unit price and the ratio of the revenue achieved at the approximate unit price to the true optimum revenue as functions of the ratio of the two path gains. The almost perfect match for most path gain ratios between the revenue gained at the approximate unit price and the revenue at the optimum one verifies our approximation. Thus, under the approximate method, the number of user equilibrations required to achieve the revenue optimality is effectively reduced to one.

V. CONCLUSIONS

In this paper, we have considered a radio resource management problem with joint user centric and network centric objectives. Specifically, we used a utility function (measured in units of bits per Joule) as the user-centric metric and for the network-centric counterpart, we considered a function of the sum of the throughputs of users in the network. The user-centric measure reflected individual user's throughput as well as the battery energy (transmit power) consumed to achieve it. The network-centric measure reflected the total revenue derived by the usage of network resources. We introduced an explicit pricing mechanism to mediate between the user-centric and network-centric resource management problems. Users adjusted their powers in a distributed fashion to maximize the difference between their utilities and their payments (measured as a product of the unit price and throughput). The network sought out the unit price that maximized its revenue (measured as

[4]This assumption is only true when the best user has much better channels than any other users, but this happens to be a very common phenomenon in the context of an air interface.

the sum of the individual payments). We showed that the distributed user-centric power control resulted in a unique Nash equilibrium. Our numerical results indicated that there existed a unique unit price that maximized the revenue of the network. We also derived a semi-analytical, computationally simple and highly accurate approximation to the optimal unit price. Our results showed that while users with better channels received better qualities of service as usual (e.g., as in waterfilling), they also made proportionally higher contributions to the network revenue.

APPENDIX A
PROOF OF EXISTENCE OF NASH EQUILIBRIUM (THEOREM III.1)

Proof: A necessary condition for the Nash equilibrium is:

$$\frac{\partial U_i^{\text{net}}}{\partial p_i} \;=\; 0, \quad \forall i \,. \tag{13}$$

We will get two solutions to the above equation. One is $p_i \;=\; 0$, which leads to zero net utility. Since the net utility $U_i^{\text{net}} \;=\; U_i - \lambda T_i \;=\; (\frac{1}{p_i} - \lambda)\, T_i$ is positive for $0 < p_i < 1/\lambda$, the stationary point $p_i \;=\; 0$ is obviously a local minimum and not the desired solution. The other solution satisfies the following transcendental equation:

$$1 - \lambda p_i \;=\; \frac{1}{M}\frac{\left[e^{\nu\gamma_i(p_i)} - 1\right]}{\nu\gamma_i(p_i)} \;, \tag{14}$$

where we have used the expression: $\text{BER}(\gamma) \;=\; \frac{1}{2}e^{-\nu\gamma_i}, \text{ with } \nu > 0$. Note that this is a generic expression for BER that captures the performances of many modulation schemes on a AWGN channel. For example, $\nu \;=\; 0.5$ and $\nu \;=\; 1$ characterize non-coherent FSK modulation and DPSK modulation respectively.

Consider a Taylor series expansion to the exponential function of the right-hand side (RHS) of equation (14):

$$\begin{aligned} 1 - \lambda p_i \;&=\; \frac{1}{M}\sum_{n=0}^{\infty}\frac{1}{(n+1)!}\left[\frac{\nu G h_i}{I_i(\mathbf{p})}\right]^n p_i^n \\ &=\; \sum_{n=0}^{\infty}\beta_n(\mathbf{p})p_i^n \;. \end{aligned} \tag{15}$$

