Small and Large Scale Fading, Co-channel Interference, Modulated Signals*

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Abstract- This article mainly discusses about co-channel interference and outage probability. It also gives a comparison of small fading components. Additionally shadowing is discussed and some terminology related to modulated signals is explained.

I. INTRODUCTION/MOTIVATION

The mobile channel is characterized mainly by three important factors: path losses larger than free space, fading typically taken as Raleigh and shadowing generally classified as lognormal. For cellular systems, in order to determine acceptable reuse distances between base stations and to compare modulation methods the probability of unacceptable co-channel interference (outage probability) has to be determined in the realistic situation where both shadowing and fading occur. [3] The quality of the service is often expressed by the outage probability experienced by the users.

II. SMALL SCALE FADING

Depending on the relation of between signal parameters (such as bandwidth, symbol period etc.) and the channel parameters (such as rms delay spread and Doppler spread different transmitted signals go different types of fading. The dispersion due to multipath causes the transmitted signal to undergo either flat or frequency selective fading. Flat fading occurs if the mobile radio channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal and if the symbol period is greater than rms delay. Such a channel is called as narrowband channel or non frequency selective channel. By flat fading the spectral characteristics of the signal are preserved by the signal. Frequency selective fading occurs if the channel possesses a constant gain and linear phase over a bandwidth smaller than the bandwidth of the transmitted signal and the symbol duration is less than the rms delay. The received signal includes multiple versions of the transmitted signal. Frequency selective fading is due to the time dispersion of the transmitted signals within the channel. Thus channel includes intersymbol interference. Frequency selective channels are also known as wideband channels. A common rule of thumb is that a channel is flatif $T_s \geq 10\,\sigma_T$ and frequency selective if $T_s \leq 10\,\sigma_T$.

Depending on how rapidly the transmitted baseband signal changes as compared to the rate of the change of a channel may be classified as a fast fading or slow fading channel. In a fast fading channel the channel impulse response changes rapidly within the symbol duration. That is the coherence time is smaller than the symbol duration. This causes frequency dispersion (also called time selective fading) due to Doppler spread, which leads to signal distortion. Therefore the signal goes fast fading if the symbol time is greater than the coherence time and signal bandwidth is smaller than the Doppler spread ($T_s > T_c$ and $B_s < B_D$).

In a slow fading channel, channel impulse changes at a very much slower rate than the transmitted baseband signal. In this case the channel can be assumed to be static over one ore several bandwidth intervals. Therefore the signal goes slow fading if $T_s << T_c$ and $B_s >> B_D$. Fast or small fading only relate to the rate of change, velocity and signal bandwidth. It doesn't depend if the channel is frequency selective or flat. The

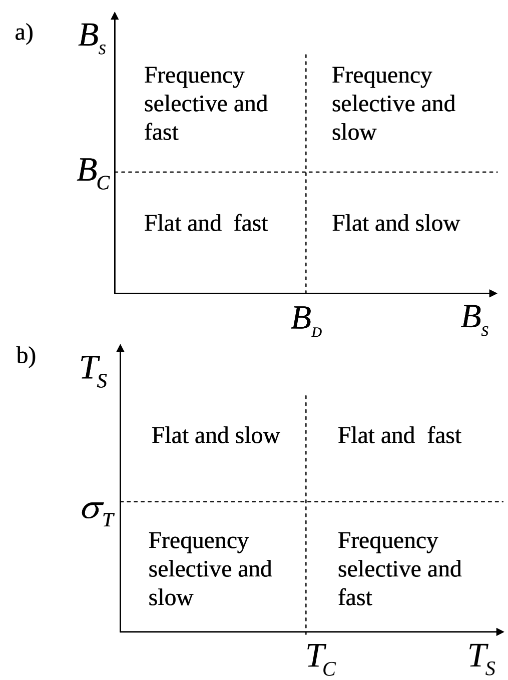

Figure 1 Matrix illustrating type of fading experienced by a signal as a function of a) baseband signal bandwidth b) symbol period

relation between various multipath parameters and the type of fading is summarized in Figure 1. Most of the fading occurs in the frequencies less than the Doppler spread.

