Impact of End-User Decisions on Pricing in Wireless Networks

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***Abstract*—We consider the impact of end-user decision-making on pricing of wireless resources when there is uncertainty in the Quality of Service (QoS) guarantees offered by the Service Provider (SP). Specifically, we consider a scenario where an SP tries to sell wireless broadband services to a potential customer when the advertised transmission rate cannot be fully guaranteed at all times. Relying on Prospect Theory (PT), a Nobel Prize winning theory developed by Kahneman and Tversky to explain people's real life decision-making that often violates the principles of the Expected Utility Theory (EUT), we formulate a game theoretic model to study the interplay between price offerings of the SP and the service choices made by the end-user. We characterize the Nash Equilibrium (NE) of the underlying game and provide insights into the impact of the deviations of decision-making from that expected under EUT and show how pricing can be used as a mechanism to mitigate such impact.**

***Index Terms*—Game Theory, Nash Equilibrium, Expected Utility Theory, Prospect Theory, Probability Weighting**

I. Introduction

The demand for wireless broadband internet service has increased dramatically over the past decade thanks to the fast advancing pace of the physical layer wireless technologies [1] and the invention of the smart phones and tablets. As an interest of the Service Providers (SPs) offering such kind of services, considerable effort has been devoted to finding a proper pricing strategy of such services, so that the SPs can maximize their profit while effectively maintaining the traffic below the congestion level [2]. On the other hand, as an interest of the users, effort has been made to study the impact that different aspects of the network's performance may have on the users [3][4]. Despite the efforts made on both ends, however, not too much work focuses on the fact that the actual performances delivered by the Service Providers (SPs) do not always meet the advertised rates, and how knowing this piece of information could affect a user's decision-making process when choosing between different retail packages.

In an attempt to measure the actual delivered service from the SPs as well as to fully inform the users about the actual wireless broadband service they are purchasing, the Federal Communications Commission (FCC) has recently launched a project in which users can send feedback to the FCC via a smart phone application, which collects information including the down link and up link rates, latency, as well as packet loss of the wireless network [5]. Meanwhile, a report published by the FCC which measured the actual rates delivered by the SPs shows that the advertised rates are not fully delivered by most of the SPs all the time even for the wired internet service [6]. In light of this, we investigate the problem where an SP tries to sell its wireless broadband service to a potential customer over a wireless channel. In particular, we focus on modeling the fact that the channel might not be capable of delivering the advertised service rate at full extent all the time and this piece of information is available to the customer when making the decision as the probability, or the portion of time in the long run, that the advertised service rate can be successfully met.

However, it has been well known in economics that, when risks are involved, people can systematically violate the principles of the Expected Utility Theory (EUT) [7]. In an effort to study the consequences of this violation in the context of wireless services, [8] investigated the scenario of data pricing in wireless random access where the users do not objectively evaluate their probability of successfully accessing the channel. In the scenario of deciding whether or not to purchase service over a channel that might not be capable of delivering the advertised service rate all the time, the user's inaccurate weighting of probability directly impacts her judgment on whether the service is worth purchasing, and thus potentially increasing the burden on the radio resources owned by the SP. In this paper, we study the impact of the end-user's inaccurate view of the service guarantee on her decision-making process through the formulation of a Stackelberg game. Specifically, we consider the case when the end-user decision making follows PT and compare and contrast it to the case of EUT based decision-making. We study the impact on the profit of the SP and also discuss the possibility of fully recovering the revenue, as well as preserving the NE of the EUT game via "prospect pricing," a term we use to describe the use of pricing as a control mechanism to overcome end-user behavior..

The rest of the paper is organized as follows: in Section II, we introduce the model of the interactions between the user and the SP as a Stackelberg game, and discuss conditions under which the existence of an NE can be guaranteed; in section III, we discuss the impact of the Probability Weighting

Effect (PWE) on the end-user's decision-making, the revenue of the SP, as well as the prospect of recovering the revenue of the EUT game or maintaining the NE of the EUT game by selecting a different pricing function; numerical results are shown in Section IV while conclusions and discussions are presented in Section V; finally, we present the proofs of the theorems in the Appendix.