We define $\Psi_L\left(\xi\right)$ as $\Psi_L\left(\xi\right) \;=\; 1 - \lambda\xi$, the function on the LHS of equation (15), which is a line with negative slope of $-\lambda$. We also define $\Psi_R\left(\xi;\, \mathbf{p}\right)$ as $\Psi_R\left(\xi;\, \mathbf{p}\right) \;=\; \sum_{n=0}^{\infty}\beta_n(\mathbf{p})\xi^n$, the function on the RHS of equation (15), which is a curve that is always upward sloped. The solution to equation (15) is p_i that satisfies $\Psi_L\left(p_i\right) \;=\; \Psi_R\left(p_i;\, \mathbf{p}\right)$. We illustrate the solution to the above equation by plotting the left hand side (LHS) function Ψ_L and the RHS function Ψ_R in Figure 6. In equation (15), note that each coefficient $\beta_n(\mathbf{p}) = \frac{1}{M(n+1)!}\left[\frac{\nu G h_i}{I_i(\mathbf{p})}\right]^n$ is positive, therefore $Psi_R(\xi)$ is a monotone increasing curve (for non-negative ξ) with vertical intercept of $\beta_0 = \frac{1}{M}$. On the other hand, $\Psi_L(\xi)$ is a monotone decreasing line with vertical intercept of 1 which is greater than the vertical intercept of Ψ_R, i.e., $1 > \frac{1}{M}$. Therefore, these two curves must meet (intersect) at one and only one point, thereby indicating that there is a unique global maximum. That every local maximum is a global maximum implies that the net utility function U_i^{net} is a quasi-concave function of p_i for any user i.

Furthermore, the strategy space of each user $S_i \triangleq \left\{p_i \,\middle|\, p_i^{\min} \le p_i \le p_i^{\max}\right\}$, is non-empty, convex and compact. Thus, using the social equilibrium existence theorem in [20], we have the existence of the Nash equilibrium proved. □

Appendix B
Proof of Positivity, Monotonicity and Scalability Properties of the Distributed Power Control Algorithm for User-centric optimization

Proof: The proof of positivity follows from the fact that the power vector is always non-negative and that zero power leads to a local minimum instead of a maximum of the objective function U_i^{net}.

Monotonicity and scalability can be illustrated easily with the help of graphs.

Monotonicity is to prove: If $\mathbf{p} > \mathbf{p}'$, then $X_i(\mathbf{p}) \ge X_i(\mathbf{p}')$.

First, replacing p_i with $X_i(\mathbf{p})$ in equation (15), we will get:

$$\begin{aligned} 1 - \lambda X_i(\mathbf{p}) &= \frac{1}{M}\sum_{n=0}^{\infty}\frac{1}{(n+1)!}\left[\frac{\nu G h_i}{I_i(\mathbf{p})}\right]^n [X_i(\mathbf{p})]^n \\ &= \sum_{n=0}^{\infty}\beta_n(\mathbf{p})\,[X_i(\mathbf{p})]^n \ . \end{aligned} \tag{16}$$

Note that the solution to equation (16) is $X_i(\mathbf{p})$ that satisfies $\Psi_L\left(X_i(\mathbf{p})\right) = \Psi_R\left(X_i(\mathbf{p});\, \mathbf{p}\right)$.

The increase of power vectors $\mathbf{p}' > \mathbf{p}$ does not influence Ψ_L since Ψ_L is independent of $\mathbf{p}$. This increase in power vector, however, leads to the growth of the interference plus noise level I_i which reduces the value of any coefficient β_n except for $n = 0$ ($\beta_0 = 1/M$) and further lowers the value of Ψ_R for any $\xi > 0$. Mathematically,

$$\begin{aligned} \mathbf{p}' > \mathbf{p} \quad &\Rightarrow \quad I_i(\mathbf{p}') > I_i(\mathbf{p}); \\ &\Rightarrow \quad \beta_n(\mathbf{p}') < \beta_n(\mathbf{p}), \quad \forall n > 0; \\ &\Rightarrow \quad \Psi_R\left(\xi;\, \mathbf{p}'\right) < \Psi_R\left(\xi;\, \mathbf{p}\right), \quad \forall \xi > 0 \ . \end{aligned} \tag{17}$$

The increased power vector $\mathbf{p}'$ results in the lowering of the curve of Ψ_R while Ψ_L remains the same. Therefore, referring to Figure 6, the $X_i(\mathbf{p}')$ satisfying $\Psi_L\left(X_i(\mathbf{p}')\right) \;=\; \Psi_R\left(X_i(\mathbf{p}');\, \mathbf{p}'\right)$ is greater than the $X_i(\mathbf{p})$ and hence monotonicity is proved.

