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Impact of End-User Decisions on Pricing in Wireless Networks under a Multiple-User-Single-Provider Setting

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***Abstract*—We consider the impact of end-user decision-making on pricing of wireless resources when there is uncertainty in the Quality of Service (QoS) guarantees offered by the Service Provider (SP). Specifically, we consider the scenario where an SP tries to sell wireless broadband services to multiple potential customers when the advertised transmission rate cannot be fully guaranteed at all times. Modeling the decision-making of the end-users using Prospect Theory (a Nobel-Prize-winning theory that captures human decision-making and its deviation from Expected Utility Theory (EUT)), we formulate a game to study the interplay between the price offerings, bandwidth allocation by the SP and the service choices made by end-users. We characterize the Nash Equilibria (NE) of the underlying game and study the impact of such decision-making on the system performance as well as revenue. We propose "prospect pricing," a pricing mechanism that can make the system robust to decision-making that deviates from EUT.**

***Index Terms*—Game Theory, Nash Equilibrium, Expected Utility Theory, Prospect Theory, Probability Weighting, Prospect Pricing**

I. INTRODUCTION

The global mobile data traffic has seen a tremendous growth in the past few years. According to a report by Cisco [1], the global mobile data traffic grew 81% in the year 2013 alone, and is expected to increase 11-fold between 2013 and 2018. To overcome the challenge and provide a high-capacity and reliable network, various technical approaches have been explored by the Service Providers (SPs), including improving spectrum efficiency, utilizing spectrum at 5GHz, placing smaller cells in crowded areas, etc.. Meanwhile, from an economical point of view, pricing mechanisms have been studied extensively, since a proper pricing mechanism can not only help an SP generate more revenue, but also provide necessary control over the mobile data traffic [2]. In addition, effort has been made to understand how people perceive the quality of the delivered service, how different aspects of the network's performance may impact a user's level of satisfactory [3], [4], so that the SPs can improve and optimize their network infrastructures accordingly. Despite all the efforts, there isn't much work that focuses on the fact that the actual rates delivered by the SPs aren't always as fast as they advertise, and that the uncertainty that lies within the statistics of the actual delivered data rate could potentially affect an end-user's decision-making process when she decides among different service offers.

In order to fully inform the consumers about the actual performance of their purchased services, the Federal Communications Commission (FCC) launched a project in which the actual performances of broadband internet delivered by different SPs are measured and compared [5]. The result shows that most SPs cannot deliver the advertised rates all the time. A later project steered the attention to wireless broadband service, in which participating users download a background application onto their smartphones which periodically reports key parameters that indicate the network's performance such as up link and down link data rates, and packet loss [6]. In light of this, we investigate the problem where an SP tries to sell her wireless broadband service to multiple potential customers. We introduce the concept of service guarantee as the probability that a user receives the advertised rate successfully, and focus on the scenario where the users are fully informed about this piece of information when they purchase the service from an SP.

However, it is well known in behavioral economics that, when risk and uncertainty are involved in a decision-making process, people's choices can systematically violate the predictions of Expected Utility Theory (EUT), the foundation for the rationality assumption in game theory. Most of the paradoxes can be successfully explained using Prospect Theory (PT), a Nobel-Prize-winning theory first introduced by D. Kahneman and A. Tversky in 1979 [7]. While PT was originally developed to model and explain decision-making in monetary transactions, it has recently found widespread use in many contexts: social sciences [8]–[10], communication networks [11]–[17], and smart energy management [18], [19]. Driven by the motivation to investigate the impact of the violation of the predictions of the EUT caused by end-users' decision-making processes in a wireless communications setting, [12] investigated the scenario of data pricing in a wireless random access game and compares the results between the game assuming that the users follow the decision-making process predicted by EUT and the game where the users follow the decision-making process predicted by PT. In particular, the authors studied the impact when the users do not accurately evaluate their probabilities of successfully accessing the channel. In our work, where the users are to decide whether or not to purchase service over a channel that might not be capable of delivering the advertised service rate at all times, a user's inaccurate weighting of probability directly impacts her judgment on whether the service is worth purchasing or not, and, as will be shown, this inaccurate weighting of probability impacts the SP in both her revenue and radio resources she owns. This impact motivates us to study the role of "prospect pricing," a term we use to describe the method that effectively minimizes the revenue loss of the SP and maintaining the radio resource allocation over the users by adjusting the price of the service.

