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WINLAB | Wireless Information Network Laboratory

When Users Interfere with Protocols

Prospect Theory in Wireless Networks

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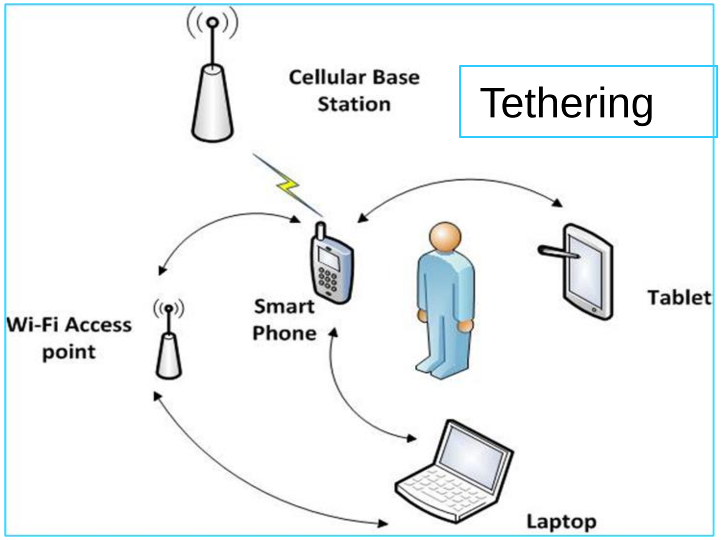
Motivation: Engineered System Design

- Current radio technologies and associated communication protocols are still mostly agnostic to the decision-making of end-users
  - "Engineered System Design" where underlying algorithms/protocols designed based on precepts of Expected Utility Theory (EUT)
  - Radio resource management algorithms and protocols are the result of optimization strategies under the framework of EUT
- Expected Utility Theory (EUT)
  - Alternatives with uncertainty are valued as their mathematical expectation
  - However, violations to it are constantly observed in real-life

Wireless: Increased End-User Influence

- End-users can influence system performance
- Cognitive radio, smart phone applications and user interfaces
  - Allow end users (people) greater degree of freedom to control devices
  - Impact underlying algorithms design and system performance
  - Example: user modifying radio cards and underlying protocols
  - Example: devices with flexible user interfaces
  - Example: end-user actions in response to link conditions, pricing



Prospect Theory: An Alternative to Expected Utility Theory

- Prospect L: a contract yields M outcomes, e.g., $\{o_{1,\dots,}o_M\}$, each occurring with probability p_i
  - ❑ How to valuate a prospect?

Expected Utility Theory (EUT)

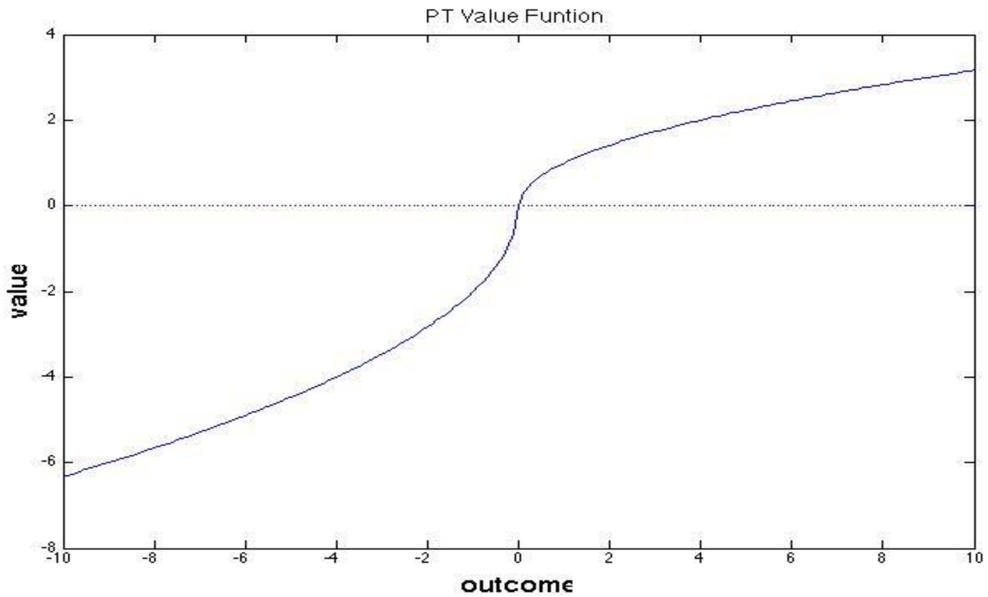
- Proposed by Bernoulli, developed by Von Neumann, Morgenstern, others
- Game Theory heavily depends on it
  - ❑ E.g. game theoretic models in radio resource management
- Value of a prospect is estimated as the mathematical expectation of values of possible outcomes
- However, violations to EUT have constantly been observed in real-life decision-making

Prospect Theory (PT)

- Proposed by Kahneman and Tversky
- A better theory in describing people's real life decisions facing alternatives with risk
- Able to successfully explain the observed violations to EUT
- People use subjective probability to weigh values of outcomes
- People valuate outcomes in terms of relative gains or losses rather than final asset position

Prospect Theory: An Alternative to Expected Utility Theory

- Framing Effect
  - People evaluate outcomes in terms of relative gains and losses regarding a reference point rather than the final asset position
  - People's value function of outcomes is concave in gains and convex in losses
  - Losses usually "loom larger" than gains