II. A Stackelberg Game Model

We investigate the preliminary scenario where there is a monopoly SP and one potential user. The interactions between the SP and the user are modeled using a Stackelberg game, where the SP makes an offer first and the user decides whether or not to accept the offer with some probability p. We define an offer made by the SP as the triple $\{b, r(b), \bar{F}_B(b)\}$, which corresponds to the rate b, the price of the service at that rate determined by a predefined function $r(b)$, and the guarantee level of the service at b, defined by

$$\bar{F}_B(b) := \mathbb{P}(B > b). \tag{1}$$

Denote the user's utility upon receiving guaranteed service at rate b with $h(b)$. If she accepts the offer at rate b, she pays a price $r(b)$, and with probability $\bar{F}_B(b)$ she receives successful service, while with probability $F_B(b) := 1-\bar{F}_B(b)$ the channel cannot successfully deliver the service at rate b and the user experiences an outage. Hence, the expected utility of the user when she chooses p as the probability of accepting the offer at rate r, can then be represented as

$$\begin{aligned} U_{user}(p,b) = p\left[-r(b) + h(b)\bar{F}_B(b) + h(0)F_B(b)\right] + \\ + (1-p)h(0). \end{aligned} \tag{2}$$

As for the SP, a cost $c(b)$ is incurred upon her when she makes an offer at rate b. Hence, the expected utility of the SP is

$$U_{SP}(p,b) = p\left[r(b) - c(b)\right] + (1-p)(-c(b)). \tag{3}$$

As natural assumptions, we assume that $r(b)$ is monotonically increasing and concave, due to the fact that a customer that buys more is usually awarded with a lower unit price; we also assume that $h(b)$ is monotonically increasing, concave, obeying the law of diminishing sensitivity. In addition, we assume that

$$c(b) = c_1 b + c_2, \tag{4}$$

indicating that the cost mainly consists of the investment in bandwidth proportional to b, and a fixed cost. As for the guarantee of the channel, we assume that

$$\lim_{b\to\infty} \bar{F}_B(b) = 0. \tag{5}$$

With these above settings, we have the following results for the game, the proofs of which are given in the Appendix.

Lemma 1. *The best response of the SP given the acceptance probability of the user p, denoted by b_p^*, is a monotonically increasing function w.r.t. p.*

Lemma 2. *Under the NE, which we denote using (p^*, b^*), we have $b^* \in [0, b_1^*]$ and $p^* \in (p_{min}^*, 1]$ when $b^* > 0$, where*

$$p_{min}^* = \frac{c_1}{\lim_{\varepsilon\to 0_+} \frac{d}{db} r(b)|_{b=\varepsilon}}, \tag{6}$$

and

$$b_1^* = \operatorname*{argmax}_{b'} \left(r(b') - c(b')\right). \tag{7}$$

Lemma 3. *For p_1, p_2 s.t., $p_{min}^* < p_1 < p_2 \le 1$, we have $U_{SP}(p_1, b_{p_1}^*) < U_{SP}(p_2, b_{p_2}^*)$.*

The above results can be intuitively explained as follows. First of all, Lemma 1 indicates that the SP should offer a lower rate when the user is less likely to accept the offer. As a consequence, the SP is likely to achieve less profit when the user is less committed, which is stated by Lemma 3. Lemma 2 implies that, in order to achieve an NE where neither the user nor the SP has the incentives to deviate unilaterally, the rate provided by the SP must not exceed b_1^*, which follows directly from Lemma 1, while the acceptance probability of the user must be greater than p_{min}^* if $b^* > 0$. This is because, when the user's acceptance probability is lower than p_{min}^*, the SP cannot make any profit by making an offer with rate $b > 0$, hence the best response for the SP in this case is to simply walk away from the table.

With the help of the above lemmas, we state the following conclusions.

Theorem 1 (The existence of multiple Nash Equilibria (NE))**.** *If $\exists b > 0$, s.t.,*

$$c(b) - c(0) < r(b) \le \left[h(b) - h(0)\right] \bar{F}_B(b), \tag{8}$$

then there exist $\mathbb{1} + |S|$ NE of the form (p^*, b^*), in which $b^* \in S$,

$$S = \left\{b \in (0, b_1^*) : \left[h(b) - h(0)\right] \bar{F}_B(b) = r(b)\right\}, \tag{9}$$

while $\mathbb{1}$ equals 1 if b_1^* satisfies (8) and 0 otherwise.