The rest of the paper is organized as follows: in Section II, we introduce the model of the interactions between the user and the SP as a Stackelberg game, and discuss conditions under which the existence of a pure strategy NE can be guaranteed; in section III, we discuss the impact of the Probability Weighting Effect (PWE) on the end-user's decision-making process, the revenue of the SP, as well as the prospect of recovering the revenue of the EUT game or maintaining the NE of the EUT game via prospect pricing; numerical results are shown in Section IV and conclusions are presented in Section V.

II. A Stackelberg Game Model

Consider the scenario where there is a monopoly SP and N potential users. The preliminary scenario involving a monopoly SP and one potential user was studied in [20]. Similar to the single user scenario, we model the interactions between the SP and the users with a Stackelberg game, where the SP invests in the data rate, makes an offer to a set of users first, based on the amount of bandwidth she has, and each user decides whether or not to accept the offer with some probability accordingly. We define an offer made by the SP under the EUT game as the triple $\{b, r_{EUT}(b), \vec{BW_{EUT}}\}$, which corresponds to the rate b, the price of the service at that rate determined by a predefined function $r_{EUT}(b)$, and a specific allocation of the SP's bandwidth $\vec{BW_{EUT}} = \{BW_{1,EUT}, ..., BW_{N,EUT}\}$, where $|\vec{BW_{EUT}}| = BW_{max,EUT}$, which is the total amount of bandwidth the SP has. On the user's side, we assume that no interference exists between the users, and that the guarantee level of the service at rate b is a function of the rate offered and the amount of bandwidth allocated to that user. In particular, for the i-th user, the service guarantee is defined by

$$\bar{F}_{B_i}(b; BW_{i,EUT}) := \mathbb{P}(B_i > b | BW_{i,EUT}), \qquad (1)$$

where B_i is the random variable representing the highest rate the channel can support, and for a fixed rate b, a larger $BW_{i,EUT}$ yields a higher service guarantee.

Denote the i-th user's benefit upon receiving guaranteed service at rate b with $h_i(b)$. Then, if the user accepts the offer, she pays a price $r_{EUT}(b)$, and with probability $\bar{F}_{B_i}(b; BW_{i,EUT})$ she receives successful service, and with probability $F_{B_i}(b; BW_{i,EUT}) := 1 - \bar{F}_{B_i}(b; BW_{i,EUT})$ the channel cannot successfully deliver the service at rate b and the user experiences an outage. Hence, if the user accepts the service with probability p_i, the expected utility of the i-th user can then be represented as

$$\begin{aligned} &U_{user,i}(p_i, b, BW_{i,EUT}) \\ &= p_i[-r_{EUT}(b) + h_i(b)\bar{F}_{B_i}(b; BW_{i,EUT}) + \\ &+ h_i(0)F_{B_i}(b; BW_{i,EUT})] + (1 - p_i)h_i(0). \end{aligned}$$

As a natural assumption, we assume that for all the users, $h_i(0) = 0$. Thus,

$$\begin{aligned} U_{user,i}(p_i, b, BW_{i,EUT}) = p_i[&-r_{EUT}(b) \\ &+ h_i(b)\bar{F}_{B_i}(b; BW_{i,EUT})]. \end{aligned}$$