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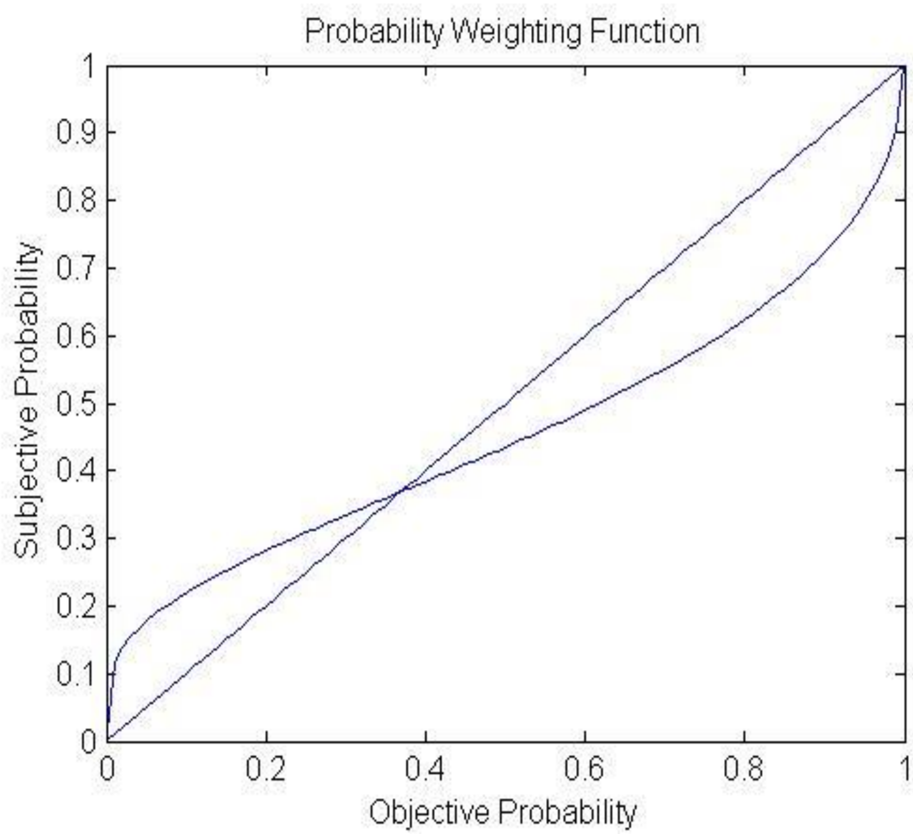
Prospect Theory: An Alternative to Expected Utility Theory

- Probability Weighting Effect
  - People “nonlinearly transform” objective probabilities to subjective probabilities



- “Overweigh” low probabilities
- “Underweigh” moderate and high probabilities
- E.g. Asymmetrically reflected at $\frac{1}{e}$, i.e., $w\left(\frac{1}{e}\right) = 1/e$
- Concave in $\left[0, \frac{1}{e}\right]$, convex in $\left[\frac{1}{e}, 1\right]$
- People are able to objectively evaluate certainty, i.e.,
- $w(0) = 0 \qquad w(1) = 1$

$$w(p) = \exp(-(-\ln p)^{a}), 0 < a £ 1$$

a characterizes deviation from EUT

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Prospect Theory: Valuation of a Prospect

- Expected Utility Theory (EUT)

$$u^{EUT}(L) = \sum_{i=1,\ldots,M} p_i v^{EUT}(o_i)$$

  - ❑ Expectation of values of all possible outcomes

- Prospect Theory (PT)

"The Psychophysics of Chance"

Probability Weighting Effect

Framing Effect

$$u^{PT}(L) = \sum_{i=1,\ldots,M} w(p_i) v^{PT}(o_i)$$

When EUT Fails, PT Explains

- A variation of Allais' paradox

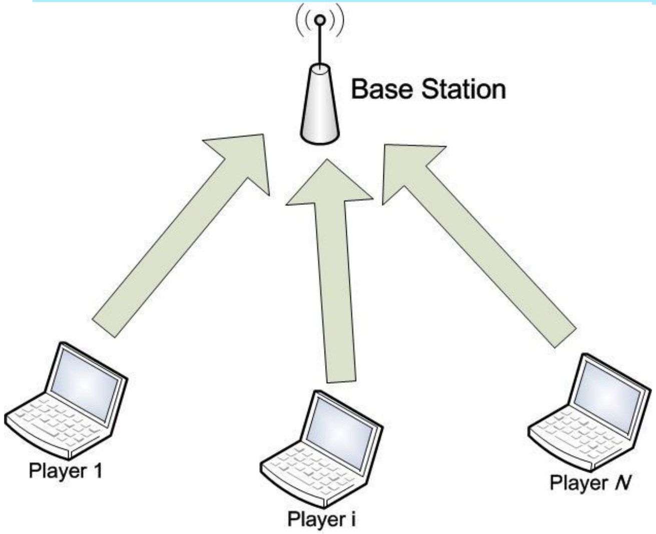
AN EXAMPLE OF EUT VIOLATION

Problem \ Prospect	A	B
1	\$2500 *with probability* 0.33 \$2400 *with probability* 0.66 \$0 *with probability* 0.01	\$2400 *with certainty*
2	\$2500 *with probability* 0.33 \$0 *with probability* 0.67	\$2400 *with probability* 0.34 \$0 *with probability* 0.66

- 61% respondents choose 1B and 2A
  - Under EUT,
    - 1B implies $0.34v^{EUT}(2400) > 0.33v^{EUT}(2500)$
    - 2A implies $0.34v^{EUT}(2400) < 0.33v^{EUT}(2500)$
  - Under PT with $\alpha = 0.5$ and linear value function with zero as the reference point, the two choices established simultaneously

Paradox

Toy Problem: Wireless Random Access



- A set of *N* selfish players accessing the same base station
- A time-slotted and synchronous system
- Each player has a saturated queue of packets

- In a time slot, a player can either transmit or wait, $a_i \in A_i = \{t, nt\}$
  - $t = transmit \quad nt = NOT\ transmit$
- Pure strategy profile: $\boldsymbol{a} = \{a_1, a_2, \dots, a_N\}$
- Collection of pure strategy profiles:
  - $\boldsymbol{A} = A_1 \times A_2 \times \cdots \times A_N$

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A Wireless Random Access Game

- If a player transmits
  - A successful transmission: obtains a unit throughput reward c_i and incurs a unit energy cost e_i
  - A failed transmission: incurs a unit delay penalty d_i and a unit energy cost e_i
- If a player waits: incurs a unit delay penalty d_i
- For both PT and EUT, we assume players use same value function
  - linear in unit throughput reward, delay penalty and energy cost with reference point zero
- Fix a pure strategy profile $\boldsymbol{a} = \{a_1, \dots, a_N\}$, a player evaluates the possible outcomes as

$$v_{i|\mathbf{a}} = \begin{cases} p_{i|\mathcal{J}(\mathbf{a})}(c_i - e_i) + (1 - p_{i|\mathcal{J}(\mathbf{a})})(-e_i - d_i) & \text{if } a_i = t \\ -d_i & \text{if } a_i = nt \end{cases}$$