In particular, if there exists a threshold $b_0 > 0$, such that the service is over-priced when $b > b_0$, i.e., $\left[h(b) - h(0)\right] \bar{F}_B(b) < r(b)$, and under-priced when $b < b_0$, i.e., $\left[h(b) - h(0)\right] \bar{F}_B(b) > r(b)$, then the NE is unique in $(p_{min}^*, 1] \times (0, b_1^*]$. We also point out that $\forall p \in [0, 1]$, $(p, 0)$ is an NE. However, we care more about the NE where $b > 0$, which predicts a stable state where an offer is actually made and accepted. Hence, we assume in the rest of this paper, that $b^* > 0$.

Among the multiple NE of the game given a fixed pricing function $r(\cdot)$, the one we are most interested in is the one that maximizes the expected utility of the SP. This particular NE can be found by following the theorem below.

Theorem 2. *Among all NE of the game, the one with the highest offered service rate yields the largest expected utilities for both the SP and the user.*

In practice, the SP can select the rate under this particular NE to maximize her profit under an NE.

III. The impact of Prospect Theory

Prospect Theory, a theory developed by D. Kahneman and A. Tversky, models people's decision process under risk. It explains some of the paradoxes of real-life decision-making that cannot be explained by EUT. We omit the general background of PT in this paper, and instead refer the readers to the original paper [7] for an introduction, [9] for a review of the recent progress on the theory, and [8] for a brief tutorial. Here we only give a brief introduction to the PWE, which we adopt in the rest of this section to model the end-user's decision-making process under PT.

A. The PWE

Among the various procedures in [7], the PWE is one of the most important elements of PT. It reveals the underlying trend of how people weight different outcomes of an alternative. Unlike EUT, which postulates that people calculate the expected utility of each alternative by multiplying the utilities of the outcomes with their corresponding probability of occurrence and then adding them up, PT postulates that people will substitute the probability of occurrence with a subjective weight, which is, for most of the time, inaccurate. Experimental results reveal that people tend to over-weight events with small occurrence probability and under-weight the events with large probability. As an analytical way of describing the relationship between the objective occurrence probability of an event and its subjective weight in a person's mind, various probability weighting functions have been proposed [7][10]. In the numerical results presented here, we consider Prelec's probability weighting function $w(p) = \exp\{-(-\ln p)^{\alpha})\}$, first proposed in [10]. However, most of the conclusions in this paper can be easily generalized to other probability weighting functions.

B. The impact of PT on the end-user's decision

Under the influence of PWE, the user now makes her decision based on her prospect rather than the actual utility. Denote the user's prospect under PWE as $U_{user,PT}(p, b)$, the expression of which is represented as follows.

$$U_{user,PT}(p,b) = p\left[-r(b) + (h(b) - h(0))w(\bar{F}_B(b))\right] + (1-p)h(0). \tag{10}$$

This yields from the result according to [7], which states that the user will focus on the difference between her options. Hence, she will accept the offer with probability 1 if

$$[h(b) - h(0)]\, w(\bar{F}_B(b)) > r(b). \tag{11}$$

She will reject the offer if

$$[h(b) - h(0)]\, w(\bar{F}_B(b)) < r(b). \tag{12}$$

She will accept the offer with probability $p \in (0, 1]$ if

$$[h(b) - h(0)]\, w(\bar{F}_B(b)) = r(b). \tag{13}$$

Theorem 3. *Assume that (p^*_{EUT}, b^*_{EUT}) is an NE under the EUT game, where p^*_{EUT} is the acceptance probability of the user corresponding to the offer $\{b^*_{EUT}, r(b^*_{EUT}), \bar{F}_B(b^*_{EUT})\}$. If the same offer is made and the user follows the decision-making process of PT, then if $p^*_{EUT} < 1$, the user will reject the offer with probability 1 when the guarantee level of the service is under-weighted, i.e., $w(\bar{F}_B(b^*_{EUT})) < \bar{F}_B(b^*_{EUT})$ and accept the offer with probability 1 when $w(\bar{F}_B(b^*_{EUT})) > \bar{F}_B(b^*_{EUT})$. If $p^*_{EUT} = 1$, then the user will accept the same offer if and only if $[h(b^*_{EUT}) - h(0)]\, w(\bar{F}_B(b^*_{EUT})) \geq r(b^*_{EUT})$.*