The user will accept the offer at rate b with probability 1 if

$$h_i(b)\bar{F}_{B_i}(b; BW_{i,EUT}) > r_{EUT}(b). \qquad (2)$$

As for the SP, a cost $c_i(b, BW_{i,EUT})$ is incurred upon her when she allocates an amount of bandwidth $BW_{i,EUT}$ and makes an offer at rate b to the i-th user. Specifically, we assume that

$$c_i(b, BW_{i,EUT}) = c_1 b + c_2 + c_3 BW_{i,EUT}, \qquad (3)$$

indicating that the cost for offering the service to a user is an affine function with respect to the rate offered and the amount of bandwidth allocated to that user. Hence, the expected utility of the SP can be expressed as

$$\begin{aligned} U_{SP}(\vec{p}, b, \vec{BW_{EUT}}) = \sum_{i=1}^{N} [&p_i \left[r_{EUT}(b) - c_i(b, BW_{i,EUT})\right] \\ &+ (1 - p_i)(-c_i(b, BW_{i,EUT}))]. \end{aligned}$$

Similar to the single user game presented in our previous work, we assume that $r_{EUT}(b)$ and $h_i(b)$ are monotonically increasing and concave functions. We also assume that the guarantee level of the service goes to 0 as the offered rate goes to infinity, and that the guarantee level of the service is a monotonically increasing function with respect to the bandwidth allocated to the user. Finally, we assume that the fixed cost for the SP to offer the service is zero, i.e., $c_2 = 0$.

With the above settings, the conditions for the existence of an NE can be characterized. For simplicity, we dub the the game involving a single user and a single SP as a Single-User-Single-Provider (SUSP) game, and dub the generalized game involving multiple users and a single SP as a Multiple-User-Single-Provider (MUSP) game. Note that the results of the SUSP game can be found in [20].

Theorem 1 (The existence of multiple Nash Equilibria (NE))**.** *Assuming that* $\forall i$,

$$r_{EUT}(b^*_{1,EUT}) > c_i(b^*_{1,EUT}, BW_{max,EUT}), \qquad (4)$$

where

$$b^*_{1,EUT} = \underset{b}{\text{argmax}}(r_{EUT}(b) - c_i(b, BW_{max,EUT})), \qquad (5)$$

then there exists a pure strategy NE for the MUSP game[1] *if and only if there exists a pure strategy NE for at least one of the SUSP games consisting of one of the N users and the SP, under which all the bandwidth of the SP is allocated to that user.*

Proof. If an SUSP game has a pure strategy NE, then that NE is a pure strategy NE for the MUSP game. If none of the SUSP games has a pure strategy NE, then $\forall i$, $r_{EUT}(b^*_{1,EUT}) > h_i(b^*_{1,EUT})\bar{F}_{B_i}(b^*_{1,EUT}; BW_{max,EUT})$. Hence, assuming the SP decides to offer the service to a set of users denoted by S_{EUT}, then $\forall i \in S_{EUT}$, we must have $BW_{i,EUT} < BW_{max,EUT}$, and hence $r_{EUT}(b^*_{1,EUT}) > h_i(b^*_{1,EUT})\bar{F}_{B_i}(b^*_{1,EUT}; BW_{i,EUT})$, which follows from the assumption that reducing the bandwidth reduces the service guarantee. Hence, the users within set S_{EUT} will not accept the offer at rate $b^*_{1,EUT}$. Finally, we complete the proof by showing that the same rate is offered under the pure strategy NE of the MUSP and SUSP games. In fact, $b^*_{1,EUT}$ is the best response of the SP given that the users within set S_{EUT} all accept with probability 1, regardless of the choice of S_{EUT}. This is because

$$\begin{aligned} b^*_{1,EUT} &= \underset{b}{\operatorname{argmax}}(r_{EUT}(b) - c_i(b, BW_{max,EUT})) \\ &= \underset{b}{\operatorname{argmax}}(r_{EUT}(b) - c_1 b) - c_3 BW_{max,EUT} \\ &= \underset{b}{\operatorname{argmax}}\, |S_{EUT}|\, (r_{EUT}(b) - c_1 b) \\ &\quad - \sum_{i \in S_{EUT}} BW_{i,EUT} \\ &= \underset{b}{\operatorname{argmax}} \sum_{i \in S_{EUT}} (r_{EUT}(b) - c_i(b, BW_{i,EUT})). \end{aligned}$$