Packet Reception Probability

Set of players who transmit

A Wireless Random Access Game: Utility Functions

- Under Expected Utility Theory

j – th player's transmission probability

$$u_i^{EUT}(\mathbf{p}) = \sum_{\mathbf{a}\in\mathbf{A}} \left(\prod_{j\in\mathcal{J}(\mathbf{a})} p_j \prod_{k\notin\mathcal{J}(\mathbf{a})} (1-p_k) v_{i|\mathbf{a}} \right)$$

  - Objective expectation of values of all possible pure strategy profiles

- Under Prospect Theory

Strategy profile where the player transmits

Strategy profile where the player NOT transmit

$$u_i^{PT}(\mathbf{p}) = \sum_{\mathbf{a}_1\in\mathbf{A}, a_{1i}=t} SP(\mathbf{a}_1) v_{i|\mathbf{a}_1} + \sum_{\mathbf{a}_2\in\mathbf{A}, a_{2i}=nt} SP(\mathbf{a}_2) v_{i|\mathbf{a}_2}$$

  - Values of all possible pure strategy profiles are weighed by subjective probabilities

Subjective transmission probability of player j viewed by player i

$$SP(\mathbf{a}_1) = p_i \prod_{j\in\mathcal{J}(\mathbf{a}_1)\setminus\{i\}} w_i(p_j) \prod_{k\in\mathcal{J}^c(\mathbf{a}_1)} w_i(1 - p_k)$$

$$SP(\mathbf{a}_2) = (1-p_i) \prod_{j\in\mathcal{J}(\mathbf{a}_2)} w_i(p_j) \prod_{k\in\mathcal{J}^c(\mathbf{a}_2)\setminus\{i\}} w_i(1-p_k)$$

Consequence of Deviation from EUT?

- 2-Player Heterogeneous Game
  - One PT player and one EUT player
- What impact does the PT player have compared to a 2-player homogeneous EUT game?
  - Performance change of the EUT player
  - Performance difference between PT and EUT player
  - Overall system performance
- Metrics Studied
  - Average Energy
  - Average Throughput
  - Average Delay

Utility Functions and Performance Metrics (Linear)

- Utility Functions $i = 1, 2$
  - PT player:

$$u_i^{PT}(\mathbf{p}) = p_i w_i(p_j) v_{i|\{t,t\}} + p_i w_i(1-p_j) v_{i|\{t,nt\}} + (1-p_i)(-d_i)$$

  - EUT player:

$$u_i^{EUT}(\mathbf{p}) = p_i p_j v_{i|\{t,t\}} + p_i(1-p_j) v_{i|\{t,nt\}} + (1-p_i)(-d_i)$$

- Communication Performance Measures $i = 1, 2$

$$T_i(\mathbf{p}) = c_i \left(p_i p_j p_{i|\{i,j\}} + p_i(1-p_j) p_{i|\{i\}}\right)$$

Throughput rewards

$$E_i(\mathbf{p}) = p_i e_i$$

Energy Costs

Delay Penalties

$$D_i(\mathbf{p}) = d_i \left(p_i p_j (1 - p_{i|\{i,j\}}) + p_i(1-p_j)(1-p_{i|\{i\}})\right) + (1-p_i) d_i, \quad \text{for } i = 1, 2$$

Existence and Uniqueness of Mixed NE

- There exists a unique mixed NE for the Heterogeneous game if

$$v_{i|\{t,nt\}} > 0$$

- The value of a collision free transmission is "positive"

$$v_{i|\{t,t\}} < -d_i$$

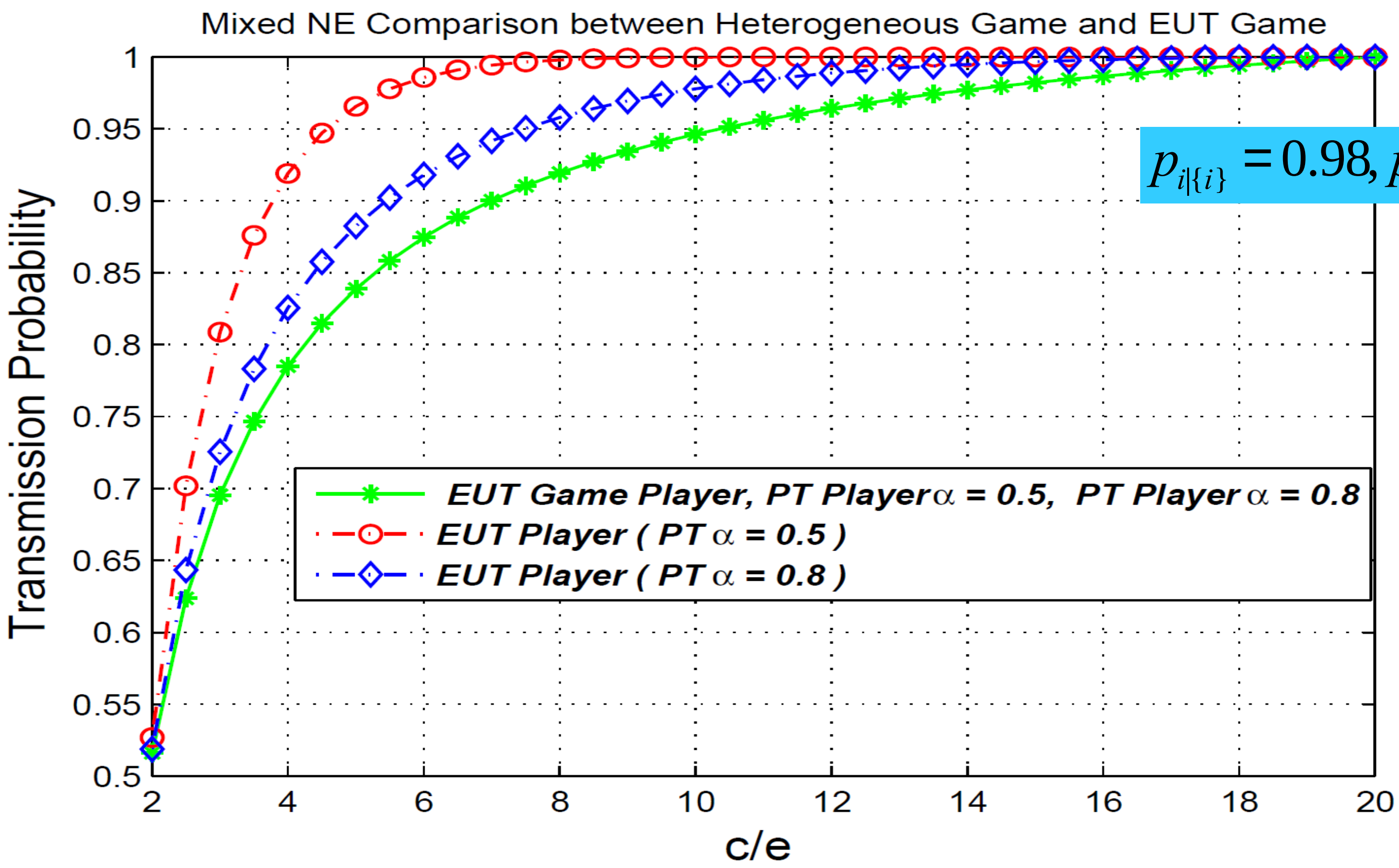
- A "negative" value results when there is a collision (simultaneous user transmission)
- The negative value is smaller than $-d_i$
  - d_i is the unit delay cost