Under EUT, the cases of $p^*_{EUT} = 1$ and $p^*_{EUT} < 1$ correspond to the scenarios where the user will accept the offer with and without certainty, respectively. As stated in the above theorem, PWE will definitely affect the eventual form of the decision of the end-user when $p^*_{EUT} < 1$. This is because when $p^*_{EUT} < 1$, under-weighting the guarantee level of the service $\bar{F}_B(b^*_{EUT})$ means that the user will experience the feeling that the service is over-priced, while over-weighting the guarantee level of the service indicates that the user will most likely feel the opposite. On the other hand, when $p^*_{EUT} = 1$, the user's prospect is less than her actual utility when $\bar{F}_B(b^*_{EUT})$ is under-weighted, thus she will only accept the offer if either (11) or (13) holds. When $p^*_{EUT} = 1$ and $\bar{F}_B(b^*_{EUT})$ is over weighted, the user will accept the offer for sure. PWE will not affect the end-user's decision when the rate can be guaranteed at all times as PWE does not affect events that happens with probability 1.

C. Prospect Pricing

In this subsection, we propose pricing as a way of mitigating PWE. However, changing the pricing function often affects the rate as well as the corresponding acceptance probability of a NE, and thus creating a different burden on the radio resources owned by the SP. Particularly, we highlight the following Radio Resource Management (RRM) constraints,

$$p^*_{EUT} = p^*_{PT}, \tag{14}$$
$$b^*_{EUT} = b^*_{PT}, \tag{15}$$

and answer the following questions.

- Given $r_{EUT}(\cdot)$, and a NE (p^*_{EUT}, b^*_{EUT}), how to choose $r_{PT}(\cdot)$ such that the RRM constraints are met?
- Can we fully recover the revenue when we are forced to meet the RRM constraints under the PT game? If not, can we fully recover the revenue when the RRM constraints are relaxed?

Proposition 1 (Preserving the RRM constraints under the PT game)**.** *Assuming that the service guarantee is under-weighted by the user, then as long as*

$$[h(b^*_{EUT}) - h(0)]w(\bar{F}_B(b^*_{EUT})) > c(b^*_{EUT}), \tag{16}$$

we can always maintain (p^*_{EUT}, b^*_{EUT}) as an NE under the PT game by applying a concave pricing function, which can be chosen as

$$r_{PT}(b) = \gamma(r_{EUT}(b) - c(b)) + c(b), \tag{17}$$

where

$$\gamma = \frac{r_{EUT}(b^*_{EUT})w(\bar{F}_B(b^*_{EUT})) - c(b^*_{EUT})\bar{F}_B(b^*_{EUT})}{\left(r_{EUT}(b^*_{EUT}) - c(b^*_{EUT})\right)\bar{F}_B(b^*_{EUT})}. \tag{18}$$

Although the choice of the pricing function is not unique, the following two conditions combine to serve as a set of necessary and sufficient conditions for constructing a concave pricing function $r_{PT}(b)$:

Condition 1. *The value of $r_{PT}(b)$ is constrained by*

$$r_{PT}(b^*_{EUT}) = r_{EUT}(b^*_{EUT})\frac{w(\bar{F}_B(b^*_{EUT}))}{\bar{F}_B(b^*_{EUT})}, \tag{19}$$

if $p^*_{EUT} < 1$. If $p^*_{EUT} = 1$, then

$$r_{PT}(b^*_{EUT}) \leq [h(b^*_{EUT}) - h(0)]w(\bar{F}_B(b^*_{EUT})). \tag{20}$$

Condition 2. *The maximum of $r_{PT}(b) - c(b)$ is constrained by*

$$b^*_{EUT} = \underset{b}{\operatorname{argmax}} \left\{p^*_{EUT} r_{PT}(b) - c(b)\right\}. \tag{21}$$

In particular, for a concave, derisive pricing function $r_{PT}(b)$, the derivative of the right hand side of the above equation must be 0.

Next, we present our result on the SP's ability to recover her revenue.

Theorem 4. *Assume that the service guarantee is underweighted, and the RRM constraints are to be preserved under the PT game. If $p^*_{EUT} < 1$, then the SP will obtain less revenue regardless of the choice of the pricing function. If $p^*_{EUT} = 1$, then the SP will be able to recover the revenue fully if and only if*

$$U_{SP,EUT}(p^*_{EUT}, b^*_{EUT}) \leq [h(b^*_{EUT}) - h(0)]w(\bar{F}_B(b^*_{EUT})) - c(b^*_{EUT}). \tag{22}$$

Finally, we observe the following conclusion when the RRM constraints are allowed to be relaxed.

