Wireless Communications Technologies (16:332:546)

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INTRODUCTION

In this lecture, we continue on radio resource management in wireless networks, and are focused on the **resource allocation algorithm**. Firstly, we discuss the goal of optimum regarding resource allocation, based on which we define the *assignment failure rate* and the *instantaneous capacity* of a wireless system. We also discuss two kinds of channel allocation: *Fixed Channel Allocation* (FCA) and *Dynamic Channel Allocation* (DCA). And then we study **power control**, including the SIR requirement and a theorem (and a corollary) for solving the problem. Distributed power control algorithm, specially the per-user version is presented. We also go through some power control models such as the minimum power assignment, macro diversity and multiple connection reception, and a unified framework for uplink power control. Finally power control in practice is discussed and the block diagram is given.

RESOURCE ALLOCATION ALGORITHM

Resource Allocation Algorithm

Resource Allocation Algorithm is to assign for each mobile

(a) an access point from set $\mathcal{A}$,

(b) a channel (pair) from set $\mathcal{C}$, and

(c) a transmitter power for RAP and MS

such that all links meet their minimum SIR requirements.

The relevant link gains of the associated system are characterized by a *link gain matrix* $\mathbf{G}$ which is given as

$$G = \begin{bmatrix} G_{11} & G_{12} & \dots & G_{1M} \\ G_{21} & G_{22} & \dots & G_{2M} \\ \dots & & & \\ G_{B1} & G_{B2} & \dots & G_{BM} \end{bmatrix}$$

where M is the number of active mobiles and B is the number of base stations.

Given that a mobile j has been assigned to port i on channel (pair) c, the following must hold:

$$\text{For uplink}: \ \gamma_j^u = \frac{PG_{ij}}{\sum_{m \in \mathcal{M}^{(c)}} PG_{im} + N_b} \geq \gamma_0$$

$$\text{For downlink}: \ \gamma_j^d = \frac{PG_{ij}}{\sum_{b \in \mathcal{B}^{(c)}} PG_{bj} + N_m} \geq \gamma_0$$

where γ_0 is the minimum SIR requirement, $\mathcal{M}^{(c)} = \{m : \text{mobile } m \text{ has been assigned channel } c, c = 1, 2, \dots, C\}$, and $\mathcal{B}^{(c)} = \{b : \text{base station } b \text{ has been assigned channel } c, c = 1, 2, \dots, C\}$.

But finding the optimum assignment is a formidable problem, and there is no known general efficient algorithm.

What is Optimum?

Define Y as the number of mobiles from M total mobiles that have an adequate link. The optimum problem becomes to maximize Y for a given G.

To handle stochastic nature of the system, we assume both M and Y are random variables, and define $Z = M - Y$ which is also a random variable.

Define the *assignment failure rate* ν as the average ratio of the number of mobiles which are not provided with a channel to the total number of mobiles, i.e., it is given as

$$\nu = \frac{E[Z]}{E[M]}$$

We usually model mobiles as being active with a 2D Possion point process with arrival rate w (mobiles/unit area).

If A is the service area, then $E[M] = wA$ and thus

$$\nu = \frac{E[Z]}{wA}$$

For large wA, ν is also a good approximation of the probability that a randomly chosen mobile at some given instant is not provided with a channel.

Define the *instantaneous capacity* of a wireless system as the maximum allowed traffic load in order to keep the assignment failure rate ν below some threshold ν_0, i.e.

$$\omega^*(\nu_0) = \{\max \omega : \ \nu \leq \nu_0\}$$

Note that satisfying the above criteria is NOT practical. So we try different ideas and see how they work.

CHANNEL ALLOCATION

Channel assignment in literature is based on simple heuristic design rules.

Fixed Channel Allocation (FCA)

FCA is kind of fixed reuse and assignment by sectorization and directional antennas. It partitions the available spectrum into channel sets. The reuse distance constraint is satisfied by assigning these channel sets to the cells in each cluster in a manner determined by a graph coloring problem [2].

Dynamic Channel Allocation (DCA)

By DCA, channels are temporarily assigned for use in cells for the duration of the call, according to the current system conditions and user needs, instead of relying on a priori information.

(1) *Traffic adaptive DCA*: to adapt allocation of spectral resources among cells according to current number of active mobiles in each cell. It is a maximum packing problem.

A new call will be blocked only if there is no possible channel allocation to calls that would result in room for the new call. This strategy is to find the minimum number of channels to carry instantaneous existing calls. For example, some traffic adaptive DCA algorithm tries to pack the cells using a given channel as compactly as possible so that the channel can be reused in the closest possible range [3].

(2) *Reuse partitioning DCA*: to use overlaid cell plans (reuse distance) with different reuse distances. The idea is based on fact that an MS which is closer to the BS can tolerate lower reuse distance than the MS at cell edge. Typically reuse partitioning DCA is combined with traffic adaptive DCA [4].