Consequence of Deviation from EUT

Proven under mild conditions

- *Consequence 1*: The PT player causes the EUT player
  - To gain higher average throughput
  - To experience lesser average delay
  - To incur higher average energy costs
- *Consequence 2*: The PT player
  - Achieves lesser average throughput
  - Experiences greater average delay
- *Consequence 3*: System level performance degraded
  - Lower total average throughput
  - Greater total average delay
  - Higher total average energy costs
- All the trends are exaggerated with lower a

Transmission Probability at Mixed NE (d=0)



$p_{i|\{i\}} = 0.98, p_{i|\{i,j\}} = 0.05$

- EUT player if forced to transmit more aggressively
- If PT behavior is increasingly exaggerated, EUT player needs to be more aggressive

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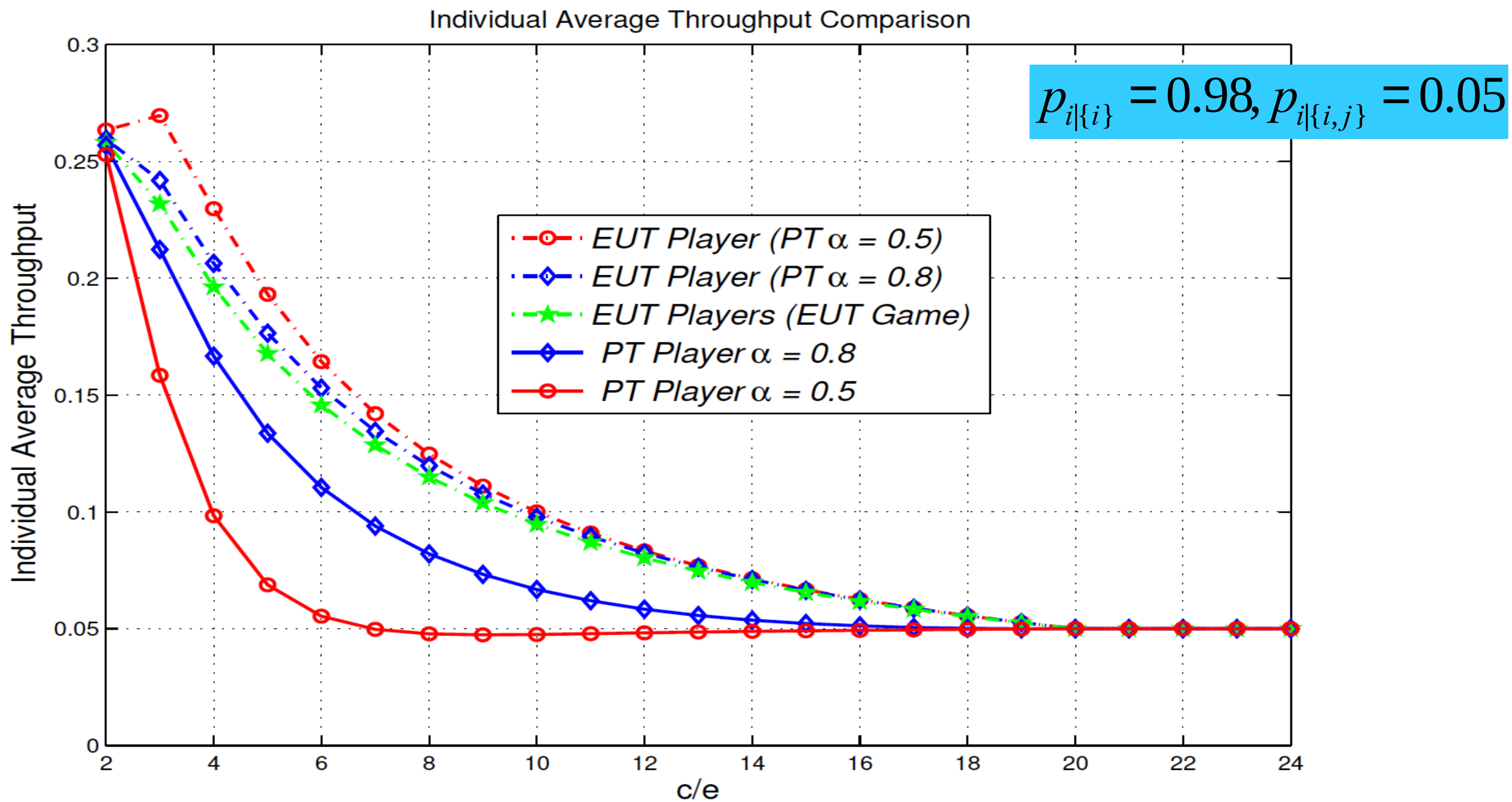
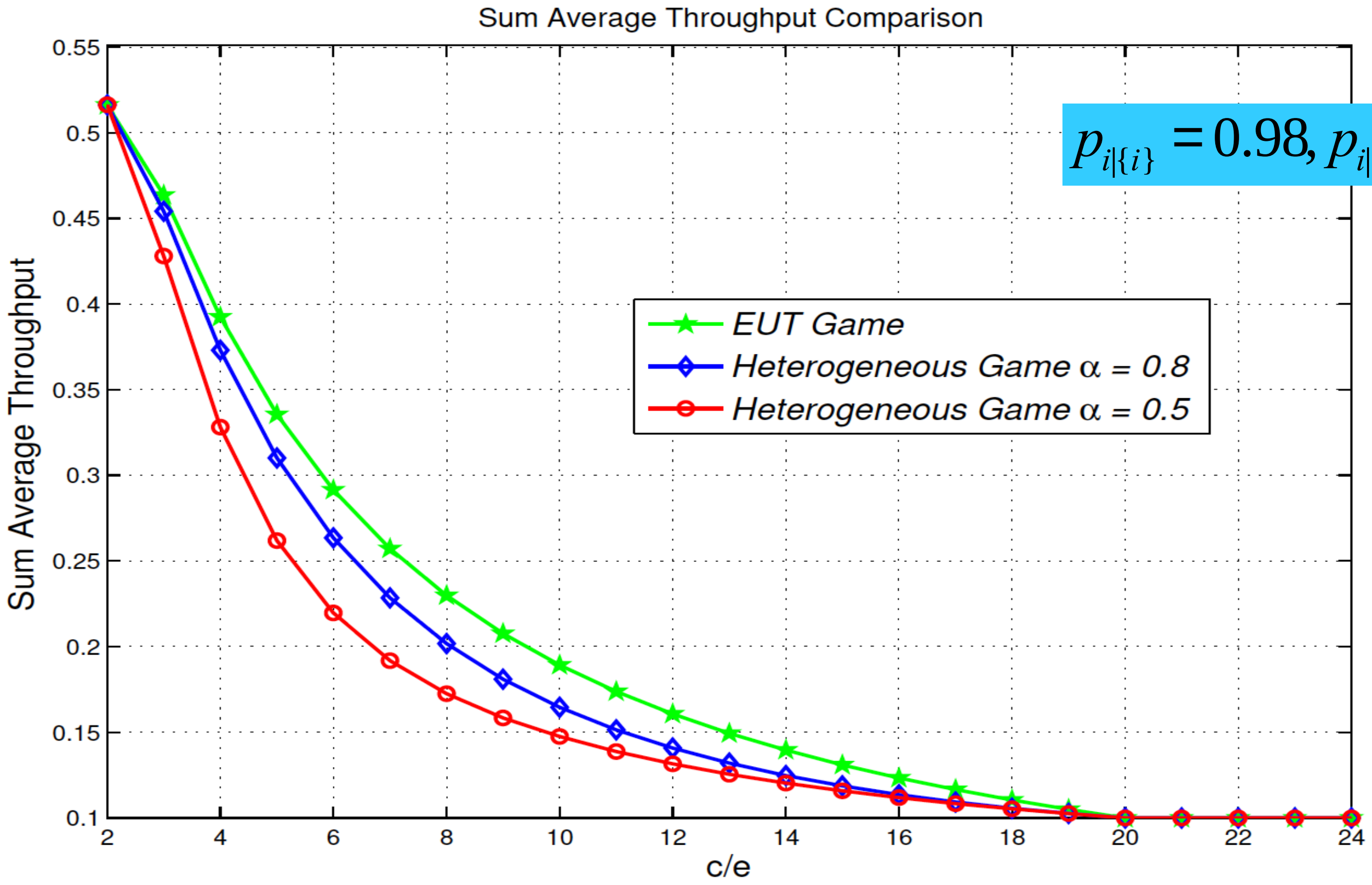
Individual Throughput Comparison (d=0)