Theorem 5. *Denote the NE under the EUT game as (p^*_{EUT}, b^*_{EUT}) and the NE under the PT game as (p^*_{PT}, b^*_{PT}), then under the PT game, the revenue $U_{SP,EUT}(p^*_{EUT}, b^*_{EUT})$ can be fully recovered if and only if*

$$U_{SP,EUT}(p^*_{EUT}, b^*_{EUT}) \leq p^*_{PT}[h(b^*_{PT}) - h(0)]w(\bar{F}_B(b^*_{PT})) - c(b^*_{PT}). \tag{23}$$

As can be easily seen, the second part of Theoreom 4 is a special case of Theorem 5. However, for any $p^*_{EUT} < 1$, the revenue cannot be fully recovered if the RRM constraints are enforced under the PT game. This is because the SP must pay for the user's skewed view of the service offer in order for the user to accept the offer with some probability p.

IV. Numerical Results

In this section, we present a demonstration of some of the conclusions drawn above. In particular, we compare the NE under the EUT game with its counterpart under the PT game; compare the behavior of the SP's utility under an NE of the EUT game and its counterpart under the PT game when the RRM constraints are met/relaxed; compare the behavior of the user's surplus under the EUT game and under the PT game when the RRM constraints are met/relaxed.

Consider a Rayleigh block fading channel, whose guarantee of service can be expressed as [11]

$$\bar{F}_B(b) = \exp\left\{-\frac{2^{\frac{b}{BW}} - 1}{P/N_0 BW}\right\}. \tag{24}$$

Here, P is the transmission power, while N_0 is the power spectral density (PSD) of the noise. Meanwhile, BW represents the bandwidth, and b represents the encoding rate of the SP.

In Figure 1, we locate the NE under the EUT and PT games by plotting the best response functions of the SP and the user. PWE is parameterized by α, and for each α, the best response of the user is plotted using a dashed line as a function of the encoding rate of the SP, while the best response of the SP is plotted using a solid line as an inverse function of the user's acceptance probability. The figure shows that, when the user follows the decision-making process of PT, her best response for the offer that induces the NE under the EUT game becomes $p^*_{PT} = 0$. In addition, for the particular choice of the pricing function used in Figure 1, PWE affects the system in a way such that the user tends to accept an offer with a lower rate with a smaller probability, and the revenue of the SP tends to decrease. In Figure 2, the same procedure is demonstrated, except that the initial pricing function under the EUT game is selected differently. Under this pricing function, PWE affects the system in a way where the user tends to accept an offer containing an higher rate with a larger probability, and the actual utilities for both the SP and the user increase.

Figure 3 shows the behavior of the revenue of the SP under the EUT game and the PT game. Firstly, we plot the SP's revenue under the NE of the EUT game as a function of the rate $b^* \in (0, b^*_1)$ under the NE when the pricing function $r_{EUT}(b)$ is predefined. This can be done because when the pricing function is fixed, each $b^* \in (0, b^*_1)$ corresponds to a unique p^*. Next, we plot the revenue of the SP under the PT game when the RRM constraints are preserved. We see from the figure that when α becomes smaller, i.e., when PWE impacts the user more severely, the SP sacrifices more revenue in the effort of maintaining the satisfaction of the RRM constraints. Finally, we plot the revenue of the SP under the NE when the RRM constraints are partially relaxed, i.e., the pricing function is changed such that $b^*_{PT} = b^*_{EUT}$ while p^*_{PT} is not constrained to the value p^*_{EUT}. From the figure, we can see that the maximum retainable revenue decreases as α decreases. It can also be seen that for each α, the SP is able to recover her revenue of the EUT game when the RRM

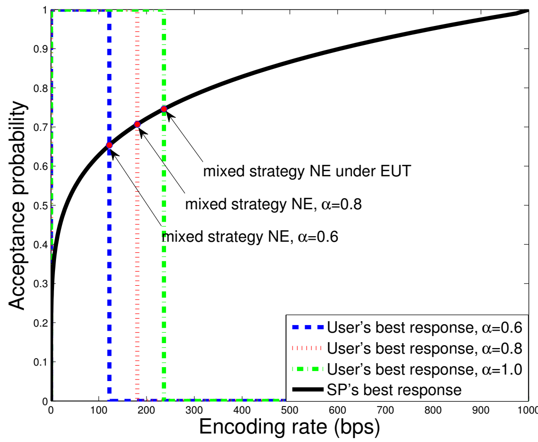

Fig. 1. NE under different levels of PWE. $N_0 = 10^{-3}W/Hz$, $BW = 0.2bHz$, $P = 10W$, $r(b) = 0.5b^{0.8}$, $c(b) = 0.5BW$, $h(b) = b^{0.7}$.