(3) *Interference based DCA*: the interference based DCA schemes include strategies for channel searching and prediction of interference levels.

TRANSMITTER POWER CONTROL

The goal of transmitter power control is to adjust the transmit powers of all users such that the SIR of each user meets a given threshold required for acceptable performance. This problem is more complicated in the uplink than in the downlink, because the channel gains may be different in the uplink.

Let us consider a channelized system. Let mobile i communicate with RAP i on some channel. The SIR of MS i at RAP i is given by

$$\gamma_i = \frac{G_{ii}P_i}{\sum_{j\neq i} G_{ij}P_j + N} \geq \gamma_0 \tag{1}$$

where P_i is the transmitter power of MS i, N is the receiver noise power, γ_0 is the threshold SIR, and the expression $\sum_{j\neq i} G_{ij}P_j$ is the cochannel interference. We let the interfering MSs be connected to BSs in other cells and there be Q in number.

Let $\mathbf{P} = (P_1, P_2, \ldots, P_Q)^T$ be the column vector of transmitter powers, and P_j is nonnegative i.e. $P_j \geq 0\ \forall j$. Let $\mathbf{N}$ be the column vector of noise power.

Eq. (1) can be rewritten as

$$\gamma_i = \frac{P_i}{\sum_{j\neq i} P_j \frac{G_{ij}}{G_{ii}} + \frac{N}{G_{ii}}} = \frac{P_i}{\sum_{j=1}^{Q} P_j Z_{ij} - P_i + N_i'} \tag{2}$$

where $N_i' = \frac{N}{G_{ii}}$ and $Z_{ij} = \frac{G_{ij}}{G_{ii}}$.

Further we define the *normalized link gain matrix* $\mathbf{Z}$ whose entries are Z_{ij}, and denote $\mathbf{N}'$ the column vector whose i^{th} element is N_i'.

We require $\gamma_i \geq \gamma_0\ \forall i$. Note that it is not possible to satisfy this requirement all the time.

Definition

The SIR γ_0 is *achievable* if $\exists$ a nonnegative $\mathbf{P}$ such that $\gamma_i \geq \gamma_0\ \forall i$.

The requirement can be represented in matrix form as follows:

If the system of linear inequality

$$(\frac{1+\gamma_0}{\gamma_0}\mathbf{I} - \mathbf{Z})\mathbf{P} \geq \mathbf{N}' \tag{3}$$

has some solution with $\mathbf{P} \geq \mathbf{0}$, the SIR γ_0 is *achievable*. The matrix inequality requires the component-wise inequalities, i.e., the components of the vector in the LHS is greater than or equal to the components of the vector in the RHS.

Theorem (Noiseless) [Zandei 1992] The inequality

$$(\frac{1+\gamma_0}{\gamma_0}\mathbf{I} - \mathbf{Z})\mathbf{P} \geq \mathbf{N}'$$

has solution in $\mathbf{P} \geq \mathbf{0}$ iff

$$\gamma_0 < \frac{1}{\lambda^* - 1} = \gamma^*$$

where λ^* is the dominant eigenvalue of the matrix $\mathbf{Z}$ (from the Perron-Frobenius theorem). The power vector satisfying the expression with equality and thus achieving the largest SIR is $\mathbf{P}^*$, which is the eigenvector corresponding to the eigenvalue λ^*.

If $\mathbf{P}^*$ achieves exactly γ^* at all links, the system is called as "*SIR-balanced*".

Corollary (Noise included) The inequality

$$(\frac{1+\gamma_0}{\gamma_0}\mathbf{I} - \mathbf{Z})\mathbf{P} \geq \mathbf{N}'$$

has solution in $\mathbf{P} \geq \mathbf{0}$ if $\gamma^* > \gamma_0$, where $\gamma^* = \frac{1}{\lambda^* - 1}$.

Distributed Power Control

Distributed power control is based on SIR-balancing described before.

The algorithm can be simplified to a per-user version. The powers of users are adjusted according to the following iterative procedure:

$$P_i^{(n+1)} = P_i^{(n)} \frac{\gamma^*}{\gamma_i^{(n)}} \tag{4}$$

for $i = 1, 2, \ldots, Q$. γ^* is the required target and $\gamma_i^{(n)}$ is the SIR measured by user i at step n. Hence, each transmitter increases power when its SIR is below the target and decreases power when its SIR exceeds its target. In practice, SIR measurements or a function of them such as BER are typically made at the RAPs, and a simple "up" or "down" command regarding transmit power can be fed back to each of the transmitters to perform the iterations [5].

It is shown that the algorithm converges as long as the set of power remains feasible (i.e., the SIR γ^* is achievable).

There are two kinds of updates for the iteration:

(1) Synchronous updates (by Foschini & Miljanic, 1993): all power updates take place simultaneously.