- Introduction of PT player makes EUT player gain more throughput rewards
- EUT player obtains more than PT player
- A more deviated PT player exaggerates the two trends

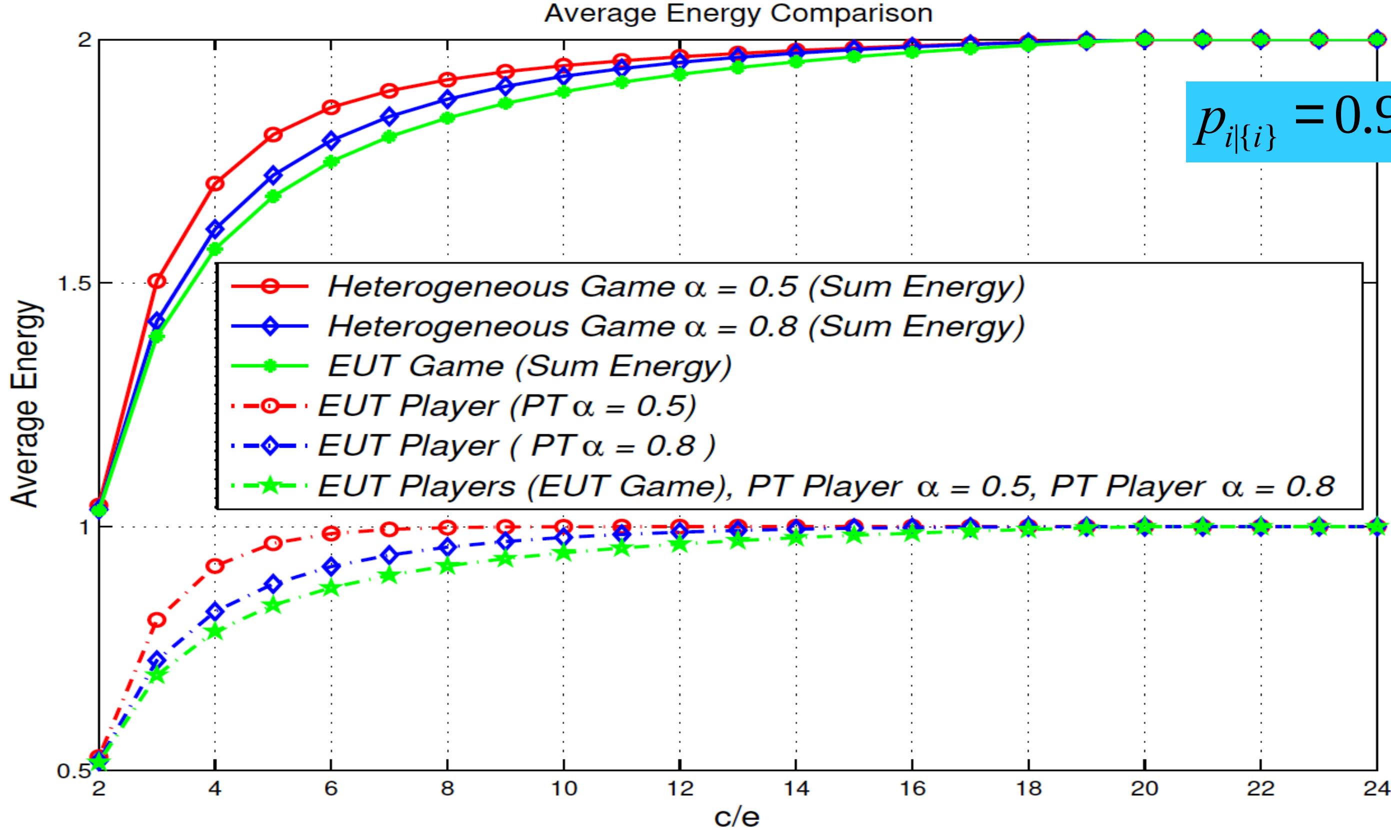
Sum Throughput Comparison (d=0)