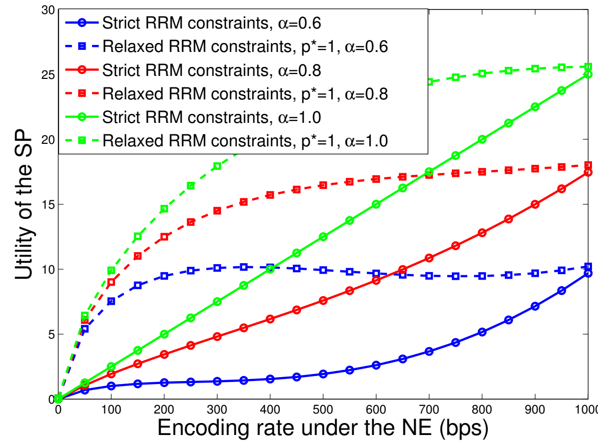

Fig. 3. SP's utility under the NE of the EUT game and the PT games with different levels of PWE. $N_0 = 10^{-3}W/Hz$, $BW = 0.2bHz$, $P = 10W$, $r_{EUT}(b) = 0.5b^{0.8}$, $c(b) = 0.5BW$.

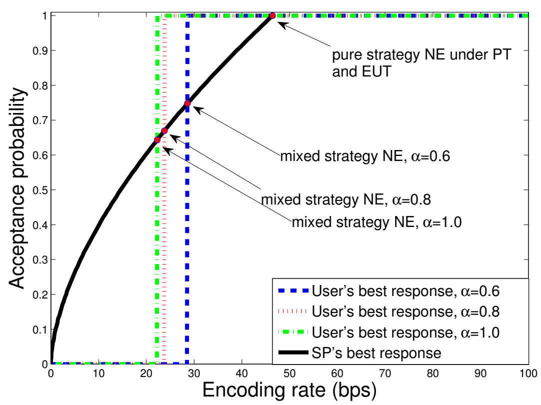

Fig. 2. NE under different levels of PWE. $N_0 = 10^{-3}W/Hz$, $BW = 0.2bHz$, $P = 10W$, $r(b) = 3b^{0.4}$, $c(b) = 0.5BW$, $h(b) = b^{0.7}$.

constraints are relaxed up to a certain rate. When α decreases, this rate also decreases, indicating that the possibility that the SP is able to recover her revenue is smaller. However, in a generalized multiple user scenario (although not modeled in this paper), the SP bears a heavier burden on the network she owns by relaxing the RRM constraints to recover the revenue. Note that in the figure, each b^* corresponds to a distinct user with a distinct utility function $h(b)$, while for the same b^*, all the curves can be regarded as for a particular user.

V. Conclusion

In this paper, we studied the impact of the end-user decision-making on pricing of the wireless resources when the QoS guarantee offered by the SP contains uncertainty. Particularly, we formulated a Stackelberg game and studied the interplay between price offerings of the SP and the decisions of the end-user. We characterized the NE under the EUT game, and studied the impact on the NE of the game when the end-users make their decisions relying on PT. We also showed that the RRM constraint can be preserved by changing the pricing function, and the revenue of the SP can be fully recovered under certain conditions.

Appendix

A. Proof of Lemma 1

Proof. Since

$$b_p^* = \arg_{b'} \left\{ \frac{d}{db} \left[r(b) \right]_{b=b'} = \frac{c_1}{p} \right\}, \tag{25}$$

where $r(b)$ is assumed to be a monotonically increasing but concave function, it can be seen that b_p^* decreases as $\frac{c_1}{p}$ increases. Hence, it is a monotonically increasing function w.r.t. p. □

B. Proof of Lemma 2

Proof. According to Lemma 1, $b^* \leq b_1^* = \text{argmax}_{b'} U_{SP}(1, b') = \text{argmax}_{b'}(r(b') - c(b'))$. For any NE with $b^* > 0$, since b^* is the best response of p^*, we must have a positive solution for equation (25), i.e.,$\frac{c_1}{p^*} < \lim_{\varepsilon \to 0_+} \frac{d}{db} r(b)|_{b=\varepsilon}$. □