□

It is worth pointing out that we do not consider mixed strategy NE in the MUSP game. This is because, assuming that under an NE the acceptance probability of the users is represented by $\vec{p}$, the offered rate is b, and the allocation of the bandwidth is represented by $\vec{BW}_{EUT}$, we have

$$\begin{aligned} U_{SP}(\vec{p}, b, \vec{BW}_{EUT}) = \sum_{i \in S_{EUT}} & p_i(r_{EUT}(b) \\ & - c_i(b, BW_{i,EUT})) \\ = \bar{p}\, |S_{EUT}|\, & r_{EUT}(b) - |S_{EUT}|\, c_1 b \\ & - c_3 BW_{max,EUT}, \end{aligned}$$

where $\bar{p}$ is the average acceptance probability of all the users within set S_{EUT}. In order to reach a mixed strategy NE, the SP must find a rate b and a corresponding bandwidth allocation $\vec{BW}_{EUT}$ such that all the users are indifferent between accepting and denying the offer. However, the expression also shows that the users' acceptance probabilities represented by $\vec{p}$ only affect the SP's decisions through their average $\bar{p}$. Hence, for any combinations of offered rate and bandwidth allocation that induce a mixed strategy NE, the acceptance probabilities of the users can be arbitrary as long as the average acceptance probability remains fixed and the SP cannot obtain a higher revenue through offering the service to a subset of S_{EUT}. Hence, the SP does not have control over the individual user's acceptance probability under a mixed strategy NE.

Among the multiple NE of the MUSP game, we assume that the SP selects the one that yields maximum profit. This particular NE contains the largest number of users that could possibly be supported by the total amount of bandwidth the SP has. There can be multiple NE that achieve this maximum profit, we here specify the procedure to find one.

1) Determine the minimum amount of bandwidth needed for each user to accept the offer at rate $b^*_{1,EUT}$ and price $r_{EUT}(b^*_{1,EUT})$, denoted by $BW_i(b^*_{1,EUT})$. This bandwidth can be represented by the inverse function of $F_{B_i}(b^*_{1,EUT}; BW_{i,EUT})$ with $BW_{i,EUT}$ as the variable, since $F_{B_i}(b^*_{1,EUT}; BW_{i,EUT})$ is monotonically increasing with respect to $BW_{i,EUT}$ and the SP knows the service guarantee of the users in advance. Specifically, the expression
$$BW_i(b^*_{1,EUT}) = \bar{F}^{-1}_{B_i}\left(\frac{r_{EUT}(b^*_{1,EUT})}{h_i(b^*_{1,EUT})}, b^*_{1,EUT}\right)$$
is equivalent to
$$\bar{F}_{B_i}(b^*_{1,EUT}; BW_i(b^*_{1,EUT})) = \frac{r_{EUT}(b^*_{1,EUT})}{h_i(b^*_{1,EUT})}.$$
2) Upon determining the minimum bandwidth needed for each user to accept the offer, the SP selects the user with the smallest $BW_i(b^*_{1,EUT})$ each time until the remaining bandwidth can no longer support an additional user.

III. The Impact of Prospect Theory on End-User Decisions and Prospect Pricing

In the remaining of this article, we consider the impact of PT on the end-users' decisions of whether or not to accept a service offer, the impact to the radio resources and the revenue of the SP, and the method the SP could utilize to mitigate these effects and maintain the system on the original designed operating status under the EUT game. PT is a theory introduced in 1979 by D. Kahneman and A. Tversky. It is commonly regarded as a better model for people's decision-making process under risky situations since it explains some of the paradoxes that cannot be explained by EUT. The introduction of the theory and a recent review can be found in [7] [21], while a brief tutorial that tailors the theory to the need of wireless communication scenarios can be found in [12]. In our model, we consider specifically the impact of the PWE, which characterizes people's inaccurate judgement of the probability of the occurrence of an event. It has been agreed that people commonly under-weight probabilities that are large and moderate, and over-weight probabilities that are small. In the scenario in our model, we assume that all the users under-weight the service guarantee. As an analytical way to describe this under-weighting effect, we adopt the notion of Prelec's probability weighting function

[1]In this article, the pure strategy NE refers to the NE where the users accept the service with probability 1.