$p_{i|\{i\}} = 0.98, p_{i|\{i,j\}} = 0.05$

- Total system throughput is degraded
- A more deviated PT player results in more severe degradation

Energy Costs Comparison (d=0)



$p_{i|\{i\}} = 0.98, p_{i|\{i,j\}} = 0.05$

- Introduction of PT player causes EUT player to incur higher energy costs
- Introduction of PT player incurs higher system sum energy costs
- A more deviated PT player exaggerate the two trends

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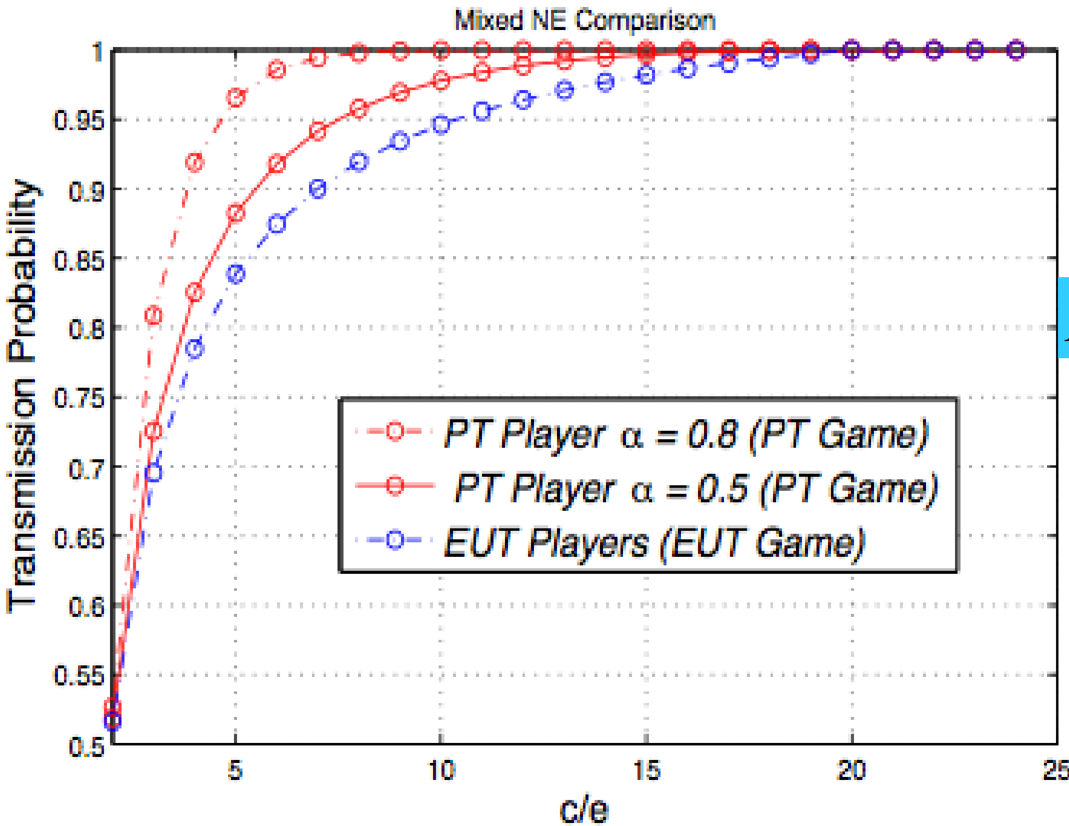
Homogeneous Game: Consequence of Deviation from EUT

- 2-Player Homogeneous Game
  - Two players are either both PT or both EUT
- *Consequence 4*: System level performance degraded
  - Lower total average throughput
  - Greater total average delay
  - Higher total average energy costs
- *Consequence 5*: The PT player deviating less from EUT
  - Achieves more average throughput
  - Suffers less average delay
  - But incurs more average energy cost

Transmission Probability at the mixed NE (d = 0)



Homogeneous PT vs EUT Game

$p_{i|\{i\}} = 0.98, p_{i|\{i,j\}} = 0.05$

- PT players in PT game transmit more aggressively than the players of EUT game
- Within PT game, PT player deviates less from EUT transmits more aggressively

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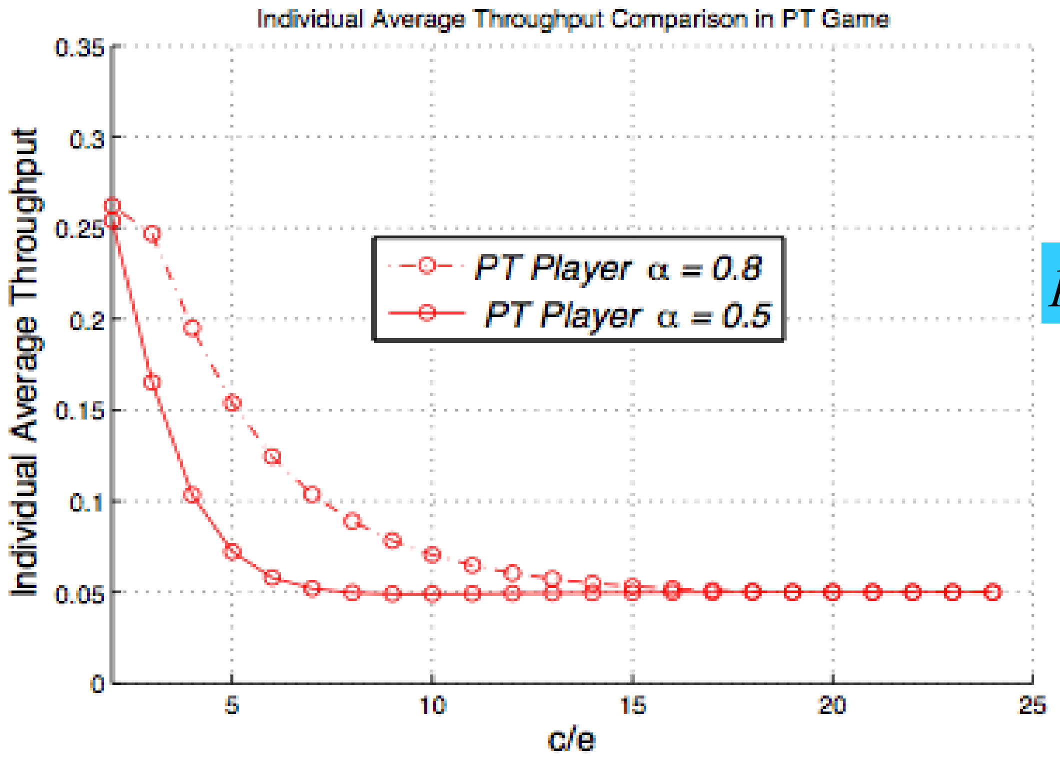
2-Player PT Game: Individual Average Throughput