Scalability $(\forall\alpha > 1,\; \alpha X_i(\mathbf{p}) > X_i(\alpha\mathbf{p}))$ can also be shown with the aid of a graph. First, it is easy to verify the following inequality:

$$I_i(\alpha\mathbf{p}) \;=\; \sum_{j\neq i} h_j \alpha p_j + \sigma^2 \;<\; \sum_{j\neq i} h_j \alpha p_j + \alpha\sigma^2 \;=\; \alpha I_i(\mathbf{p}), \qquad \forall\alpha > 1\;. \tag{18}$$

Replacing $\mathbf{p}$ with $\alpha\mathbf{p}$ and $X_i(\mathbf{p})$ with $X_i(\alpha\mathbf{p})$ in equation (16), we obtain:

$$1 - \lambda X_i(\alpha\mathbf{p}) \;=\; \frac{1}{M}\sum_{n=0}^{\infty} \frac{\left[\nu G X_i(\alpha\mathbf{p})/I_i(\alpha\mathbf{p})\right]^n}{(n+1)!}\;. \tag{19}$$

Using the inequality (18), we obtain:

$$1 - \lambda X_i(\alpha\mathbf{p}) > \frac{1}{M}\sum_{n=0}^{\infty} \frac{\left[\nu G X_i(\alpha\mathbf{p})/(\alpha I_i(\mathbf{p}))\right]^n}{(n+1)!}\;, \tag{20}$$

which is equivalent to the following inequality:

$$\begin{aligned} 1 - \lambda\alpha\left[\frac{X_i(\alpha\mathbf{p})}{\alpha}\right] \;&>\; \frac{1}{M}\sum_{n=0}^{\infty} \frac{\left\{\nu G\left[\frac{X_i(\alpha\mathbf{p})}{\alpha}\right]/I_i(\mathbf{p})\right\}^n}{(n+1)!} \\ &=\; \sum_{n=0}^{\infty} \beta_n(\mathbf{p}) \left[\frac{X_i(\alpha\mathbf{p})}{\alpha}\right]^n \quad \forall\alpha > 1\;. \end{aligned} \tag{21}$$

If we use the variable Y_i to replace $\frac{X_i(\alpha\mathbf{p})}{\alpha}$, we will get:

$$1 - \alpha\lambda Y_i \;>\; \sum_{n=0}^{\infty} \beta_n(\mathbf{p}) Y_i^n, \quad \forall\alpha > 1\;. \tag{22}$$

Note that the RHS of inequality (22) is the same as $\Psi_R(Y_i;\mathbf{p})$. We define $\Psi'_L(\xi)$ as $1 - \alpha\lambda\xi$, the function on the LHS of inequality (22). Suppose that Y'_i satisfies $\Psi'_L(Y'_i) \;=\; \Psi_R(Y'_i;\mathbf{p})$. $\Psi'_L(\xi)$ has a steeper slope ($-\alpha\lambda$) than $\Psi_L(\xi)$, with both of their vertical intercepts equal to 1. Hence Y'_i, the intersection of $\Psi'_L(\xi)$ and $\Psi_R(\xi;\mathbf{p})$, must be located to the left of $X_i(\mathbf{p})$, the intersection of $\Psi_L(\xi)$ and $\Psi_R(\xi;\mathbf{p})$, i.e.,

$$Y'_i \;<\; X_i(\mathbf{p})\;. \tag{23}$$

Further, we can see from Figure 6 that Y_i that satisfies the inequality (22), must be located to the left of the Y'_i, i.e.,

$$Y_i \;<\; Y'_i\;. \tag{24}$$

The scalability follows from the two inequalities (23) and (24):

$$Y_i \;=\; \frac{X_i(\alpha\mathbf{p})}{\alpha} \;<\; X_i(\mathbf{p}), \qquad \forall\alpha > 1 \; . \tag{25}$$

APPENDIX C
PROOF OF PROPERTIES OF NETWORK OBJECTIVES (THEOREM III.3).