$w(p) = \exp\{-(-\ln p)^{\alpha})\}$ [22], and assume that all the users share the same parameter α.

A. Impact of PWE

We study the condition under which the system is robust to the PWE in the sense of retaining all the users without having to change the service offer. The result is summarized as follows.

Theorem 2. *If all the users under-weight the service guarantee, and the same offer inducing the pure strategy NE under the EUT game is offered to the same set of users, then the NE is preserved under PWE if and only if* $\forall i \in S_{EUT}$,

$$BW_{i,EUT} > \bar{F}_{B_i}^{-1}\left(\lambda_i, b_{1,EUT}^*\right), \tag{6}$$

where

$$\lambda_i = w^{-1}\left(\bar{F}_{B_i}\left(b_{1,EUT}^*; BW_i(b_{1,EUT}^*)\right)\right). \tag{7}$$

Proof. For the i-th user, the necessary and sufficient condition for him to accept an offer at rate $b_{1,EUT}^*$ and price $r_{EUT}(b_{1,EUT}^*$ under the impact of PWE is $h_i(b_{1,EUT}^*)w(\bar{F}_{B_i}(b_{1,EUT}^*; BW_{i,EUT})) > r_{EUT}(b_{1,EUT}^*)$, i.e., $w(\bar{F}_{B_i}(b_{1,EUT}^*; BW_{i,EUT})) > \bar{F}_{B_i}(b_{1,EUT}^*; BW_i(b_{1,EUT}^*))$. Since the probability weighting function is monotonically increasing, and thus has an inverse, we have $BW_{i,EUT} > \bar{F}_{B_i}^{-1}\left(w^{-1}\left(\bar{F}_{B_i}\left(b_{1,EUT}^*; BW_i(b_{1,EUT}^*)\right)\right), b_{1,EUT}^*\right)$. ☐

The above result indicates that, in order to retain all the users without changing the service offer, the total amount of bandwidth of the SP must satisfy

$$\begin{aligned} BW_{max,EUT} &= \sum_{i\in S_{EUT}} BW_{i,EUT} \\ &> \sum_{i\in S_{EUT}} \bar{F}_{B_i}^{-1}\left(\lambda_i, b_{1,EUT}^*\right). \end{aligned} \tag{8}$$

When $\alpha = 1$, $w(p) = p$, and the PT game reduces to EUT game. As α decreases, $w^{-1}(p)$ increases for every fixed p that satisfies $w(p) < p$, and hence the right hand side of the above inequality increases, indicating that when PWE is introduced and the users under-weight the service guarantee, the SP must invest in more bandwidth than the amount required under the EUT game in order to retain all the users with the same offer.

B. Prospect Pricing

When the bandwidth of the system does not satisfy the condition specified in equation (8), we resort to the method of prospect pricing, which changes the pricing function to achieve the following goals.

- The first aspect of the goal is to retain the Radio Resource Management (RRM) constraints when PWE is introduced. Similar to the RRM constraints we introduced in the SUSP game, the RRM constraints for the MUSP game are as follows.

$$S_{EUT} = S_{PT}, \tag{9}$$

$$b_{1,EUT}^* = b_{1,PT}^*, \tag{10}$$

$$BW_{max,EUT}^* = BW_{max,PT}^*, \tag{11}$$

$$\vec{BW_{EUT}} = \vec{BW_{PT}}. \tag{12}$$

  The constraints restrict the SP to offer a service package containing the same rate to the same set of users when PWE is introduced. They also restrict the SP to allocate the same amount of bandwidth to each user within the set.
- The second aspect of the goal is to retain the revenue when PWE is introduced. Similar to the results obtained in the SUSP game, in order to retain the revenue, the SP must violate the RRM constraints, assuming that all the users under-weight the service guarantee.

We answer the questions of whether prospect pricing can be used to retain strict RRM constraints and the question of whether prospect pricing can be used to retain the same amount of revenue with or without strict RRM constraints being held.