III. LARGE SCALE FADING (SHADOWING)

Long term energy variability in multipath fading channels is widely been accepted as being well described by lognormal statistics. This phenomenon is commonly referred as shadowing. [4]

Recall from the small scale fading components the mean envelope level of the received signal which is either Rayleigh or Ricean faded is defined as:

$$\Omega_v = E[Z(t)] \qquad (1)$$

Ω_v is called as local mean since it represents the envelope level averaged over a distance of a few wavelengths. Actually Ω_v is itself is a random variable due to shadow variation that are caused by large terrain features like hills, buildings between the mobile station and base station .

The distribution of Ω_v is determined using the results empirical measurements. Thus the distribution of Ω_v is given as a lognormal distribution.

$$p(\Omega_v) = \frac{\xi_\rho}{\Omega_v\, \sigma_\Omega \sqrt{2\pi}} \exp\left\{ \frac{-\left(10 \log \Omega_v - \mu_{\Omega_v}\right)}{2\sigma_\Omega^2} \right\} \qquad (2)$$

where $\mu_{\Omega_v} = E[\Omega_v(dB)]$ and $\xi = \frac{10}{\ln 10}$.

The distribution of $\Omega_v(dB) = 10 \log \Omega_v$ is Gaussian

$$p(\Omega_v(dB)) = \frac{1}{\sigma_\Omega \sqrt{2\pi}} \exp\left\{ \frac{-\left(\Omega_v(dB) - \mu_{\Omega_v}\right)}{2\sigma_\Omega^2} \right\} \qquad (3)$$

σ_Ω is typically 8 dB in macro cellular applications and varies between 5 to 12 dB. In reality all these different types of propagation losses (path loss, small scale fading, large scale fading) exists and effect the signal, you need to select appropriate model according to application. Figure 2 shows the effect of these components on the signal envelope.

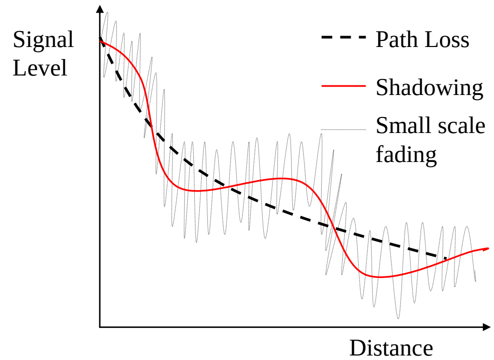

Figure 2 Effects of path loss shadowing and small scale fading on the signal envelope

IV. COMPOSITE SHADOW FADING DISTRIBUTIONS

Suzuki distribution [4] is a mixed distribution compromising the lognormal and Rayleigh distributions. One approach is to condition on local mean Ω_v and then to integrate the conditional power density of the envelope over the density Ω_v. The composite density distribution is:

$$p_{Z_C}(x) = \int_0^\infty p_{Z|\Omega_v}(x \mid w)\; p_{\Omega_v}(w)\, dw \qquad (4)$$

The local mean is

$$\Omega_v = E[Z(t)] = \sqrt{\frac{\pi}{2}}\sigma \qquad (5)$$

and the conditional power density function for Rayleigh fading on local mean Ω_v is given by:

$$p_{Z|\Omega_v}(x) = \frac{\pi\ x}{2w^2} \exp\left(\frac{-\pi\ x^2}{4\pi^2} \right) \qquad (6)$$

Hence the Suzuki distribution can be written as

$$p_{Z_C}(x) = \int_0^\infty \frac{\pi\, x}{2w^2} \exp\left(\frac{-\pi\, x^2}{4\pi^2} \right) \frac{\xi_\rho}{w\, \sigma_\Omega \sqrt{2\pi}} \exp\left\{ \frac{-\left(10 \log w - \mu_{\Omega_v}\right)}{2\sigma_\Omega^2} \right\} dw \qquad (7)$$

The distribution of the path strength within a site tends to be a Nakagami distribution at initial paths and lognormal distribution as the excess delay becomes large. [5]

V. Effect of Co-Channel Interference

In wireless communication systems the same frequency channels are used many times due to limited spectrum resources. These reuse of channels increase the spectrum efficiency of the system but meanwhile the co-channel interference is increased which is a major source of the performance impairment. In wireless systems where the small scale fading effects are averaged out, the co channel interference considerations are more strongly dependent on the large scale signal variations caused by shadowing. Thus, computing wireless system performance under the effects of lognormal shadowing is of interest. [6]