C. Proof of Lemma 3

Proof. Since $\forall b \in [0, b_1^*]$, $U_{SP}(p_1, b) < U_{SP}(p_2, b)$, we have $U_{SP}(p_1, b_{p_1}^*) < U_{SP}(p_2, b_{p_1}^*) \leq \text{argmax}_b U_{SP}(p_2, b) = U_{SP}(p_2, b_{p_2}^*)$. □

D. Proof of Theorem 1

Proof. If $r(b_1^*) \in \left(c(b) - c(0), \left[h(b_1^*) - h(0) \right] \bar{F}_B(b) \right]$, then $(1, b_1^*)$ is an NE as $U_{user}(1, b_1^*) \geq U_{user}(p, b_1^*)$, and $U_{SP}(1, b_1^*) \geq U_{SP}(1, b)$ for all $p \in [0, 1]$ and $b > 0$ respectively. For $b \in (0, b_1^*)$, we show that (p, b) can only be an NE if $b \in S$. This is because, if $[h(b) - h(0)] \bar{F}_B(b) > r(b)$, then $p_b^* = 1$. Thus, as a best response, the SP should provide the rate b_1^*, which contradicts the assumption that $b \in (0, b_1^*)$. On the other hand, if $[h(b) - h(0)] \bar{F}_B(b) < r(b)$, then $p_b^* = 0$, the best response of the SP for which is 0, contradicting the assumption that $b \in (0, b_1^*)$. Next, we show that if $b \in S$, then there exists a unique p_b^* s.t. (p_b^*, b) is an NE. This is because when $b \in S$, $\forall p \in [0,1]$ is a best response of the user. Hence, the only p that ensures that (p,b) is an NE is the one such that $b = \text{argmax}_{b'} U_{SP}(p, b')$. The solution to this equation exists and is unique according to Lemma 1 and Lemma 2. □

E. Proof of Theorem 2

Proof. If $(p^*_{(1)}, b^*_{(1)})$ and $(p^*_{(2)}, b^*_{(2)})$ are two NE under the same pricing function of the EUT game, and $b^*_{(1)} < b^*_{(2)}$, then $p^*_{(1)} < p^*_{(2)}$. This is because, according to Lemma 1, if $p^*_{(1)} > p^*_{(2)}$, then $b^*_{(1)} > b^*_{(2)}$, contradicting the assumption. Hence, the proof follows by further using the result of Lemma 3. □

F. Proof of Theorem 3

Proof. When $p^*_{EUT} < 1$, the user receives 0 utility (assuming $h(0) = 0$) as she is indifferent between accepting the offer and rejecting it, i.e., $[h(b^*_{EUT}) - h(0)]\,\bar{F}_B(b^*_{EUT}) = r(b^*_{EUT})$. Hence, when the guarantee level of the service is underweighted, $[h(b^*_{EUT}) - h(0)]\,w(\bar{F}_B(b^*_{EUT})) < r(b^*_{EUT})$. The user will thus reject the offer.

When $p^*_{EUT} = 1$, the user receives positive utility under the EUT game, thus $(1, b^*_{EUT})$ will be an NE if under PWE $[h(b^*_{EUT}) - h(0)]\,w(\bar{F}_B(b^*_{EUT})) \geq r(b^*_{EUT})$. The sufficiency of the condition can be proved as follows. If the condition is satisfied, then $p^*_{PT} = 1$ is a best response to the offer $\{b^*_{EUT}, r_{EUT}(b^*_{EUT}), \bar{F}_B(b^*_{EUT})\}$. However, we know that b^*_{EUT} is the best response under the EUT game when the acceptance probability is 1, and the pricing function is the same, hence $(1, b^*_{EUT})$ is still an NE under PWE. □

G. Proof of Proposition 1 and the Conditions

Proof. The function given in the Proposition satisfies the two Conditions, the proof of which is as follows.