Theorem 3. *When (8) is not satisfied, and when all the users under-weight the service guarantee, prospect pricing can be used to retain strict RRM constraints, at the cost of the SP losing revenue of at least*

$$\begin{aligned} L_{RRM} = \max_{i\in S_{EUT}} \{ & r_{EUT}(b_{1,EUT}^*) \\ & - h_i(b_{1,EUT}^*)w(\bar{F}_{B_i}(b_{1,EUT}^*; BW_{i,EUT}))\} \end{aligned} \tag{13}$$

per user. Furthermore, the revenue loss can be reduced, but not fully recovered, when the SP is allowed to violate the RRM constraints by reallocating the bandwidth.

Proof. In order to retain strict RRM constraints, all the users must accept the same offer containing the same rate and bandwidth, i.e., $\forall i \in S_{EUT}$,

$$r_{PT}(b_{1,EUT}^*) < h_i(b_{1,EUT}^*)w(\bar{F}_{B_i}(b_{1,EUT}^*; BW_{i,EUT})). \tag{14}$$

Hence,

$$\begin{aligned} & r_{PT}(b_{1,EUT}^*) \\ & < \min_{i\in S_{EUT}} \left\{h_i(b_{1,EUT}^*)w(\bar{F}_{B_i}(b_{1,EUT}^*; BW_{i,EUT}))\right\}. \end{aligned} \tag{15}$$

However,

$$\begin{aligned} & \min_{i\in S_{EUT}} \left\{h_i(b_{1,EUT}^*)w(\bar{F}_{B_i}(b_{1,EUT}^*; BW_{i,EUT}))\right\} \\ & < r_{EUT}(b_{1,EUT}^*). \end{aligned} \tag{16}$$

Hence, in order to retain strict RRM constraints, the SP must take a revenue loss of at least

$$\begin{aligned}
&L_{RRM} \\
&\quad = r_{EUT}(b^*_{1,EUT}) \\
&\qquad - \min_{i\in S_{EUT}} \left\{ h_i(b^*_{1,EUT}) w(\bar{F}_{B_i}(b^*_{1,EUT}; BW_{i,EUT})) \right\} \\
&\quad = \max_{i\in S_{EUT}} \left\{ r_{EUT}(b^*_{1,EUT}) \right. \\
&\qquad\qquad \left. - h_i(b^*_{1,EUT}) w(\bar{F}_{B_i}(b^*_{1,EUT}; BW_{i,EUT})) \right\}. \qquad (17)
\end{aligned}$$

Allowing reallocation of the bandwidth will reduce the revenue loss, since the revenue loss allowing bandwidth allocation L_{BA} is the minimum revenue loss over all possible bandwidth allocation, and the bandwidth allocation under strict RRM constraints is only one instance. We next show that allowing reallocation of the bandwidth cannot help the SP to fully recover the revenue by contradiction. Since the service is offered to the same set of users and the offered rate remains the same, the price must be the same in oder to retain the revenue, i.e., $r_{EUT}(b^*_{1,EUT}) = r_{PT}(b^*_{1,EUT})$. Assume that there exists a bandwidth allocation such that $\forall i \in S_{EUT}$,

$$r_{PT}(b^*_{1,EUT}) < h_i(b^*_{1,EUT}) w(\bar{F}_{B_i}(b^*_{1,EUT}; BW_{i,PT})). \qquad (18)$$

Then we must have $\forall i \in S_{EUT}$,

$$\begin{aligned}
BW_{i,PT} &> \bar{F}^{-1}_{B_i}\left(w^{-1}\left(\frac{r_{PT}(b^*_{1,EUT})}{h_i(b^*_{1,EUT})} \right); b^*_{1,EUT} \right) \\
&= \bar{F}^{-1}_{B_i}\left(\lambda_i; b^*_{1,EUT}\right). \qquad (19)
\end{aligned}$$

Hence, the summation over the set S_{EUT} yields the condition specified in (8), contradicting the assumption that the bandwidth is inefficient in the first place. □

The above results show that the price $r_{PT}(b^*_{1,EUT})$ is lower than the price $r_{EUT}(b^*_{1,EUT})$. The choice of the pricing function is not considered here as we are only interested in the value of that pricing function at rate $b^*_{1,EUT}$, which remains the same when strict RRM constraints are enforced.