VI. Multiple Lognormal Interferers

A lognormal random variable is characterized by the property that the logarithm of the random variable has a Gaussian distribution. Let the lognormal random variable be L_K and the Gaussian distributed variable be Ω_K (dB) with variance $\sigma^2_{\Omega_K}$ and mean μ_{Ω_k}. Consider there is N_I lognormally shadowed interferers in the region. The sum of these interferers is:

$$L = \sum_{k=1}^{N_I} L_K = \sum_{k=1}^{N_I} 10^{\Omega_k (dB)/10} \approx \tilde{L} \tag{8}$$

Using the general consensus that the sum of the independent lognormal variables is another lognormal variable:

$$\tilde{L} = 10^{Z(dB)/10} \tag{9}$$

The accuracy of this approximation varies with the range of Ω_K (dB) and N_I. Different approaches have been proposed to determine the variance σ_Z^2 and mean μ_Z of $Z(dB)$. Most well known approaches are: Fenton-Wilkinson, Schwartz Yeh's and Farley's method.

VII. Fenton Wilkinson Method

The variance σ_Z^2 and mean μ_Z of $Z(dB)$ are obtained by matching the first two moments of the power sum L with the first 2 moment of approximation $\tilde{L}$.
Let

$$L_k = 10^{\Omega_k (dB)/10} = e^{\xi \Omega_k (dB)/10} = e^{\hat{\Omega}_k} \tag{10}$$

where $\xi = \frac{\ln 10}{16} = 0.23026$ and $\hat{\Omega}_k$ is a Gaussian variable with the mean $\mu_{\hat{\Omega}_k} = \xi \mu_{\Omega_k}$ and variance $\sigma_{\hat{\Omega}k} = \xi^2 \sigma_{\Omega_k}$.

The r-th moment of L_K is :

$$E\left[L_k^r\right] = E\left[e^{r\hat{\Omega}_k}\right] = e^{r\mu\hat{\Omega}_k + \frac{1}{2}r^2\sigma^2_{\hat{\Omega}_k}} \tag{11}$$

To find the appropriate moments of approximation, we equate moments on both sides of the equation

$$L \approx e^{\hat{z}} = \tilde{L} \tag{12}$$

where $\hat{z} = \xi z\, dB$. Let $\hat{\Omega}_1, \hat{\Omega}_2 \cdots \hat{\Omega}_{N_I}$ be independent with means $\mu_{\hat{\Omega}_1}, \mu_{\hat{\Omega}_2} \cdots \mu_{\hat{\Omega}_{N_I}}$ and identical variance $\sigma^2_{\Omega_K}$. Identical variance is assumed because the variance is not effected by radio path length for lognormal shadowing. Then

$$LHS = \mu_L = \sum_{k=1}^{N_I} E\left[L_k\right] = \left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right) e^{\frac{1}{2}\hat{\Omega}^2} \tag{13}$$

and

$$RHS = E\left[e^{\hat{z}}\right] = e^{\mu_{\hat{z}} + \frac{1}{2}\sigma^2_{\hat{z}}} \tag{14}$$

Setting LHS and RHS equal we get

$$\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right) e^{\frac{1}{2}\hat{\Omega}^2} = e^{\mu_{\hat{z}} + \frac{1}{2}\sigma^2_{\hat{z}}} \tag{15}$$

Similarly equating second moments:

$$\begin{aligned} LHS &= \left(\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}\right)\left(e^{\sigma^2_{\hat{\Omega}}}\right)\left(e^{\sigma^2_{\hat{\Omega}}} - 1\right) e^{\frac{1}{2}\hat{\Omega}^2} = \cdots \\ RHS &= e^{2\mu_{\hat{z}}}\left(e^{\sigma^2_{\hat{z}}}\right)\left(e^{\sigma^2_{\hat{z}}} - 1\right) \end{aligned} \tag{16}$$

Squaring equation (15) and dividing by (16) the expression becomes:

$$\mu_{\hat{Z}} = \frac{\sigma_{\hat{\Omega}}^2 - \sigma_{\hat{Z}}^2}{2} + \ln\left[\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right] \quad (17)$$

$$\sigma_{\hat{Z}}^2 = \ln\left[\left(e^{\sigma_{\hat{\Omega}}^2} - 1\right)\frac{\sum_{k=1}^{N_I} e^{2\mu_{\hat{\Omega}_k}}}{\left(\sum_{k=1}^{N_I} e^{\mu_{\hat{\Omega}_k}}\right)^2} + 1\right] \quad (18)$$