Proof: Property 1 follows directly from the definition of revenue $\rho(\lambda)$. Property 2 follows from the fact that throughputs are finite at any λ, therefore zero λ leads to zero payment by any user and zero revenue for the network. For property 4, writing the net utility of the i^{th} user as $U_i - \lambda T_i \;=\; (1/p_i - \lambda)T_i$, we see that the net utility is positive for $0 < p_i < 1/\lambda$. Hence, the maximum of of the net utility must be positive, which means $U_i \;>\; \lambda\, T_i$ at the maximum net utility. Each payment is upper bounded by its utility function, which is finite by definition. Thus, payment $\lambda\, T_i$ must be finite. Property 3: When the network charges the users excessively, the users cannot afford to transmit. We know from the analysis of property 4 that user i's optimum power lies between zero and $\min\{p_i^{\max}, 1/\lambda\}$. With an excessive unit price λ, the optimum power vanishes, leading to zero optimum utility. Since the optimum payments cannot exceed their corresponding utilities, optimum payments vanish together with optimum revenue.

APPENDIX D
PROOF OF USER'S ASYMPTOTIC BEHAVIOR (PROPOSITION IV.1)

We know that the Nash equilibrium of the user optimization satisfies equation (14):

$$1 - \lambda p_i \;=\; \frac{1}{M}\frac{e^{\nu\gamma_i(p_i)} - 1}{\nu\gamma_i(p_i)} \; . \tag{26}$$

We can see that the LHS of equation (14) converges to 1 when $\lambda \to 0$. The low unit price asymptotic behavior becomes $\gamma_i(p_i) \;=\; \tilde{\gamma}$, where $\tilde{\gamma}$ is the equilibrium SINR at zero pricing. The throughput $T_i(\gamma_i)$ at a low unit price is independent of λ since $\gamma_i(p_i) \;=\; \tilde{\gamma}$ is independent of λ. The payment $\rho_i \;=\; \lambda T_i$ is a linear function of the unit price at low unit price.

In order to show the high unit price asymptote, we use equation (15):

$$1 - \lambda p_i = \frac{1}{M}\sum_{n=0}^{\infty}\frac{1}{(n+1)!}\left[\frac{\nu G h_i}{I_i(\mathbf{p})}\right]^n p_i^n = \sum_{n=0}^{\infty}\beta_n(\mathbf{p})p_i^n \; . \tag{27}$$

From the argument in Appendix C, the equilibrium power p_i must satisfy $0 \;<\; p_i \;<\; 1/\lambda$. Hence when $\lambda \;\to\; \infty$, $p_i \;\to\; 0$ and RHS$\to\; \frac{1}{M}$. Therefore, we can obtain: $p_i(\lambda) \;\approx\; \left(1 - \frac{1}{M}\right)\frac{1}{\lambda}$ for the high unit price asymptote. Using the fact that γ_i vanishes as $p_i \;\to\; 0$, we can apply Taylor series expansion to the exponential function in the payment function, $\rho_i \;=\; \lambda T_i \;=\; \lambda R_i \frac{L}{M}\left(1 - e^{-\gamma_i/2}\right)^M$, and derive that the high unit price asymptotic payment is in proportion to $\lambda^{-(M-1)}$.

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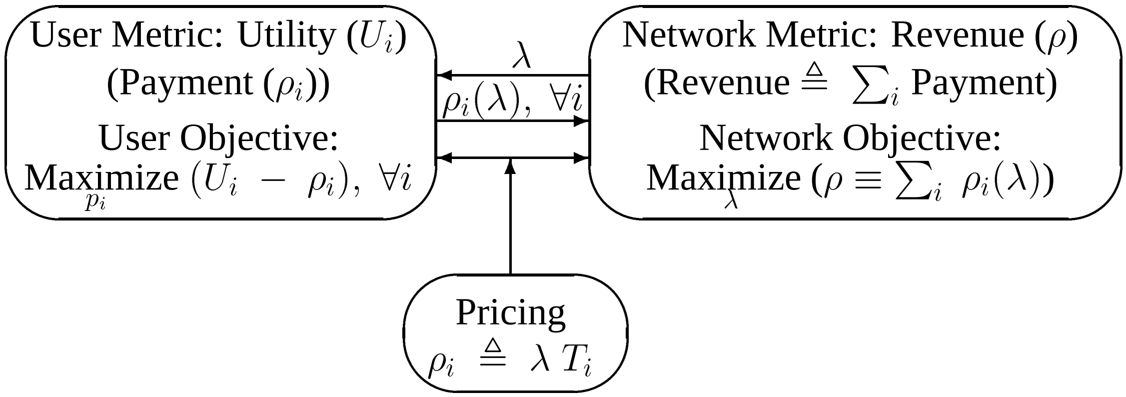