IV. Numerical Results

In this section, we demonstrate some of the conclusions drawn above. We consider a scenario where $N = 10$ users are spread uniformly within a single cell with a radius of 800 meters. There are no interference between different users, and we assume that the SP offers the service to all the users. Each user experiences a combination of path loss, shadowing, and Rayleigh fading. The guarantee of the service for each user in this setup is one minus the outage probability of the fading channel between the user and the base station and the rate offered is the encoding rate at the transmitter. The path loss and shadowing are calculated using a simplified model [23]

$$P_{r_i} = P_t + K - 10\gamma \log_{10}\frac{d}{d_0} + \varphi_{i,dB}, \qquad (20)$$

where P_t and P_{r_i} are the transmitted signal power and the received signal power at the i-th user in decibels, K is a constant taking the value $-20\log_{10}(4\pi d_0/\lambda)$. γ is the path

TABLE I
Parameters used for simulation

Parameter	Meaning	Value
P_t	Transmission power	10 W
K	Antenna dependent constant	-64.5 dB
N_0	PSD of thermal noise	-174 dBm
d_0	Reference distance for the antenna far-field	20 m
γ	Path loss exponent	4
σ	Standard deviation for $\varphi_{i,dB}$	4
r	Cell radius	800 m

loss exponent, d is the distance between the user and the base station antenna, and d_0 is the reference distance for the antenna far-field. In addition, $\varphi_{i,dB}$ is a Gaussian random variable that captures the effect of shadow fading. Finally, the guarantee function for each user can be expressed as

$$\bar{F}_{B_i}(b) = \exp\left\{ -\frac{2^{\frac{b}{BW_{i,EUT}}} - 1}{P_{r_i}/(N_0 BW_{i,EUT})} \right\}. \qquad (21)$$

where N_0 is the power spectral density (PSD) of the noise, $BW_{i,EUT}$ represents the bandwidth allocated to the i-th user, and b represents the encoding rate of the SP.

A list of the values for the parameters can be found in the following table.

In Figure 1, we show the revenue loss of the SP when no adjustment is made to the price of the service offer, and compare it to the revenue loss when prospect pricing is allowed (with/without bandwidth reallocation). Two cases are considered, in which the initial bandwidth owned by the SP is 10% (solid line) or 50% (dashed line) more than the minimum that is necessary to make all the users accept the offer under the EUT game, respectively, and is being allocated to all the users such that each user receives 10% or 50% more than the minimum needed for her to accept the offer. The horizontal axis represents different values of α, the parameter that captures the level of probability weighting of the users, while the vertical axis represents the revenue loss normalized by the revenue the SP makes under the EUT game (prices paid by the users minus the cost).

The beige curves show the revenue loss of the SP when prospect pricing is not implemented, which quickly increases as α decreases. Notice that each increment represents a loss of at least one user, and that the part of the curve above 1 indicates that the SP is getting negative revenue in reality, since not enough users are retained to recover the cost. The blue curves show the minimum revenue loss of the SP when prospect pricing is allowed, but when strict RRM constraints are enforced, while the red curve corresponds to the case where the SP is allowed to implement prospect pricing and violate the RRM constraints by reallocating the bandwidth among the users. As can be seen, the solid and dashed blue curves greatly reduce the revenue loss of their counterpart beige curves, and the red curves reduces the revenue loss further, which corresponds to our result in Theorem 3. It can also be seen from the graph that adding more bandwidth into the system provides more robustness against α, i.e., all the