Approximation is not so good in terms of moments themselves. Approximation works very well for evaluating the probability:

$$P_Y(L > x) \approx P_r\left(e^{\hat{z}} \geq x\right)\sigma_{\hat{Z}}^2 = Q\left(\frac{\ln x - \mu_{\hat{Z}}}{\sigma_{\hat{Z}}}\right) \quad (19)$$

VIII. SCHWARZ YEH'S METHOD

This method equates LHS and RHS by evaluating exact expression for first two moments of the sum of two lognormal random variables. Then recursion is used to evaluate for general N_I number of interferers.

IX. PROBABILITY OF OUTAGE

Probability of outage is a measure by evaluating system performance. It is defined in relation to signal to interference ratio (SIR) achieved on that which is minimum acceptable SIR. The target SIR will be $\Lambda_{th}(dB)$. Then the probability of outage is defined as:

$$P_{out} = \Pr\{SIR(dB) < \Lambda_{th}(dB)\} \quad (20)$$

Example-Forward Link: Let MS be at a distance d_0 from the desired BS at distances $d_1 \ldots d_{N_I}$ from the co-channel base stations. Let $\Lambda_{dB}(d)$ be the SIR achieved at BS. Then

$$\begin{aligned}\Lambda_{dB}(d) &= \Omega_{dB}(d_0) - 10\log\left(\sum_{k=1}^{N_I} 10^{\Omega_{d_0}\frac{d_k}{10}}\right) \\ \Rightarrow P_{out}(d) &= \Pr\{\Lambda_{d\mathbf{B}}(d) < \Lambda_{th}(dB)\}\end{aligned} \quad (21)$$

Using lognormal approximation

$$\left(\sum_{k=1}^{N_I} 10^{\Omega_{d_0}\frac{d_k}{10}}\right) \approx e^{\hat{z}} \quad (22)$$

where the mean and variance of the approximation is defined as

$$z_{(dB)} = \frac{\hat{z}}{\xi} \Rightarrow \mu_z = \frac{\mu_{\hat{z}}}{\xi} \quad \sigma_z^2 = \frac{\sigma_z^2}{\xi^2} \quad (23)$$

we get

$$\Lambda_{dB}(\mathbf{d}) = \Omega_{dB}(d_0) - z_{(dB)}\left(d_1 \ldots d_{N_I}\right) \quad (24)$$

This is a Gaussian with the mean and variance:

$$\mu_{\Lambda_{dB}}(\mathbf{d}) = \mu_{\Omega_{dB}(d_0)} - \mu_z \qquad \sigma_{\Lambda_{dB}}^2 = \sigma_{\Omega}^2 + \sigma_Z^2 \quad (25)$$

Finally the outage probability is defined as:

$$P_{out} = Q\left(\frac{\mu_{\Omega}(d_0) - \mu_z - \Lambda_{th}(dB)}{\sqrt{\sigma_{\Omega}^2 + \sigma_z^2}}\right) \quad (26)$$

X. MULTIPLE RICEAN/ RAYLEIGH INTERFERERS

In the case of multiple Ricean/Rayleigh interferers SIR is:

$$SIR = \frac{s_0}{\sum_{k=1}^{N_I} s_k} \quad (27)$$

s_0 is the desired signal power. This signal has LOS components and therefore affected by Ricean fading. s_k is the interfering signal power of the *k*th BS. The co-channel BS signals undergo Raleigh Fading because of non-LOS components. s_0 has non-central chi-square distribution and s_k has exponential distribution.

The outage probability is defined as:

$$P_{out} = P\{SIR < \lambda_{th}\} = P\left\{s_0 < \lambda_{th}\sum_{k=1}^{N_I} s_k\right\} \quad (28)$$

If there is only one interferer it can be written as:

$$P_{out} = \frac{\lambda_{th}}{\lambda_{th}+b_1}\exp\left\{\frac{-kb_1}{\lambda_{th}+b_1}\right\} \quad (29)$$

where k is the Rice-factor and $b_1 = \frac{\Omega_0}{(k+1)\Omega}$

If the desired user is also Rayleigh we set k=0.