Fig. 1. Pricing mediates between the user and network objectives

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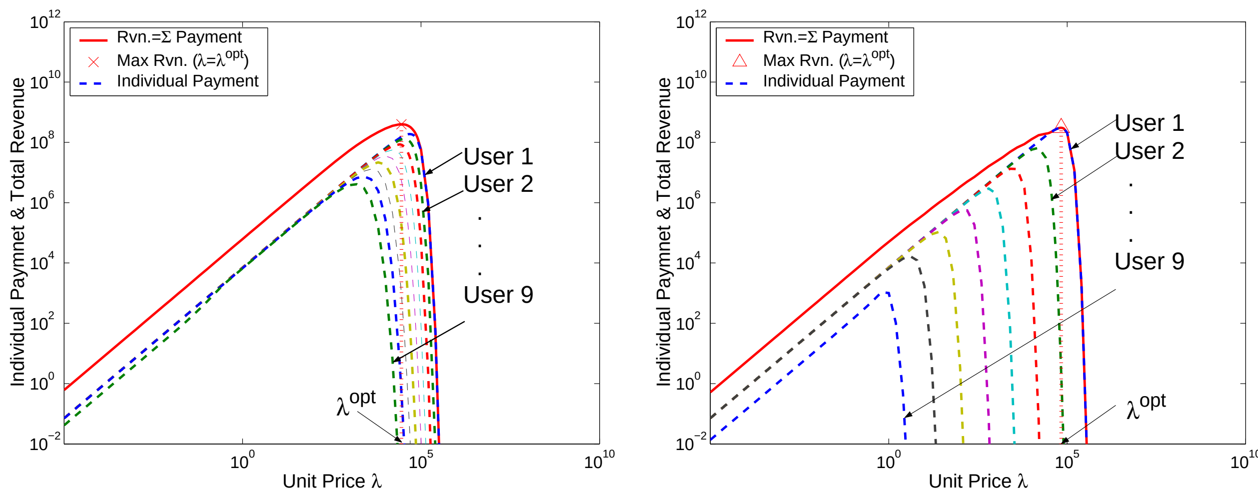

(a) Scenario (c): $\Delta h = 1$dB

(b) Scenario (d): $\Delta h = 3$dB

Fig. 2. Network optimization: seeking the unit price λ^{opt} that maximizes sum of payments (revenue), which is the highest point on the revenue vs. unit price curve. Users with higher path gains pay more at λ^{opt}.

