Estimating the Doppler Spectrum of a Short-Range Fixed Wireless Channel

Andrej Domazetovic, *Student Member, IEEE*, Larry J. Greenstein, *Fellow, IEEE*,
Narayan B. Mandayam, *Senior Member, IEEE*, and Ivan Seskar, *Member, IEEE*
Wireless Information Network Laboratory (WINLAB), Rutgers University,
73 Brett Road, Piscataway, NJ 08854-8060, USA †

***Abstract*— Time variations of fixed wireless channels result from the relative movements of scatterers in the propagating environment. We consider the temporal gain variations of short-range channels due to scattering from wind-blown leaves. In particular, we present a method for estimating the Doppler spectrum from the received signal's power samples, without requiring the phase information. We then apply the method to the results of measurements we recently conducted on fixed, short-range paths.**

***Index Terms*— Wireless data, Radio propagation, Channel models, Doppler Spectrum, Power Spectrum Estimation**

I. Introduction

In recent measurements of fixed-position, short-path (distance between antennas less than 20 m), outdoor wireless channels, we observed channel time variations which are, most probably, caused by scatter from blowing leaves on trees and bushes in the proximity of the transmitter and receiver [1]. We found these measured variations to be describable as Ricean processes with K-factors ranging from 20 to 30 dB. In this letter, we present their spectral analysis subject to the assumption that they are also wide sense stationary processes.

In general, there are two primary aspects of wireless channel time variations, and thus two different types of Doppler spectra that can be observed by the user. In the first type (which has been extensively studied), the user is moving with respect to base, or *vice versa*. In this case, the time variations of the received signal are related to the spatial variations of the electromagnetic field through a constant (i.e., the relative velocity). Assuming a uniform angle-of-arrival distribution for the received echoes of the transmitted signal, the Doppler spectrum will be the well-known U-shaped spectrum of Clarke [2]. In the second type, the base and user are both stationary, but reflectors in the environment are moving, causing time variations in the channel response. This case is studied here.

The estimation of a complex signal's power spectrum from a finite number of samples has been addressed in almost every book on digital signal processing [3], [4]. However, these methods deal with power spectrum estimation from complex signal amplitude samples. Here, we present a method for signal power spectrum estimation from *power* samples, i.e., the discrete-time measurements of the signal power, without phase information. This case is relevant because power measurements can be done simply, without requiring division of the signal into in-phase and quadrature-phase components. In cases where power gain measurements are the main goal, as in our experiments, the proposed method allows additional information (Doppler spectra) to be obtained with no additional equipment. Moreover, the use of power samples to this end is not affected by oscillator phase noise, in contrast to using complex signal samples.

In Section II we present a method for nonparametric power spectrum estimation from the signal power samples. In Section III we apply the method to the spectral analysis of short-path, wireless channel time variations. We conclude in Section IV.

II. Power Spectrum Estimation from Power Samples

Consider a sinusoidal signal transmitted through a fixed wireless channel. The received complex envelope can be represented as [5]

$$g(t) = V + v(t), \tag{1}$$

where V is a fixed (complex) component and $v(t)$ is a zero-mean, time-varying, complex Gaussian process. The envelope of $g(t)$ is known to be Rice-distributed (Rayleigh-distributed if $V = 0$) [6]. For convenience, we assume $g(t)$ is scaled so that $E[|v(t)|^2] = 1$, in which case V is related to the Ricean K-factor[1] via $K = |V|^2$.

Then, the received instantaneous power is:

$$P(t) = |g(t)|^2 = |V|^2 + |v(t)|^2 + Vv^*(t) + V^*v(t), \tag{2}$$

where $|\,|$ is the modulus operator and * denotes the complex conjugate. Using $K = |V|^2$, the mean received power is then:

$$\bar{P}(t) = E[P(t)] = K + E[|v(t)|^2] = K + 1. \tag{3}$$

If we define the autocorrelation function of $v(t)$ as:

$$r(\tau) = E[v^*(\tau)v(t+\tau)], \tag{4}$$

then the autocorrelation of the power process can be written as:

$$E[P(t)P(t+\tau)] = K^2 + 2K + E[|v(t)|^2|v(t+\tau)|^2] + Kr(\tau) + Kr^*(\tau). \tag{5}$$

[1] The Ricean K-factor is defined here as $K = |V|^2/\sigma^2$, where σ^2 is the power of the *total* time-varying process rather than just the real part, which is why the customary factor of 2 is not present in analysis.