$p_{i|\{i\}} = 0.98, p_{i|\{i,j\}} = 0.05$

- The PT player that deviates less from EUT obtains more average throughput

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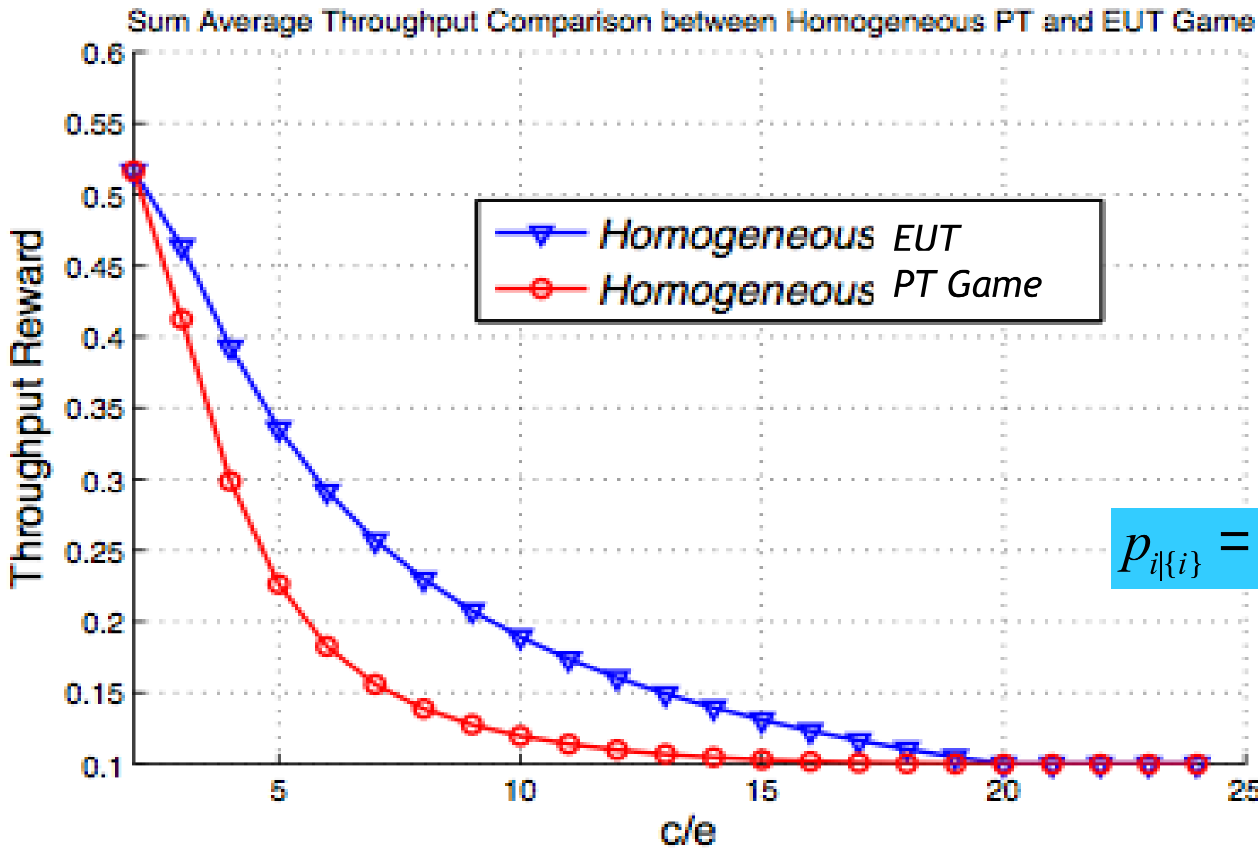
PT vs. EUT Game: Sum Average Throughput



$p_{i|\{i\}} = 0.98, p_{i|\{i,j\}} = 0.05$

- Players in homogeneous PT game achieve less sum average throughput in the EUT game