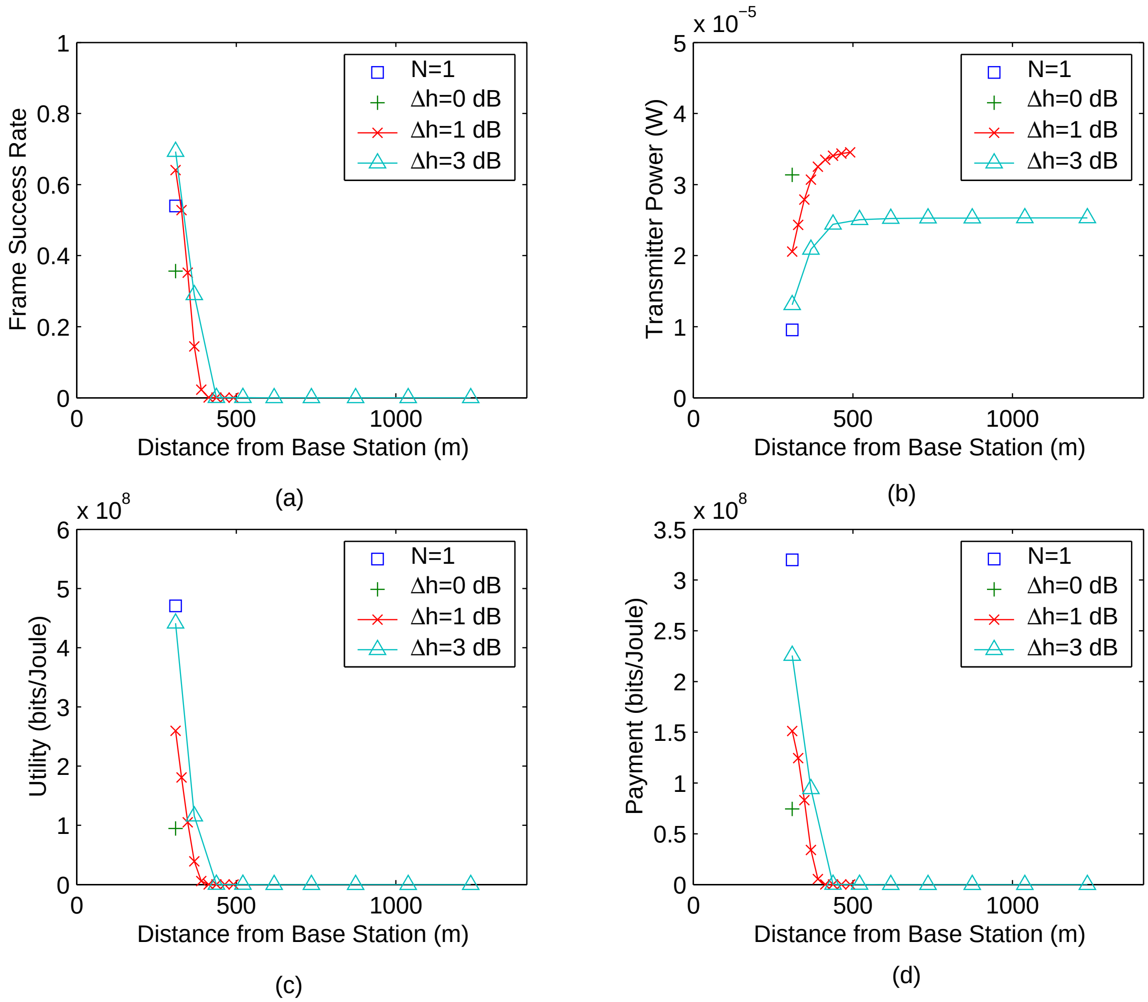

Fig. 3. Comparisons of the user metrics at λ^{opt}. (a) Frame success rate (can be regarded as normalized throughput); (b) Transmitter power; (c) Utility; (d) Payment.