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If we apply the following identity for zero-mean complex Gaussian variates[2]:

$$E[|x_1|^2\,|x_2|^2] = E[|x_1|^2]E[|x_2|^2] + |E[x_1 x_2^*]|^2, \qquad (6)$$

this can further be simplified as:

$$E[P(t)P(t+\tau)] = K^2 + 2K + 1 + |r(\tau)|^2 + K[r(\tau) + r^*(\tau)]. \qquad (7)$$

The autocorrelation function of $[P(t) - \bar{P}(t)]$ is defined as:

$$A(\tau) = E[P(t)P(t+\tau)] - E^2[P(t)]. \qquad (8)$$

Evaluating $A(\tau)$ from Equations 7 and 8, we get:

$$A(\tau) = |r(\tau)|^2 + K[r(\tau) + r^*(\tau)]. \qquad (9)$$

The power spectrum, $S(f)$, of the random part of $g(t)$ is the Fourier transform of the autocorrelation function $r(\tau)$. However, we still do not have the autocorrelation function of the signal, $r(\tau)$, but the autocorrelation function, $A(\tau)$, of the received power process. To find $r(\tau)$, we make the plausible assumption that the spectrum $S(f)$ is an even function about $f = 0$. (There is no reason to expect Nature to favor positive Doppler fluctuations over negative ones, or *vice versa*.) In that case, $r(\tau)$ is real, $r(\tau) = r^*(\tau)$, and Equation 9 becomes

$$|r(\tau)|^2 + 2Kr(\tau) - A(\tau) = 0. \qquad (10)$$

Solving for $r(\tau)$ yields

$$r(\tau) = \sqrt{K^2 + A(\tau)} - K. \qquad (11)$$

Then, as mentioned, the power spectrum, $S(f)$, is the Fourier transform of $r(\tau)$. Note that, since $r(0) = 1$, the area of $S(f)$ is 1.

III. Spectral Analysis of Wireless Channel Time Variations

Background - So far, the proposed method has been exact in the sense that we assumed perfect estimation of the autocorrelation functions. Note that both terms in Equation 11 can be evaluated from the signal power samples. For example, K can be evaluated by the method of [8], and $A(\tau)$ can be estimated in many ways. We have a finite number of samples, which implies the use of a windowing function to estimate $A(\tau)$. We have examined four of the nonparametric standard methods for the power spectrum estimation (Periodogram, Bartlett, Welch and Blackman-Tukey). Although, in our analyses, all of them lead to almost identical results, we have chosen to modify the Blackman-Tukey method since it leads to the largest quality factor estimate [3]. For more details on these methods, one can refer to [3], [4]. Here, we will present the method with just a brief description of each step and concentrate more on the results.

Measurements - To characterize channel time variations, we performed continuous waveform (CW) measurements at a frequency of 5.3 GHz. The receiver was a spectrum analyzer set to measure the received power. We used omnidirectional antennas (azimuth plane). The receiver antenna was mounted at a height of 1.8 m, while the transmitter antenna was mounted at a height of 2 meters. During measurements we kept the positions of both antennas fixed while recording received power over time periods of 15 minutes. We did this at 21 different positions, in an environment having a high density of trees and bushes close to both the transmitter and receiver, the distance between antennas ranging from 1.5 to 16 meters.

Processing - Let us denote the collection of the measured power samples by $x(n)$, $n = 0, 1, ..., N'$. Since the sampling rate was 1.4 Hz on the average, with a small jitter in sampling frequency due to equipment imperfections, we used the cubic spline interpolation to translate the measured data to a uniform $\Delta T = 0.25$ s time scale. This increases the frequency scale to $[-2 : 2]$ Hz, placing zeroes at the frequency points not included in the sampled signal. Now, we can estimate the mean power, $\bar{P}$, as an arithmetic mean of the samples. Then, we subtract the $\bar{P}$ from the samples, $x(n)$, to obtain a zero-mean set of power samples, say $y(n)$, $n = 0, 1, ..., N$. The zero-mean power process autocorrelation estimate is then:

$$\hat{A}(m) = \begin{cases} \frac{1}{N}\sum_{i=0}^{N-m} y(i)\,y(i+m) & \text{for } 0 \le m \le M \\ \frac{1}{N}\sum_{i=|m|}^{N} y(i)\,y(i+m) & \text{for } -M \le m \le -1, \end{cases} \qquad (12)$$

where M is the maximum lag ($M \le N$). At this point, we use Equation 11 to transform the power process autocorrelation function, $\hat{A}(m)$, to the autocorrelation function of the received signal, $\hat{r}(m)$. Then we proceed, as the Blackman-Tukey method suggests, by applying a window to the autocorrelation function. We chose the Blackman window, $w_B(m)$, since it leads to the highest sidelobe suppression (compared to other standard windows) [3].