For multiple interferers the outage power is given by

$$P_{out} = 1 - \left(\sum_{k=1}^{N_I} 1 - \frac{\lambda_{th}}{\lambda_{th}+b_k}\exp\left\{-\frac{Kb_k}{\lambda_{th}+b_k}\right\}\right)\prod_{j=k}^{N_I}\frac{b_j}{b_j-b_k} \quad (30)$$

This equation holds only if $\Omega_i \neq \Omega_k$. If all interferers have same mean power then $s_m = \sum_{k=1}^{N_I} s_k$ has a gamma distribution:

$$P_{S_M}(x) = \frac{x^{N_I-1}}{\Omega_j^{N_I}(N_I-1)}\exp\left(\frac{-x}{\Omega_1}\right) \quad (31)$$

The outage power is given by:

$$P_{out} = \frac{\lambda_{th}}{\lambda_{th}+b_1}\exp\left(\frac{-kb_1}{\lambda_{th}+b_1}\right)\ldots$$
$$\sum_{k=0}^{N_I-1}\left(\frac{b_1}{\lambda_{th}+b_1}\right)^k \sum_{m=0}^{k}\binom{k}{m}\frac{1}{m!}\left(\frac{-k\lambda_{th}}{\lambda_{th}+b_1}\right)^m \quad (32)$$

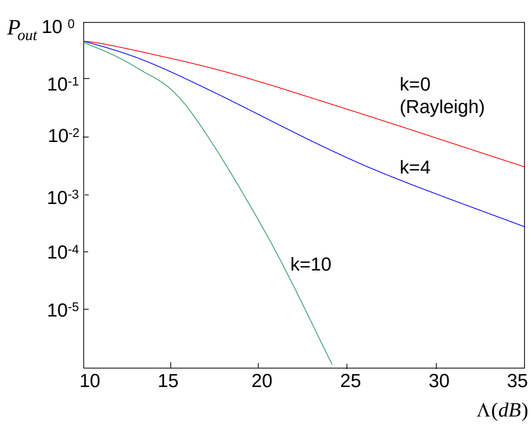

Figure 3 Outage probabilities for different Ricean factors (One user-one interferer $\lambda_{th} = 10\,dB$)

XI. Modulated Signals & Power Spectral Densities

Some factors affecting the choice of a digital modulation scheme are the demand for low BER at low SNR, bandwidth and cost efficiency.

Power efficiency is a measure of the tradeoff between the bit error rate achieved by a modulation scheme and the signal power required to achieve that BER. Formally η_p is defined as the ratio of the signal energy per bit to the noise power density achieved at a certain BER (e.g. 10^{-6})

$$\eta_p = \frac{E_b}{N_0} \quad (33)$$

Bandwidth Efficiency describes the ability of a modulation scheme to accommodate the data within certain bandwidth. In general increasing the data rate implies increasing bandwidth of the transmission. However some modulation formats have a better trade off than the others. Bandwidth efficiency reflects how well the allocated bandwidth is utilized.

η_b is defined as the ratio of the signal energy per bit to the noise power density achieved at a certain BER (e.g. 10^{-6}).

$$\eta_B = \frac{R}{B} \qquad [bits/\sec/Hz] \quad (34)$$

Fundamental upper band in achievable band is

$$\eta_B \leq \frac{C}{B} = B\frac{ld\left(1+\frac{S}{N}\right)}{B} = ld\left(1+\frac{S}{N}\right) \quad (35)$$

where *C* is Shannon capacity. Usually there is a trade off between bandwidth efficiency and power efficiency in all practical communication systems. For example channel coding increases the bandwidth occupancy, bandwidth efficiency goes down. However it increases power efficiency and coding gain allows lowering SNR.

There are different definitions for the definition of the modulated signal. *Absolute bandwidth* of a power spectral density function *S(f)* is the range of frequencies where the S(f) is nonzero. *Null-Null bandwidth* width is the width of the main spectral lobe. *Half power bandwidth* which is also called as *3dB bandwidth* is defined as the interval between frequencies at which power PSD falls to the half power. *FCC* defines the bandwidth as that band which leaves exactly 0.5% above the band and 0.5 % below the band. 99% signal power is contained in the occupied bandwidth.

XII. Conclusion

In this article small fading components and shadow fading distributions are summarized. Co-channel interference and outage probability is discussed. Additionally some factors affecting the choice of digital modulation scheme are explained.

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