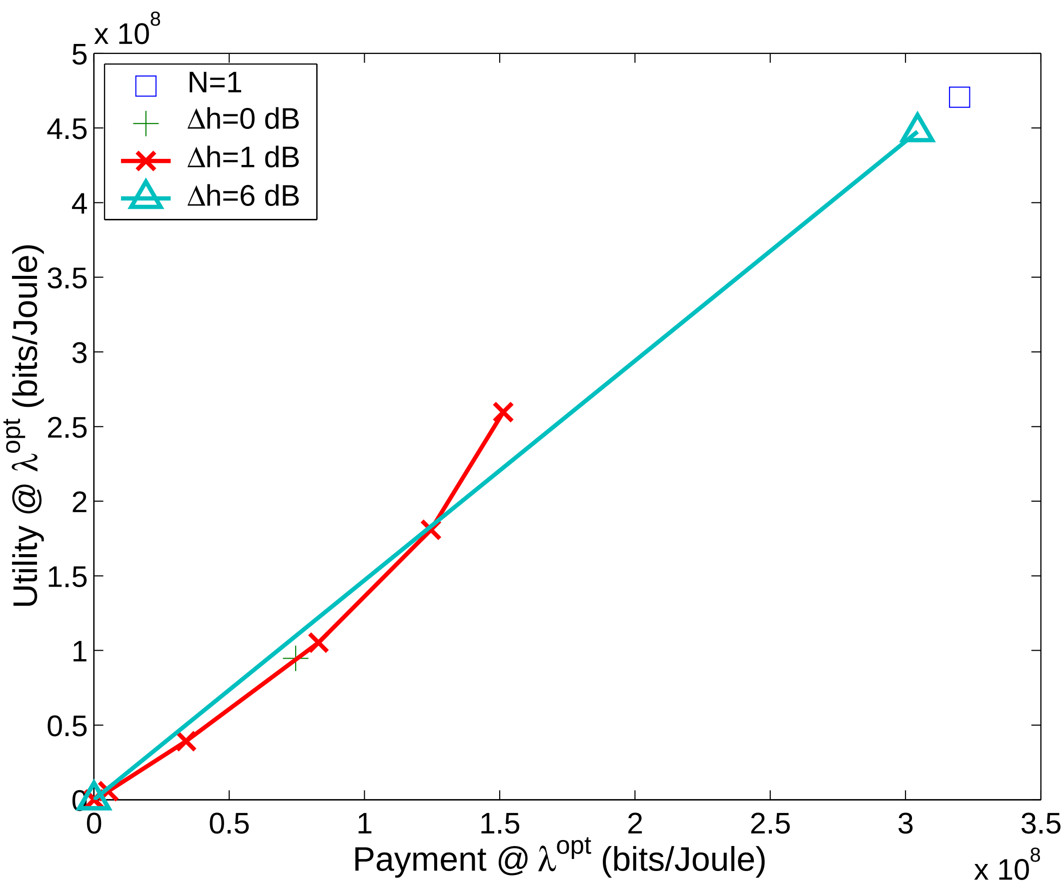

Fig. 4. Proportionality between the user metrics (Utility) and its payment.