Finally, we perform the Fourier transform:

$$\hat{S}(f) = \sum_{m=-(M)}^{M} w_B(m)\,\hat{r}(m)\,e^{-j2\pi f m}. \qquad (13)$$

It is suggested, and we used, a Blackman window length (maximum lag M) equal to 20% of the autocorrelation sequence length [4].

Results - The resulting power spectrum estimation is presented in Figures 1 and 2. In the first figure, we plot all 21 spectra (one for each measuring position), one on top of the other. The spectra are shifted on a dB scale by the same value, such that the maximum value in figure is 0 dB. It is seen that the 21 spectra are quite similar.

In the second figure, we plot the average spectrum, meaning the average of the 21 spectra on a linear (not dB) scale. Again, the result is converted to a dB scale and shifted to have a 0-dB maximum. In contrast to the U-shaped Doppler spectrum generally associated with user motion [2], the Doppler spectrum due to scatter motion is peaked at zero frequency. This is consistent with theoretical predictions for this kind of process [9]. In addition, Figure 2 shows the empirical result that $S(f) \sim 1/f^{0.78}$ down to very low f. It also shows that the spectral density falls by 14 dB between $f = 0$ and $f = 0.1$

[2]This can be derived using Equation 8-121 of [7]: $E[x_1x_2x_3x_4] = E[x_1x_2]E[x_3x_4] + E[x_1x_3]E[x_2x_4] + E[x_1x_4]E[x_2x_3]$, where the x's are real and Gaussian. Now writing $|x_1|^2|x_2|^2 = x_1x_1^*x_2x_2^*$ where the x's are complex Gaussian, and invoking this identity, we arrive at Equation 6.

Hz. In contrast, a typical Doppler spectrum for a user moving at 3 km/h, at the same frequency as our measurement (5.3 GHz), will have significant content out to around 15 Hz.

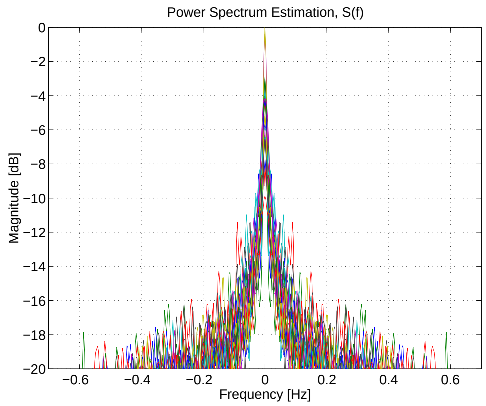

Fig. 1. Estimated power spectra for all twenty one positions

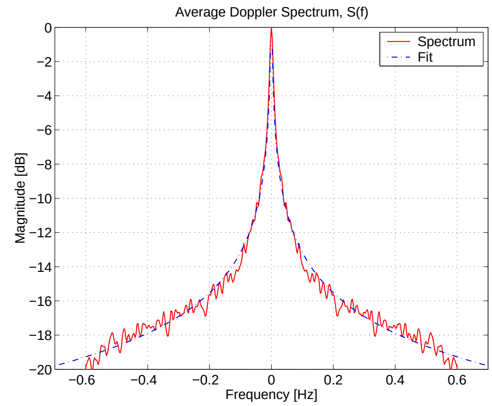

Fig. 2. Average of the estimated power spectra over all twenty one positions

IV. Conclusion

We have presented a method for estimating the power spectrum of a radio wave from a finite number of signal power samples. We illustrated the method by analysing the time variations of a short-path, fixed-position, outdoor wireless channel. We found that the Doppler spectrum arising from wind-blown leaves in the environment does not significantly change across positions, and that it is quite different from the U-shaped Doppler spectrum generally assumed in mobile radio channels.

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