(2) Asynchronous updates (by Mitra, 1993): transmitters may update their powers at different time instances.

Note that the above results on distributed power control assume (implicitly) a fixed base station assignment.

Minimum Power Assignment Algorithms

Minimum power assignment algorithms can be thought of as a generalization of soft handoff. At each step of the iterative procedure, user is assigned to the base station at which its SIR is maximized. This is equivalent to minimizing the sum of the total transmit power in the system.

The convergence of the algorithm has been proved and verified by Yates & Huang (1995) [7].

And there are two kinds of adjustments:

(1) Continuous adjustments (by Hanly, 1995)

(2) Discrete adjustments (by Stolyar & Fleming, Song & Mandayam , 2001)

Macro Diversity (Hanly, 1993)

The idea of macro diversity is combining of received signals of a user at all base stations (or at least more than one base station). Therefore, SIR is the sum of SIRs at different base stations (reminiscent of MRC discussed in previous lectures).

Multiple Connection Reception

User is required to maintain an acceptable SIR $\gamma_j > \gamma_0$ at more than one base station.

A Unified Framework for Uplink Power Control (Yates, 1995)

Yates formulated a unified framework for uplink power control and its convergence [8].

All the above classes of power control algorithms can be viewed to be of the form

$$\mathbf{P} \geq \mathbf{I}(\mathbf{P}) = (I_1(\mathbf{P}), I_2(\mathbf{P}), \ldots, I_N(\mathbf{P})) \tag{5}$$

where $I_j(\mathbf{P})$ is the effective interference from other users that user j must overcome.

We will say that a power vector $\mathbf{P} \geq \mathbf{0}$ is a feasible solution if $\mathbf{P}$ satisfies the constraints (5) and that an interference function $\mathbf{I}(\mathbf{P})$ is feasible if (5) has a feasible solution. On other words, if $P_j \geq I_j(\mathbf{P})$, then user j has an acceptable connection.

For a system with interference constraint (5), all iterative power control algorithms have the form

$$\mathbf{P}(t+1) = \mathbf{I}(\mathbf{P}(t)) \tag{6}$$

Any algorithm of the above form will converge as long as $\mathbf{I}(\mathbf{P})$ is a "*standard interference function*", which has the following properties:

(1) *Positivity* $\mathbf{I}(\mathbf{P}) > \mathbf{0}$

(2) *Monotonicity* If $\mathbf{P} \geq \mathbf{P}'$, then $\mathbf{I}(\mathbf{P}) \geq \mathbf{I}(\mathbf{P}')$.

(3) *Scalability* For all $\alpha > 1$, $\alpha\mathbf{I}(\mathbf{P}) > \mathbf{I}(\alpha\mathbf{P})$.

The result of course assume that the system is feasible.

Power Control in Practice

In practice power updates occur in discrete steps. For example, IS-95 power control adjusts power in $\pm\Delta\ dB$ ($\Delta = 1$ or 0.5). This kind of control power is also called as *inner loop*.

IS-95 power control adjusts power about 800 times a second, while 3G-WCDMA about 1600 times a second.

Figure. 1 shows the block diagram of the inner loop power control.

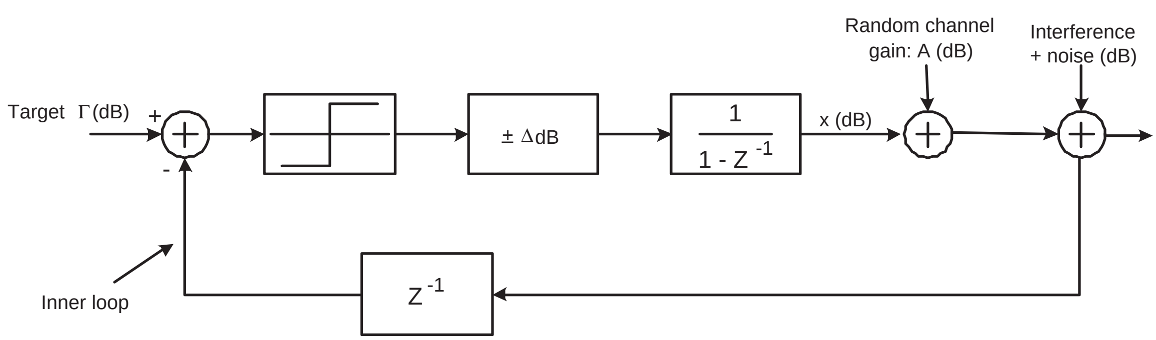

Fig. 1. Block diagram of power control

An analysis via "statistical linearization" of nonlinear feedback control system was presented by Song-Mandayam-Gajic (2001).

Outer loop controls SIR target γ^* whenever system encounters infeasibility. It operates on a slower time scale where Γ^{target} is adjusted.

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