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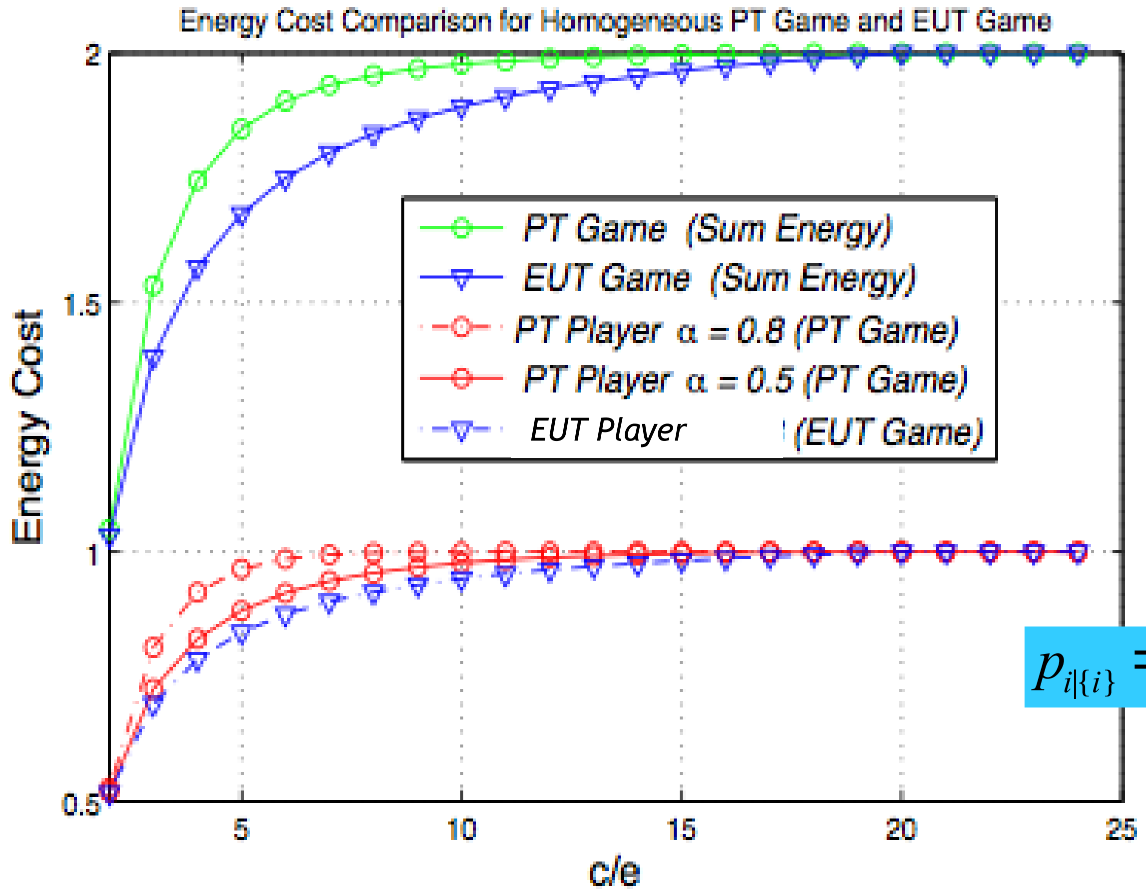
PT vs. EUT Game: Energy Costs



$p_{i|\{i\}} = 0.98, p_{i|\{i,j\}} = 0.05$

- Players in PT game incur higher energy costs than players in EUT game

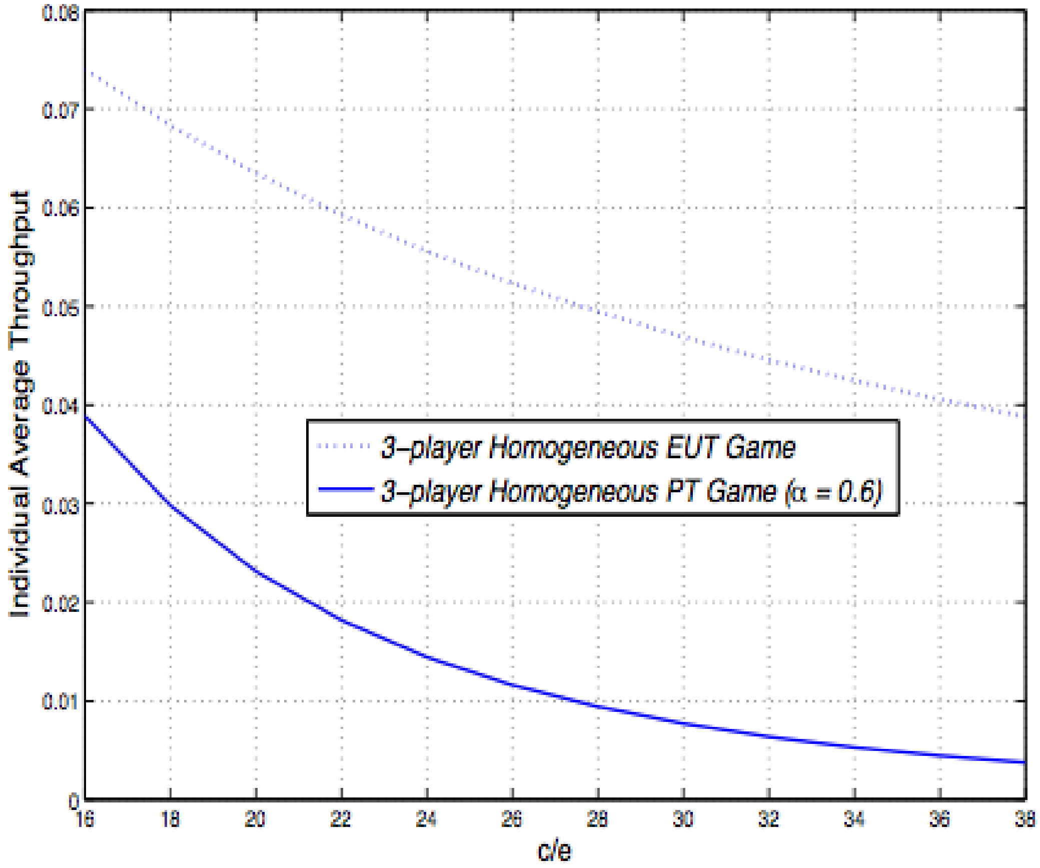
N-Player Homogeneous Game

- Symmetric: All players have identical utility functions and experience the same channel conditions

- Reflects a scenario where every player has a collective view of the set of players
  - "Collective" view of interference
  - Analyzing each of the other N-1 player's utilities and actions is beyond a single user's feasibility

- There exists a unique mixed NE for a symmetric N-player homogeneous game under mild conditions

3-Player Homogeneous Game: Average Throughput



- Fixed unit energy cost and unit delay penalty
- Degradation of average throughput

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Prospect Theory: Wireless Applications

- Differentiated Pricing of Data Services for Network Congestion
  - User preferences, biases and perceived values
- SoNs – “organization/action” of people?
- Jamming in Wireless Networks
  - Biases and perceptions
- Robust Mechanisms for mitigating “user interference”
- Psychophysics experiments of wireless users
  - Design appropriate weighting and framing effects based on “wireless” experience

References

- T. Li and N. B. Mandayam, **"Prospects in a Wireless Random Access Game"** Proceedings of CISS'2012, Princeton NJ, March 2012
- T. Li and N. B. Mandayam, **"When Users Interfere with Protocols: Prospect Theory in Wireless Networks using Random Access as an Example"** under revision in IEEE Transactions in Wireless Communications, 2013