We first show the sufficiency of the Conditions. If $p^*_{EUT} = 1$, and (20) is satisfied, then $p^*_{PT} = 1$ is the best response for the offer $\{b^*_{EUT}, r_{PT}(b^*_{EUT}), \bar{F}_B(b^*_{EUT})\}$ under the PT game. Since the second Condition is also satisfied at the same time, $b^*_{PT} = b^*_{EUT}$ is the best response of the SP under the PT game. Hence (p^*_{EUT}, b^*_{EUT}) is preserved as an NE under the PT game. Similarly, when $p^*_{EUT} < 1$, we have $r_{EUT}(b^*_{EUT}) = [h(b^*_{EUT}) - h(0)]\bar{F}_B(b^*_{EUT})$. Thus, when (19) is satisfied, p^*_{EUT} is a best response for the offer $\{b^*_{EUT}, r_{PT}(b^*_{EUT}), \bar{F}_B(b^*_{EUT})\}$ because $r_{PT}(b^*_{EUT}) = [h(b^*_{EUT}) - h(0)]w(\bar{F}_B(b^*_{EUT}))$. Since the second Condition is also satisfied, $b^*_{PT} = b^*_{EUT}$ is the best response for the user's acceptance probability under the PT game. Hence (p^*_{EUT}, b^*_{EUT}) is preserved as an NE under the PT game. To ensure that the existing pricing function is concave, we must have $\frac{r^*_{PT}(b^*_{EUT})}{b^*_{EUT}} > \frac{d}{db}c(b)|_{b^*_{EUT}}$, which is guaranteed when $r_{PT}(b^*_{EUT}) \geq c(b^*_{EUT})$, a condition that is automatically satisfied under our general assumption that $p^*_{EUT} > 0$, and when the second Condition is satisfied.

We finally prove the necessity of the Conditions 1 and 2. In order for (p^*_{EUT}, b^*_{EUT}) to be an NE under the PT game, we must have b^*_{EUT} be a best response of the SP towards the user's acceptance probability p^*_{EUT}. Since we assumed that the pricing function is concave, it must be the only best response. Hence, we must have the second Condition. In order for p^*_{EUT} to be the user's best response, the first Condition must be satisfied. We do not need further constraints to ensure the existence of a concave pricing function as the concavity is already guaranteed by the second Condition. □

H. Proof of Theorem 4

Proof. When $p^*_{EUT} < 1$, according to Criterion 1, if $w(\bar{F}_B(b^*_{EUT})) < \bar{F}_B(b^*_{EUT})$, we have $r_{EUT}(b^*_{EUT}) > r_{PT}(b^*_{EUT})$. Thus, $U_{SP,PT}(p^*_{EUT}, b^*_{EUT}) < U_{SP,EUT}(p^*_{EUT}, b^*_{EUT})$. When $p^*_{EUT} = 1$, the result can be proved by plugging $p^*_{PT} = p^*_{EUT} = 1$ into Theorem 5. □

I. Proof of Theorem 5

Proof. Under the NE (p^*_{PT}, b^*_{PT}) under the PT game, the user accepts the offer with a non-zero probability. Hence, $r_{PT}(b^*_{PT}) \leq [h(b^*_{PT}) - h(0)]w(\bar{F}_B(b^*_{PT}))$, where the maximum the SP could earn under that NE is the right hand side of (23), proving the necessity of the inequality.

The sufficiency of the inequality holds as there exists a concave function that crosses both the origin, and the point $(b^*_{PT}, [h(b^*_{PT}) - h(0)]w(\bar{F}_B(b^*_{PT})))$. Furthermore, we can ensure the p^*_{PT} is the best response of the user by letting the function satisfy $\frac{d}{db}r_{PT}(b)|_{b^*_{PT}} = \frac{1}{p^*_{PT}}\frac{d}{db}c(b)|_{b^*_{PT}}$. The concavity of the function is guaranteed when $\frac{d}{db}r_{PT}(b)|_{b^*_{PT}} < \frac{[h(b^*_{PT})-h(0)]w(\bar{F}_B(b^*_{PT}))}{b^*_{PT}}$, which must hold when (p^*_{PT}, b^*_{PT}) is an NE under the PT game and when $p^*_{PT} > 0$. This is because $b^*_{PT}\frac{d}{db}r_{PT}(b)|_{b^*_{PT}} = \frac{1}{p^*_{PT}}(b^*_{PT}\frac{d}{db}c(b)|_{b^*_{PT}}) < \frac{1}{p^*_{PT}}c(b^*_{PT}) < \frac{1}{p^*_{PT}}(p^*_{PT}r_{PT}(b^*_{PT})) \leq [h(b^*_{PT}) - h(0)]w(\bar{F}_B(b^*_{PT}))$. □

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