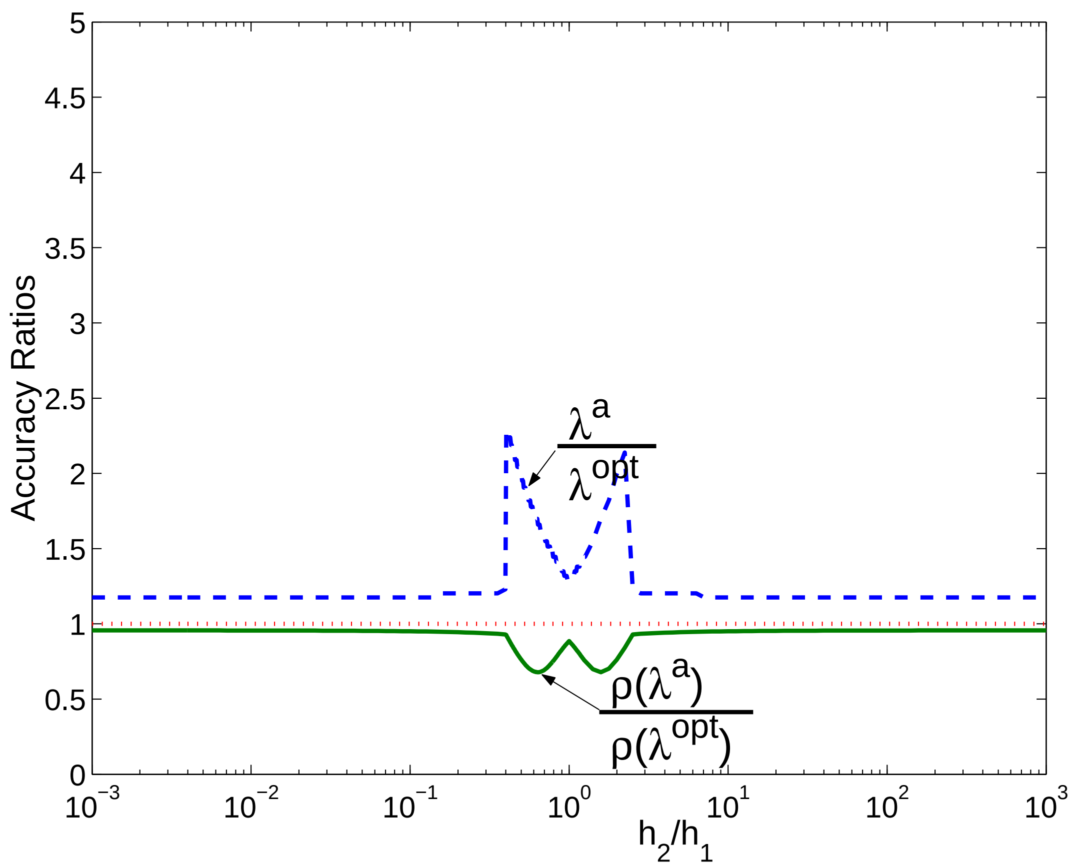

Fig. 5. Comparison of the optimum unit price and the approximate one.

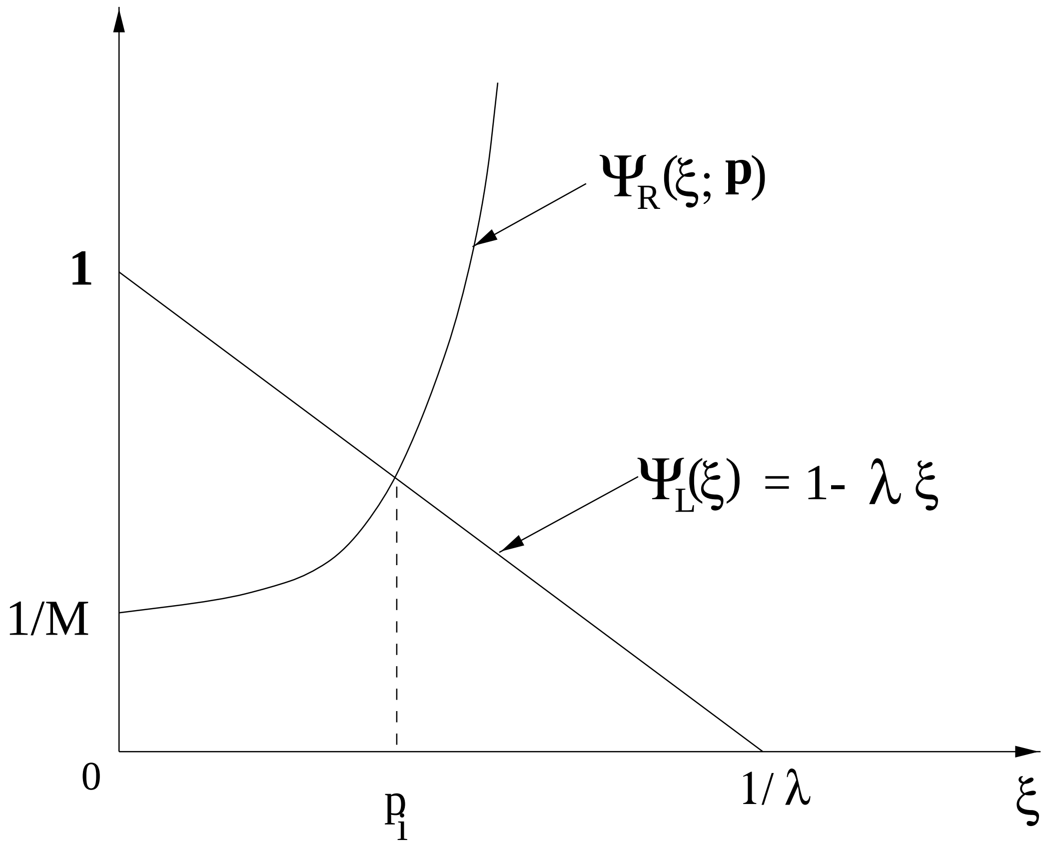

Fig. 6. Illustration of the equilibrium power: the intersection of curve corresponding to the LHS and the RHS of equation (15). The LHS is a line with a negative slope. The RHS is a function of interference and is always upward sloped. The optimum exists and is unique.