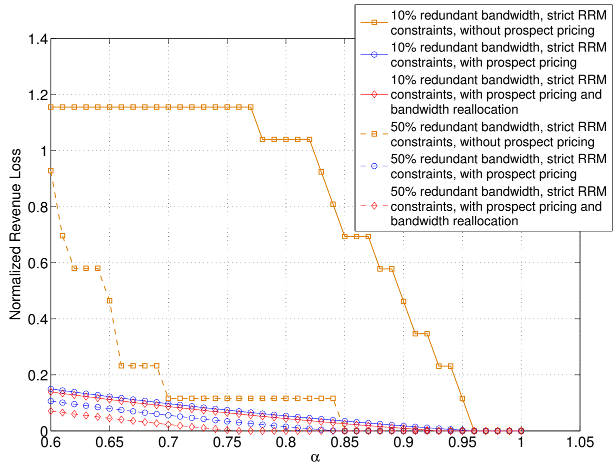

Fig. 1. Revenue loss of the SP normalized by the revenue under EUT game. $N = 10$, $h_i(b) = 10^{-2} \times (b \times 10^{-3})^{0.65}$, $r_{EUT}(b) = 2 \times 10^{-3} \times (b \times 10^{-3})^{0.82}$, $c_1 = \frac{1}{3} \times 10^{-6}$, $c_3 = 10^{-7}$, $c_i(b; BW_{i,EUT}) = c_1 b + c_3 BW_{i,EUT}$ $b^*_{1,EUT} \approx 7Mbps$.

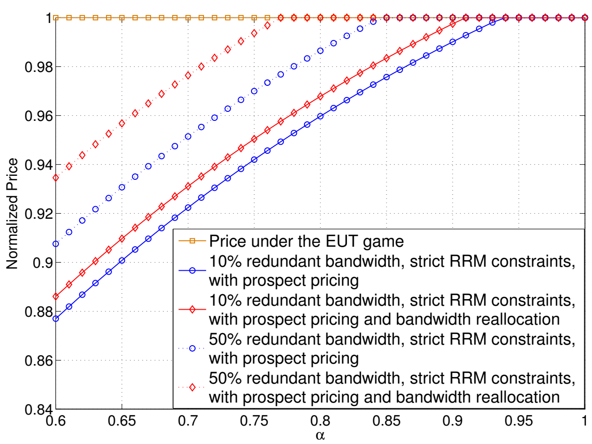

Fig. 2. Price of the SP at rate $b^*_{1,EUT}$ normalized by the price under the EUT game. $N = 10$, $h_i(b) = 10^{-2} \times (b \times 10^{-3})^{0.65}$, $r_{EUT}(b) = 2 \times 10^{-3} \times (b \times 10^{-3})^{0.82}$, $c_1 = \frac{1}{3} \times 10^{-6}$, $c_3 = 10^{-7}$, $c_i(b; BW_{i,EUT}) = c_1 b + c_3 BW_{i,EUT}$, $b^*_{1,EUT} \approx 7Mbps$.

curves are pushed towards the left hand side of the graph. Note that adding more bandwidth increases the cost for the SP, which reduces revenue in turn.

The price at rate $b^*_{1,EUT}$ under the EUT and PT game is shown in Figure 2. We can see that the price reduction is smaller when the SP is allowed to reallocate her bandwidth compared to the case when strict RRM constraints are enforced, and is smaller when the initial amount of bandwidth owned by the SP is larger.

V. Conclusion

In this work, we considered the impact of end-users' decision-making process on the resource allocation in a wireless network when there is uncertainty in the QoS guarantees offered by the SP. We formulated a Stackelberg game to study the interplay between the price offerings, bandwidth allocation by the Service Provider and the service choices made by end-users. We characterized the Nash Equilibria of the game, and studied the impact on the system performance and the SP's revenue when end-users follow the decision-making process predicted by Prospect Theory. Finally, we proposed "prospect pricing", a pricing mechanism that can make the system robust to decision-making that deviates from Expected Utility Theory. We described the minimum revenue loss of the SP under the strict Radio Resource Management constraints with and without the help from prospect pricing, we also characterized the minimum revenue loss when the SP is allowed to reallocate the bandwidth allocated among the users. We showed that, when the amount of bandwidth of the SP is not large enough, the SP cannot fully retain her revenue under the EUT game as the end-users start to under-weight the service guarantee, even when she is allowed to reallocate bandwidth and implement prospect pricing. However, prospect pricing does reduce the revenue loss of the SP under the strict RRM constraints, while further allowing bandwidth reallocation reduces the loss further more.

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